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Parameter reconstruction of materials with off-diagonal anisotropy using the state transition matrix method

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ABSTRACT

An effective non-iterative method is developed for determination electromagnetic parameters of complex materials with anisotropy. Unlike the existing methods, the proposed method can extract the electromagnetic tensor parameters of materials with off-diagonal anisotropy using co- and cross-polarized reflection and transmission without using iterative procedures. Useful analytical expressions are derived for extracting the medium parameters of materials with off-diagonal anisotropy. The advantage of the method is that it uses state transition matrix and its properties in order to avoid nonlinearity and complexity of the problem. The method can work very well for dispersive materials since it is based on frequency-by-frequency extraction. The proposed method is validated by extraction of the complex permittivity and permeability tensors of two typical anisotropic materials.

1. Introduction

Electromagnetic characterization of materials is an important issue in many fields such as microwave engineering, bioengineering, electronics, remote sensing, medical treatments, civil engineering and concrete industry. Recently, increasing attention on metamaterials has been paid due to their exciting physical behaviors and potential applications. Generally, due to the nature of the engineered composites, metamaterials exhibit complex, dispersive, and anisotropic electromagnetic material properties. Such materials find various applications in antenna, microwave devices, and radar absorbers [1–3]. It is important for the anisotropic properties of these materials to be accommodated when characterizing constitutive parameters for use in the design process.

Different methods, each with its unique advantages and limitations, have been introduced to characterize the materials' properties [4]. Due to the relative simplicity, broad frequency coverage, and higher accuracy, transmission–reflection techniques are the most widely used broadband measurement techniques. A variety of retrieval methods for obtaining isotropic permittivity and permeability through the inversion of measured scattering-parameters are available in literature [5–9]. A review of different approaches has been presented in [10]. These methods make use of analytic formulas connecting measured S-parameters to material parameters or optimization schemes. However, in more complex materials (e.g. anisotropic or bianisotropic), deriving analytic formulas connecting measured S-parameters to material parameters is generally difficult and even impossible.

The interaction of electromagnetic fields with anisotropic materials has been well described in the literature [11], and several techniques for the characterization of these materials have been introduced [12–22]. However, most these methods such as free-space and waveguide fixture have been designed and implemented for characterizing materials with diagonal permittivity and permeability tensors and the existence of off-diagonal entries in the material tensors has not been addressed. In addition, recent attempts at measuring electromagnetic parameters of materials with off-diagonal anisotropy have mainly relied on fully numerical optimization techniques, wherein the constitutive material parameters are determined by minimizing the difference between the measured and the theoretically computed reflection and transmission coefficients. These techniques, though, are often time consuming due to slow convergence and existence of spurious solutions. Therefore, for general anisotropic specimens, where the material tensor parameters are complex, a new approach is needed.

Within this framework, the aim of this study is to present an analytic methodology based on the state transition matrix method to the characterization of materials with off-diagonal anisotropy. Transition matrix method has been well described in forward scattering problems anisotropic and bianisotropic media, over the years [23–25]. Recently, its application in the formulation for inverse scattering problems including isotropic chiral layers has

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been reported [26]. In [26], only normal incidence is considered, which prevents one from taking into account spatial dispersion. In the present work, some of constitutive parameters of off-diagonal anisotropic layer are only active in oblique illuminations, and so scattering parameters corresponding to oblique illuminations are also required. The proposed retrieval method is mainly based on the properties of the state transition matrix of an anisotropic layer and allows avoiding nonlinearity and complexity of the inverse scattering problem.

Section 2 is devoted to review of the forward problem analysis using state transition matrix method. Two properties of the state transition matrix of an anisotropic layer with off-diagonal anisotropy are presented in Section 3. Detailed formulation of parameter retrieval technique is provided in Section 4. In Section 5, material parameters determined from the proposed method are compared to the known properties to assess the approach validity. Finally, in Section 6 the conclusions are drawn. Time convention is assumed as $e^{j\omega t}$ and omitted from now on.

2. Theoretical backgrounds of state transition matrix method

The geometry of the material under study is depicted in Fig. 1. The material under test is assumed to be linear and homogeneous, with permittivity and permeability tensors that are biaxial along the orthogonal axes x, y, and z, and have off-diagonal anisotropy

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \quad \bar{\bar{\mu}} = \mu_0 \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0 \\ \mu_{yx} & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \tag{1}$$

where ε_0 and μ_0 are the permittivity and permeability of vacuum, respectively. The reciprocity conditions on the material parameters, i.e. $\bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}^T$ and $\bar{\bar{\mu}} = \bar{\bar{\mu}}^T$, are the familiar requirements on the symmetry of the permittivity and permeability tensors leading to $\varepsilon_{xy} = \varepsilon_{yx}$ and $\mu_{xy} = \mu_{yx}$. For lossless anisotropic media, the constitutional tensors are hermittian, i.e. $\bar{\bar{\varepsilon}}^* = \bar{\bar{\varepsilon}}^T$ and $\bar{\bar{\mu}}^* = \bar{\bar{\mu}}^T$.

The objective is to calculate the reflection and transmission coefficients when the slab is illuminated by an incoming plane wave, as illustrated in Fig. 1. The planar structure is of infinite extent along the y-direction, and so there is no y-dependence of the fields. In addition, according to phase matching, we write $\partial/\partial x = -jk_0 \sin\theta_0$ where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is vacuum wave number. Substituting (1) into curl Maxwell's equations and eliminating z-components of electric and magnetic fields leads to the following expressions for the transverse fields

$$\frac{d}{dz}\begin{bmatrix} \bar{E}_T \\ \bar{H}_T \end{bmatrix} = \Gamma_\omega \begin{bmatrix} \bar{E}_T \\ \bar{H}_T \end{bmatrix} \tag{2}$$

where $E_T = (E_x, E_y)$ and $H_T = (H_x, H_y)$ are the transverse components of electric and magnetic fields, respectively; and the elements of the 4×4 Γ_ω-matrix are given by:

$$\Gamma_\omega = \frac{\omega}{c}\Gamma = \begin{bmatrix} 0 & 0 & -j\omega\mu_0\mu_{xy} & -j\omega\mu_0\left(\mu_{yy} - \dfrac{1}{\varepsilon_{zz}}\sin^2\theta_0\right) \\ 0 & 0 & j\omega\mu_0\mu_{xx} & j\omega\mu_0\mu_{xy} \\ j\omega\varepsilon_0\varepsilon_{xy} & j\omega\varepsilon_0\left(\varepsilon_{yy} - \dfrac{1}{\mu_{zz}}\sin^2\theta_0\right) & 0 & 0 \\ -j\omega\varepsilon_0\varepsilon_{xx} & -j\omega\varepsilon_0\varepsilon_{xy} & 0 & 0 \end{bmatrix} \tag{3}$$

where ω and c are the angular frequency and the speed of light in free space, respectively.

Consider a 4×4 state transition matrix Φ that relates the transverse components of electric and magnetic fields at the two boundaries of the slab as the follows

$$\begin{bmatrix} \bar{E}_T(0) \\ \bar{H}_T(0) \end{bmatrix} = [\Phi]\begin{bmatrix} \bar{E}_T(d) \\ \bar{H}_T(d) \end{bmatrix} = \begin{bmatrix} [\Phi_1]_{2\times2} & [\Phi_3]_{2\times2} \\ [\Phi_3]_{2\times2} & [\Phi_4]_{2\times2} \end{bmatrix}\begin{bmatrix} \bar{E}_T(d) \\ \bar{H}_T(d) \end{bmatrix} \tag{4}$$

where the state transition matrix is $\Phi = e^{-\Gamma_\omega d}$. Defining wave impedance matrix as

$$Z_0 = \begin{bmatrix} 0 & \eta_0\cos\theta_0 \\ -\eta_0\sec\theta_0 & 0 \end{bmatrix} \tag{5}$$

where η_0 is the intrinsic impedance of free space, (4) can be simplified as follows [23]

$$\bar{E}_T^i(0) + \bar{E}_T^r(0) = \Phi_1\bar{E}_T^t(d) + \Phi_2 Z_0^{-1}\bar{E}_T^t(d) \tag{6}$$

$$Z_0^{-1}\bar{E}_T^i(0) - Z_0^{-1}\bar{E}_T^r(0) = \Phi_3\bar{E}_T^t(d) + \Phi_4 Z_0^{-1}\bar{E}_T^t(d). \tag{7}$$

Using (6) and (7), one can obtain co-and cross-polarized free-space reflection and transmission coefficients [23].

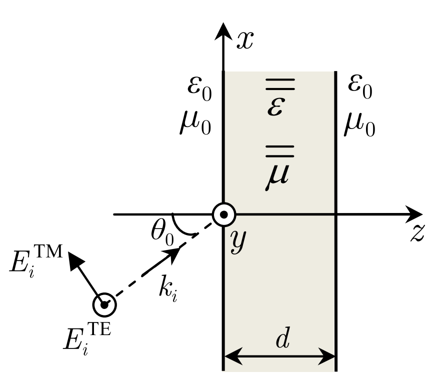

Fig. 1. An anisotropic layer with off-diagonal anisotropy exposed to a linearly polarized plane wave.

3. Properties of the state space matrix of an anisotropic layer

Prior to the discussion of the material parameter extraction methodology, it is required to discuss some useful properties of state transition matrix of an anisotropic layer with off-diagonal anisotropy. The following two theorems are introduced and proved here.

Theorem 1. *The determinant of state transition matrix Φ of an anisotropic layer with off-diagonal anisotropy is equal to unity.*

Proof. Let Γ_ω be a square matrix with complex elements. There is a theorem for determinants that says that the determinant of the exponential function of a matrix is equal to the exponential function of the trace of the matrix, which the trace of a matrix is equal to the sum of its eigenvalues [27]. As proved in [26], we may write

$$\det(\Phi) = \prod_{i=1}^{4} \exp\left(\Gamma_{\omega}(i,\, i)\right) = \exp\left[\sum_{i=1}^{4} \Gamma_{\omega}(i, i)\right]. \tag{8}$$

Finally, the sum of the diagonal elements of Γ_{ω}-matrix of an anisotropic layer with off-diagonal anisotropy is zero, and so determinant of its Φ-matrix is equal to unity.

Theorem 2. *State transition matrix of an anisotropic layer with off-diagonal anisotropy could be written as*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & -\Phi_{13} \\ \Phi_{31} & \Phi_{32} & \Phi_{22} & -\Phi_{12} \\ \Phi_{41} & -\Phi_{31} & -\Phi_{21} & \Phi_{11} \end{bmatrix} \tag{9}$$

which has only ten distinct elements: Φ_{11}, Φ_{12}, Φ_{13}, Φ_{14}, Φ_{21}, Φ_{22}, Φ_{23}, Φ_{31} Φ_{32} and Φ_{41} given in Appendix A. In addition, the Φ^{-1}-matrix could be written as the following

$$\Phi^{-1} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & -\Phi_{13} & -\Phi_{14} \\ \Phi_{21} & \Phi_{22} & -\Phi_{23} & \Phi_{13} \\ -\Phi_{31} & -\Phi_{32} & \Phi_{22} & -\Phi_{12} \\ -\Phi_{41} & \Phi_{31} & -\Phi_{21} & \Phi_{11} \end{bmatrix}. \tag{10}$$

4. Extraction methodology

This section deals with the detailed formulation of proposed methodology for retrieving the constitutive parameters of anisotropic layer with off-diagonal anisotropy using S-parameters and based on the state transition matrix.

It is evident that six tensor parameters ε_{xx}, ε_{yy}, ε_{xy}, ε_{yx}, μ_{xx}, μ_{yy}, μ_{xy} and μ_{yx} (i.e. in-plane elements of tensors) are active when the slab is normally illuminated, while two other parameters ε_{zz} and μ_{zz} (i.e. out-of-plane elements of tensors) are active when the slab is obliquely illuminated. Thus, determination of the material parameters requires normal and oblique illuminations and measurements.

4.1. Determination of state transition matrix

First, considering the perpendicular polarization incidence, where the TE incident wave is assumed to be incident from free space to the slab at angle θ_0, (6) and (7) leads to the following expressions

$$\begin{cases} E^r_{x-TE} = (\Phi^o_{11} + \dfrac{\Phi^o_{14}}{\eta_0 \cos\theta_0})E^t_{x-TE} \\ \qquad + (\Phi^o_{12} - \dfrac{\Phi^o_{13}\cos\theta_0}{\eta_0})E^t_{y-TE} \\ 1 + E^r_{y-TE} = (\Phi^o_{21} - \dfrac{\Phi^o_{13}}{\eta_0 \cos\theta_0})E^t_{x-TE} \\ \qquad + (\Phi^o_{22} - \dfrac{\Phi^o_{23}\cos\theta_0}{\eta_0})E^t_{y-TE} \end{cases} \tag{11}$$

$$\begin{cases} -\dfrac{\cos\theta_0}{\eta_0}(1 - E^r_{y-TE}) = (\Phi^o_{31} - \dfrac{\Phi^o_{12}}{\eta_0 \cos\theta_0})E^t_{x-TE} \\ \qquad + (\Phi^o_{32} - \dfrac{\Phi^o_{22}\cos\theta_0}{\eta_0})E^t_{y-TE} \\ -\dfrac{1}{\eta_0 \cos\theta_0}E^r_{x-TE} = (\Phi^o_{41} + \dfrac{\Phi^o_{11}}{\eta_0 \cos\theta_0})E^t_{x-TE} \\ \qquad + (-\Phi^o_{31} + \dfrac{\Phi^o_{21}\cos\theta_0}{\eta_0})E^t_{y-TE} \end{cases} \tag{12}$$

where "o" superscript denotes oblique incidence. Then, consider the TM incident wave, for which (6) and (7) may be rewritten as

$$\begin{cases} 1 + E^r_{x-TM} = (\Phi^o_{11} + \dfrac{\Phi^o_{14}}{\eta_0 \cos\theta_0})E^t_{x-TM} \\ \qquad + (\Phi^o_{12} - \dfrac{\Phi^o_{13}\cos\theta_0}{\eta_0})E^t_{y-TM} \\ E^r_{y-TM} = (\Phi^o_{21} - \dfrac{\Phi^o_{13}}{\eta_0 \cos\theta_0})E^t_{x-TM} \\ \qquad + (\Phi^o_{22} - \dfrac{\Phi^o_{23}\cos\theta_0}{\eta_0})E^t_{y-TM} \end{cases} \tag{13}$$

$$\begin{cases} \dfrac{\cos\theta_0}{\eta_0}E^r_{y-TM} = (\Phi^o_{31} - \dfrac{\Phi^o_{12}}{\eta_0 \cos\theta_0})E^t_{x-TM} \\ \qquad + (-\Phi^o_{32} - \dfrac{\Phi^o_{22}\cos\theta_0}{\eta_0})E^t_{y-TM} \\ \dfrac{1}{\eta_0 \cos\theta_0}(1 - E^r_{x-TM}) = (\Phi^o_{41} + \dfrac{\Phi^o_{11}}{\eta_0 \cos\theta_0})E^t_{x-TM} \\ \qquad + (-\Phi^o_{31} + \dfrac{\Phi^o_{21}\cos\theta_0}{\eta_0})E^t_{y-TM} \end{cases}. \tag{14}$$

Note that using the scattering parameters, (11)–(14) provide a simple set of eight linear equations, while the number of distinct unknown elements of the state transition matrix Φ^o is ten. The proposed method for balancing the number of unknowns and equations is to use properties of the state transition matrix of an anisotropic material discussed as two theorems in prior section. In fact, the material parameters are then found by solving the obtained system of ten equations in the ten unknown elements of Φ^o-matrix. Similarly, considering above equations in the case of normal incidence, i.e. $\theta_0 = 0°$, the state transition matrix Φ^n, where "n" superscript denotes normal incidence, can be also determined.

4.2. Determination of Γ-matrix

Once the oblique and normal state transition matrices are found, the Γ-matrices (Γ^o and Γ^n) of the system are given by

$$\Gamma = -\frac{c}{\omega d}\ln(\Phi) = -\frac{\lambda_0}{2\pi d}\ln(\Phi) \tag{15}$$

where λ_0 is the free-space wavelength. The source of probable ambiguity and uncertainty in the presented parameter retrieval technique has been discussed in [26].

4.3. Determination of material parameters

As previously mentioned, in this retrieval method, the ten tensor parameters are divided into two groups: the constitutive parameters ε_{xx}, ε_{yy}, ε_{xy}, ε_{yx}, μ_{xx}, μ_{yy}, μ_{xy} and μ_{yx} which are active when the slab is normally illuminated, and ε_{zz} and μ_{zz} which are active when the slab is obliquely illuminated. Once the Γ^n-matrix is determined, the first group of parameters is fully identified. Determination of the other constitutive parameters is relative more complex, because the elements of Γ^o are also required. Eventually, the constitutive tensor parameters are found to be

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} j\eta_0\Gamma^n_{41} & j\eta_0\Gamma^n_{42} & 0 \\ j\eta_0\Gamma^n_{42} & -j\eta_0\Gamma^n_{32} & 0 \\ 0 & 0 & \dfrac{j\eta_0\sin^2\theta_0}{\Gamma^o_{14} - \Gamma^n_{14}} \end{bmatrix} \tag{16}$$

Table 1
Retrieved material parameters for biaxial layer.

f (GHz)	Retrieved tensor parameters	
2.5	$\varepsilon_{xx}=1.9999-j0.1000$, $\varepsilon_{yy}=3.9998-j0.4999$, $\varepsilon_{zz}=3.0004+j0.0009$	$\mu_{xx}=1.0000-j0.2000$, $\mu_{yy}=2.5002-j0.0001$, $\mu_{zz}=2.0025-j1.0040$
3	$\varepsilon_{xx}=2.0002-j0.1000$, $\varepsilon_{yy}=4.0004-j0.5000$, $\varepsilon_{zz}=3.0098-j0.0009$	$\mu_{xx}=0.9999-j0.2000$, $\mu_{yy}=2.4997-j0.0001$, $\mu_{zz}=1.9985-j0.9969$
4	$\varepsilon_{xx}=2.0001-j0.1003$, $\varepsilon_{yy}=4.0005-j0.5000$, $\varepsilon_{zz}=3.0098-j0.0009$	$\mu_{xx}=0.9999-j0.2000$, $\mu_{yy}=2.4998-j0.0001$, $\mu_{zz}=1.9999-j0.1000$

$$\bar{\bar{\mu}}=\begin{bmatrix} -j\eta_0^{-1}\Gamma_{23}^{n} & -j\eta_0^{-1}\Gamma_{24}^{n} & 0 \\ -j\eta_0^{-1}\Gamma_{24}^{n} & j\eta_0^{-1}\Gamma_{14}^{n} & 0 \\ 0 & 0 & \dfrac{-j\eta_0^{-1}\sin^2\theta_0}{\Gamma_{32}^{o}-\Gamma_{32}^{n}} \end{bmatrix} \tag{17}$$

To conclude, there are two procedures for implementing the described retrieval method. One procedure makes use of the scattering parameters at normal incidence to retrieve the eight tensor components in the x-y plane. The two remaining tensor parameters in the z direction can be retrieved by including the scattering parameters at another oblique angle. The second procedure retrieves all ten electromagnetic tensor quantities of the slab directly from the scattering parameters collected at two different oblique angles. Clearly, the first procedure can be regarded as a simplified case of the second procedure.

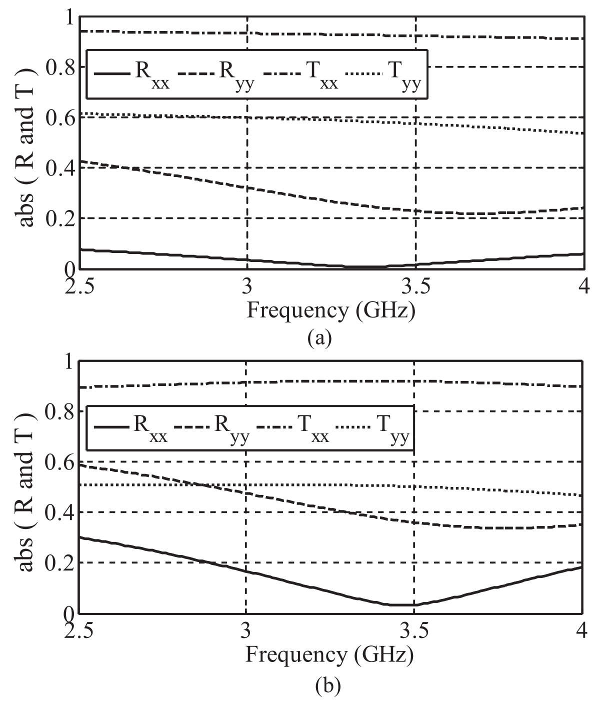

Fig. 2. Amplitudes of reflection and transmission coefficients of biaxial bianisotropic slab at (a) normal, and (b) oblique incident with $\theta_0=45^\circ$.

5. Numerical examples and results

To assess the performance and accuracy of the extraction procedure in the reconstruction of constitutive tensor parameters of anisotropic materials, two examples are provided in this section.

5.1. Biaxial anisotropic material

It is important to validate the theoretical model of extraction procedure before employing it in parameter extraction. To do this, a biaxial test material is considered, with thickness 2 cm and permittivity and permeability tensor parameters $\bar{\bar{\varepsilon}}=\varepsilon_0\, diag(2-j0.1,\ 4-j0.5,\ 3)$ and $\bar{\bar{\mu}}=\mu_0\, diag(1-j0.2,\ 2.5,\ 2-j1)$. Many example specimens with different thicknesses and material parameters were simulated and analyzed, but this case was chosen due to its difficulty level and similarity to the discussed example in [22].

The scattering parameters of such an anisotropic material in the normal and oblique incidences at $\theta_0=0$ and 45° are calculated over the portion of S-band from 2.5 to 4 GHz using the described approach in Sec. II and are shown in Fig. 2. These calculated S-parameter data are fed into the extraction algorithm to extract the constitutive tensor parameters. The normal incidence data are utilized for the extraction of in-plane terms, and the oblique incidence data are utilized for the out-of-plane terms extraction. Results are shown in Fig. 3. The results in some frequencies are reported in Table 1. A very good agreement between the true values and the calculated ones is obtained. In addition, the results are in good agreement with the measured ones has been presented in [22].

5.2. Material with off-diagonal anisotropy

As the second example, consider a more complex material with off-diagonal anisotropy and thickness $d=1.6$ mm whose constitutive parameters have the following relations

$$\bar{\bar{\varepsilon}}=\varepsilon_0\begin{bmatrix} 5-j0.4 & 2 & 0 \\ 2 & 2-j0.5 & 0 \\ 0 & 0 & 1-j0.3 \end{bmatrix} \tag{18a}$$

$$\bar{\bar{\mu}}=\mu_0\begin{bmatrix} 1-j0.1 & 0.5 & 0 \\ 0.5 & 1-j0.1 & 0 \\ 0 & 0 & 2-j1 \end{bmatrix}. \tag{18b}$$

Table 2
Retrieved material parameters for anisotropic layer with off-diagonal anisotropy.

f (GHz)	Retrieved out-of-plane terms using oblique S-parameters corresponding to $\theta_0=60^\circ$	Retrieved out-of-plane terms using oblique S-parameters corresponding to $\theta_0=30^\circ$	Retrieved in-plane terms using normal S-parameters	
3	$\varepsilon_{zz}=1.0005-j0.3007$, $\mu_{zz}=1.9898-j0.9896$	$\varepsilon_{zz}=1.0059-j0.2965$, $\mu_{zz}=1.9892-j0.9890$	$\varepsilon_{xx}=5.0003-j0.4003$, $\varepsilon_{yy}=1.9997-j0.4993$, $\varepsilon_{xy}=1.9999+j0.0000$	$\mu_{xx}=1.0005-j0.0998$, $\mu_{yy}=0.9996-j0.1000$, $\mu_{xy}=0.0000-j0.0000$
4	$\varepsilon_{zz}=0.9997-j0.2994$, $\mu_{zz}=1.9964-j0.9890$	$\varepsilon_{zz}=0.9981-j0.3008$, $\mu_{zz}=1.9856-j0.9818$	$\varepsilon_{xx}=5.0002-j0.3998$, $\varepsilon_{yy}=1.9999-j0.5000$, $\varepsilon_{xy}=2.0005+j0.0001$	$\mu_{xx}=0.9999-j0.1001$, $\mu_{yy}=1.0004-j0.1002$, $\mu_{xy}=0.0000+j0.0000$

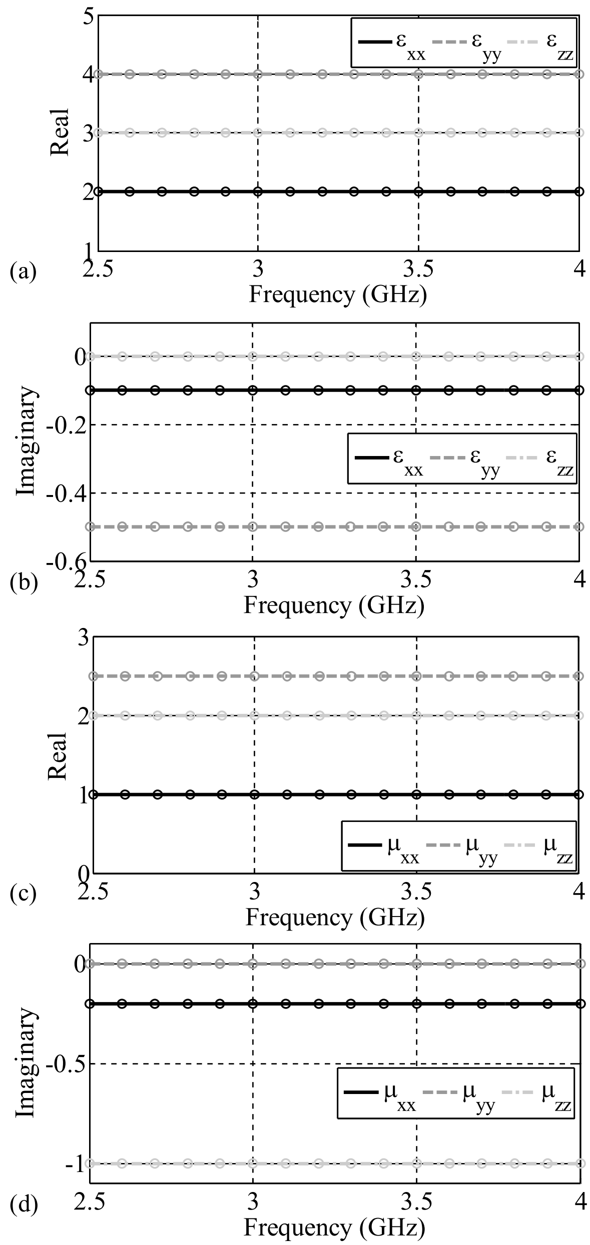

Fig. 3. Real and imaginary parts of constitutive parameters of anisotropic material. (a) and (b) Relative permittivity tensor. (c) and (d) Relative permeability tensor parameters. True values are shown with circles.

Fig. 4 shows reflection and transmission coefficients at normal and oblique incidence at $\theta_0 = 60^\circ$. Observe that the material yielded cross-polarized transmission and reflections. Scattering parameters corresponding to normal incidence at frequency 3 GHz and 4 GHz are fed to the reconstruction procedure and the in-plane terms of constitutive tensor parameters are extracted and reported in Table 2. In addition, out-of-plane terms of constitutive tensor parameters are extracted using scattering parameters calculated at different oblique incidence angles. The retrieval process is first performed using the 30° incidence angle scattering data. A separate inversion is also performed using the 60° scattering data. Observe that these results agree well with each other. Finally, these results, when compared with the true values, demonstrate the capability of the proposed extraction procedure to the characterization of materials with off-diagonal anisotropy from scattering data.

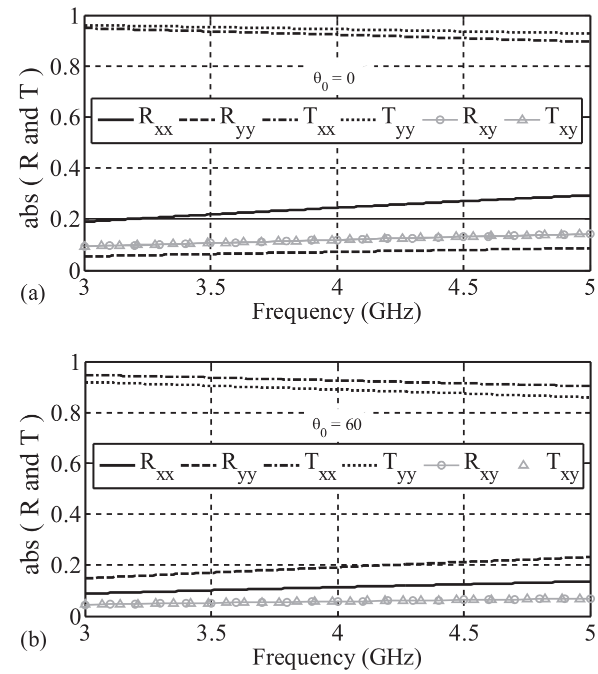

Fig. 4. Amplitudes of reflection and transmission coefficients of biaxial bianisotropic slab at (a) normal, and oblique incident with (b) $\theta_0 = 60^\circ$.

6. Conclusions

A frequency domain method based on the state-space approach has been introduced for retrieving the permittivity and permeability tensors of materials with off-diagonal anisotropy. There are three main advantages of the presented method over the methods in the literature. First, it can extract the electromagnetic tensor parameters of materials with off-diagonal anisotropy without using iterative procedures. Second, it uses state transition matrix and its properties to avoid nonlinearity and complexity of the inverse scattering problem. Third, the probable cause of ambiguity in the proposed method can be fully identified and removed based on advanced techniques such as using Kramers–Kronig relations [26]. Finally, the proposed method has been validated by extraction of the complex permittivity and permeability tensors of two typical anisotropic materials. In these numerical examples, we use MATLAB to perform numerical computation. Some deviation amongst the correct and retrieved constitutive parameters is related to numerical accuracy with which the operations are carried out on a computer. From the numerical results one can find the capability of the proposed extraction procedure to the characterization of complex materials with off-diagonal anisotropy.

Appendix A. Appendix A

For the computation of exponential function of a square matrix, many methods have been proposed such as expansion in power series, Cayley–Hamilton theorem and Leverrier's algorithm [28]. Based on Cayley–Hamilton theorem, for every 4×4 matrix W with distinct eigenvalues w_1, w_2, w_3 and w_4, there exist four scalar functions a_0, a_1, a_2, and a_3 for which

$$e^W = \sum_{q=0}^{3} a_q W^q. \tag{A1}$$

and

$$e^{w_p} = \sum_{q=0}^{3} a_q w_p^q, \qquad for\, p = 1,\ 2,\ 3\, and\, 4. \tag{A2}$$

By solving these four equations, unknown coefficients a_q are fully determined in terms of eigenvalues w_p [26]. For an anisotropic layer, considering W = $-\Gamma_\omega d$, g_1 = W(1,4), g_2 = W(2,3), g_3 = W(3,2), g_4 = W(4,1), g_5 = W(1,3), and g_6 = W(3,1), the eigenvalues of W are

$$w_{13} = -w_{24} = \left\{ \frac{g_1g_4 + g_2g_3}{2} + g_5g_6 \pm \left[\left(\frac{g_1g_4 - g_2g_3}{2} \right)^2 + g_5g_6\,(g_1g_4 + g_2g_3) - (g_1g_2g_6^2 + g_3g_4g_5^2) \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}. \tag{A3}$$

Then, after some matrix manipulations in (A1), the matrix exponential $\Phi = e{-}^{\Gamma\omega d}/^{c}$ may be written as

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & -\Phi_{13} \\ \Phi_{31} & \Phi_{32} & \Phi_{22} & -\Phi_{12} \\ \Phi_{41} & -\Phi_{31} & -\Phi_{21} & \Phi_{11} \end{bmatrix} \tag{A4}$$

wherein

$$\begin{cases} \Phi_{11} = c_0 + c_2(g_1g_4 + g_5g_6), \quad \Phi_{12} = -c_2(g_1g_6 - g_3g_5) \\ \Phi_{13} = c_1g_5 - c_3\,[g_1(g_2g_6 - g_4g_5) - g_5(g_2g_3 + g_5g_6)] \\ \Phi_{14} = c_3\,[g_1(g_1g_4 + g_5g_6) + g_5(g_1g_6 - g_3g_5)] + c_1g_1 \\ \Phi_{21} = c_2(g_2g_6 - g_4g_5), \quad \Phi_{22} = c_0 + c_2(g_2g_3 + g_5g_6) \\ \Phi_{23} = c_3\,[g_2(g_2g_3 + g_5g_6) + g_5(g_2g_6 - g_4g_5)] + c_1g_2 \\ \Phi_{31} = c_3\,[g_3(g_2g_6 - g_4g_5) + g_6(g_1g_4 + g_5g_6)] + c_1g_6 \\ \Phi_{32} = c_3\,[g_3(g_2g_3 + g_5g_6) - g_6(g_1g_6 - g_3g_5)] + c_1g_3 \\ \Phi_{41} = c_3\,[g_4(g_1g_4 + g_5g_6) - g_6(g_2g_6 - g_4g_5)] + c_1g_4 \end{cases} \tag{A5}$$

For the computation of the inverse matrix Φ^{-1}, $e^{\,+\Gamma\omega d/c}$ can be computed. In this case, the following should be considered

$$e^{-W} = \sum_{q=0}^{3} a_q{}'(-W)^q. \tag{A6}$$

It can be easily seen that

$$a_0{}' = a_0, \quad a_1{}' = -a_1, \quad a_2{}' = a_2, \quad a_3{}' = -a_3. \tag{A7}$$

After some straightforward matrix manipulations, one can obtain Φ^{-1} as presented in (10).

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