Chapter 7

ASSUMPTIONS AND AIMS OF PLASTIC DESIGN

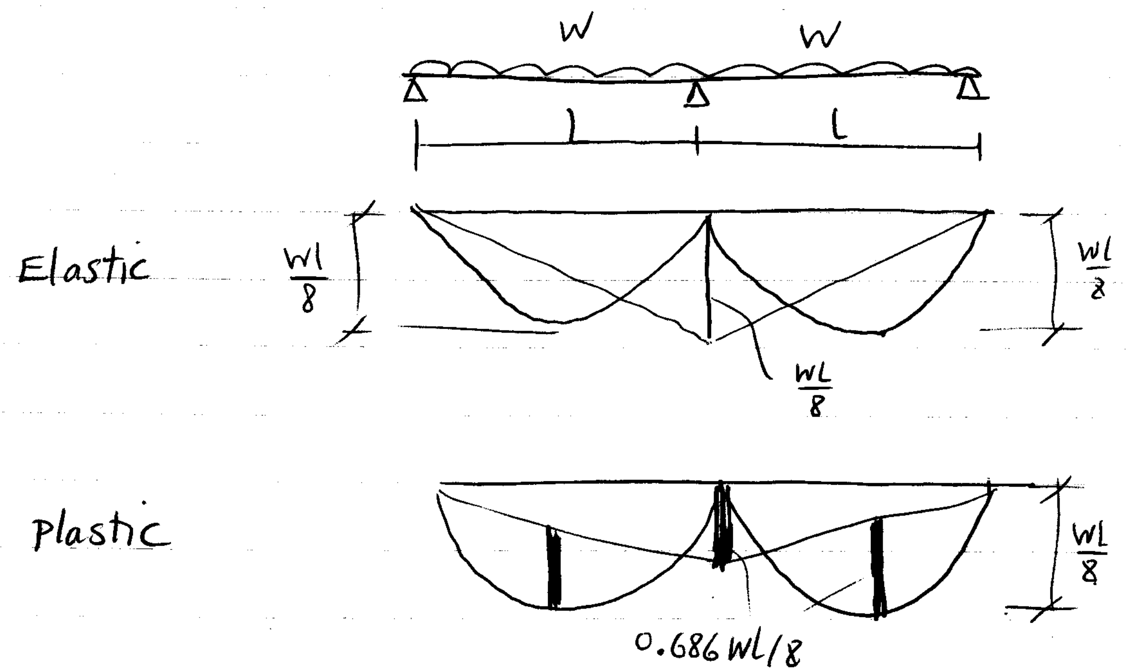
- Plastic Theory has been used up to now for framed structures, in which the action of external loads is resisted by bending of the members. The Theory can also be applied to other types of structures, such as plates, grids, etc.
- Plastic Theory mainly concentrates on calculating the collapse load. The main design criteria considered is strength. In its simple form (first order) it makes no attempts to assess deflexions, nor to enquire into the stability of individual members or of the structure as whole. Of course having designed for strength, the designer can check the deflexions, stability or any other calculations
- Plastic design is only suitable for structures for which deflexions are small in a very definite technical scence and danger of instability is neglegible. The ability of the members to form plastic hinges, which undergo very large rotations without fall-off in the value of the full plastic moment is also important.
- Similar to elastic design it is assumed that deflexions are small, in the sense that, in all calculations, original undeformed dimensions will be used in the equilibrium equations (first order analysis)
- A large proportion of typical steel building frames has deflexions which are negligible in this sense. Arches and pitched roof frame (with high-strength steel)

may suffer rather large deflexion, and one has to be careful. One can make a simple estimate of reduction in load carrying capacity due to deformation. If it is less than 5% one can use simple plastic theory. For more than 10% one may forget about first order rigid plastic analysis.

- Fortunately deflexions can be calculated by a double integration method if a reasonable guss can be made of the B.M. distribution. Thus designer can first neglect the deflexion, design his frame by plastic method, and then check them for being small.
- Similarly stability of columns can be checked after plastic design. Every member should be stable at all stages of loading. Since a single unstable member can threaten the safty of the whole structure.
- The prime requirement, infact, is that of ductility. Not only the material should be ductile so that, in bending a plastic hinge can form and suffer large rotations without drop in bending moment. The overall bending behaviour must also be "ductile" in the sense that there must be no fall-off in strength, for example due to instability.

Suppose everything is all right and simple plastic design is made. Certainly the plastic designer does not proportion a structure so that it collapses under the working load; the use of load factor ensures that there is an adquate margin of safty.

Elastic designer does not make a good estimate of the working state of an actual frame, unless the structure is statically determinate one. consider the following example.



For elastic analysis $M_{max} = \frac{wL}{8}$ and usually the beam is designed for this M_{max} uniformally. It is assumed that supports do not have differential stettlments otherwise M_{max} will be reduced or increased. It is this single value on which elastic design is based.

This is typical behaviour of such structure. If all variable factors, selllements, imperfect fabrication stresses induced on errection and so on could be calculated, then perhaps an "actual" B.M.D. could be calculated.

It is obvious that structural defects can not have real influence on the overall strength of the structure. From this point of view, plastic theory is simply a way of obtaining the max possible advantage from the situation.

- Both B.M.Ds. are drawn for working loads. Elastic designer will arrange that corresponding σ_x does not exceed a certain value, say 165 N/mm^2 for steel with a yield stress $\sigma_0 = 250$ N/mm^2. The plastic designer will arrange the largest working value of B.M. (0.686WL/8) also produces a stress which does not exceed 165 N/mm^2 (or a different permitted value if he is working to a load factor other than 1.75). In doing so, the designer is sure that the beam cannot collapse until the working load is multiplied by a hypothetical load factor of value 250/165 times the shape factor, that is 1.75 for the I-section.
- Elastic B.M.D (b) which is believed to be correct one is not "correct". It is merely a possible equilibrium distribution of B.M. in the beam. Plastic B.M.D (c) is another possible equilibrium distribution, and either distribution can be used by the plastic theorems, as a basis for design.
- For steel frames the conventional elastic analysis bears no relation to the actual state under working loads. By contrast plastic calculations, while also giving no information about the structure under working loads, do predict very accurately the collapse load, and the designer can be confident, if he is using an adequate load factor, of the "strength" safety of his structure under working load.

- In minimizing the largest bending moment, the plastic designer will achieve economics in his structure, in Fig. (c) the largest B.M. has been made as small as possible, with %30 reduction from the elastic fig (b). This equalization of moments at the direction of the designer is the key, not only for economy, but also to direct design instead of analysis. In our example which is $\gamma(S)=1$, two B.Ms. are equalised and economy is obtained (instead of one peak value). This is general and for a frame with $\gamma(S)=R$ one can equalize $R+1$ B.Ms.
- Furthermore this equalization does not depend on the flexibilities of the members. For stat. indet. structure elastic designer needs to do a lot of analysis and should know section properties, while plastic designer constructs a reasonable set of B.Ms without knowing section properties.
- Plastic design can be used for other types of structures as far as strength is the main criterion. Although reinforced concrete can have "drooping" moment-curvature characteristic, so that plastic theorems cannot strictly apply nevertheless a "plastic" estimate of working conditions will at least be as good as 'elastic' estimate. A plastic design can be used for reinforced concrete with as much confidence as an elastic design. Reinforced concrete has proved itself to have sufficient ductility for the

construction of safe structures; a reinforced concrete frames does not fail prematurely by excessive hinge rotation at critical section.

- perhaps the most important contribution of plastic theory is that it has led to this concept of design, based on considerations of equilibrium alone, and rejecting those of compatibility and deformations. The lower bound (safe) theorem is specific: If an equilibrium state can be found which does not violate the yield condition, then however "unlikely" that state may seem to be, the structure is safe.

The theory applies strictly only to ductile materials like steel. However, if strength is the main design criterion and if deflexions are small and the danger of instability negligible, then plastic theory is the fundamental tool for design of a structure made of any non-brittle material.