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The Exponent 3/2 at Pyramidal Nanoindentations

G. KAUPP[1] AND M. R. NAIMI-JAMAL[2]

[1]University of Oldenburg, Faculty 5, Edewecht, Germany
[2]Iran University of Science and Technology, Faculty of Chemistry, Tehran, Iran

Summary: The analysis of published loading curves reveals the exponent 3/2 to the depth for nanoindentations with sharp pyramidal or conical tips. This has geometric reasons, as it occurs independent on the bonding states and indentation mechanisms. Nevertheless, most mathematical deductions and finite element simulations of nanomechanical parameters in the literature continue using the experimentally not supported Hertzian exponent 2. Therefore, numerous published loading curves of various authors are plotted using the experimental exponent 3/2 to present unbiased proof for its generality with metals, oxides, semiconductors, biomaterials, polymers, and organics. Linearity is independent of equipment and valid for load controlled, or depth controlled, or continuous stiffness, or AFM force measurements. The linearity with exponent 3/2 often extends from the nano- into the microindentation ranges. The tip rounding and taper influence of the "geometrical similar" indenters are discussed. When kinks occur in such linear plots through the origin, these indicate change of the materials' mechanical properties under pressure by phase transition. These events are discussed for nanoindentations with respect to the known hydrostatic transformation pressures that are, of course, always higher than the necessary indentation mean pressure. Numerous Raman, as well as X-ray and electron diffraction results from the literature support the phase transitions that are now easily detected. Nanoporous materials first fill the pores upon indentation. Published loading curves exhibit more information than hitherto assumed. SCANNING 32: 265–281, 2010. © 2010 Wiley Periodicals, Inc.

Address for reprints: G. Kaupp, University of Oldenburg, Faculty 5, Diekweg 15, D-26188 Edewecht, Germany
E-mail: gerd.kaupp@uni-oldenburg.de

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Introduction

In 1882, Hertz (1882) set up a mathematical hypothesis for the contact of solids claiming an exponential relationship between normal force and penetration into a flat surface. He suggested exponent 1 for flat, exponent 3/2 for spherical, and exponent 2 for conical indenters. The now mostly used pyramidal indenters in micro- and nanoindentations are geometrical similar to cones; therefore, the Hertzian exponent 2 would apply for pyramidal indenters, which has been widely accepted for microindentations and even for nanoindentations. However, there were also cases with varying exponents in micro- and macro-indentations (Meinhard *et al.* 1997), and experimental proof of the exponent 2 for the loading curves (normal force F_N vs. indentation depth h) in nanoindentations has never been achieved. Nevertheless, exponent 2 has been mostly used in simulations and mathematical deductions of mechanical parameters, although without checking the experimental exponent of the loading curve in question. It was however shown since 2004 (Kaupp and Naimi-Jamal 2004, 2005a; Kaupp 2006; Naimi-Jamal and Kaupp 2007) that a universal exponent 3/2 applies for 60°, 90°, and 142.3° three-sided pyramidal tips in nanoindentations of virtually all different kinds of materials, independent of their bonding state. While numerous excellent linear plots through the origin according to Equation (1) (which exhibit more fine details than double logarithmic plots) were published (notwithstanding slight deviations from cutting at the origin due to water layers or oxide layers etc. on the surface), there were frequent judgments that the exponent 3/2 would only occur in this authors' hands, even when we had started to cite published literature data of