Adaptive Fuzzy Predictive Hybrid Position/Velocity Control of Nonholonomic Wheeled Mobile Robots

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Abstract

This paper presents Nonlinear Model-based Predictive Control (NMPC) of Wheeled Mobile Robots (WMRs) based on a discrete-time fuzzy model to approximate the dynamics of the robot and the actuator. The parameters of the fuzzy model are adjusted on-line by using gradient descent algorithm and recursive least square estimation method in order to cope with uncertainties in the system. Moreover, by tuning the weights in the cost function of the NMPC, better tracking error of the WMR can be obtained. The simulation results show that the proposed method can effectively control a type (2,0) WMR with a good performance.

Keywords: Mobile robots, Trajectory tracking, Nonlinear model predictive control, Fuzzy modeling, Adaptive control.

1. INTRODUCTION

Nowadays, the robots are inserted more and more in dynamic environments such as robotic soccer, manufacturing plants, etc. Various approaches have been proposed to follow a path or to avoid obstacles. In some applications, the desired path is accompanied by a desired velocity. Recently, intelligent algorithms such as Neural Networks (NNs) and fuzzy logic have been used in the controller designs to deal with various uncertainty problems in the system.

In recent decades, fuzzy logic control strategies have been used by many researchers to overcome disturbances and dynamic uncertainties of mobile robots [1-3]. Most controllers, including PID controllers, don't include future state of the plant to calculate the current control input. The Model-based Predictive Control (MPC) predicts the future state using the open-loop plant dynamics over a prediction horizon. Features of MPC are well explained in many references such as [4] and [5]. Properties that set MPC apart from other control laws are its on-line optimization and constraints. Over the last few decades, MPC has been widely used for controlling chemical process plants, but its applications are expanding to robot controls as well. The class of MPC, which was extended to nonlinear systems, is called the Nonlinear MPC (NMPC) [5]. A reactive trajectory tracking controller based on NMPC has been presented in [6]. An NMPC scheme with obstacle avoidance for tracking trajectory of mobile robots is proposed in [7]. More examples can be found in [8--11].

Reference [12] presents a tracking method for a mobile robot that combines predictive control and NNs, where a multilayer back-propagation NN is employed to model non-linear kinematics of the robot. In [13] a multilayer perceptron NN has been trained to reproduce the NBPC behavior in a supervised way.

A path tracking scheme for mobile robot based on fuzzy logic and predictive control is presented in [14], where the predictive control is used to predict the position and the orientation of the robot, while the fuzzy control is used to deal with the non-linear characteristics of the system.

The main contribution of this paper is to develop an NMPC for hybrid position-velocity control of WMRs. In the proposed controller, the parameters of the fuzzy model are adapted on-line to better estimate the dynamic of the robot. For adaptation scheme, the gradient descent and recursive least square estimation methods are employed in order to cope with uncertainties in both kinematic and dynamic parameters as well as actuator parameters. The proposed method is applied to a type (2,0) WMR.

The rest of this paper is organized as follows. In Section 2, the WMR dynamics, the NMPC strategy and the fuzzy structure are presented. Section 3 describes the adaptive NMPC design. Section 4 shows simulation results, and finally, conclusions are given in Section 5.

2. PRBLEM FORMULATION

2.1. Dynamic model of WMR

Using the Euler-Lagrange formulation, the dynamics of WMRs can be described by [15-17]:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+F(\dot{q})+G(q)+\tau_d=B(q)\tau-A^T(q)\lambda \tag{1}$$

where $M(q)\in\Re^{n\times n}$ is the symmetric and positive definite inertia matrix, $C(q,\dot{q})\in\Re^{n\times n}$ is the centripetal and Coriolis matrix, $F(\dot{q})\in\Re^{n\times 1}$ is the vector of surface friction, $G(q)\in\Re^{n\times r}$ is the gravitational vector, τ_d denotes the bounded unknown disturbances including unstructured unmodeled dynamics, $B(q)\in\Re^{n\times r}$ is the input transformation matrix, $\tau\in\Re^{r\times 1}$ is the input vector,

$A(q) \in \Re^{m \times n}$ is the matrix associated with the constraints, and $\lambda \in \Re^{m \times 1}$ is the vector of constraint forces.

Surface friction is considered as:

$$f(\dot{q}) = F_v \dot{q}_i + F_d \,\mathrm{sgn}(\dot{q}_i) \tag{2}$$

where F_v and F_d are the coefficients of the viscous and dynamic frictions, respectively.

The dynamics of the DC servomotors, which drive the wheels of the robot, can be expressed as

$$\begin{aligned} \tau_s &= K_T i_a \\ L\dot{i}_a + Ri_a + K_e \dot{\varphi}_e &= u \end{aligned} \tag{3}$$

where $\boldsymbol{\tau}_e \in R^n$ is the vector of torque generated by the motor, $\mathbf{K}_T \in R^{n \times n}$ is the positive definite constant diagonal matrix of the motor torque constant, $\mathbf{i}_a \in R^n$ is the vector of armature currents; L, R, and K_e are the diagonal matrix of armature inductance, armature resistance and back electromotive force constant of the motors, respectively; and $\dot{\varphi}_e$ is the angular velocities of the actuator motors.

The motor torque τ_s and the wheel torque τ are related by the gear ratio N as

$$\tau = N\tau_s \tag{4}$$

where N is a positive definite and constant diagonal matrix. The angular velocities of the actuators $\dot{\varphi}_e$ is related to the wheel angular velocities v_w as

$$v_w = N^{-1}\dot{\varphi}_e \tag{5}$$

By ignoring the armature inductance and considering relations (4)-(5), Eq. (3) can be written as

$$\tau = K_1 u - K_2 v_w \tag{6}$$

where $K_1 = (NK_T / R_a)$ and $K_2 = NK_e K_1$. The relation between the wheel angular velocities v_w and the velocity vector v is

$$v_w = \begin{pmatrix} \omega_r \\ \omega_l \end{pmatrix} = \begin{pmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{pmatrix} v \equiv \Sigma v \tag{7}$$

Substituting (6) and (7) in (1), the equation of WMR, including actuator dynamics, can be obtained as

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = \\ B(q)\,(K_1 u - K_2 \Sigma v) - A^T(q)\lambda \end{aligned} \tag{8}$$

The kinematic model of WMR can be expressed as

$$\dot{q} = S(q)v \tag{9}$$

By taking time derivative of the kinematic model (8), the robot dynamics (8) can be transformed as

$$\overline{M}\dot{v} + \overline{C}v + \overline{F} + \overline{\tau}_d = K_1 \overline{B}\, u \tag{10}$$

where

$$\overline{M} = S^T M S\,,\ \overline{C} = S^T M \dot{S} + S^T C S + K_2 \overline{B}\Sigma \tag{11}$$

and

$$\overline{\mathrm{B}} = \mathrm{S}^{\mathrm{T}} B\ ,\ \overline{\mathrm{F}} = \mathrm{S}^{\mathrm{T}} F\ , \overline{\tau}_d = \mathrm{S}^{\mathrm{T}} \tau_d$$

According to (11), the input voltages of the wheel actuators are considering as the control inputs.

2.2. Model Predictive Control

The MPC is an optimal control that uses predictions of the system output to calculate the control law [18].

At each sampling instant, the model of the system is used to predict the output of the system over the prediction horizon N_p, and by minimizing a predefined objective function, the future sequence of control inputs is computed. By using the receding horizon strategy, only the first control action in the predicted input sequence is applied to the system until the next sampling time [18]. The horizons are moved one sample period towards the future, and the optimization process is repeated.

Consider the following nonlinear state-space model:

$$x_{t+1} = f(x_t, u_t) \tag{12}$$

where $x_t \in R^n$ and $u_t \in R^m$ are the system state and the control input, respectively. In this paper, it is assumed that function f in (12) is continuous over $R^n \times R^m$. By defining error vectors $\tilde{x} = x - x_r$ and $\tilde{u} = u - u_r$, the cost function can be formulated as

$$\begin{aligned} J(k) = \sum_{j=1}^{N_p} \tilde{x}^T(k+j-1 \mid k)\, Q\, \tilde{x}(k+j-1 \mid k) \\ + \sum_{j=1}^{N_c} \tilde{u}^T(k+j-1)\, R\, \tilde{u}(k+j-1) \end{aligned} \tag{13}$$

where N_p and N_c are the prediction and control horizons, respectively; and $Q \geq 0$ and $R \geq 0$ are the weighting matrices for the error vectors of the state and control variables, respectively.

The constraints on the amplitude of the control variable is defined as

$$u_{\min} \leq u(k+j \mid k) \leq u_{\max} \tag{14}$$

Hence, the nonlinear optimization problem can be expressed as

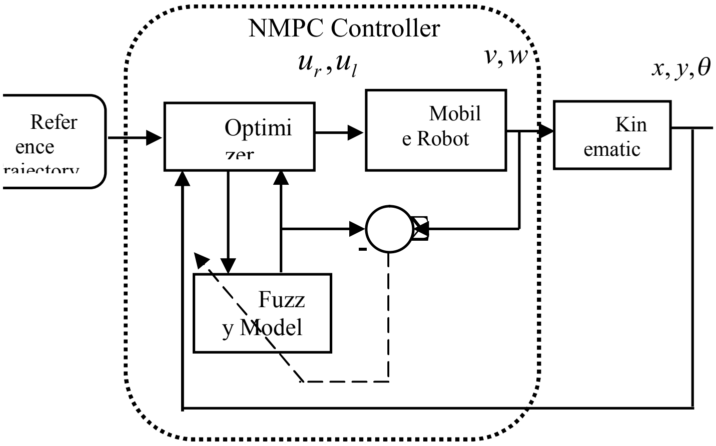

Fig. 1. Block diagram of proposed NMPC

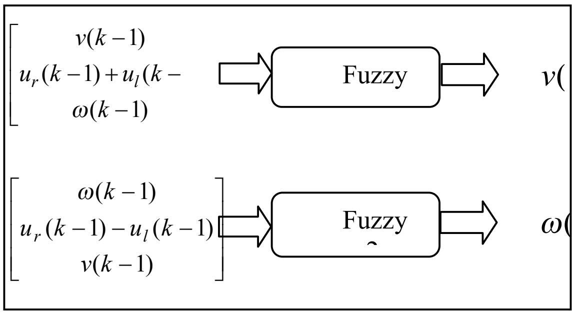

Fig. 2. Block diagram of Fuzzy model

$$u^* = \arg\min_{x,u}\{J(k)\} \qquad (15)$$

At each sampling time k, the optimization problem (15) is solved. Then, the first element of the sequence of optimal control $u^*(k\,|\,k)$ is applied to the system. This procedure is repeated at time $k+1$.

2.3. Fuzzy Model Structure

Fuzzy systems are appropriate candidates for modeling and control of nonlinear systems. An adaptive fuzzy system is defined as fuzzy logic systems whose rules are adapted through the training process. The fuzzy system adopted in this paper includes singleton fuzzifier, product inference engine, and the center-average defuzzifier. The parameters of the fuzzy model are adjusted on-line by using gradient descent algorithm. The fuzzy system can be expressed as

$$y = \sum_{i_1, i_2 \ldots i_n \in I} \xi_{i_1, i_2, \ldots i_n}(x)\, y_{i_1, i_2, \ldots i_n} \qquad (16)$$

$$\xi_{i_1, i_2, \ldots i_n}(x) = \frac{\prod_{j=1}^{n} A^j_{i_j}(x_j)}{\sum_{i_1 i_2 \ldots i_n \in I} \prod_{j=1}^{n} A^j_{i_j}(x_j)} \quad \begin{cases} i_j = 1,2,\ldots,N_j \\ j = 1,2,\ldots,n \end{cases} \qquad (17)$$

The fuzzy membership functions are of Gaussian type

$$A^j_{i_j}(x_j, p_{i_j}, q_{i_j}) = \exp\left[-\frac{\left(x_j - p_{i_j}\right)^2}{2q^2_{i_j}}\right] \qquad (18)$$

with the center p_{i_j} and the width q_{i_j}.

3. ADAPTIVE FUZZY NMPC DESIGN

The purpose of trajectory tracking of WMRs is to obtain a control law based on an adaptive fuzzy NMPC technique. The overall control structure is shown in Fig. 1. The proposed fuzzy model is used to approximate the model of the mobile robot, including the actuator dynamics, in order to predicting the future output. The gradient descent algorithm is employed to adapt the parameter's uncertainties.

The fuzzy model consists of two parallel fuzzy systems as shown in Fig.2. Each fuzzy system has three inputs and one output. The vectors

$$[v(k-1) \quad \omega(k-1) \quad u_r(k-1)+u_l(k-1)]^T$$

and

$$[w(k-1) \quad v(k-1) \quad u_r(k-1)-u_l(k-1)]^T$$

are the first and the second fuzzy system input variables, respectively. The parameters $u_r(k-1)$ and $u_l(k-1)$ denote the right and the left wheel voltages of the robot, respectively. The outputs are the linear velocity $v(k)$, and the angular velocity $\omega(k)$ at time instant k. Fuzzy membership functions are shown in Fig.3. The fuzzy rule base contains rules covering all combinations of membership functions of the 3 input variables, giving a total of 45 rules. These rules are obtained using data gathered from the input-output of the system.

According to the NMPC algorithm, the cost function is given by (13). Constraints in the optimization problem can be considered as (14).

4. SIMULATION RESULTS

The parameters of the WMR and control parameters are summarized in Table 1. The parameter m_c is the mass of the platform without the driving wheels and the rotors of the DC motors, m_ω denotes the mass of each driving wheel plus the rotor of its motor, I_c denotes the moment of inertia of the platform without the driving wheels and the rotors of the motors and I_m denotes the moment of inertia of each wheel and the motor rotor about a wheel diameter.

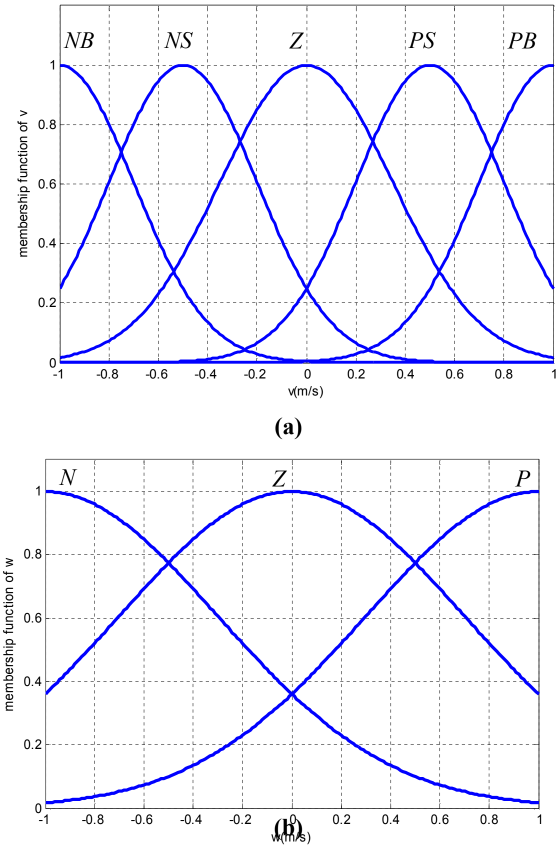

Figure 3. membership functions of (a) first input and output, (b) second and third inputs of fuzzy systems

It is assumed that the value of parameters, such as mass (m), the moment of inertia (I), the wheel radius (r), the distance between two wheels ($2b$), and the actuator parameters (K_1 and K_2), are uncertain.

The kinematic and dynamic matrices in Eq. (11) are expressed as

$$\bar{C}=\begin{bmatrix}\frac{2K_2}{r^2} & m_c d\dot{\theta} \\ -m_c d\dot{\theta} & \frac{2b^2K_2}{r^2}\end{bmatrix},$$

$$S(q)=\begin{bmatrix}\cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1\end{bmatrix}, \overline{M}=\begin{bmatrix}m & 0 \\ 0 & I\end{bmatrix}, \qquad (19)$$

where $m=m_c+2m_\omega$ and $I=I_c+2I_m+m_c d^2+2m_\omega b^2$.

The motor voltages bounds are [-12 12] V. The horizon parameters of the output system and the controller are selected as $N_p=5$ and $N_c=1$. The sampling time is 0.1 sec. The weighting matrices are

$$\mathbf{Q}=\text{diag}\{50,30,1\}, \quad \mathbf{R}=\text{diag}\{0.005,0.005\}.$$

For simulation purposes, a smooth desired trajectory is chosen as follows:

$$x_r(t)=10+7.5\cos(\omega_r t), \\ y_r(t)=25+7.5\sin(v_r t) \qquad (20)$$

where $\omega_r(t)=0.2$ and $v_r(t)=1.5$. The initial position of WMR is selected as $q_0(t)=[19 \quad 25 \quad pi/2]^T$.

Simulation results of the proposed fuzzy NMPC are shown in Figs. 4 to 6. As these figures show, the WMR can follow the desired path with good accuracies in position as well as in velocity.

5. CONCLUSION

To achieve better path tracking for WMRs, an adaptive fuzzy NMPC method was designed in this paper. The proposed controller can solve the integrated kinematic and dynamic tracking problem in presence of both parametric and nonparametric uncertainties. To this end, a fuzzy model, whose parameters updated on-line by gradient descent algorithm, has been employed in this paper. While this fuzzy system can provide appropriate model of the robot to the NMPC, it can cope with any changes in robot parameters. Simulation results on a type (2,0) WMR illustrate the effectiveness of the proposed control scheme. Future work will focus on the stability analysis of the proposed method.

Table 1. WMR parameters

Parameter	Simulation value	Parameter	Simulation value
r(m)	0.15	I_m (Kg.m^2)	0.0025
b (m)	0.75	I_c (Kg.m^2)	15.625
d(m)	0.3	I_ω (Kg.m^2)	0.005
L(m)	0.1	dt(s)	0.02
m_ω (m)	1	K_1	7.2
m_c (m)	36	K_2	2.592

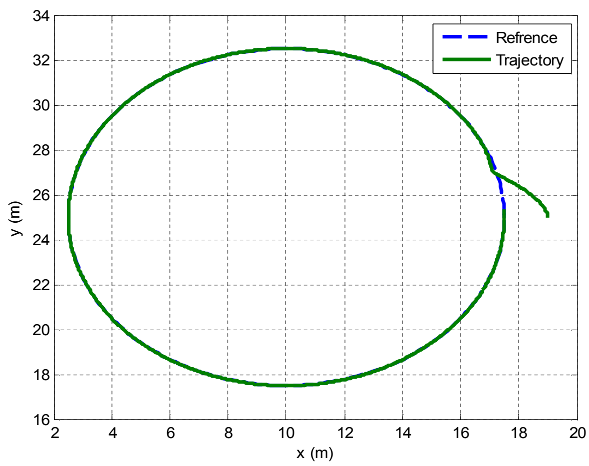

Figure 4. Desired and actual trajectories for WMR

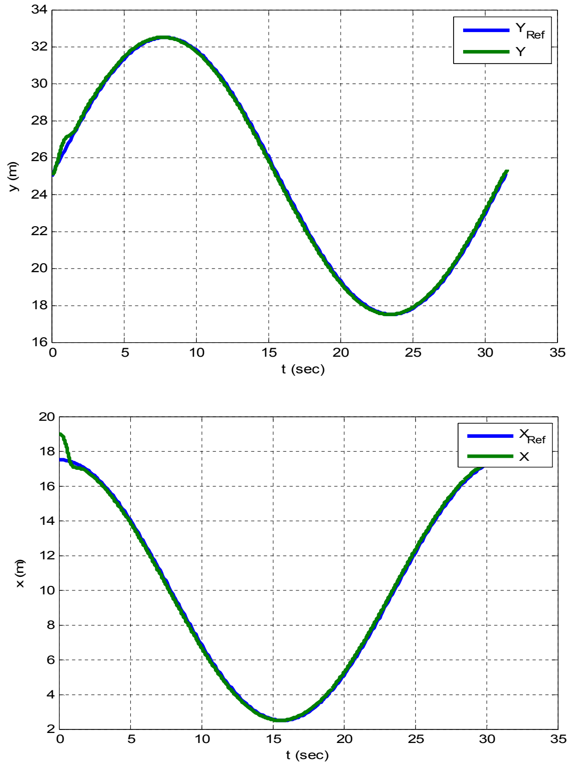

Figure 5. Desired and actual positions of WMR

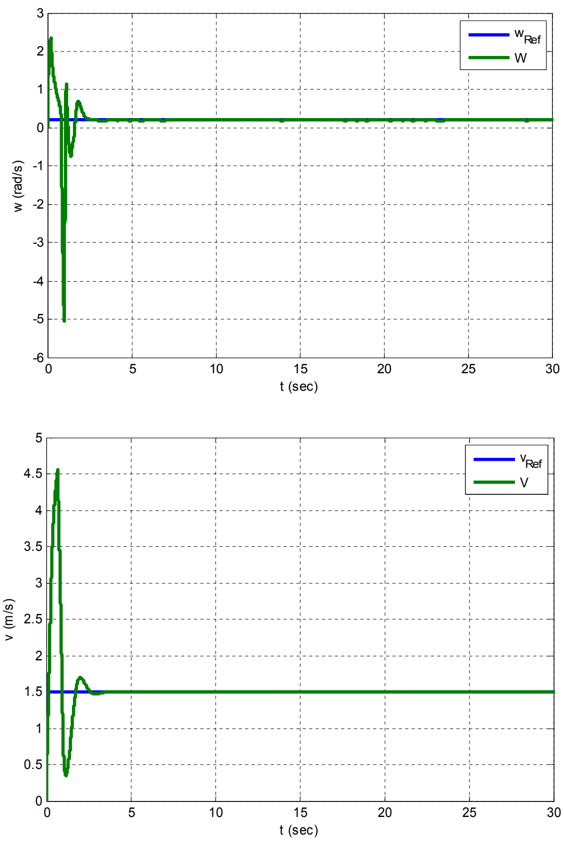

Figure 6. Desired and actual velocities of WMR

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