Designing a Hybrid Controller for Non-minimum Phase Quadruple Tank System with Model Uncertainties

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Abstract

This paper is concerned with the design of a hybrid state-feedback sliding-mode controller using fuzzy logic for a multivariable laboratory process of quadruple tank system. This apparatus is set to operate in its non-minimum phase mode which is more challenging to control as compared to the minimum phase mode. In the proposed control strategy, the consequent part of the fuzzy rules consists of either a sliding-mode controller (SMC) or a state-feedback controller (SFC). The proposed method takes advantages of the fast transient response of the SMC and the zero steady-state errors in SFC. Experimental results confirm the effectiveness of the proposed method as compared to the standalone SMC and SFC methods, especially when there are uncertainties in the model of the system.

Keywords: Non-minimum phase system; Quadruple tank system; Sliding-mode control; State-feedback control; Fuzzy hybrid systems

1. INTRODUCTION

The well known multivariable laboratory process called quadruple tank system (QTS) consists of four interconnected water tanks, two pumps and two valves. This system has been widely used as bench mark problem in multivariable control to show its performance limitations, especially when it operates in non-minimum phase mode. I.e., the linearized dynamics of the process exhibits a multivariable zero on the right-hand side of the *s*-plane. This situation is achieved by adjusting the position of valves on the system [1]. This feature has attracted many researchers to control this process in both minimum and non-minimum phase modes.

Shneiderman and Palmor have extended the QTS to include multivariable dead times, which may introduce infinite, finite or not any non-minimum phase zeros [2]. They have shown that the existence of the non-minimum phase zero depends on particular combination of multivariable dead times.

Malar and Thyagarajan have proposed decentralized fuzzy pre-compensated PI controller for QTS in both minimum and non-minimum phase modes [3]. They have employed relative-gain array analysis [4] for decentralized control of this process and have shown that in non-minimum phase mode the input-output pairing should be reversed and the controller yields smoother output without oscillation, which would increase the actuator life time.

Biswas et al. have developed a sliding-mode controller (SMC) for QTS in non-minimum phase mode in which the controller was based on feedback linearization method [5]. Although the proposed method provides robust control of the process, the presence of discontinuous function in the controller creates chatterings, which is undesirable for system performances. To reduce this effect, they have considered the well-known boundary layer around the sliding surface that creates steady-state errors. Gareli et al. have proposed a collective SMC for QTS in minimum phase mode [6]. An inherent property of the multivariable systems is the interaction between their deferent inputs and outputs. In this Gareli have presented a partial decoupling method for MIMO systems and implemented it to the non-minimum phase QTS and have shown that the switching is carried out at very high frequencies [7]. Another approach, which prevents chattering in SMC, has been proposed by Alfi and Farrokhi [8] and Hosseini et al. [9]. In this method, a combination of SMC and SFC controllers using fuzzy logic has been implemented to a SISO system.

In this paper, the objective is to combine the SMC and SFC by means of fuzzy logic for applying to the non-minimum phase MIMO QTS when there are uncertainties in the model of the system.

This paper is organized as follows: Section 2 discusses features of the nonlinear QTS. In Section 3, modelling and parameter estimation of QTS will be given. Sections 4 and 5 describe the design of the SMC and the SFC, respectively. In Section 6, the combined controller will be designed and implemented to the QTS. Section 7 shows simulation results followed by conclusion in Section 8.

2. PROCESS DESCRIPTION

The quadruple tank system (QTS) consists of four interconnected water tanks with two pumps [1]. The schematic diagram of this system is shown in Fig. 1. One of the interesting characteristics of this system is placing one of its multivariable zeros on either half of the “*s*” plane by changing the position of two valves. The manipulated variables of QTS are voltages applied to the pumps and its controlled variables are the water levels in two lower tanks (i.e., tanks 1 and 2). The output of each pump is divided into two tanks, one in the lower part and another in the upper part, diagonally opposite. In other words, the outflow of pump 1 splits between tank 1 and tank 4; similarly, the outflow of pump 2 splits between tank 2 and tank 3. The split ratio is determined by the position of the valves. The quadruple tank process has two transmission zeros. The position of one of these zeros depends on the split fraction γ_1 and γ_2 in valves 1 and 2, respectively. The minimum and non-minimum phase modes can be achieved as

$$\begin{aligned} &\text{Minimum phase:} && 1<(\gamma_1+\gamma_2)<2 \\ &\text{Non-minimum phase:} && 0<(\gamma_1+\gamma_2)<1 \end{aligned} \tag{1}$$

The non-minimum phase mode (i.e., when there exists right-half-plane zeros) of this system imposes serious limitations on the performance of the controller.

3. MODELLING AND PARAMETER ESTIMATION

The governing dynamical equations of QTS is [1]

$$\begin{aligned}
\frac{dh_1}{dt} &= \frac{-a_1}{A_1}\sqrt{2gh_1}+\frac{a_3}{A_1}\sqrt{2gh_3}+\frac{\gamma_1 k_1}{A_1}v_1 \\
\frac{dh_2}{dt} &= \frac{-a_2}{A_2}\sqrt{2gh_2}+\frac{a_4}{A_2}\sqrt{2gh_4}+\frac{\gamma_2 k_2}{A_2}v_2 \\
\frac{dh_3}{dt} &= \frac{-a_3}{A_3}\sqrt{2gh_3}+\frac{(1-\gamma_2)k_2}{A_3}v_2 \\
\frac{dh_4}{dt} &= \frac{-a_4}{A_4}\sqrt{2gh_4}+\frac{(1-\gamma_1)k_1}{A_4}v_1 \\
y &= \begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix}\begin{bmatrix}h_1\\h_2\end{bmatrix}
\end{aligned} \tag{2}$$

where h_i, a_i and A_i $(i=1,\ldots,4)$ are the water level, the cross section of the outlet hole and the cross section of the corresponding tank, respectively, γ_1 and γ_2 are the split coefficients of valves 1 and 2, respectively, v_1 and v_2 are the voltage applied to the pumps, respectively, k_1 and k_2 are constants relating the control voltages with water flow from the pumps and g is the gravitational constant.

For designing SFC, all parameter values of system are needed because this method depends on the model of the system. To this end, parameters k_1, k_2, γ_1, and γ_2 in (2) need to be estimated. For parameter estimation, data have been collected from the laboratory system by applying step inputs to the pumps. The parameter values are estimated using the *Idnlgrey* model in MATLAB software. This model is a Grey-Box and describes the system behaviour as a set of nonlinear differential equation with unknown parameters. Model validation is shown in Fig. 2. As this figure shows, the black line describes the measured values and the blue line describes the *Idnlgrey* model. Estimated parameters are presented in Table 1, where $0<\gamma_1+\gamma_2\approx 0.76<1$, which shows that the system is in its non-minimum phase mode.

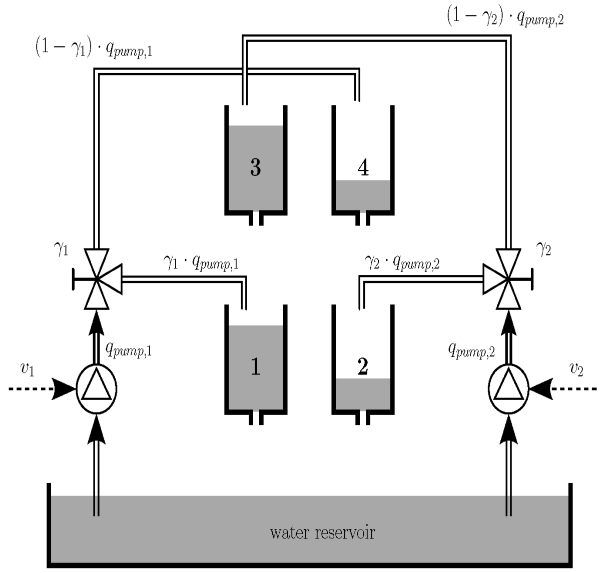


Fig.1. Schematic diagram of QTS.

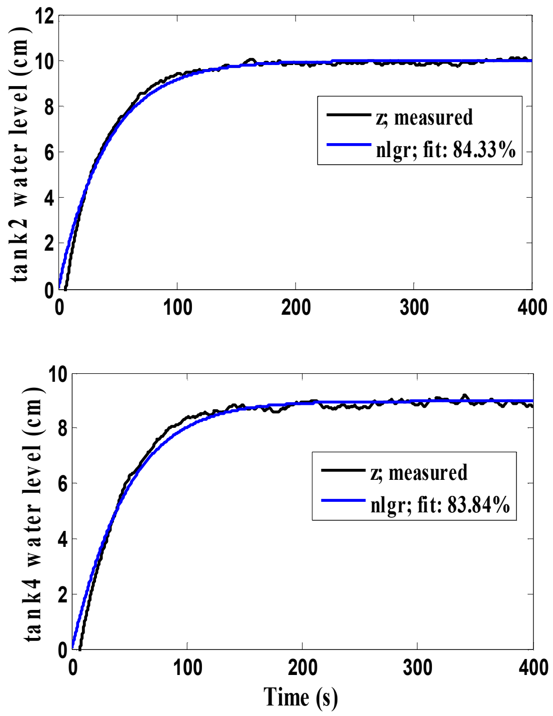


Fig.2. Model validation

Table 1. Estimated parameters

γ_1	γ_2	k_1	k_2
0.42	0.34	27.43	19.55

4. DESIGN OF SMC

The standard normal form for a 2×2 MIMO system is [5]

$$\begin{aligned} \dot{x}_1 &= x_2^1 \\ \dot{x}_2^1 &= \ddot{x}_1^1 = f_1^1(x) + g_1^1(x)u_1 + g_2^1(x)u_2 \\ \dot{x}_1^2 &= x_2^2 \\ \dot{x}_2^2 &= \ddot{x}_1^2 = f_1^2(x) + g_1^2(x)u_1 + g_2^2(x)u_2 \end{aligned} \tag{3}$$

where $\mathbf{x}=[x_1^1\ x_2^1\ x_1^2\ x_2^2]^T$ is the state vector and $\mathbf{y}=[x_1^1\ x_1^2]^T$ is the output vector.

Equation (2) is not in the form of (3) and hence, should be transformed to the standard normal MIMO form.

In QTS, in non-minimum phase mode, the manipulated variables $[u_1\ u_2]^T$ have little effects on the levels of the bottom two tanks since their dynamics are mainly controlled by the water flow from their respective upper tanks. Hence, the flow ratio from their direct pump can be ignored. Thus, for determining the relative degree of the system for designing the SMC, inputs u_1 and u_2 must appear in the controlled variables h_1 and h_2 [10]. Therefore, by taking derivatives of $\dot{h}_1$ and $\dot{h}_2$ in (2), it gives

$$\begin{aligned} \dot{y}_1 &= \dot{x}_1^1 = x_2^1 \\ \dot{x}_2^1 &= \ddot{h}_1 = \left(\frac{a_1^2 g}{A_1^2} - \frac{a_1 a_3 g\sqrt{h_3}}{A_1^2\sqrt{h_1}} - \frac{a_3^2 g}{A_1 A_3}\right) \\ &\quad + \left(-\frac{a_1 g\gamma_1 k_1}{A_1^2\sqrt{2gh_1}}\right)u_1 + \left(\frac{a_3 g(1-\gamma_2)k_2}{A_1 A_3\sqrt{2gh_3}}\right)u_2 \\ \dot{y}_1 &= \dot{x}_1^2 = x_2^2 \\ \dot{x}_2^2 &= \ddot{h}_2 = \left(\frac{a_2^2 g}{A_2^2} - \frac{a_2 a_4 g\sqrt{h_4}}{A_2^2\sqrt{h_2}} - \frac{a_4^2 g}{A_2 A_4}\right) \\ &\quad + \left(\frac{a_4 g(1-\gamma_1)k_1}{A_2 A_4\sqrt{2gh_4}}\right)u_1 + \left(-\frac{a_2 g\gamma_2 k_2}{A_2^2\sqrt{2gh_2}}\right)u_2 \end{aligned} \tag{4}$$

According to (4), the relative degree of the QTS in non-minimum phase mode is equal to two. Based on the sliding surface equation

$$s = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \tag{5}$$

The sliding surfaces for this MIMO system can be written as

$$\begin{aligned} s_1 &= \lambda e_1 + \dot{e}_1 \\ s_2 &= \lambda e_2 + \dot{e}_2 \end{aligned} \tag{6}$$

where λ is a positive constant and $e = x - x_d$ is the tracking error. Sufficient condition for reaching error trajectories on the sliding surfaces and staying on them is that the manipulated variables u_1 and u_2 are designed such that the following sliding condition is satisfied :

$$\frac{1}{2}\frac{d}{dt}\left(s_1^2 + s_2^2\right) \le -\eta_1|s_1| - \eta_2|s_2| \qquad \forall t \tag{7}$$

where η_1 and η_2 are small positive constants.

By considering the uncertainties $\hat{f}_1^1$ and $\hat{f}_1^2$ for f_1^1 and f_1^2 in (3), respectively, the upper bound of uncertainties can de defined as

$$|\hat{f}_1^i - f_1^i| \le F_i \qquad i = 1,2\,. \tag{8}$$

The uncertainties on the input vector can be considered as

$$G(\mathbf{x}) - \hat{G}(\mathbf{x}) = \Delta\hat{G}(\mathbf{x}) \tag{9}$$

where $G(x) = \begin{bmatrix} g_1^1(x) & g_2^1(x) \\ g_1^2(x) & g_2^2(x) \end{bmatrix}$.

It can be shown that the sliding control law can be derived as [10]

$$u = \begin{pmatrix} -\dfrac{a_1 g\gamma_1 k_1}{A_1^2\sqrt{2gh_1}} & \dfrac{a_3 g(1-\gamma_2)k_2}{A_1 A_3\sqrt{2gh_3}} \\ \dfrac{a_4 g(1-\gamma_1)k_1}{A_2 A_4\sqrt{2gh_4}} & -\dfrac{a_2 g\gamma_2 k_2}{A_2^2\sqrt{2gh_2}} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \ddot{h}_{1d} - \left[\left(\dfrac{a_1^2 g}{A_1^2} - \dfrac{a_1 a_3 g\sqrt{h_3}}{A_1^2\sqrt{h_1}} - \dfrac{a_3^2 g}{A_1 A_3}\right) + \lambda\dot{e}_1\right] - k_{s1}\,\mathrm{sgn}(s_1) \\ \ddot{h}_{2d} - \left[\left(\dfrac{a_2^2 g}{A_2^2} - \dfrac{a_2 a_4 g\sqrt{h_4}}{A_2^2\sqrt{h_2}} - \dfrac{a_4^2 g}{A_2 A_4}\right) + \lambda\dot{e}_2\right] - k_{s2}\,\mathrm{sgn}(s_2) \end{pmatrix} \tag{10}$$

where $\mathbf{k}_s = [k_{s1}\ k_{s2}]^T$ satisfies the reaching condition (7) with the sign function defined as

$$\mathrm{sgn}(s) = \begin{cases} 1 & \text{if } s > 0, \\ 0 & \text{if } s = 0, \\ -1 & \text{if } s < 0. \end{cases} \tag{11}$$

5. DESIGN OF SFC

By linearizing (2) around the equilibrium point using Taylor series expansion, the state-space realization of QTS can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \tag{12}$$

where

$$\mathbf{A}=\begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{1}{T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{1}{T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix}, \ \mathbf{B}=\begin{bmatrix} \frac{\gamma_1 k_p}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_2} \\ \frac{(1-\gamma_1)k_1}{A_1} & 0 \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad T_i=\frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}} \quad (i=1,\ldots,4)$$

in which h_i^0 ($i=1,\ldots,4$) are the equilibrium points. The objective of implementing the state-feedback controller is to minimize the following performance index:

$$J=\int_0^\infty (\mathbf{x}^T\mathbf{Q}\mathbf{x}+\mathbf{u}^T\mathbf{R}\mathbf{u})dt$$

where $\mathbf{Q}$ and $\mathbf{R}$ are constant and positive-definite matrices. The optimal control law is

$$\mathbf{u}_{\mathrm{SFC}}=-\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{x} \tag{13}$$

where $\mathbf{P}$ is a symmetric positive-definite matrix that satisfies the following algebraic Riccati equation:

$$-\mathbf{P}\mathbf{A}-\mathbf{A}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}-\mathbf{Q}=0.$$

Hence, the SFC for the QTS is

$$\mathbf{u}_{\mathrm{SFC}}=-\mathbf{K}^{\mathrm{T}}\mathbf{x} \tag{14}$$

where the gain matrix $\mathbf{K}$ is equal to

$$\mathbf{K}=-\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} \tag{15}$$

6. HYBRID CONTROLLER DESIGN

In this section, a combination of the SMC and the SFC with the aid of the fuzzy logic will be presented. It is well known that the SMC has a fast transient response and is robust against uncertainties in the system. However, when the system trajectories are near the sliding surfaces the chattering phenomenon occurs. By introducing a boundary layer around the operating point, as several researchers perform, the chattering can be avoided, but there will be steady-state errors. In order to eliminate the chattering and at the same time obtaining zero steady-state response, the SFC will be used when the system trajectories are in the neighbourhood of the sliding surfaces. The switching between these two controllers is performed using a fuzzy system.

The fuzzy IF-THEN rules for the combined controller are defined as

$$\begin{array}{l}\text{Rule1: IF } |e| \text{ is H, THEN } u=u_{\mathrm{SMC}} \\ \text{Rule2: IF } |e| \text{ is L, THEN } u=u_{\mathrm{SFC}}\end{array} \tag{16}$$

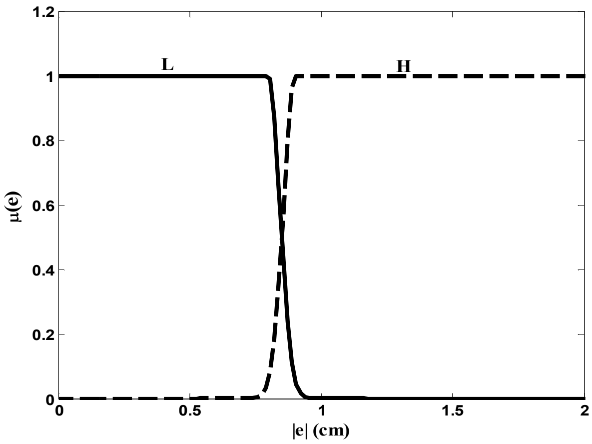


Fig. 3. Input membership functions

where $|e|$ is the absolute value of the tracking error and H and L are fuzzy variables standing for high and low, respectively, with the membership functions shown in Fig. 3. Since there are two manipulated variables in QTS, two fuzzy systems are needed, where $|e|$ is defined as the corresponding fuzzy input variable. As discussed in Section 4, when the system is in the non-minimum phase mode, the water level of the lower tanks is mainly controlled by the outflow of their respective upper tanks. Thus, in the fuzzy system, $|e_1|$ is the input of the fuzzy controller for generating u_2 and $|e_2|$ is the input of the fuzzy system for generating u_1. When the states of the system are far from the operating point, the first rule in (16) is triggered and hence, the SMC is applied to the system. On the other hand, when the states of the system are near the sliding surface (i.e., near the operating points) the second rule is activated and the SFC is applied to the system. Finally, when the states of the system are neither far from the operating point and nor near them, a combination of the SMC and the SFC is applied to the system. By using the weighted-sum defuzzification method, the inputs to the pumps $[u_1 \ u_2]$ can be obtained as

$$u_i(t)=\frac{\mu_H(e_i)u_{\mathrm{SMC}(i)}+\mu_L(e_i)u_{\mathrm{SFC}(i)}}{\mu_H(e_i)+\mu_L(e_i)} \quad (i=1,2). \tag{17}$$

7. EXPERIMENTAL RESULTS

The parameter values of the QTS in the non-minimum phase mode, are represented in Tables 1 and 2.

The SMC is applied to the system with the switching parameters $k_{s1}=k_{s2}=5$ and $\lambda=0.1$. In addition, the gain matrix $\mathbf{K}$ in SFC has been determined as

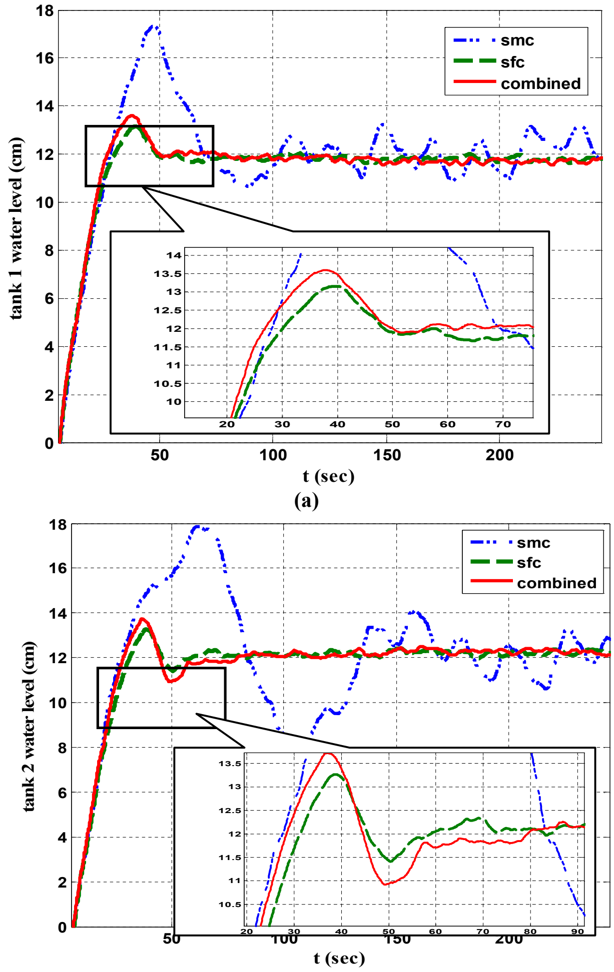
$$\mathbf{K}=\begin{bmatrix} 0.2422 & 0.1954 & 0.0522 & 0.1541 \\ 0.1932 & 0.1409 & 0.1675 & -0.0144 \end{bmatrix}$$

It should be mentioned that the same parameters are used in experiments for the combined fuzzy SMC-SFC controller proposed in this paper.

As depicted in Figs. 4 and 5, the combined controller has better performance as compared to the SMC and SFC. The SMC has undesirable overshoots and chattering in control signals, which can damage the pumps in a short time. Moreover, the SFC responses are not as fast as the SMC and have larger rise times. On the other hand, the proposed controller has better response as compared to both controllers. Table 3 summarizes the performance of different controllers.

Next, by changing the cross section of the outlet hose of pump 1, the performance of the SFC and the hybrid controller are compared with each other. As Fig. 6 shows, the effect of this uncertainty on the SFC is much larger than the proposed controller during steady-state case. This is mainly due to the fact that the SFC is depends on the model of system. Fig. 7 shows that the hybrid controller continuously switches between the SFC and the SMC in order to overcome the large uncertainty.

The block diagram of the proposed controller is shown in Fig. 9.



(a)

(b)

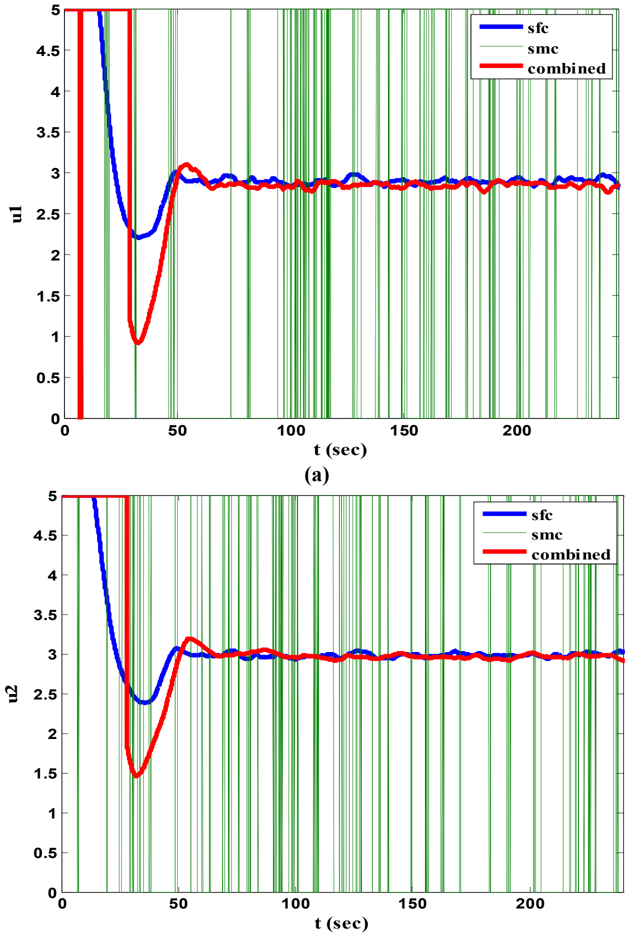
Fig. 4. Performance of three controllers. The desired value for both tanks is 12 cm, (a) Water level in tank 1 and (b) Water level in tank 2.

Table 2. Parameter values of QTS

Parameter	Value
Cross section of tanks $A_i \quad (i = 1,\ldots,4)$	(cm^2) 138.9
Cross section of outlet hose $a_i \quad (i = 1,\ldots,4)$	0.50265 (cm^2)
G	(cm/s^2) 981

8. CONCLUSION

In this paper, a combination of the state-feedback and the sliding mode controller using the fuzzy logic was presented for better performance of nonlinear and non-minimum phase quadruple tank system. The proposed controller has the advantages of both SMC and SFC. In other words, fast transient response of the SMC and zero steady-state error of the SFC. It has been shown in experimental results that the proposed method offers fast response as well as insignificant steady-state errors. Moreover, the combined controller can cope with system uncertainties better than SFC.



(a)

(b)

Fig. 5. Input u_1 and u_2 for different controllers.

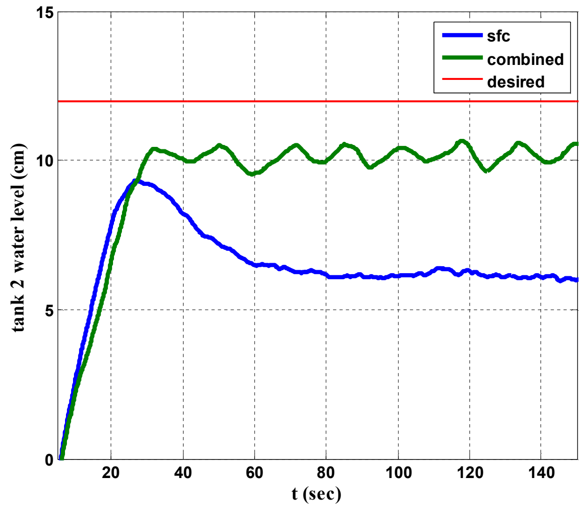


Fig. 6. Comparison of SFC and combined controller in presence of system uncertainty.

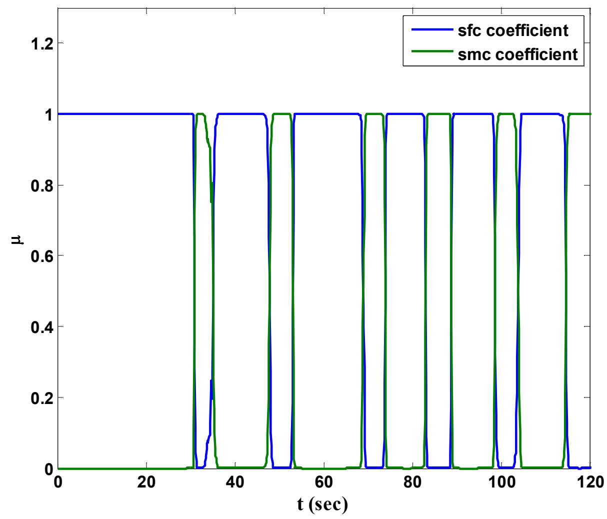


Fig. 7. Contribution of SFC and SMC to the hybrid control law for the case of Fig. 6.

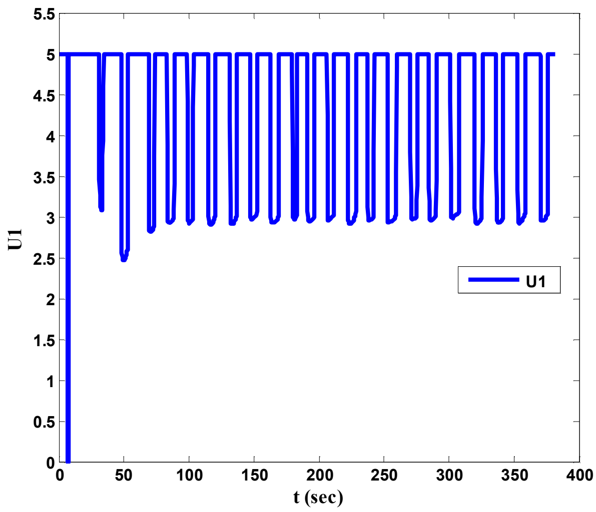


Fig. 8. Control input u_1 for the case of Fig. 7.

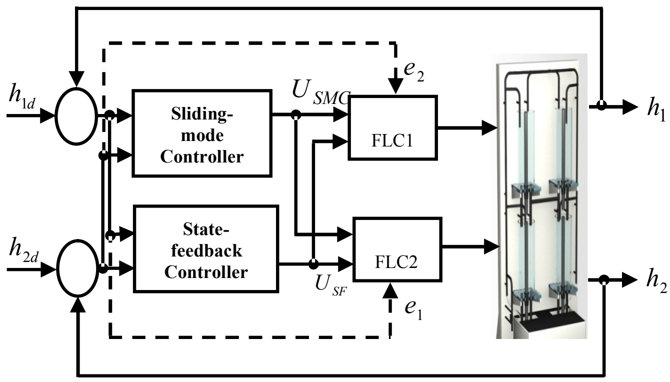


Fig. 9. Block diagram of the fuzzy SMC-SFC controller.

Table 3. Quantitative comparison of controllers for tank 1

Controller	Overshoot	Rise time (s)	Settling time (s)
SMC	44.16%	66	-
SFC	10%	80	120
Comb. Controller	13.33%	65	90

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