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Sliding-Mode State-Feedback Control of Non-Minimum Phase Quadruple Tank System Using Fuzzy Logic

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Abstract: In this paper, a combined state-feedback sliding-mode controller for quadruple tank system using fuzzy logic is presented. The quadruple system is set to operate in its non-minimum phase mode, which is more challenging as compared to the minimum phase mode. The Sliding-Mode Control (SMC) method is employed to achieve fast transient response, while the state-feedback controller (SFC) can provides zero steady-state errors. Simulation results show effectiveness of the proposed method as compared to the standalone SMC and SFC methods.

Keywords: Non minimum phase systems, Sliding-mode control, State feedback, Fuzzy hybrid systems

1. INTRODUCTION

The quadruple tank system (QTS) has been widely used in multivariable control to show the performance limitations of system especially when it operates in non-minimum phase mode, i.e. when one of its multivariable zero is positioned on the right-half of the *s*-plane by adjusting the position of valves (Johansson, 2000). This feature has attracted many researchers for control of this process in both minimum and non-minimum phase modes. Biswas et al. (2009) have developed a sliding-mode controller (SMC) for QTS in non-minimum phase mode in which the controller was based on feedback linearization method. Although the proposed method provides robust control of the process, the presence of discontinuous function in the controller created chatterings, which is undesirable for system performances. To reduce this effect, they have considered the well-known boundary layer around the sliding surface that creates steady-state errors. Gareli et al. (2008) have proposed a collective SMC for QTS in minimum phase mode.

An Inherent property of the multivariable systems is the interaction between their deferent inputs and outputs. In this regard, Gareli et al. (2006a and 2006b) have presented a partial decoupling for MIMO systems and implemented it to the non-minimum phase QTS and have shown that the switching is carried out at very high frequencies. Malar et al. (2008) have proposed decentralized fuzzy controller for QTS in both minimum and non-minimum phase modes. They have employed relative-gain-array analysis (Moaveni and Khaki-Sedigh, 2007) for decentralized control of this process and have shown that in non-minimum phase mode the input-output paring should be reversed. Another approach, which prevents occurrence of chattering in SMC, has been proposed by Alfi and Farrokhi (2008) and Hosseini et al. (2002). In this method, a combination of SMC and SFC controllers via fuzzy logic has been implemented to a SISO system.

In this paper, the objective is to combine the SMC and SFC by means of fuzzy logic for applying to a non-minimum phase MIMO system.

The remainder of this paper is organized as follows: Section 2 discusses the features of nonlinear QTS. In Section 3, the SMC will be designed. Section 4 represents the design of the SFC. In Section 5, the combined controller will be designed and implemented to the QTS. Section 6 shows simulation results followed by conclusion in Section 7.

2. QUADRUPLE TANK SYSTEM

The quadruple tank system (QTS) consists of four interconnected water tanks with two pumps (Johansson, 2000). The schematic diagram of this system is shown in Fig. 1. One of the interesting characteristics of this system is placing one of its multivariable zeros on either half of the "*s*" plane by changing the position of two valves. The manipulated variables of QTS are voltages applied to the pumps and its controlled variables are the water levels in two lower tanks (i.e tank 1 and 2). The output of each pump is divided into two tanks, one in the lower part and another in the upper part, diagonally opposite. In other words, the outflow of pump 1 splits between tank 1 and tank 4; similarly, the outflow of pump 2 splits between tank 2 and tank 3. The split ratio is determined by the position of the valves. The quadruple tank process has two transmission zeros. The position of one of these zeros depends on the split fraction γ_1 and γ_2 in valves 1 and 2, respectively. The minimum and non-minimum phase modes can be achieved as

$$\begin{aligned} &\text{Minimum phase:} && 1 < (\gamma_1 + \gamma_2) < 2 \\ &\text{Non-minimum phase:} && 0 < (\gamma_1 + \gamma_2) < 1 \end{aligned} \tag{1}$$

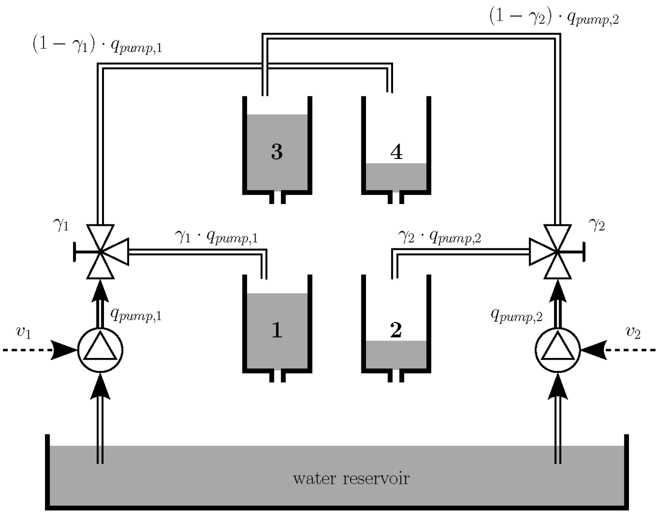


Fig.1. Schematic diagram of QTS.

The non-minimum phase mode (i.e., when there exists right-half-plane zero) of this system imposes serious limitations on the performance of the controller. The governing dynamical equations of QTS is given by (Johansson, 2000)

$$
\begin{aligned}
\frac{dh_1}{dt} &= \frac{-a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1 \\
\frac{dh_2}{dt} &= \frac{-a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2 \\
\frac{dh_3}{dt} &= \frac{-a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3} v_2 \\
\frac{dh_4}{dt} &= \frac{-a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} v_1 \\
y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}
\end{aligned} \tag{2}
$$

where h_i, a_i and A_i $(i = 1, \dots, 4)$ are the water level, the cross section of the outlet hole and the cross section of the corresponding tank, respectively, γ_1 and γ_2 are the split coefficients of valves 1 and 2, respectively, v_1 and v_2 are the voltage applied to the pumps, respectively, k_1 and k_2 are constants relating the control voltages with water flow from the pumps and g is the gravitational constant.

3. SMC DESIGN

The standard normal form for a 2×2 MIMO system is (Biswas at al. 2009):

$$
\begin{aligned}
\dot{x}_1^1 &= x_2^1 \\
\dot{x}_2^1 &= \ddot{x}_1^1 = f_1^1(x) + g_1^1(x)u_1 + g_2^1(x)u_2 \\
\dot{x}_1^2 &= x_2^2 \\
\dot{x}_2^2 &= \ddot{x}_1^2 = f_1^2(x) + g_1^2(x)u_1 + g_2^2(x)u_2
\end{aligned} \tag{3}
$$

where $\mathbf{x} = [x_1^1 \; x_2^1 \; x_1^2 \; x_2^2]^{\mathrm{T}}$ is the state vector and $\mathbf{y} = [x_1^1 \; x_1^2]$ is the output vector.

Equation (2) is not in the form of (3) and hence, should be transformed to the standard normal MIMO form.

In QTS, in non-minimum phase mode, the manipulated variables $[u_1 \; u_2]$ have little effects on levels of bottom two tanks since their dynamics are mainly controlled by the water flow from their respective upper tanks; hence, the flow ratio from their direct pump can be ignored. Thus, for determining the relative degree of system for designing the SMC, inputs u_1 and u_2 must appear in the controlled variables h_1 and h_2 (Slotine and Li, 1991). Therefore, by taking derivatives of $\dot{h}_1$ and $\dot{h}_2$ in (2), it gives

$$
\begin{aligned}
\dot{y}_1 &= \dot{x}_1^1 = x_2^1 \\
\dot{x}_2^1 &= \ddot{h}_1 = \left(\frac{a_1^2 g}{A_1^2} - \frac{a_1 a_3 g\sqrt{h_3}}{A_1^2\sqrt{h_1}} - \frac{a_3^2 g}{A_1 A_3}\right) + \left(-\frac{a_1 g\gamma_1 k_1}{A_1^2\sqrt{2gh_1}}\right)u_1 \\
&\quad + \left(\frac{a_3 g(1-\gamma_2)k_2}{A_1 A_3\sqrt{2gh_3}}\right)u_2 \\
\dot{y}_1 &= \dot{x}_1^2 = x_2^2 \\
\dot{x}_2^2 &= \ddot{h}_2 = \left(\frac{a_2^2 g}{A_2^2} - \frac{a_2 a_4 g\sqrt{h_4}}{A_2^2\sqrt{h_2}} - \frac{a_4^2 g}{A_2 A_4}\right) + \left(\frac{a_4 g(1-\gamma_1)k_1}{A_2 A_4\sqrt{2gh_4}}\right)u_1 \\
&\quad + \left(-\frac{a_2 g\gamma_2 k_2}{A_2^2\sqrt{2gh_2}}\right)u_2
\end{aligned} \tag{4}
$$

According to (4), the relative degree of the QTS in non-minimum phase mode is equal to two. Based on the sliding surface equation

$$s = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \tag{5}$$

The sliding surfaces for this MIMO system can be written as

$$
\begin{aligned}
s_1 &= \lambda e_1 + \dot{e}_1 \\
s_2 &= \lambda e_2 + \dot{e}_2
\end{aligned} \tag{6}
$$

where λ is a positive constant and $e = x - x_d$ is the tracking error. Sufficient condition for reaching error trajectories on the sliding surfaces and staying on them is that the manipulated variables u_1 and u_2 are designed such that

$$\frac{1}{2}\frac{d}{dt}\left(s_1^2 + s_2^2\right) \le -\eta_1 |s_1| - \eta_2 |s_2| \tag{7}$$

where η_1 and η_2 are small positive constants.

By considering the uncertainties $\hat{f}_1^1$ and $\hat{f}_1^2$ for f_1^1 and f_1^2 in (3), respectively, the upper bound of uncertainties can de defined as

$$|\hat{f}_1^i - f_1^i| \le F_i \qquad i = 1, 2 \tag{8}$$

The uncertainties on the input vector can be considered as

$$G(\mathbf{x}) - \hat{G}(\mathbf{x}) = \Delta\hat{G}(\mathbf{x}) \tag{9}$$

where $G(x)=\begin{bmatrix} g_1^1(x) & g_2^1(x) \\ g_1^2(x) & g_2^2(x) \end{bmatrix}$.

It can be shown that the sliding control law can be derived as (Slotine and Li 1991)

$$u=\begin{pmatrix} -\dfrac{a_1 g\gamma_1 k_1}{A_1^2\sqrt{2gh_1}} & \dfrac{a_3 g(1-\gamma_2)k_2}{A_1A_3\sqrt{2gh_3}} \\ \dfrac{a_4 g(1-\gamma_1)k_1}{A_2A_4\sqrt{2gh_4}} & -\dfrac{a_2 g\gamma_2 k_2}{A_2^2\sqrt{2gh_2}} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \ddot{h}_{1d}-\left[\left(\dfrac{a_1^2 g}{A_1^2}-\dfrac{a_1a_3 g\sqrt{h_3}}{A_1^2\sqrt{h_1}}-\dfrac{a_3^2 g}{A_1A_3}\right)+\lambda\dot{e}_1\right]-k_{s1}\,\mathrm{sgn}(s_1) \\ \ddot{h}_{2d}-\left[\left(\dfrac{a_2^2 g}{A_2^2}-\dfrac{a_2a_4 g\sqrt{h_4}}{A_2^2\sqrt{h_2}}-\dfrac{a_4^2 g}{A_2A_4}\right)+\lambda\dot{e}_2\right]-k_{s2}\,\mathrm{sgn}(s_2) \end{pmatrix} \quad (10)$$

where $\mathbf{k}_s=[k_{s1}\ k_{s2}]^{\mathrm{T}}$ satisfies the reaching condition (7) with the sign function defined as

$$\mathrm{sgn}(s)=\begin{cases} 1 & \text{if } s>0, \\ 0 & \text{if } s=0, \\ -1 & \text{if } s<0. \end{cases} \quad (11)$$

4. SFC DESIGN

By linearizing (2) around the equilibrium point using Taylor series expansion, the state-space realization of QTS can be written as

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}, \quad (12)$$

where

$$\mathbf{A}=\begin{bmatrix} -\dfrac{1}{T_1} & 0 & \dfrac{1}{T_3} & 0 \\ 0 & -\dfrac{1}{T_2} & 0 & \dfrac{1}{T_4} \\ 0 & 0 & -\dfrac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\dfrac{1}{T_4} \end{bmatrix},\ \mathbf{B}=\begin{bmatrix} \dfrac{\gamma_1 k_p}{A_1} & 0 \\ 0 & \dfrac{\gamma_2 k_2}{A_2} \\ 0 & \dfrac{(1-\gamma_2)k_2}{A_2} \\ \dfrac{(1-\gamma_1)k_1}{A_1} & 0 \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\quad T_i=\frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}\quad (i=1,\dots,4)$$

in which h_i^0 $(i=1,\dots,4)$ are the equilibrium points. The objective of implementing the state-feedback controller is to minimize the following performance index:

$$J=\int_0^\infty (\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}+\mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u})dt$$

where $\mathbf{Q}$ and $\mathbf{R}$ are constant matrices. The optimal control law is

$$\mathbf{u}_{\mathrm{SFC}}=-\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{x} \quad (13)$$

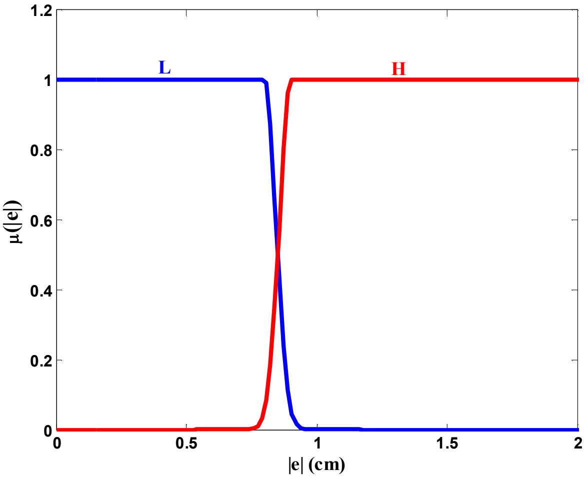


Fig. 2. Input membership functions

where $\mathbf{P}$ is a symmetric positive matrix that satisfies the following algebraic Riccati equation:

$$-\mathbf{P}\mathbf{A}-\mathbf{A}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}-\mathbf{Q}=0$$

Hence, the SFC for the QTS is

$$\mathbf{u}_{\mathrm{SFC}}=-\mathbf{K}^{\mathrm{T}}\mathbf{x} \quad (14)$$

where the gain matrix $\mathbf{K}$ is equal to

$$\mathbf{K}=-\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} \quad (15)$$

5. COMBINED CONTROL LAW

In this section, a combination of the SMC and the SFC with the aid of the fuzzy logic will be presented. It is well known that the SMC has a fast response and is robust against uncertainties in the system; however, when the system trajectories are near the sliding surfaces the chattering phenomenon occurs. By introducing a boundary layer around the operating point, as several researchers perform, there will be steady-state errors. In order to eliminate the chattering and at the same time obtaining zero steady-state response, the SFC will be used when the system trajectories are in the neighborhood of the sliding surfaces. The switching between these two controllers is via a fuzzy system.

The fuzzy IF-THEN rules for the combined controller are defined as

$$\begin{aligned} &\text{Rule 1: IF } |e| \text{ is H, THEN } u=u_{SMC} \\ &\text{Rule 2: IF } |e| \text{ is L, THEN } u=u_{SFC} \end{aligned} \quad (16)$$

where e is the tracking error and H and L are fuzzy variables standing for high and low, respectively, with the membership functions shown in Fig. 2. Since there are two manipulated variables in QTS, two fuzzy systems are needed, where e is defined as the corresponding fuzzy input variable. As discussed in Section 3, when the system is in non-minimum phase mode, the water level of lower tanks are mainly controlled by the outflow of their respective upper tanks. Thus, in the fuzzy controllers, e_1 is the input of the fuzzy controller for generating u_2 and e_2 is the input of fuzzy controller for generating u_1. When the states of the system are far from the operating point, the first rule in (16) is triggered and hence, the SMC is applied to the system; when the states of the system are near the sliding surface (i.e., the operating point) the second rule is activated and the SFC is applied to the system; and finally, when the states of the system are neither far from the operating point and nor near them, a combination of the SMC and the SFC is applied to the system. By using weighted-sum defuzzification method, the inputs to the pumps $[u_1 \;\; u_2]$ can be obtained as

$$u_i(t) = \frac{\mu_H(|e_i|)u_{SMC(i)} + \mu_L(|e_i|)u_{SFC(i)}}{\mu_H(|e_i|) + \mu_L(|e_i|)} \quad (i = 1,2). \qquad (17)$$

6. SIMULATION RESULTS

The value of parameters of the QTS in non-minimum phase mode, given in (2), are represented in Table 1.

The SMC is applied to the system with switching parameters $k_{s1} = k_{s2} = 10$ and $\lambda = 0.1$. In addition, the gain matrix K in SFC has been determined as

$$K = \begin{bmatrix} 53.45 & 73.69 & -9.9 & 14.48 \\ 73.83 & -19.05 & 17.07 & -22.36 \end{bmatrix}.$$

It should be mentioned that the same parameters are used in simulations for the combined fuzzy SMC-SFC controller proposed in this paper.

As depicted in Fig. 3 and 4, the combined controller has better performance as compared with SMC and SFC.

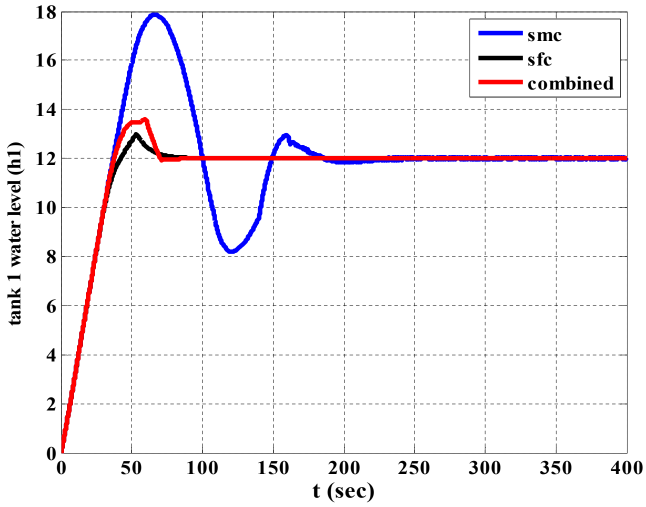
SMC has undesirable overshoots and chattering of control signals, according to Fig. 4, is very large, which can damage the pumps in a short time. Moreover, the SFC responses are not as fast as the SMC and have large rise times. On the other hand, the proposed controller has better response as compared to both controllers (Fig. 3. b). Table 2 summarizes performance of different controllers.

Next, The disturbance rejection ability of different controllers is tested by applying a step disturbance to the output of tank 1 h_1. At $t = 300$ s the water level of tank 1 is increased by 3 cm. As Fig. 5 shows this disturbance also affects the water level in tank 2 (h_2). The SMC has some overshoots and undershoots before it can stabilize the outputs. The SFC exhibits steady-state errors. On the other hand, the combined controller has the best disturbance rejection as compared to the other two controllers.

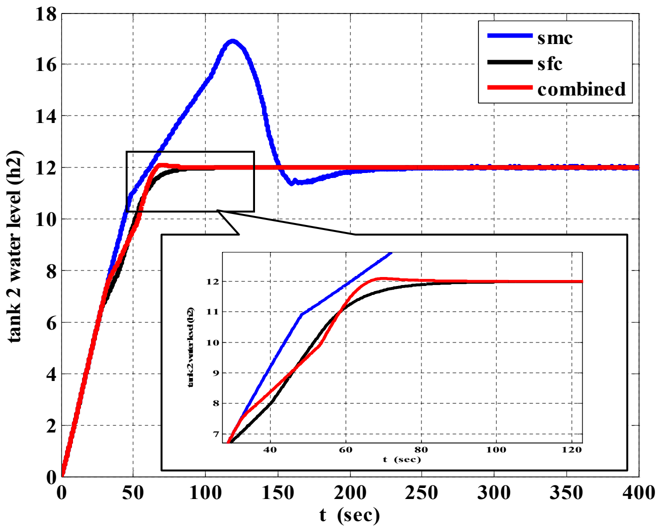
The block diagram of the proposed controller is shown in Fig. 6.

Table 1. Parameter values of QTS

Description	Value
Cross section of tank $A_i \quad (i = 1,\ldots,4)$	$138.9 \ (\mathrm{cm}^2)$
Cross section of outlet hole $a_i \quad (i = 1,\ldots,4)$	$0.50265 \ (\mathrm{cm}^2)$
G	$981 \ (\mathrm{cm/s^2})$
k_1, k_2	27.44, 27.97
γ_1, γ_2	0.23 , 0.12



(a)



(b)

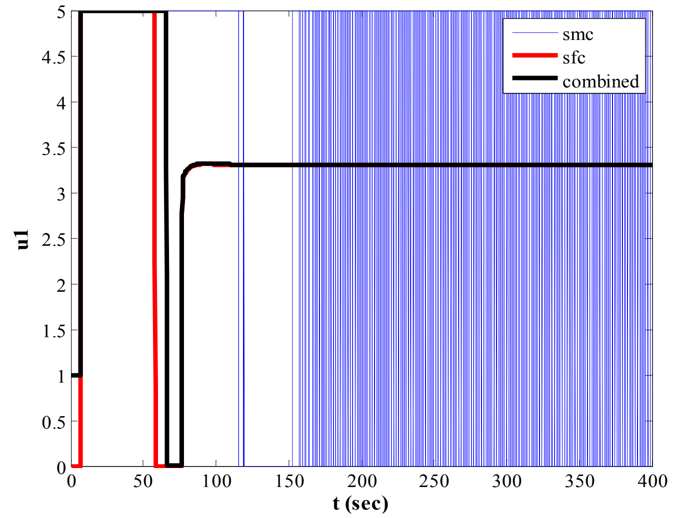
Fig. 3. Performance of three controllers applied to QTS. (a) Water level in tank 1. (b) Water level in tank 2.

Table 2. Quantitative comparison of controllers

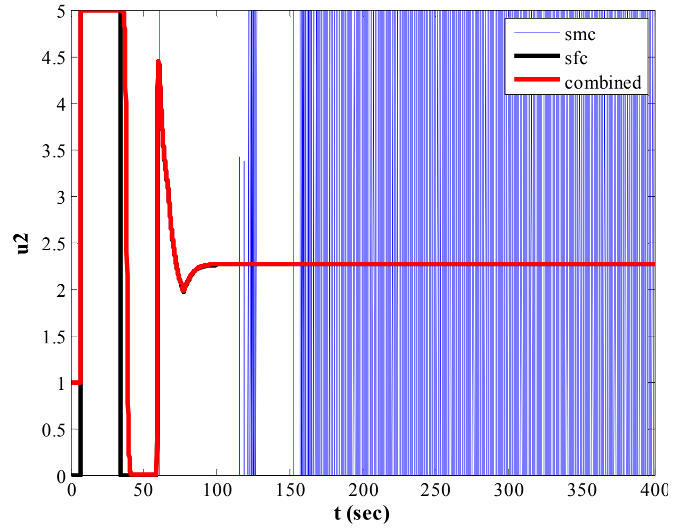
	Overshoot	Rise time (s)	Settling time (s)
SMC	41%	61	240
SFC	0	80	120
Comb. Controller	0	66	90

7. CONCLUSION

In this paper, a combination of the state-feedback and the sliding mode controller using the fuzzy logic was presented for better performance of nonlinear and non-minimum phase quadruple tank system. The proposed controller has the advantages of both SMC and SFC. In other words, fast transient response of the SMC and zero steady-state error of the SFC. It has been shown in simulations that the proposed method offers fast response as well as insignificant steady-state errors. Moreover, the combined controller has better disturbance rejection as compared to the SMC and the SFC.

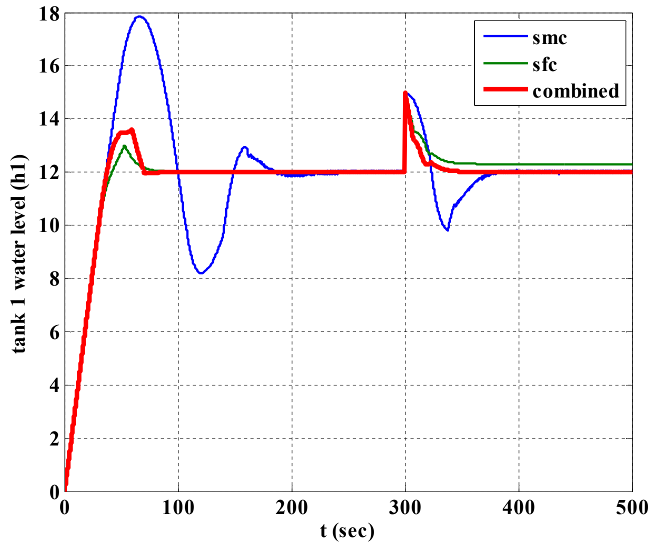


(a)

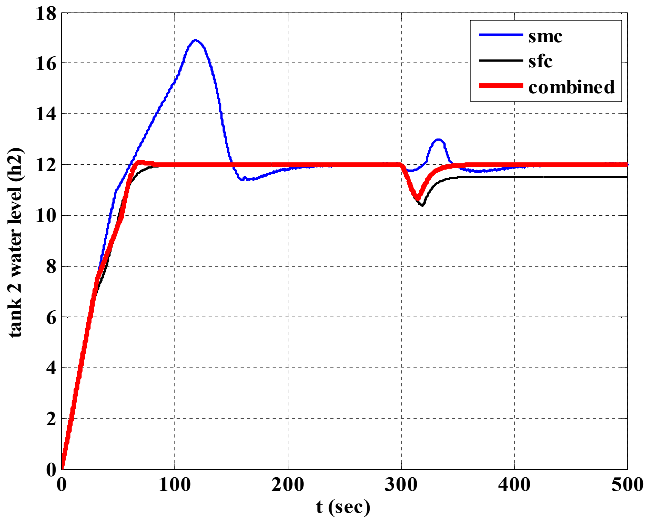


(b)

Fig.4. Manipulated variables u_1 and u_2 .



(a)



(b)

Fig. 5. Comparison of three controllers in presence of disturbance.

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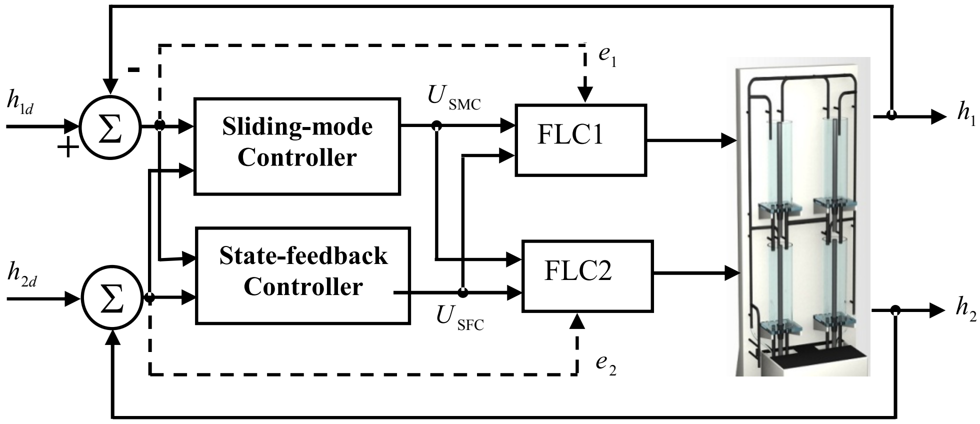

Fig. 6. Block diagram of the fuzzy SMC-SFC controller.