DECENTRALIZED DECOUPLED SLIDING-MODE CONTROL FOR TWO- DIMENSIONAL INVERTED PENDULUM USING NEURO-FUZZY MODELING

Mohammad Farrokhi and Ali Ghanbarie

Iran University of Science and Technology, Tehran 16844, IRAN

Abstract: In this paper, a new method has been proposed to control a two-dimensional inverted pendulum. The proposed method in this paper is as follows: First, the system of a two-dimensional inverted pendulum is divided into two subsystems using decentralized control theory. Then, using decoupling method, each subsystem is decoupled into two surfaces for applying sliding-mode control. Those portions of the controllers, which are directly dependent on the dynamic of the system and its parameters, are modeled with two neuro-fuzzy (ANFIS) networks. Simulations show a high performance for the proposed method as compared to the existing methods.

Keywords: decentralized control, sliding-mode control, neuro-fuzzy modeling

1. INTRODUCTION

One-dimensional inverted pendulum is a nonlinear problem, which has been considered by many researchers (Omatu and Yashioka, 1998; Magana and Holzapfel, 1998; Nelson and Kraft, 1994; Anderson, 1989), most of which have used linearization theory in their control schemes. In general, the control of this system by classical methods is a difficult task (Lin and Sheu, 1992). This is mainly because this is a nonlinear problem with two degrees of freedom (i.e. the angle of the inverted pendulum and the position of the cart), and only one control input. When this problem is extended into a two-dimensional inverted pendulum (e.g. an inverted pendulum, whose maneuver is not restricted in a plane, and can move in three-dimensional space, and also its cart does not move along one axis, but in x-y plane), then the system becomes a MIMO and very complicated nonlinear system. The control of this system, which is a more realistic model of the launched missile, is the subject of this paper. In order to solve this problem, if the multivariable classical control methods are used, then the model of the system must be linearized. But, because of the highly nonlinear behavior of the two-dimensional inverted pendulum in large deviation angles, these methods can control this system only for small deviation angles. On the other hand, the use of nonlinear classical methods for a MIMO system with differential equation of order 8 can be extremely difficult. In contrast, the decentralized control theory has opened a horizon for controlling of complicated and nonlinear systems (Ghanbarie, 2000a). According to this theory, a complicated problem will be divided into a few simpler subsystems and each subsystem is controlled separately. Therefore, instead of solving one complicated problem, a few simpler sub problems are solved. The use of this method simplifies the control of a two-dimensional inverted pendulum. In the proposed method, in this paper, each subsystem is further decoupled into two-sub subsystems. Then, with defining two sliding surface, which are related to each other, the sliding-mode control will be applied to it (Ji-Change and Ya-Hui, 1998). In this way, the problem of controlling a system of order 8 can be accomplished with sliding-mode controllers for systems of order 2. To control each subsystem, dynamic-dependent portions and their parameters are modeled with two neuro-fuzzy networks. This makes the controllers independent from the system model. Moreover, due to the nature of the neuro-fuzzy networks, it is possible to design an adaptive controller using on-line training mechanism for these networks. In the next section, the model of the tow-dimensional inverted pendulum will be given. In section III, the decentralized control theory and its application to the two-dimensional inverted pendulum will be briefly explained. The design of the controller will be brought in section IV, followed by neuro-fuzzy modeling in section V. Section VI shows the simulation results. The conclusion is given in section VII.

2. THE MODEL OF TWO-DIMENSIONAL INVERTED PENDULUM

The system of a two-dimensional inverted pendulum consists of an inverted pendulum connected to a cart, and can move in the three dimensional space. That is, it can deviate from vertical direction (i.e. parallel to the z axis) towards both x and y directions. The cart also can move in x-y plane (Fig.1). The dynamic of this system is as follows (Esmailie-Khatier, 1994):

$$\begin{aligned}
&\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \\
&\mathbf{x} = [x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6 \;\; x_7 \;\; x_8]^T \\
&\quad = [\alpha \;\; \dot{\alpha} \;\; x \;\; \dot{x} \;\; \beta \;\; \dot{\beta} \;\; y \;\; \dot{y}]^T \qquad (1) \\
&\mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{G}^{-1}(\mathbf{x}).\mathbf{h}(\mathbf{x},\mathbf{u}) \\
&\mathbf{u} = [F_x \;\; F_y]^T
\end{aligned}$$

where $\mathbf{h}$ is a 8×1 vector, influenced by the state vector of the system $\mathbf{x}$ and the input vector $\mathbf{u}$, and $\mathbf{G}$ is a 8×8 matrix, which is a function of the state vector $\mathbf{x}$. If the deviation angle of the inverted pendulum from the z-axis is assumed to be γ, then α and β are the projections of γ on the x-z and y-z planes, respectively, and $\dot{\alpha}$ and $\dot{\beta}$ are the corresponding angular velocities. Also, x and y are the coordinates of the position of the cart in x-y plane, and $\dot{x}$ and $\dot{y}$ are the velocities of the cart along x-axis and y-axis, respectively. F_x and F_y are the applied forces to the cart along x and y-axis, respectively. After some matrix manipulations, the dynamic of the system can be rewritten as follows:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_{11}(\mathbf{x}) + b_{11}(\mathbf{x})F_x + v_{11}(\mathbf{x})F_y \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_{12}(\mathbf{x}) + b_{12}(\mathbf{x})F_x + v_{12}(\mathbf{x})F_y \\
\dot{x}_5 &= x_6 \qquad\qquad\qquad\qquad (2) \\
\dot{x}_6 &= f_{21}(\mathbf{x}) + b_{21}(\mathbf{x})F_y + v_{21}(\mathbf{x})F_x \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= f_{22}(\mathbf{x}) + b_{22}(\mathbf{x})F_y + v_{22}(\mathbf{x})F_x
\end{aligned}$$

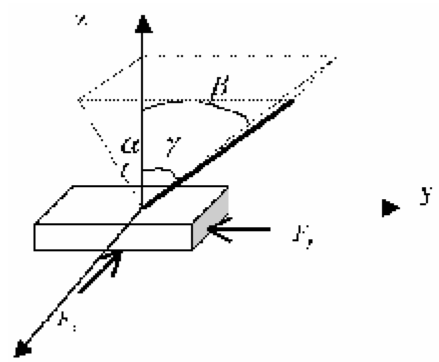

Fig. 1. The schematic diagram of a two-dimensional inverted pendulum

where $f_{ij}(\mathbf{x}), b_{ij}(\mathbf{x})$, and $v_{ij}(\mathbf{x})(i=1,2$ and $j=1,2)$ are nonlinear functions of the state variables (Ghanbarie, 2000b).

3. THE USE OF THE DECENTRALIZED CONTROL THEORY IN TWO-DIMENSIONAL INVERTED PENDULUM

What are mostly common in control theory are the centralized control methods: dada are received from the plant and all of them are sent to the controller. Then, the decision is made by the central control and the appropriate commands are sent to the plant. Here, the controller considers the plant as a whole. But in complicated systems the design of the controllers can encounter difficulties. One of the methods to control these kinds of systems is the decentralized control scheme. The main idea in this theory is the distribution of tasks. That is, the process under control is appropriately divided into several sub processes. Then, the controllers are designed locally and the processing is made in a decentralized fashion. In other words, the local controllers generate the control commands. Obviously, because of the interactions between subsystems on each other, the control commands, which are applied to the corresponding subsystems, affects other subsystems as well. Therefore, a third kind of disturbance, in addition to two well-known disturbances (i.e. the disturbances due to external signals and the disturbances due to the unmodelled dynamics), which is the effects of all subsystems on every subsystem, is defined (Ghanbarie, 2000a). If the dynamic equations of a two-dimensional inverted pendulum are linearized around $\mathbf{x}=0$, then its state equations become diagonal. In other words, in the process of linearization, the decoupling occurs along the x any y-axis. It should be mentioned that the linearized model of the two-dimensional inverted pendulum has not been used in this paper. The goal is only to conclude from the above comments that considering the x-axis as one subsystem and the y-axis as the other can be appropriate because the interactions between these two subsystems are relatively small. Moreover, because of the similarities between the x-axis and the y-axis dynamics, one needs only to design the controller for one axis. As a result, the MIMO system of order 8 is converted into two SIMO subsystems of order 4. Next, considering the disturbances of the third kind, one controller is designed for each subsystem. The closer the pendulum gets to the operating point ($\mathbf{x}=0$), the smaller the magnitude of the third kind of disturbance (i.e. at $\mathbf{x}=0$ this disturbance is equal to zero and at the deviation angle equal to 90 degrees it has its maximum value. Therefore, eqs. (2) become

$$\dot{x}_1 = x_2 \tag{3}$$

$$\dot{x}_2 = f_{11}(\mathbf{x}) + b_{11}(\mathbf{x}) F_x + d_{11}(t) \tag{4}$$

$$\dot{x}_3 = x_4 \tag{5}$$

$$\dot{x}_4 = f_{12}(\mathbf{x}) + b_{12}(\mathbf{x}) F_x + d_{12}(t) \tag{6}$$

$$\dot{x}_5 = x_6 \tag{7}$$

$$\dot{x}_6 = f_{21}(\mathbf{x}) + b_{211}(\mathbf{x}) F_y + d_{21}(t) \tag{8}$$

$$\dot{x}_7 = x_8 \tag{9}$$

$$\dot{x}_8 = f_{22}(\mathbf{x}) + b_{22}(\mathbf{x}) F_y + d_{22}(t) \tag{10}$$

In these equations $d_{ij}(t)$ $(i=1,2$ and $j=1,2)$ are the disturbances of the third kind. Later it will be shown how to convert each subsystem of order 4 into two sub subsystems of order 2 using the decoupled sliding-mode method. The result is a simple design procedure and a controller with good performance.

4. DESIGN OF THE CONTROLLER

The goal of the control is to bring the inverted pendulum to the vertical position (in the z-axis direction) while the cart is brought to the origin of the coordinates. The state equations (3)-(6) are for the subsystem of the x-axis and the state equations (7)-(10) are for the subsystem of the y-axis. Hence, there are two subsystems of order 4, which are not in canonical form and must be controlled independently.

4.1. The Decoupled Sliding-Mode Control Method

Consider the following four state equations, similar to the subsystems of the two-dimensional inverted pendulum:

$$\dot{x}_1 = x_2 \tag{11}$$

$$\dot{x}_2 = f_1(\mathbf{x}) + b_1(\mathbf{x})\, u + d_1(t) \tag{12}$$

$$\dot{x}_3 = x_4 \tag{13}$$

$$\dot{x}_4 = f_2(\mathbf{x}) + b_2(\mathbf{x})\, u + d_2(t) \tag{14}$$

where $\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ is the state vector, and $f_1(\mathbf{x})$, $f_2(\mathbf{x})$, $b_1(\mathbf{x})$, and $b_2(\mathbf{x})$ are nonlinear functions equal to $f_{ij}(\mathbf{x})$ and $b_{ij}(\mathbf{x})$ $(i=1,2$ and $j=1,2)$, respectively. Also, $\mathbf{u}$ is control input, and $d_1(t)$ and $d_2(t)$ are disturbances of the third kind. It is assumed that these disturbances have a higher limit: $|d_1(t)| \le D_1(t), |d_2(t)| \le D_2(t)$.

Now, two sliding surfaces are defined as follows:

$$s_1 = c_1 x_1 + x_2, \quad s_2 = c_2 x_3 + x_4 \tag{15}$$

According to the sliding-mode control theory, the control laws can be defined as follows (Slotine and Li, 1991):

$$\begin{aligned} u_1 &= \hat{u}_1 - k_1 \operatorname{sat}(s_1\, b_1(\mathbf{x}) / \varphi_1) \\ k_1 &> D_1 / |b_1(\mathbf{x})| \\ \hat{u}_1 &= (-c_1 x_2 - f_1(\mathbf{x})) / b_1(\mathbf{x}) \end{aligned} \tag{16}$$

$$\begin{aligned} u_2 &= \hat{u}_2 - k_2 \operatorname{sat}(s_2 b_2(\mathbf{x}) / \varphi_2) \\ k_2 &> D_2 / |b_2(\mathbf{x})| \\ \hat{u}_2 &= (-c_2 x_4 - f_2(\mathbf{x})) / b_2(\mathbf{x}) \end{aligned} \tag{17}$$

Clearly, if $u = u_1$ in Eqs. (12) and (14), only states x_1 and x_2 along with the hyper surface s_1 will converge to zero. On the other hand, if $u = u_2$, only states x_3 and x_4 along with the hyper surface s_2 will converge to zero. In other words, the sliding-mode controller is able to control either the inverted pendulum or the cart, while the goal is the simultaneous control of both the inverted pendulum and the cart. One method might be as follows: converting the above fourth order system into a canonical form and then applying the sliding-mode control theory. But, this conversion has some conditions (Khalil, 1992). Moreover, its computations are complicated and laborious. Using the decoupled sliding-mode control scheme (Ji-Change and Ya-Hui, 1998), one can employ the sliding-mode theory without converting the system into canonical form. The main idea of the decoupled sliding-mode controller is as follows:

The existing fourth order system is divided into two subsystems A and B of order two. Subsystem A consists of the state variables x_1 and x_2, and the sliding surface s_1; subsystem B consists of the state variables x_3 and x_4, and the sliding surface s_2. The main goal of the controller is to guide the state variables of subsystem A to the surface $s_1 = 0$ such that x_1 and x_2 approach exponentially to zero.

The secondary goal is the same thing for the state variables of subsystem B and the corresponding sliding surface $s_2 = 0$. Since the main goal is to bring subsystem A into stable conditions, the information of subsystem B is considered as secondary data. These secondary data must be transferred via a mechanism to the primary data. For this reason, a dummy variable z is defined, which transfers the secondary data to the primary data. Therefore, the sliding surface s_1 changes into $s_1 = c_1(x_1 - z_1) + x_2$. Now with this modified s_1, the main goal changes from $x_1 = 0$ and $x_2 = 0$ into $x_1 = z = 0$ and $x_2 = 0$, such that z is a function of s_2. Hence, the main goal and the secondary goal are linked together through the dummy variable z, and both of these goals will be controlled simultaneously. The dummy variable z can be found as follows:

According to the above statements

$$s_1 = c_1(x_1 - z_1) + x_2 \tag{18}$$

4and s_2 can be define as before

$$s_2 = c_2 x_3 + x_4 \tag{19}$$

The control input is the sliding-mode control of subsystem A (Eq. (16)). Since in sliding-mode control theory it is assumed that $u = u_1$ to control the entire system, the boundedness of x_1 can be assured with $0 < z_u < 1$ and $|z| \le z_u$. In other words, the maximum absolute value of x_1 is always bounded. Here, z_u is the upper limit of $|z|$. Therefore, z can be defined as follows:

$$\begin{gathered} z = \mathrm{sat}(s_2 / \varphi_z) z_u \\ 0 < z_u < 1 \end{gathered} \tag{20}$$

Therefore, z is a decaying signal, since z_u is less than one. The control action is accomplished as follows: The main object of Eq. (16) is to make x_1 and x_2 equal to zero according to the sliding-mode control theory. But when $s_2 \neq 0$, then $z \neq 0$ in Eq. (18). This causes Eq. (16) to apply an input such that z is decreased. When z is decreased, s_2 will be decreased too. Moreover, when $s_2 \to 0$, then $x_3 \to 0$ and $x_4 \to 0$, which satisfies the secondary goal as well. In summary, with the introduction of dummy variable z in s_1, the system is decoupled into two sliding surfaces. Then, with appropriate changes in u_1, s_1 and s_2 will converge simultaneously to zero. The same procedure will be performed for the y-direction. The overall structure of the proposed control system is shown in Fig. 2. A few points are worth to be mentioned here:

1. All nonlinear functions $f_{ij}(\mathbf{x})$ and $b_{ij}(\mathbf{x})$ $(i = 1,2 \text{ and } j = 1,2)$, are independent of state variables x_3 and x_7.
2. All nonlinear functions $b_{ij}(\mathbf{x})$ $(i = 1,2 \text{ and } j = 1,2)$, are independent of state variables x_2, x_4, x_6, and x_8.
3. It can be shown that b_{11} is less affected by x_5 than by x_1, and f_{11} is less affected by x_4, x_5, x_6, and x_8 than by x_1 and x_2. Similarly, it can be shown that b_{21} is less affected by x_1 than by x_5, and f_{21} is less affected by x_1, x_2, x_4, and x_5 than by x_6 and x_8.
4. According to Eq. (16), k_1 must be selected such that $k_1 > D_1 / |b_1(\mathbf{x})|$, where D_1 is the upper limit of the third kind disturbance.

5. NEURO-FUZZY MODELING

According to Eq. (16), each local controller needs $f_1(\mathbf{x})$ and $b_1(\mathbf{x})$ to control its own subsystem. In other words, the control system depends on $f_{i1}(\mathbf{x})$ and $b_{i1}(\mathbf{x})$ $(i = 1,2)$ during the control process. In addition, because of the dependency of the control system to the parameters of the dynamic of the system, the system makes errors when these parameters change. Moreover, because of the ideal dynamic model, the controller might encounter difficulties when it is used for a real system. Here, those sections of the controller, which are dependent on the dynamic of the system, are modeled with two ANFIS networks (Jyh-Shing and Jang, 1993). The advantages of this kind of modeling is as follows:

- Because of a good approximation, the modeling accuracy is high.
- Due to the simplicity of these networks, their training is very fast.
- The trained network has a very fast on-line response.
- The controller is able to control the system without any need to the dynamic equations of the system.
- Because of the trainability nature of these networks, the controller can adapt itself to the changes in the dynamic of the system, by adding an on-line training mechanism. In this way, the controller becomes adaptive and robust.
- Due to the existence of the physical interpretation for $f_{ij}(\mathbf{x})$

$(i = 1,2 \text{ and } j = 1,2)$ and the fuzzy nature of the ANFIS network, one can use the expert knowledge in defining the fuzzy rules in the corresponding networks.

Hence, two ANFIS networks are constructed to model $f_{i1}(\mathbf{x})$ and $b_{i1}(\mathbf{x})$ $(i = 1,2)$ for each local

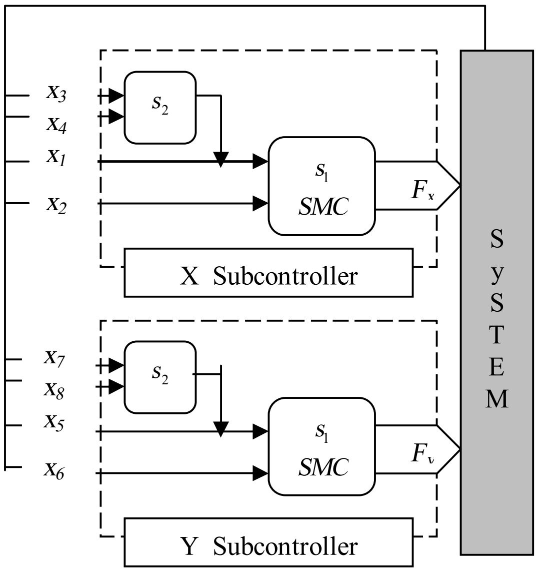

Fig. 2. The control system structure

controller. The inputs to the ANFIS networks for f_{11}, f_{21}, b_{11}, and b_{21} are $\{x_1, x_2\}$, $\{x_5, x_6\}$, $\{x_1\}$, and $\{x_5\}$, respectively. The elimination of the less-effective state variables not only does not deteriorate the accuracy of the modeling, but also increases the accuracy of the modeling due to the smaller and simpler structure of ANFIS networks.

5.1 Defining Fuzzy Rules

In this paper the subtractive clustering method has been used to define fuzzy rules (Chiu, 1996). The subtractive clustering method is a scheme for extraction and classification of fuzzy rules. The advantages of this method, as compared to similar methods, are (Chiu, 1994): 1) there is no need to determine the number of clusters beforehand, 2) the complexity of the computations increases linearly with the dimension of the problem, which yields a higher computational speed, 3) the ability to consider each data, not each section, as the candidate for the center of the cluster, 4) extraction of fewer rules along with higher performance. Using this method for 656 input-output data points, which have been obtained for $f_{i1}(\mathbf{x})$ and $b_{i1}(\mathbf{x})$ $(i = 1,2)$ based on their maximum excitation, 8 and 3 cluster centers have been obtained, respectively. Therefore, $f_{i1}(\mathbf{x})$ $(i = 1,2)$ is modeled with eight fuzzy rules and $b_{i1}(\mathbf{x})$ $(i = 1,2)$ with three fuzzy rules.

5.2 The Structure of the ANFIS Networks

Two ANFIS networks, one for $b_{i1}(\mathbf{x})$ with three first order rules and one for $f_{i1}(\mathbf{x})$ with eight first order fuzzy rules, for each local controller, are constructed using singleton fuzzifier, center average defuzzifier, and product inference engine. The membership functions of the input variables are gaussian.

5.3 Training of the ANFIS Networks

The trainable parameters of the ANFIS networks are the primary and the tally parameters of the fuzzy rules. Primary parameters are the adjustable variables of the input membership functions (i.e. the center and the width of the gaussian membership functions) and the tally parameters are the coefficients of the linear equations of the tally part of the first order fuzzy rules. These adjustable parameters in the ANFIS network for $f_{i1}(\mathbf{x})$ $(i = 1,2)$, for the primary and the tally parts of the fuzzy rules, are 24 and 34, respectively. Therefore, the total number of adjustable parameters for this network is 56. In the ANFIS network for $b_{i1}(\mathbf{x})$ $(i = 1,2)$ the primary and the tally parts of the fuzzy rules have 6 and 6 adjustable parameters, respectively; hence, a total of 12 adjustable parameters. The hybrid method, which is a combination of gradient descent and least square estimation, has been employed to train the networks (Jyh-Shing and Jang, 1993). Therefore, this method has the advantages of both gradient descent and least square estimation schemes. Reduction of the search space dimensions and hence a faster convergence speed, and less possibility of falling into local minima are two advantages of this method. The average errors of the ANFIS networks for $f_{i1}(\mathbf{x})$ and $b_{i1}(\mathbf{x})$ after the completion of the training phase (45 and 40 epochs) are 0.00856 and 8.5637×10^{-5}, respectively. The reasons for such small errors are the proper structure of the ANFIS networks and the high performance of the hybrid training method.

6. SIMULATION RESULTS

In this section the simulation results of the proposed controller, which is performed on the model of a two-dimensional inverted pendulum, are presented. The initial values of the state variables are as follows:

$$\alpha(0) = 85 \text{ deg}, \quad \dot{\alpha}(0) = 0 \text{ rad/s},$$
$$\beta(0) = -78 \text{ deg}, \quad \dot{\beta}(0) = 0 \text{ rad/s},$$
$$x(0) = 20 \text{ m}, \quad \dot{x}(0) = 0 \text{ m/s},$$
$$y(0) = -30 \text{ m}, \quad \dot{y}(0) = 0 \text{ m/s},$$

Every local controller controls its corresponding sub system. While the inverted pendulum comes to the vertical position, the cart reaches the origin of the coordinates. Figs. 3 and 4 show the results.

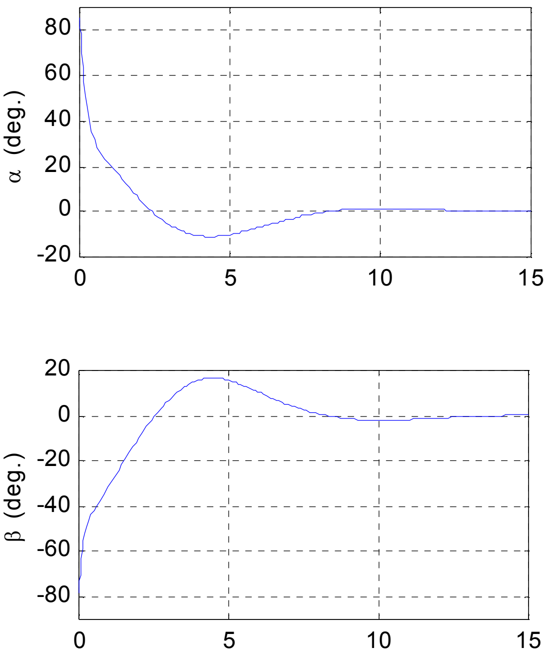

Fig. 3. Simulation results of α and β

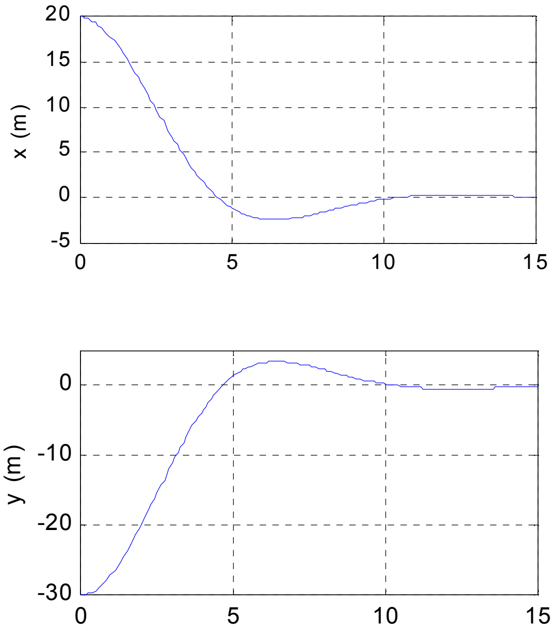

Fig. 4. Simulation results of x and y

Despite the large initial values for angles ($\alpha(0) = 85^\circ$ and $\beta(0) = 78^\circ$) the proposed controller is able to bring the pendulum to the vertical position. Also, the responses have acceptable overshoot and undershoot. It should be mentioned, however, that such large initial values for angles are not practical because the initial amount of forces would be also very large in order to bring the pendulum to the vertical position. These initial

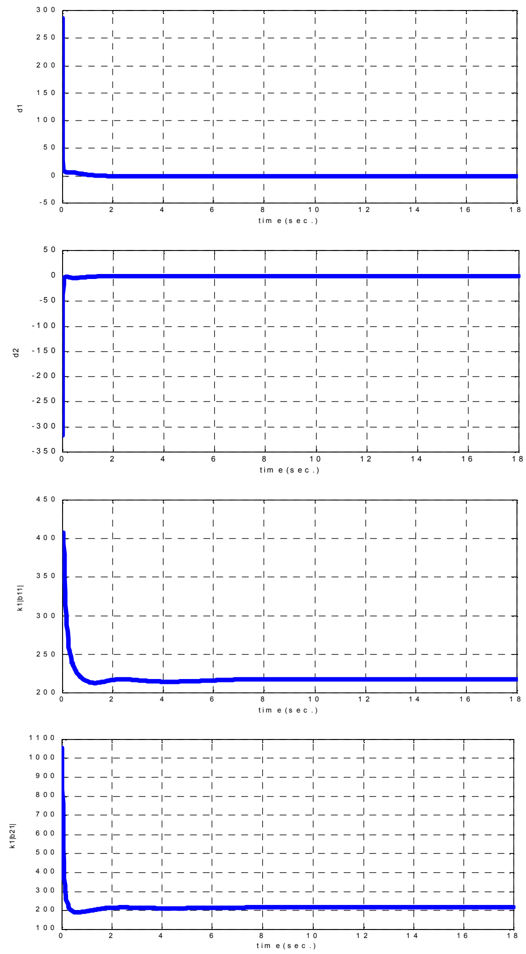

Fig. 5. Inequality (16) for each local controller

values for angles have been only considered to show the performance limitations of the proposed method. According to the inequality in Eqs. (16), for the disturbances of the third kind

$$D_1 < 10|b_1(\mathbf{x})|$$

where D_1 is the upper limit of d_1. Fig. 5 shows d_1, d_2, $k_1|b_{11}(\mathbf{x})|$, and $k_1|b_{21}(\mathbf{x})|$. As it can be observed from these graphs, the above inequality is always valid. Therefore, it can be concluded that despite large initial deviation of the inverted pendulum from the vertical position, the disturbance of the third kind is situated in such a range that the local controllers are able to overcome that. Moreover, Fig. 5 shows that the disturbance of the third kind decreases as the inverted pendulum reaches the vertical position and is equal to zero at the balanced position. As a comparison with other methods, the proposed method in (Asgarie-Raad, 1998) is able to control the same pendulum as in this paper only for much smaller initial values of the inverted pendulum with larger overshoots and undershoots.

6.1 Applying External Disturbances

The above simulations are repeated with external disturbances applied to both x and y directions. This disturbance, which is shown in Fig. 6, has relatively large amplitude and has been created using several sinusoidal waveforms with different frequency and amplitude along with random coefficients. The same initial values have been considered for α and β as before. The simulation results are shown in Fig. 7. As it is clear from the graphs, the proposed controller can bring the inverted pendulum in the vertical position and hold it there, despite large amount of external disturbances and large initial value for angles.

6.2 On-line Change of the Parameters of Inverted Pendulum

As it was mentioned before, the two-dimensional inverted pendulum is the model of a balanced missile. Due to the consumption of the fuel, the mass of a launched missile decreases continuously. In addition, the gravity acceleration changes according to the following equation as the missile is distancing from the surface of the earth:

$$g = g_0\left(\frac{d_s}{d}\right) \tag{20}$$

where g_0 is equal to 9.81 m/s, d_s is the radius of earth, and d is the distance of the missile from the center of the earth. Moreover, some portions of the missiles, which are not needed anymore, are separated from it. This brings some sudden changes in the mass of the missile. These changes are simulated in the two-dimensional inverted pendulum as on-line. For this reason, it is assumed that the mass of the cart changes in two steps as follows:

$$M = \begin{cases} 0.8\,\text{kg} & 0 \le t < 0.8 \text{ s} \\ 0.6\,\text{kg} & 0.8 \le t < 5 \text{ s} \\ 0.1\,\text{kg} & t > 5 \text{ s} \end{cases} \tag{21}$$

In fact, here the mass of the cart has been decreased by 25% and 78.5% in the first and the second steps, respectively. To simulate the change of the mass due the reduction of the mass of the fuel, the mass of the inverted pendulum has been changed according to the following equation:

$$m = 0.5\times10^{-0.05\,t} \text{ kg} \tag{22}$$

where t is time in s. And finally the gravity acceleration changes as follows:

$$g = \frac{9.8}{(1+0.02\,t)^2} \text{ m/s}^2 \tag{23}$$

where t is also time in s. The same initial values have

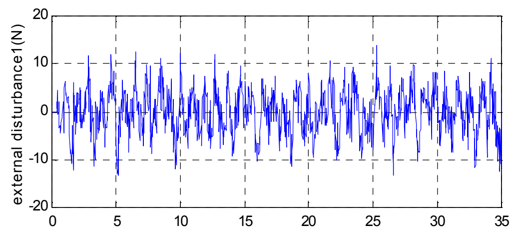

Fig. 6. Applied external disturbance to each axis

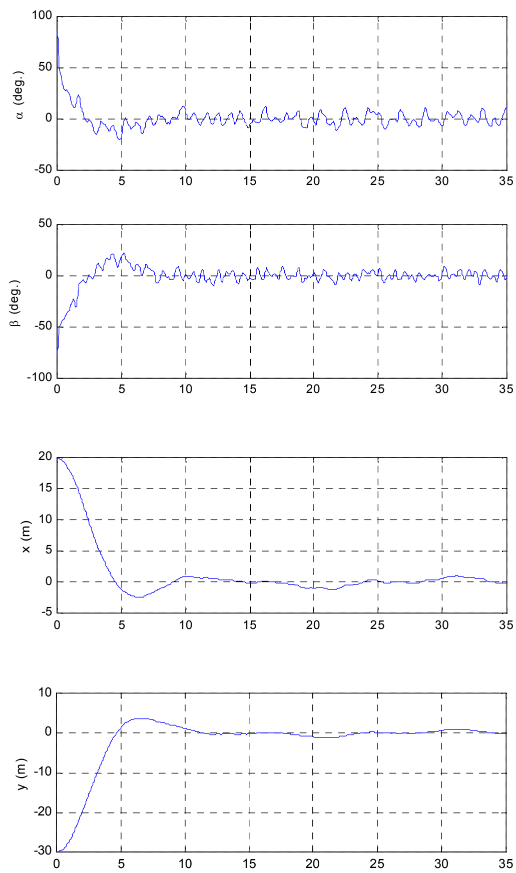

Fig. 7. Simulation results in the presence of noise

been used for the inverted pendulum as before. Fig. 8 shows the simulation results. As the graphs show, despite considerable changes in the parameters of the system, the proposed control method can bring the inverted pendulum to the vertical position. This can be an indication of the high degree of the robustness of the controller. The run time of the simulations is very fast, even in the MATLAB environment. A personal computer with Pentium II 400 processor and 96 MB of RAM performed a 15 second simulation in just 6 seconds. Hence, one can use the proposed method as an on-line controller, without any need to write the required codes in low level programming languages.

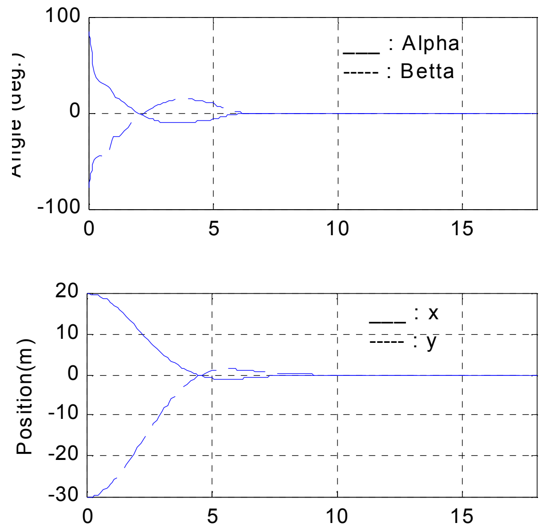

Fig. 8. Simulation results for the case of on-line changes of the parameters of the system

7. CONCLUSION

A new method, based on synthesis of the decentralized control theory and the decoupled sliding-mode method, was presented for controlling two-dimensional inverted pendulum. First, the dynamic equation of the system with order 8 was divided into two sub systems with order 4. Second, two sliding surfaces have been assigned to each sub system, and the sliding-mode control was performed using an intermediate variable. In this way, a nonlinear dynamic system was controlled in a simple fashion. Then, the dynamic-dependent portions of each controller were modeled with two ANFIS networks, with high degree accuracy. This made the controllers independent of the system parameters. Moreover, the inverted pendulum was controlled with on-line changes in the parameters of the system. In addition, due to the parallel processing nature of ANFIS networks, the simulations were performed much faster than the cases with no such networks. The simulation results show that the performed control method is robust against external noises too. Also, theoretically the controller is able to bring the inverted pendulum from large initial deviations to the vertical position.

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