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Discrete Sliding Control of Nonlinear Non affine Systems Using Neural Network and Approximate Models

Saeed Mohammad-Hoseini Mohammad Farrokhi
Iran University of Science and Technology
sm_hoseini@iust.ac.ir farrokhi@iust.ac.ir

Abstract: *In this paper, an indirect combined controller is proposed for non-affine nonlinear systems based on affine approximate input-output model of the system. Here, the NARMA model, which is an exact representation of the input-output model, is approximated with an affine input-output model. The conventional controllers which are based on this approximated model dose not guarantee good performance of the closed-loop system for large magnitude control signal so, in this paper a robust combined controller is proposed which guarantees a reliable performance for the closed-loop system, even for considerable model approximation errors.*

Keywords: Sliding mode, nonlinear systems, neural networks, model approximation.

1 Introduction

The NARMA model is an exact representation of the input–output behavior of finite-dimensional nonlinear discrete-time dynamical systems in the neighborhood of equilibrium states [1]. However, it is not convenient for purposes of adaptive control using neural networks due to its nonlinear dependence on the control input. As a results control system designer are often forced to approximate plant dynamics with linear models such as ARMA model, or use mathematical approximations to overcome the computational complexity of neural controller problems [2, 3].
Recently, NARMA-L1 and NARMA-L2 have been introduced as approximated models of NARMA model [3].
These models are nonlinear with respect to past outputs and inputs but linear with respect to the current input and therefore, suitable for control design. However, they are still restricted to small input magnitude. To eliminate the small magnitude requirement, two other approximate models NARMA-L1B and NARMA-L2B has been proposed in the literature, which require small input changes rather than small input magnitude [2]. To relax the input magnitude restriction, in this paper we propose a hybrid controller which combines sliding-mode and feedback linearization laws through a fuzzy system. This controller will guarantee the stability and performance of the closed-loop system against modeling error.
This paper is organized as follows. In section 2, the input-output models of a non-affine nonlinear plant are presented; section 3 presents an MLP neural network with suitable structure to identify the nonlinear non-affine plant as affine NARMA-L2 model. Moreover, the conventional feedback linearization and sliding-mode design will be given in this section. Also, two strategies for elimination of the chattering, inherent in the sliding-mode control law is proposed in section 3. Section 4 shows the simulation results, and finally conclusions are given in section 5.

2 Input-Output Models

Consider a single-input single-output (SISO) nonlinear discrete-time system of the form

$$\Sigma: \quad x(k+1) = f\left(x(k), u(k)\right) \qquad y(k) = h\left(x(k)\right) \tag{1}$$

Where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}$, and $y(k) \in \mathbb{R}$ are states, input, and output of system at time k, respectively; f is a vector field and h is functions which are smooth on their domains, with $f(0,0) = 0$ and $h(0) = 0$.
Let the system Σ has a relative degree of r at $(0, 0)$. Then, there exists a smooth local change of

coordinates $z = \Phi(x)$ with $\Phi(0) = 0$ such that in z-coordinates, the system becomes [5]:

$$\Sigma_N: \quad \begin{aligned} z_i(k+1) &= z_{i+1}(k) \\ z_r(k+1) &= F(\mathbf{z}(k), \boldsymbol{\eta}(k), u(k)) \\ \boldsymbol{\eta}(k+1) &= D(\mathbf{z}(k), \boldsymbol{\eta}(k), u(k)) \\ y(k) &= z_1(k) \end{aligned} \tag{2}$$

where r is relative degree of system which is defined in [4]. Additionally, assume

$$\left.\frac{\partial F(z(k), \eta(k), u(k))}{\partial u(k)}\right|_{(0,0,0)} \neq 0$$

From the normal form of Σ_N we have [5]

$$y(t+r) = F(Y_r(k), \boldsymbol{\eta}(k), u(k)) \tag{3}$$

where

$$Y_r(k) \triangleq [y(k), y(k-1), \ldots, y(k-r+1)] \tag{4}$$

Let Σ_N be observable around the equilibrium state $(0, 0, 0)$. Then, there exist a neighborhood around the equilibrium state where $\eta(k)$ can be written as

$$\eta(k) = \Gamma(Y_n(k-n+r), U_{n-r}(k-n+r))^{n-r} \tag{5}$$

by substituting (5) into (4)

$$\begin{aligned} y(k+r) &= F(Y_r(k), \Gamma(Y_n(k-n+r), U_{n-r}(k-n+r)), u(k)) \\ &= F(y(k+r-1), \ldots, y(k-n+r), u(k), \ldots, u(k-n+r)) \end{aligned} \tag{6}$$

with $\left.\frac{\partial F(\cdot)}{\partial u(k)}\right|_{(0,\ldots,0)} \neq 0$. The nonlinear auto regressive moving average (NARMA) model can be derived by iterating (6) r-1 times backwards, as follows [3]:

$$y(k+r) = F'(y(k), \ldots, y(k-n+1), u(k), u(k-n+1)) \tag{7}$$

with $\left.\frac{\partial F'(\cdot)}{\partial u(k)}\right|_{(0,\ldots,0)} \neq 0$

Zero dynamics: If the output of system Σ is to be identically zero, (that is, $z(k) = 0$ for $k > k_0$) it follows that:

$$\begin{aligned} 0 &= F(0, \ldots, 0, \eta(k), u(k)) \\ \eta(k+1) &= D(0, \ldots, 0, \eta(k), u(k)) \end{aligned} \tag{8}$$

since $\left.\frac{\partial F(z(k), \eta(k), u(k))}{\partial u(k)}\right|_{(0,0,0)} \neq 0$ the implicit function theorem implies that a state feedback law $u(k) = g_s(\eta(k))$ exists such that

$$F(0, \ldots, 0, \eta(k), g_s(\eta(k))) = 0$$

for $\eta(k)$ in some neighborhood of $N \subset R^{n-r}$, and the closed-loop becomes

$$\eta(k+1) = D(0, \ldots, 0, \eta(k), g_s(\eta(k))) \tag{9}$$

If the zero dynamic of Equation (9) is asymptotically stable, then it can be shown by the implicit function theorem that there is a control law as [3]

$$\begin{aligned} u(k) = G(&y(k), \ldots, y(k-n+1), y^*(k+r), \\ &u(k-1), \ldots, u(k-n+1)) \end{aligned} \tag{10}$$

such that $y(k+r) = y^*(k+r)$.

This suggests that a separate Neural Network (NN) can be used as the controller. Since the input of this NN contains the history of the output, so a dynamic algorithm must be used for training, which is all quite slow and computationally intensive [6]. In addition to that, this feedback may causes instability is training algorithm. Therefore, the NARMA model is not convenient for the computation of a control input to the plant, in order to track the desired reference signals. In view of this, a NARMA-L2 model has been proposed in [3]. The main feature of this method is that the control input $u(k)$ occurs linearly in equation relating inputs to the outputs. This in turn permits easy algebraic computation of control inputs without requiring a separate neural network controller.

By using a Taylor expansion of $F'(y(k), \ldots, y(k-n+1), u(k), u(k-n+1))$ around the scalar $u(k)$, the first two components are

$$P_1(W, u) = F'(W, 0) + \left.\frac{\partial F'}{\partial u(k)}\right|_{(W,0)} u(k) \tag{11}$$

where $W = [y(k), \ldots, y(k-n+1), u(k-1), \ldots, u(k-n+1)]$ The reminder $R(W, u) \triangleq F'(W, u) - P_1(W, u)$ is bounded by

$$R(W, u) \leq \frac{M \cdot u^2}{2} = \Delta_R \tag{12}$$

where M is the maximum value of $\partial^2 / \partial u^2 \, F'(W, u)$.

Since a continuous function attains a maximum in a compact set, the value of M is bounded and hence the accuracy of approximation is a function of the amplitude of the control signal. Hence, the approximated NARMA-L2 model can be presented as follows:

$$y(t+r) = f_0(W) + g_0(W) \cdot u(k) \tag{13}$$

where $f_0(\cdot) = F'(W, 0)$ and $g_0(\cdot) = \left.\frac{\partial F'}{\partial u(k)}\right|_{(W,0)}$

3 Control Using NARMA-L2 Model

3.1 Identification Using NN

Consider the NARMA-L2 model in Equation (13). A Multi-Layer Perceptron (MLP) with suitable structure, as in Figure 1, can be used to estimate f_0 and g_0 in NARMA-L2 model of the plant. Also, training of the network can be performed offline, by back-propagation algorithm. Moreover, the recursive Gauss-Newton algorithm may be used online to derive f_0 and g_0 [7].

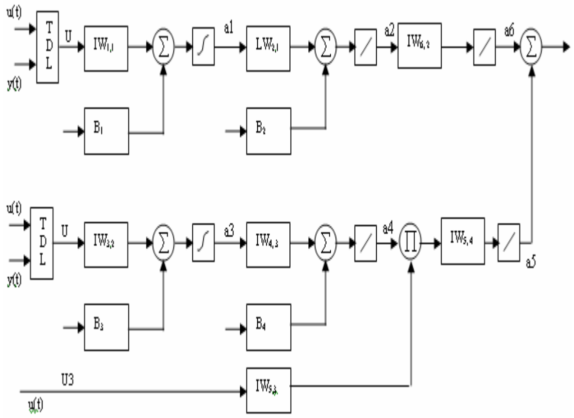

Figure 1: MLP structure to identify NARMA-L2 models

3.2 Feedback Linearization

Consider the NARMA-L2 model of Equation (13), the conventional feedback linearization control law

$$u_{FL}(k)=\frac{y^*(k+d)-f_0(.)}{g_0(.)} \tag{14}$$

can be used such that the output of system follows the desired signal $y^*(k)$. Using the above control in plant dynamic equation, the closed-loop equation can be given as

$$y(t+r)=y^*(t+r) \tag{15}$$

This means that the output of system will track the desired signal after r steps.

Because of approximation error in modeling, one can't obtain a perfect performance as the magnitude of the input signal increases. In addition, some modeling error may occur during training procedure such as sticking in local minima. In such a case, the above control law doesn't perform properly.

3.3 Discrete Sliding-Mode Control

NARMA-L2 model can be represented by considering the modeling error as

$$\begin{aligned} y(t+r)=F'(W,u)&=f_0(W)+g_0(W)u(k)\\ &+R(W,u) \end{aligned} \tag{16}$$

where R is the bounded modeling error. In addition, let $\hat{f}_0$ and $\hat{g}_0$ be the estimation of f_0 and g_0, respectively, which satisfy the following bounds

$$\left|f_0-\hat{f}_0\right|\le\Delta_f \quad \text{and} \quad \beta^{-1}g_0\le\hat{g}_0\le\beta g_0 \tag{17}$$

When the internal dynamics is stable, according to Equation (2), the (16) can be represented in state-space form as

$$\begin{aligned} &z_i(k+1)=z_{i+1}(k)\\ &z_r(k+1)=f_0(W)+g_0(W)\cdot u(k)+R(W,u)\\ &y(k)=z_1(k) \end{aligned} \tag{18}$$

The problem of tracking an r-dimensional vector $[y(k), y(k-1),..., y(k-r+1)]$ can in effect be replaced by a first order stabilization problem in s, which is defined as [8]

$$\begin{aligned} &s(k)=e(k)+\lambda_1\cdot e(k-1)+\cdots+\lambda_{r-1}e(k-r+1)\\ &e(k)=y(k)-y_d(k) \end{aligned} \tag{19}$$

Hence, the tracking problem is treated as a stabilization problem with the following simplified first order dynamic system $\Delta s(k)=0$. By defining a Lyapunov function as

$$V(k)=s(k)^2 \tag{20}$$

The condition for asymptotically stability is

$$\Delta V(k)=V(k)-V(k-1)<0 \tag{21}$$

when s is a smooth function. Here, we assume ΔV be represented as

$$\Delta V(k)=\left(s(k)-s(k-1)\right)s(k)=\Delta s(k)\cdot s(k)<0 \tag{22}$$

Moreover, in order to simplify the computations, we replace stability condition in Equation (22) with a more conservative equation

$$\Delta V(k)=\left(s(k)-s(k-1)\right)s(k)<-\eta|s| \tag{23}$$

Proposition: Let the control law be defined as follows:

$$u_{SL}=u_{FL}-\frac{k}{\hat{g}_0}\operatorname{sgn}(s)=\frac{y(t+r)-\hat{f}_0}{\hat{g}_0}-\frac{k}{\hat{g}_0}\operatorname{sgn}(s) \tag{24}$$

Then, the closed-loop system is stable, provided that k is large enough. In other words, we propose a law that assures the stability of the system in the presence of model approximation error R.

Proof: Let define the sliding surface s as follow:

$$\begin{aligned} s(k)=e(k)+\lambda e(k-1)=\left(y(k)-y_d(k)\right)+\\ \lambda\left(y(k-1)-y_d(k-1)\right) \end{aligned} \tag{25}$$

Then, according to (22) we can write

$$\begin{aligned} \Delta s(k)=e(k)+(\lambda-1)e(k-1)+\lambda e(k-2)=\\ y(k)-y_d(k)+(\lambda-1)e(k-1)+\lambda e(k-2) \end{aligned} \tag{26}$$

Now, using (16), yields

$$\begin{aligned} \Delta s(k)=f_0+g_0u(k-2)+R-y_d(k)\\ +(\lambda-1)e(k-1)+\lambda e(k-2) \end{aligned} \tag{27}$$

In the above derivation, for the sake of equation simplicity, we have assumed that the relative degree of the system described by Equation (16) is r=2. But, the proof of the proposition can be extended to any desired rank of the system.

Now, by substituting from Equation (24) we have

$$\begin{aligned}\Delta s(k) &= (f_0 - g_0\hat{g}_0^{-1}\hat{f}_0) + (g_0\hat{g}_0^{-1}-1)y_d(k)\\ &-g_0\hat{g}_0^{-1}k\ \mathrm{sgn}(s) + R + (\lambda-1)e(k-1) + \lambda e(k-2)\end{aligned} \tag{28}$$

To satisfy the stability condition in Equation (22)

$$\begin{aligned}&s\left((f_0 - g_0\hat{g}_0^{-1}\hat{f}_0) + (g_0\hat{g}_0^{-1}-1)y_d(k)\right)\\ &+s\left(R+(\lambda-1)\,e(k-1)+\lambda\, e(k-2)\right)\\ &-g_0\hat{g}_0^{-1}k|s| \le -\eta|s|\end{aligned} \tag{29}$$

Since $f_0 = \hat{f}_0 + \left(f_0 - \hat{f}_0\right)$, then

$$\begin{aligned}&\Big|(g_0^{-1}\hat{g}_0-1)\left(\hat{f}_0 - y_d\right) + g_0^{-1}\hat{g}_0\left(f_0-\hat{f}_0\right)\\ &+g_0^{-1}\hat{g}_0\left(R+(\lambda-1)\,e(k-1)+\lambda e(k-2)\right)\Big| + g_0^{-1}\hat{g}_0\eta \le k\end{aligned} \tag{30}$$

by using Equations (12) and (17) in (30) it yields

$$k \ge (\beta-1)\left|\hat{f} - y_d\right| + \beta\left|\Delta_f + \eta\right| + \beta\left|\Delta_R + \Gamma\right| \tag{31}$$

3.3.1 Chattering Elimination

The drawback of the sliding-mode control is the chattering phenomenon. In this paper, we present two techniques to eliminate the chattering.

A. filtering

By employing an appropriate low pass filter, a smooth control signal can be generated. In this approach, dynamic of low pass filter should be considered in designing the sliding-mode controller. Suppose the low pass filter has the following transfer function

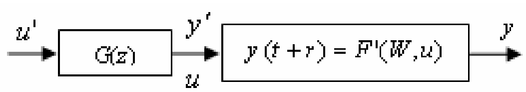

Figure 2: Using a low pass filter to eliminate chattering

$$G(z^{-1}) = \frac{\gamma}{1+\alpha z^{-1}} \tag{32}$$

with $\gamma = 1+\alpha$. According to Figure 2 we can write

$$y'(k) = -\alpha y'(k-1) + \gamma u'(k) \tag{33}$$

The output y can be written as NARMA model in terms of input u' as

$$\begin{aligned}y(t+r) &= F_1\left(y(k),.., y(k-n+1), u'(k),.., u'(k-n+1)\right)\\ &= F_1\left(y(k),.., y(k-n+1), u(k)+\Delta_0,...\right.\\ &\qquad \left., u(k-n+1)+\Delta_{n-1}\right)\end{aligned} \tag{34}$$

where

$$\Delta_i = u(k-i)\left(\frac{1}{\gamma}-1\right) + \frac{\alpha u(k-i-1)}{\gamma}$$

Since $\gamma = 1+\alpha$, then

$$\Delta_i = \left(\frac{1}{\gamma}-1\right)\left(u(k-i) - u(k-i-1)\right)$$

If the changes in the input signal are small, then by using the Taylor expansion of Equation (34) we have

$$y(t+r) = F_1\left(y(k),..., y(k-n+1), u(k),.., u(k-n+1)\ \right) + \Delta_G$$

where

$$\Delta_G = \frac{\partial F_1}{\partial u(k)}\Delta_0 + \cdots + \frac{\partial F_1}{\partial u(k-n+1)}\Delta_{n-1} + \mathrm{HOT}$$

and $F_1(W, u(k)) = F'(W, u(k))$. Hence, the effect of the low pass filter can be considered as an additional model error. As a result, the new condition for stability can be achieved by substituting $\Delta_R \to \Delta_R + \Delta_G$ in Equation (31).

B. Combination of feedback linearization and sliding-mode

To remove the chattering phenomenon without loosing the advantages of sliding-mode control, a fuzzy combination of feedback linearization and sliding-mode has been given in [9]. In this approach, a fuzzy system decides between the feedback linearization and sliding-mode control methods. That is, a continuous fuzzy switch makes smooth changes between these two controllers with the following fuzzy IF-THEN rules:

Rule 1: IF s is P, THEN $u_P = u_{FL} - \dfrac{k}{\hat{g}_0}$

Rule 2: IF s is Z, THEN $u_Z = u_{FL}$

Rule 3: IF s is N, THEN $u_N = u_{FL} + \dfrac{k}{\hat{g}_0}$

where P, Z and N are fuzzy sets defined on input fuzzy variable s, witch is applied to fuzzy controller. Also u_P, u_Z, u_N are the outputs of the fuzzy inference engine for the above three fuzzy rules. In the above fuzzy rules, there exist at least one non-zero degree of membership $\mu_0(s) \in [0,1]$ for each rule as depicted in Figure 3. Appling the weighted sum defuzzification method, the overall output of the fuzzy controller can be written as

$$u = \frac{\mu_P(s)u_P + \mu_Z(s)u_Z + \mu_N(s)u_N}{\mu_P(s) + \mu_Z(s) + \mu_N(s)}$$

The sufficient condition for stability of the system under this control law has been given in [10]. To guarantee the stability, system under each of two feedback linearization and sliding control law should be stable; in addition to that, the control signal u should remain bounded for any input s.

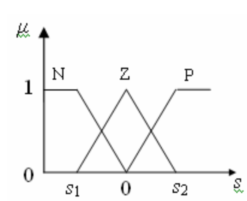

Figure 3: Membership function of fuzzy variable s

4 Simulation Results

A second-order plant is chosen for simulation

$$x_1(k+1) = 0.1x_1(k) + 2\frac{u(k)+x_2(k)}{1+\left(u(k)+x_2(k)\right)^2}$$

$$x_2(k+1) = 0.1x_2(k) + u(k)2 + \frac{u^2(k)}{1+x_1^2(k)+x_2^2(k)}$$

$$y(k) = x_1(k) + x_2(k)$$

It is clear that by one iteration on $y(t)$, the control signal will appear in the output. Therefore, the relative degree of this system is one. Hence, this plant can be represented as a NARMA model as follows

$$y(k+1) = F\left(y(t), y(t-1), u(t), u(t-1)\right)$$

and NARMA-L2 model as

$$y(k+1) = f_0\left(y(t), y(t-1), u(t-1)\right) + g_0\left(y(t), y(t-1), u(t-1)\right)u(k)$$

An MLP neural network, as in Figure 1, has been used for identification. This MLP contains 9 and 3 neurons in the first and second hidden layers, respectively. When $|u| \le 0.4$ the modeling error is small and the feedback linearization control law exhibits good performance. This is shown in Figures 4 and 5. As the magnitude of the control signal increases, the ability of feedback linearization control to achieving an acceptable performance decreases. Figures 6 and 7 show this situation for $|u| \le 1$. To correct the tracking quality, the sliding-mode control law is employed. The results are presented in Figures 8 and 9. As it was expected, the chattering phenomenon appears. To eliminate the chattering, two approaches, presented in this paper, are used together to achieve a good performance. Figures 10 and 11 show the results. As these Figures show, the system has a very good performance although the approximation error is relatively large.

5 Conclusion

An indirect combined controller is proposed for non-affine nonlinear system based on NARMA-L2 approximate input-output model of plant. Due to the approximation error, the conventional feedback linearization can't guarantee the perfect performance. To overcome this problem, a hybrid control law has been proposed, witch assures the stability and good performance of the close-loop system in the presence of approximation error. The stability of the system has been proved as a proposition in this paper.

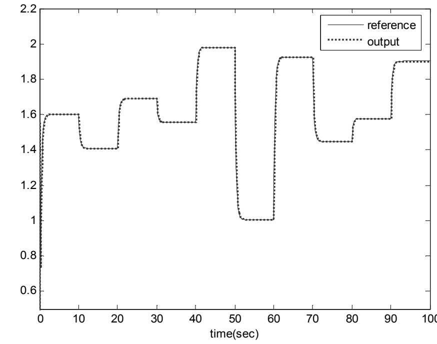

Figure 4: Tracking of system with feedback linearization control when $|u| \le 0.4$

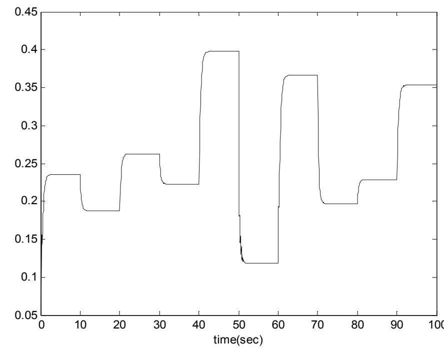

Figure 5: Control signal (feedback linearization)

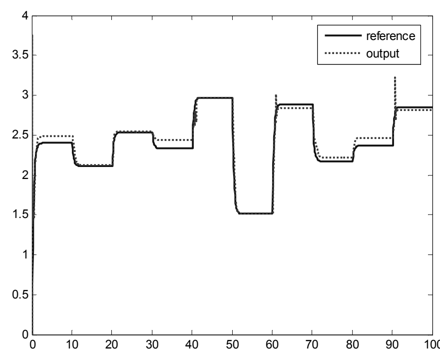

Figure 6: Tracking of system with feedback linearization control and $|u| \le 1$

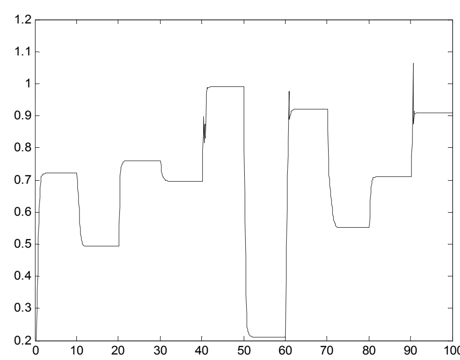

Figure 7: Control signal (feedback linearization)

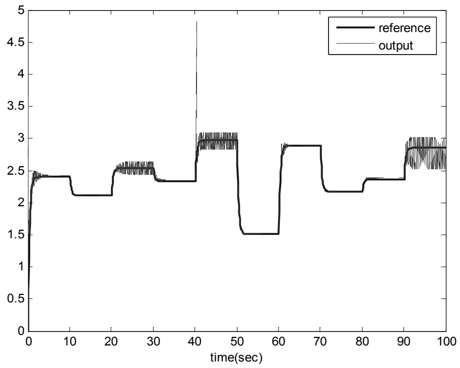

Figure 8: Tracking of system with sliding control and $|u| \leq 1$

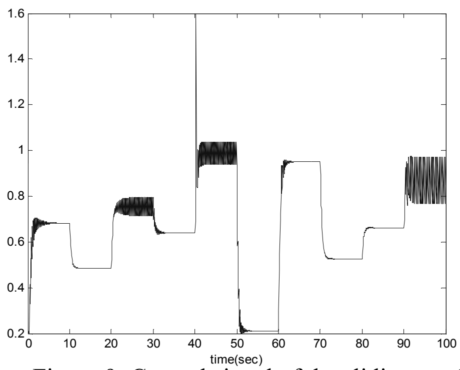

Figure 9: Control signal of the sliding-mode control

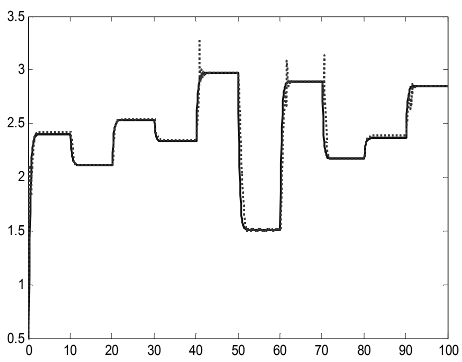

Figure 10: Tracking of the system with combined control law and filtering $|u| \leq 1$

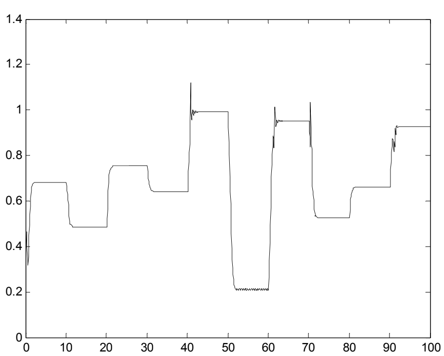

Figure 11: Control signal of the proposed controller

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