Decentralized Neuro-Fuzzy Controller Design Using Decoupled Sliding-Mode Structure for Two-Dimensional Inverted Pendulum

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***Abstract*- In this paper, a new method has been proposed to control a two-dimensional inverted pendulum. First, the system of a two-dimensional inverted pendulum is divided into two subsystems using decentralized control theory. Then, using decoupling method, each subsystem is decoupled into two surfaces for applying sliding-mode control. Next, this controller has been used to train two neuro-fuzzy ANFIS (Adaptive-Network-Based Fuzzy Inference System) networks. Due to the high accuracy of the ANFIS networks, these two networks can learn the controlling abilities of the teacher. Each trained network controls its own subsystem as a local controller. The trained networks not only have the properties of their teacher, but also due to the parallel processing property of the ANFIS networks, they response much faster than their teacher. Moreover, the neuro-fuzzy controller doesn't need any model of the plant or its parameters. Simulation results show a high performance for the proposed method as compared to the existing methods.**

***Keywords*- two-dimensional inverted pendulum, decentralized control, ANFIS networks, sliding-mode control, neuro-fuzzy control**

I. Introduction

One-Dimensional inverted pendulum is a nonlinear problem, which has been considered by many researchers [1]-[4], most of which have used linearization theory in their control schemes. In general, the control of this system by classical methods is a difficult task [5]-[9]. This is mainly because this is a nonlinear problem with two degrees of freedom (i.e. the angle of the inverted pendulum and the position of the cart), and only one control input. Moreover, when the deviation angle of the inverted pendulum becomes large, then the system shows highly nonlinear behavior. Hence, the linearization theory encounters difficulties. When this problem is extended into a two-dimensional inverted pendulum (i.e. an inverted pendulum, whose maneuver is not restricted in a plane, and can move in three-dimensional space, and also its cart does not move along one axis, but in x-y plane), then the system becomes a very complicated MIMO (Multiple-Input Multiple-Output) nonlinear system. The control of this system, which is a more realistic model of a launched missile, is the subject of this paper.

In order to solve this problem, if the multivariable classical control methods are used, then the model of the system must be linearized. But, because of the highly nonlinear behavior of the two-dimensional inverted pendulum in large deviation angles, these methods can control this system only for small deviation angles. On the other hand, the use of nonlinear classical methods for a MIMO system with differential equation of order 8 can be extremely difficult. In contrast, the decentralized control theory has opened a horizon for controlling of complicated and nonlinear systems [10]. According to this theory, a complicated problem will be divided into a few simpler subsystems considering some conditions and each subsystem is controlled separately. Therefore, instead of solving one complicated problem, a few simpler sub problems will be solved. The use of this method simplifies the control of a two-dimensional inverted pendulum. In the proposed method, in this paper, each subsystem is further decoupled into two-sub subsystems. Then, with defining two sliding surface, which are related to each other, the sliding-mode control will be applied to it [11]. In this way, the problem of controlling a system of order 8 can be accomplished with sliding-mode controllers for systems of order 2. Next, the designed controller, which has a very good performance to control the system, will be used as a teacher. One ANFIS (Adaptive-Network-Based Fuzzy Inference System) network will be considered for each local controller. These ANFIS networks will be trained with the teacher. After the training phase, each ANFIS network controls its own subsystem. The advantages of this method are: 1- high accuracy in modeling, 2- fast response due to the parallel structure of the ANFIS networks, 3- robustness against external disturbances, 4- robustness against on-line changes in the parameters of the system, 5- ability to control the inverted pendulum with large initial deviation angles. Moreover, due to the nature of the ANFIS networks, the controllers can be adaptive. In this way, the parameters of the controllers can be changed with an on-line scheme, in order to compensate the changes in the parameters of the system. The rest of this paper is organized as follows. In the next section, the model of the two-dimensional inverted pendulum will be given. In section III, the decentralized control theory and its application to the two-dimensional inverted pendulum will be briefly explained. The design of the controller will be brought in section V, followed by the design of two ANFIS networks in section IV. Section VI shows the simulation results. The conclusion is given in section VII.

II. The Model of Two-Dimensional Inverted Pendulum

The system of a two-dimensional inverted pendulum consists of an inverted pendulum connected to a cart, and can move in the three dimensional space. That is, it can deviate from vertical direction (i.e. parallel to the z axis) towards both x and y directions. The cart also can move in x-y plane as in Fig.1. The dynamic equations of this system are as follows [12]:

$$\begin{aligned}
&\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \\
&\mathbf{x} = [x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6 \;\; x_7 \;\; x_8]^T \\
&\;\;= [\alpha \;\; \dot{\alpha} \;\; x \;\; \dot{x} \;\; \beta \;\; \dot{\beta} \;\; y \;\; \dot{y}]^T \\
&\mathbf{f}(\mathbf{x},\mathbf{u}) = \mathbf{G}^{-1}(\mathbf{x}).\mathbf{h}(\mathbf{x},\mathbf{u}) \\
&\mathbf{u} = [F_x \;\; F_y]^T
\end{aligned} \tag{1}$$

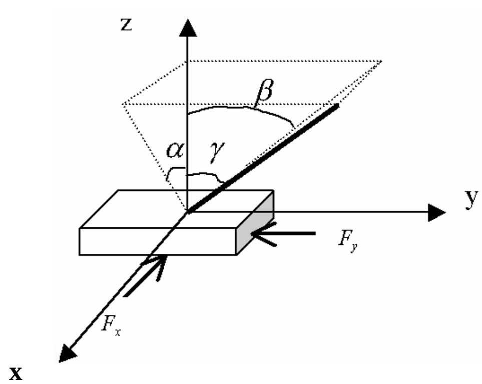

Fig. 1. The schematic diagram of a two-dimensional inverted pendulum

Where $\mathbf{h}$ is a 8×1 vector, influenced by the state vector of the system $\mathbf{x}$ and the input vector $\mathbf{u}$, and $\mathbf{G}$ is a 8×8 matrix, which is a function of the state vector $\mathbf{x}$. If the deviation angle of the inverted pendulum from the z-axis is assumed to be γ, then α and β are the projections of γ on the x-z and y-z planes, respectively. $\dot{\alpha}$ and $\dot{\beta}$ are the corresponding angular velocities. Also, x and y are the coordinates of the position of the cart in x-y plane, and $\dot{x}$ and $\dot{y}$ are the velocities of the cart along the x-axis and the y-axis, respectively. F_x and F_y are the applied forces to the cart along x and y-axis, respectively. After some matrix manipulations, the dynamic of the system can be rewritten as follows:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_{11}(\mathbf{x}) + b_{11}(\mathbf{x})F_x + v_{11}(\mathbf{x})F_y \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_{12}(\mathbf{x}) + b_{12}(\mathbf{x})F_x + v_{12}(\mathbf{x})F_y \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= f_{21}(\mathbf{x}) + b_{21}(\mathbf{x})F_y + v_{21}(\mathbf{x})F_x \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= f_{22}(\mathbf{x}) + b_{22}(\mathbf{x})F_y + v_{22}(\mathbf{x})F_x
\end{aligned} \tag{2}$$

Where $f_{ij}(\mathbf{x}), b_{ij}(\mathbf{x}),$ and $v_{ij}(\mathbf{x})(i=1,2 \text{ and } j=1,2)$ are nonlinear functions of the state variables [18].

III. THE DECENTRALIZED CONTROL THEORY

What are mostly common in control theory are the centralized control methods: dada are received from the plant and all of them are sent to the controller. Then, the decision is made by the central control and the appropriate commands are sent to the plant. Here, the controller considers the plant as a whole. But in complicated and/or distributed systems the design of the controllers can encounter difficulties. One of the methods to control these kinds of systems is the decentralized control scheme. The main idea in this theory is the distribution of tasks. That is, the process under control is appropriately divided into several sub processes. Then, the controllers are designed locally and the processing is made in a decentralized fashion. In other words, the local controllers generate the control commands. Obviously, because of the interactions between subsystems, the control commands, which are applied to the corresponding subsystems, affects other subsystems as well. Therefore, a third kind of disturbance, in addition to two well-known disturbances (i.e. the disturbances due to external signals and the disturbances due to the unmodelled dynamics), which is the effects of all subsystems on every subsystem, is defined [10].

A. The Use of The Decentralized Control Theory in Two-Dimensional Inverted Pendulum

If the dynamic equations of a two-dimensional inverted pendulum are linearized around $\mathbf{x}=0$, then its state equations become diagonal. In other words, in the process of linearization, the decoupling occurs along the x and y-axis [12]. That is, the system of the two-dimensional inverted pendulum can be considered as two separated subsystems. It should be mentioned that the linearized model of the two-dimensional inverted pendulum has not been used in this paper. The goal is only to conclude from the above comments that it is appropriate to consider the x-axis as one subsystem and the y-axis as the other because the interactions between these two subsystems are relatively small. Moreover, because of the similarities between the x-axis and the y-axis dynamics, one needs only to design the controller for one axis. As a result, the MIMO system of order 8 is converted into two SIMO (Single-Input Multiple-Output) subsystems of order 4. Therefore, one local controller needs to be designed for every subsystem, such that the disturbances of the third kind are also considered. The closer the pendulum gets to the operating point ($\mathbf{x}=0$), the smaller the magnitude of the third kind of disturbance (i.e. at $\mathbf{x}=0$ this disturbance is equal to zero and at the deviation angle near 90 degrees it has its maximum value). Therefore, (2) can be written as:

$$\dot{x}_1 = x_2 \tag{3}$$

$$\dot{x}_2 = f_{11}(\mathbf{x}) + b_{11}(\mathbf{x})F_x + d_{11}(t) \tag{4}$$

$$\dot{x}_3 = x_4 \tag{5}$$

$$\dot{x}_4 = f_{12}(\mathbf{x}) + b_{12}(\mathbf{x}) F_x + d_{12}(t) \quad (6)$$

$$\dot{x}_5 = x_6 \quad (7)$$

$$\dot{x}_6 = f_{21}(\mathbf{x}) + b_{21}(\mathbf{x}) F_y + d_{21}(t) \quad (8)$$

$$\dot{x}_7 = x_8 \quad (9)$$

$$\dot{x}_8 = f_{22}(\mathbf{x}) + b_{22}(\mathbf{x}) F_y + d_{22}(t) \quad (10)$$

In these equations $d_{ij}(t)$ $(i = 1,2$ and $j = 1,2)$ are the disturbances of the third kind. Later it will be shown how to convert each subsystem of order 4 into two sub-subsystems of order 2 using the decoupled sliding-mode method. The result is a simple design procedure and a controller with good performance.

IV. DESIGN OF THE CONTROLLER

The goal of the control is to bring the inverted pendulum to the vertical position (in the z-axis direction) while the cart is brought to the origin of the coordinates. The state equations (3)-(6) are for the subsystem of the x-axis and (7)-(10) are for the subsystem of the y-axis. Hence, there are two subsystems of order 4, which are not in canonical form and must be controlled independently.

A. *The Decoupled Sliding-Mode Control Method*

Consider the following four state equations, similar to the subsystems of the two-dimensional inverted pendulum:

$$\dot{x}_1 = x_2 \quad (11)$$

$$\dot{x}_2 = f_1(\mathbf{x}) + b_1(\mathbf{x})\, u + d_1(t) \quad (12)$$

$$\dot{x}_3 = x_4 \quad (13)$$

$$\dot{x}_4 = f_2(\mathbf{x}) + b_2(\mathbf{x})\, u + d_2(t) \quad (14)$$

Where $\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$ is the state vector, and $f_1(\mathbf{x})$, $f_2(\mathbf{x})$, $b_1(\mathbf{x})$, and $b_2(\mathbf{x})$ are nonlinear functions equal to $f_{ij}(\mathbf{x})$ and $b_{ij}(\mathbf{x})$ $(i = 1,2$ and $j = 1,2)$, respectively. Also, **u** is control input, and $d_1(t)$ and $d_2(t)$ are disturbances of the third kind. It is assumed that these disturbances have an upper limit: $|d_1(t)| \le D_1(t), |d_2(t)| \le D_2(t)$. Now, two sliding surfaces will be defined as

$$s_1 = c_1 x_1 + x_2, \quad s_2 = c_2 x_3 + x_4 \quad (15)$$

According to the sliding-mode control theory, the control laws can be defined as follows [13]:

$$\begin{cases} u_1 = \hat{u}_1 - k_1 \operatorname{sat}(s_1\, b_1(\mathbf{x}) / \varphi_1) \\ k_1 > D_1 / |b_1(\mathbf{x})| \\ \hat{u}_1 = (-c_1 x_2 - f_1(\mathbf{x})) / b_1(\mathbf{x}) \end{cases} \quad (16)$$

$$\begin{cases} u_2 = \hat{u}_2 - k_2 \operatorname{sat}(s_2 b_2(\mathbf{x}) / \varphi_2) \\ k_2 > D_2 / |b_2(\mathbf{x})| \\ \hat{u}_2 = (-c_2 x_4 - f_2(\mathbf{x})) / b_2(\mathbf{x}) \end{cases} \quad (17)$$

Clearly, if $u = u_1$ in (12) and (14), only states x_1 and x_2 along with the hyper surface s_1 will converge to zero. On the other hand, if $u = u_2$, only states x_3 and x_4 along with the hyper surface s_2 will converge to zero. In other words, the sliding-mode controller is able to control either the inverted pendulum or the cart, while the goal is the simultaneous control of both the inverted pendulum and the cart. One method might be as follows: converting the above fourth order system into a canonical form and then applying the sliding-mode control theory. But, this conversion has some conditions [14]. Moreover, its computations are complicated and laborious. Using the decoupled sliding-mode control scheme [11], one can employ the sliding-mode theory without converting the system into canonical form. The main idea of the decoupled sliding-mode controller is as follows: The existing fourth order system is divided into two subsystems A and B of order two. Subsystem A consists of the state variables x_1 and x_2, and the sliding surface s_1; subsystem B consists of the state variables x_3 and x_4, and the sliding surface s_2. The main goal of the controller is to guide the state variables of subsystem A to the surface $s_1 = 0$ such that x_1 and x_2 approach exponentially to zero. The secondary goal is the same thing for the state variables of subsystem B and the corresponding sliding surface $s_2 = 0$. Since the main goal is to bring subsystem A into stable conditions, the information of subsystem B is considered as secondary data. These secondary data must be transferred via a mechanism to the primary data. For this reason, a dummy variable *z* is defined, which transfers the secondary data to the primary data. Therefore, the sliding surface s_1 changes into $s_1 = c_1(x_1 - z_1) + x_2$. Now with this modified s_1, the main goal changes from $x_1 = 0$ and $x_2 = 0$ into $x_1 = z = 0$ and $x_2 = 0$, such that *z* is a function of s_2. Hence, the main goal and the secondary goal are linked together through the dummy variable *z*, and both of these goals will be controlled simultaneously. The dummy variable *z* can be found as follows:

According to the above statements

$$s_1 = c_1(x_1 - z_1) + x_2 \quad (18)$$

and s_2 can be define as before

$$s_2 = c_2 x_3 + x_4 \quad (19)$$

The control input is the sliding-mode control of subsystem A (16). Since in sliding-mode control theory it is assumed that $u = u_1$ to control the entire system, the boundedness of x_1 can be assured with $0 < z_u < 1$ and $|z| \le z_u$. In other words, the maximum absolute value of x_1 is always bounded. Here, z_u is the upper limit of $|z|$. Therefore, *z* can be defined as follows:

$$z = \operatorname{sat}(s_2 / \varphi_z) z_u \qquad 0 < z_u < 1 \quad (20)$$

Therefore, *z* is a decaying signal, since z_u is less than one. The control action is accomplished as follows: The main object of (16) is to make x_1 and x_2 equal to zero according to the sliding-mode control theory. However, when $s_2 \neq 0$, then $z \neq 0$ in (18). This causes (16) to apply an input such that z is decreased. When z is decreased, s_2 will be decreased too. Moreover, when $s_2 \to 0$, then $x_3 \to 0$ and $x_4 \to 0$, which satisfies the secondary goal as well. In summary, with the introduction of dummy variable z in s_1, the system is decoupled into two sliding surfaces. Then, with appropriate changes in u_1, s_1 and s_2 will converge simultaneously to zero. The same procedure will be performed for the y-direction. The overall structure of the proposed control system is shown in Fig. 2.

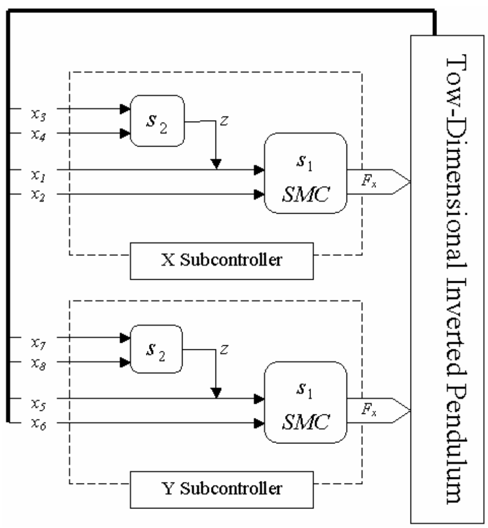

Fig. 2. The control system structure

V. DESIGN OF THE NEURO-FUZZY CONTROLLER

In this section, two ANFIS networks will be designed to control the two-dimensional inverted pendulum. To do this, the ANFIS networks will be trained in order to capture the behavior of the local controllers, defined in section V.

A. *Inputs to the ANFIS Networks*

The inputs to each network are the states of the subsystem, which the local controller has to control it. Therefore, using 7392 input-output pairs obtained from the decoupled sliding controller, the ANFIS networks will be trained. The subtractive clustering method yields 42 centers of clusters. $\{x_1 \quad x_2 \quad x_3 \quad x_4\}$ and $\{x_5 \quad x_6 \quad x_7 \quad x_8\}$ are the inputs to the x-direction and y-direction ANFIS networks, respectively. Fig. 3 shows the structure of the controller.

Since the subsystems have similar structure, the trained ANFIS network for one direction can be used for the other direction as well. Therefore, it suffices to train one ANFIS network with 4 inputs and 1 output, using the data obtained from the controller of the previous section.

B. *Defining Fuzzy Rules*

In this paper the subtractive clustering method has been used to define fuzzy rules [15]. The subtractive clustering method is a scheme for extraction and classification of fuzzy rules. The advantages of this method, as compared to similar methods, are [16]: 1) there is no need to determine the number of clusters beforehand, 2) the complexity of the computations increases linearly with the dimension of the problem, which yields a higher computational speed, 3) the ability to consider each data, not each section, as the candidate for the center of the cluster, 4) extraction of fewer rules along with higher performance. The range of influence, the squash factor, the accept rate and the reject rate are as follows:

$$r_a = 0.5, \quad r_a / r_b = 1.25, \quad \bar{\varepsilon} = 0.5, \quad \underline{\varepsilon} = 0.15 \qquad (21)$$

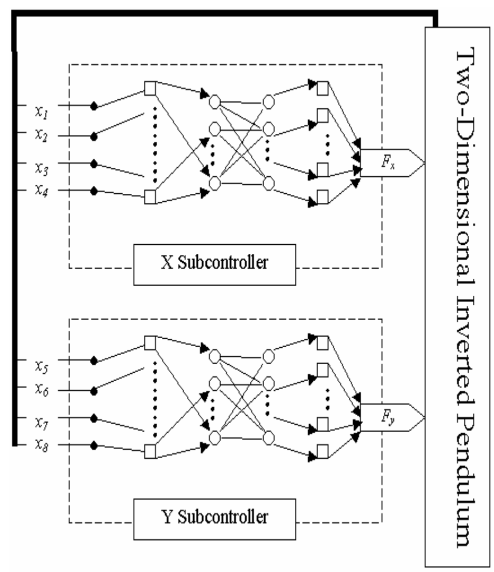

Fig. 3. The decentralized structure of the control system using two ANFIS networks

C. *Structure of the ANFIS networks*

The ANFIS networks are constructed using 42 fuzzy rules of the first order, obtained from the subtractive clustering method. Each input variable has 42 membership functions. The singleton fuzzifier, center average defuzzifier and product inference engine have been employed in the ANFIS networks. The membership functions of the input variables are gaussian. Total number of network nodes is 427.

D. *Training of the ANFIS Networks*

Each ANFIS network has 336 trainable nonlinear parameters as the center and width of the membership functions, and 210 trainable linear parameters as the coefficients in the linear combination of the fuzzy rules in the tally part of the fuzzy system. Therefore, there are 546 adjustable parameters in every ANFIS network. A hybrid method has been used to train these networks [17]. The average of the error after 206 epochs of training is 0.059661. The required time for this training was 72 hours.

VI. SIMULATION RESULTS

The simulation of the proposed control method is performed on a two-dimensional inverted pendulum with the following specifications:

$$\begin{array}{ll} \alpha(0) = 85 \text{ degree}, & \dot{\alpha}(0) = 0 \text{ rad/s} \\ x(0) = 21 \text{ meter}, & \dot{x}(0) = 0 \text{ m/s} \\ \beta(0) = 78 \text{ degree}, & \dot{\beta}(0) = 0 \text{ rad/s} \\ y(0) = 26 \text{ meter}, & \dot{y}(0) = 0 \text{ m/s} \end{array} \qquad (22)$$

Each local controller controls its corresponding subsystem. While the inverted pendulum comes to the vertical position, the cart reaches the origin of the coordinates. Fig. 4 shows the simulation results. The ANFIS controllers are able to control the system as good as their teachers. Despite the large initial values for angles the proposed controller is able to bring the pendulum to the vertical position. Also, the responses have acceptable overshoot and undershoot.

The run time of the simulations is very fast, even in the MATLAB environment. Due to the parallel processing nature of the ANFIS networks, the proposed controller is able to control the system 12 times faster than the teacher (the sliding-mode controller, defined in section IV, takes 62 seconds to perform 15 seconds of simulations on the same computer whereas the neuro-fuzzy controller performs the same simulation in 5 seconds). Hence, one can use the proposed method as an on-line controller, without any need to write the required codes in low level programming languages. Also, this method has some advantages over the method, which has been proposed by the authors of this paper in [18]. In Table I result of these two methods has been compared when the initial condition is the same.

TABLE I

Decentralized Controller	A	B	C	D
Neuro-Fuzzy using ANFIS Networks	86	7	14	5
Adaptive Robust with Fuzzy Modeling	35	0.6	2	42

A: Maximum controllable angle deviation (degree)
B: Maximum controllable cart distance from origin (meter)
C: Time of approaching to balance point (second)
D: Necessary time for simulation of 15 seconds (second)

A. *Applying External Disturbances*

The above simulations are repeated with external disturbances applied to both x and y directions. This disturbance, which is shown in Fig. 5, has relatively large amplitude and has been created using several sinusoidal waveforms with different frequency and amplitude along with random coefficients. The same initial values have been

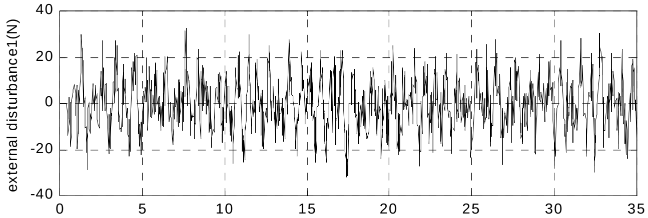

Fig. 5. Applied external disturbance to each axis

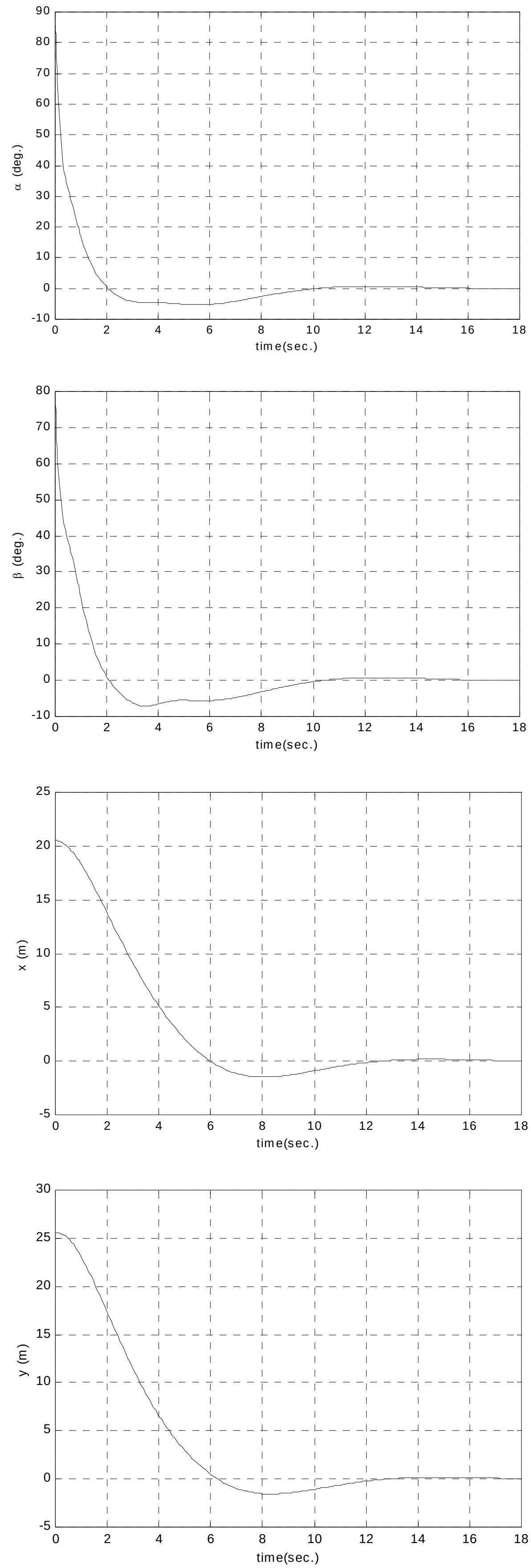

Fig. 4. Simulation results

considered for α and β as before. The simulation results are shown in Fig. 6. As it is clear from the graphs, the proposed controller can bring the inverted pendulum in the vertical position and hold it there, despite large amount of external disturbances and large initial value for angles.

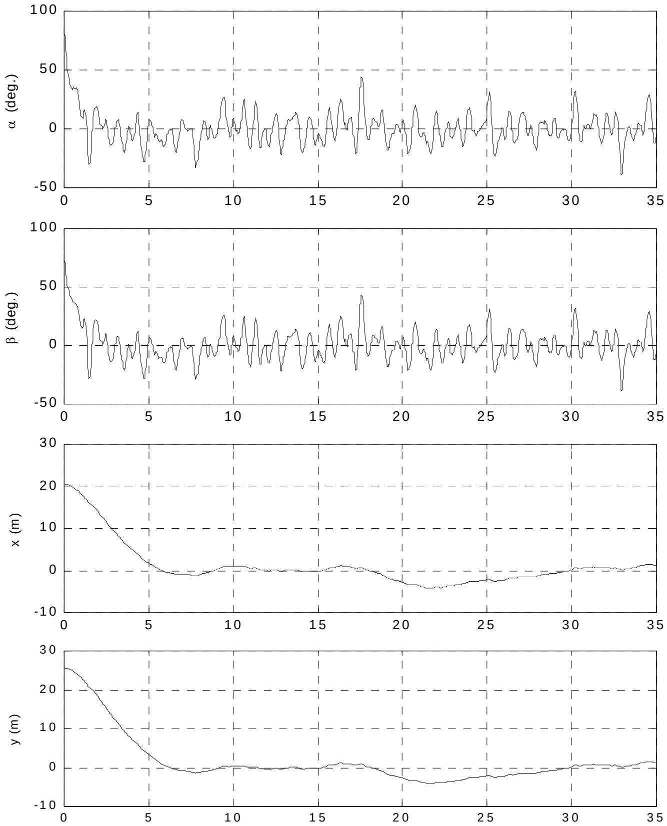

Fig. 6. Simulation results in the presence of external disturbances

VII. CONCLUSION

A new method, based on synthesis of the decentralized control theory and the decoupled sliding-mode method, was presented for controlling two-dimensional inverted pendulum. First, the dynamic equation of the system with order 8 was divided into two sub systems with order 4. Second, two sliding surfaces have been assigned to each sub system, and the sliding-mode control was performed using an intermediate variable. Next, this controller has been used to train two neuro-fuzzy ANFIS networks. Due to the high accuracy of the ANFIS networks, these two networks can learn the controlling properties of the teacher. That is, the ability to control the inverted pendulum with the large initial deviation angles. Moreover, due to the parallel structure of the ANFIS networks, the proposed controller responds much faster than its teacher (i.e. the decoupled sliding-mode controller). Also, the simulation results show that the performed control method is robust against external disturbances as well. In addition, the neuro-fuzzy structure of the ANFIS networks makes it needful from the system parameters, which yields a more robust controller against the changes in the parameters of the system. It has been showed in [18] by the authors that the proposed controller is robust against on-line changes of the parameters of the Inverted Pendulum, as a model of a launched missile, such as mass (to model the consumption of the fuel), gravity acceleration (as the missile is distancing from the surface of the earth) and sudden mass changes (as some sections of the missile will be separated). Also, since the neuro-fuzzy systems are trainable, the controller can be adaptive as well. Moreover, due to the fuzzy nature of ANFIS networks, the use of the expert knowledge in control strategy is possible. Although, the proposed method in this paper was used to control two-dimensional inverted pendulum, however it can be used to control a large class of nonlinear dynamic systems.

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