# On The Closed-Loop Stability Analysis of Transparent Teleoperation Systems with Time-Varying Delay Using a New Structure

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*Abstract*: In this paper, a new structure for teleoperated systems with time-varying delay in communication channel will be proposed, which provides transparency for the system. The key features of this structure are its simple design as well as the ability to analyse the stability of the closed-loop system using the property of the stable scalar functions and the small gain theorem. In the proposed structure, two local controllers will be designed, such that the transparency of the teleoperated system as well as the local stabilities is guaranteed. One local controller will be designated for position tracking of the slave system and the other one, whilst ensuring the stability of the closed-loop system in presence of time-varying delay in communication channel, performs the force tracking.

Keywords: Bilateral Teleportation, Transparency, Time delay

## 1. Introduction

The remote control of telerobotic manipulators has gained considerable attention in recent years. Teleoperated mobile robots are widely used in order to carry out complex tasks in hazardous environments, such as handling radioactive materials and maintenance of power units in nuclear plants; or to perform tasks in unreachable places, such as exploring and exploiting the seas and sea beds [1]. A teleoperated system consists of five different parts, as shown in Figure 1: master robot, communication channel, slave robot, human operator and task environment. The master is directly driven by the human operator in the local environment, whereas the slave is located in the remote environment, ready to follow commands that human operator orders by moving the master. The communication channel and interactions between the remote environment and the slave are of important matter. If the force exerted on the slave by the remote environment can be fedback to the master robot and applied to the human operator, which is called force reflecting control in teleoperation systems, the overall performance can be improved [2]. When the distance between the master robot and slave robot is too long, a significant time delay in communication channel appears that can not be ignored. This time delay can destabilize the bilateral teleoperation system [3], [4]. To solve this problem, different control schemes have been proposed in literature. The most widely used control schemes are the passivity theory [5], compliance control [6], wave variables [7], adaptive control [8] and robust control [9]. In each method, transparency is a major criterion for performance of telerobotic systems in presence of time delay in communication channel. If the slave accurately reproduces the master's commands and the master correctly feels the slave forces, the human operator experiences the same interaction as the slave would. This is called complete transparency in teleoperation system.

In this paper a novel control method of bilateral teleoperation systems with variable time delay in communication channel and complete transparency is proposed. In this structure, to achieve transparency, force measurement is used at the slave site (i.e. compliance control method), and force feedback (i.e. direct force-measurement force-reflecting control method) has been used at the master site.

The rest of this paper is organized as follows. Section 2 briefly describes general definitions of teleoperation systems. In section 3 and 4, the proposed control method in this paper is discussed. Section 5 analyses the stability of the proposed structure. In section 6, modelling of teleoperation system is described. Section 7 shows the simulation results. And finally, section 8 draws conclusions and gives some suggestions for the future work.



Figure 1: The General Structure of a Teleoperation System

## 2. General Teleoperation Definitions

A two-port network can be used to model a teleoperation system by using the equivalence between mechanical systems and electrical circuits. In Figure 2, the teleoperation system is modelled as a two-port network, where the operator-master interface is designated as the master port and the slave-environment interface as the slave port. The environment is considered as an impedance  $Z_e$ . The relationship between efforts ( $f_h$  and  $f_e$ ) and flows ( $\dot{x}_m$  and  $\dot{x}_s$ ) of the two ports can be described in terms of the so-called hybrid matrix. The hybrid matrix for the teleoperation system and its parameters are as follows [10]:

$$\begin{bmatrix} F_h(s) \\ V_s(s) \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_m(s) \\ -F_e(s) \end{bmatrix}$$
(1)

where  $F_h(s)$ ,  $F_e(s)$ ,  $V_m(s)$  and  $V_s(s)$  are the Laplace transforms of  $f_h(t)$ ,  $f_e(t)$ ,  $\dot{x}_m(t)$  and  $\dot{x}_s(t)$ , respectively. The equation relating the contact force to the slave position can be derived as

$$F_e = Z_e V_s \tag{2}$$



Figure 2: Two-part model of teleoperation systems

If the operator feels as if the task environments were being handled directly, one would say "the teleoperation system is ideal" or "the master-slave pair is transparent to human-task interface". Using the scaling factors, the position/velocity command to the slave and the force command to the master are modified such that

$$x_{m} = K_{p} x_{s}$$

$$f_{h} = K_{f} f_{e}$$
(3)
(4)

where  $K_p$  and  $K_f$  are the position and force scaling factors, respectively. Then, for ideal one-degree-of-freedom teleoperation system, the **H** matrix is

$$\mathbf{H}_{\text{ideal}} = \begin{bmatrix} 0 & K_f \\ K_p & 0 \end{bmatrix}$$
(5)

## 3. The Proposed Control Scheme

The proposed control scheme for teleoperation systems (with varying time delay in communication channels and uncertainty in task environment), has been shown in Figure 3, where G and C denote the transfer function of the controller, subscript *m* and *s* denote the master and slave, respectively,  $T_{ms}$  and  $T_{sm}$  denote the forward time delay (master to slave) and backward time delay (slave to master) in communication

channel, respectively;  $f_e$  is the force exerted on the slave by its environment,  $f_h$  is the force applied at the master by the human operator and  $f_r$  is the force reflected. In our proposed method, we combined the compliance control (i.e. contact forces are used at the slave robot), and direct force-measurement-force reflecting control. Direct force-measurement-force reflecting control is a simple form of force-reflecting scheme using a force sensor in which the contact force is reflected to the human operator. The main goal of this control scheme is to achieve transparency and stability. This has been done by designing two local controllers; one in remote site (slave robot)  $C_s$  and the other one in local site (master robot)  $C_m$ . The remote controller guarantees the position/velocity tracking. That is, the position/velocity slave has to follow the position/velocity, and the local controller guarantees the force tracking. Furthermore, the local controller guarantees the stability of the overall system. Here we assume that the scaling factors are identical, and also  $f_e$  is measurable. In next sections, the design of local controllers will be described.

## 4. Design of Control Schemes

#### 4.1. Local Slave Controller

According to Fig. 3, if the output of the master and the slave robots is velocity, then the transfer function from slave to master can be written as

$$\frac{V_s}{V_m} = \frac{C_s G_s}{1 + C_s G_s + Z_e C_c G_s} e^{-sT_{ms}}$$
(6)

Since the forward time delay doesn't appear in the denominator of the above equation, time delay will not have any effect on the stability. Also, we can use the classical control methods for linear systems like PID, to design a local slave controller  $C_s$  in the remote site such that system in (6) is stable. So, the velocity of the slave robot will follow the velocity of the master robot in such a way that the tracking error for velocity is satisfactory.

#### 4.2. Local Master Controller

Based on direct force-measurement force-reflecting control, we propose the local master controller, which can assure the stability of the closed-loop system as well as the force-tracking problem. The force tracking means the reflecting force has to follow the human operator force. Now, let define the following variables:

$$\hat{G}_{s}(s) = \frac{C_{s}G_{s}Z_{e}}{1 + C_{s}G_{s} + Z_{e}C_{c}G_{s}}$$
(7)

$$G(s) = \hat{G}_s(s)G_m(s)$$

$$T = T_{ms} + T_{cm}$$
(8)
(9)

$$F_{r}(s) = F_{e}(s)e^{-sT}$$
(10)

Using these variables, the control scheme, shown in Figure 3, can be simplified as in Figure 4. We notice that the local slave controller  $C_s$  is designed such that the velocity tracking is satisfied (i.e., the poles of  $\hat{G}_s$  are in the left-hand side of the S-Plane.)

Now, for considering the force tracking, the contact force has to follow the human operator force. In most literatures, the forward and backward time delays are assumed to be identical [11], [12]. Based on this assumption, the closed-loop transfer function of system given in Figure 4 can be written as

$$M_{c}(s) = \frac{C_{m}(s)G(s)e^{-Ts}}{1 + C_{m}(s)G(s)e^{-2Ts}}$$
(11)



Figure 3: Structure of the proposed control method



Figure 4: New Control Scheme

Notice that the roles of  $M_c(s)$  are the stability of the overall system as well as the force tracking. From (11), it can be seen the delay time is in the denominator of the closed-loop transfer function and can affect the overall performance of the system and may cause instability. The Smith predictor is an effective method to solve this problem [13]. This predictor can effectively cancel out time delays from the denominator in the transfer function of the closed-loop system. In other words, using the Smith predictor, the system output is simply the delayed value of the delay-free portion of the system. So, we can use the classical control methods for local master controller. Figure 5 shows the general structure of a Smith predictor.

The main drawback of the Smith predictor is that 1) the time delay must be constant, and 2) the model must be known precisely [14]. It is well known that it is not easy to obtain a precise model for a teleoperation system. Moreover, the time delay in communication channel is not constant.



Figure 5: Smith predictor control method

These problems, in this paper, have been dealt with using linear scalar systems and small gain theory as follows. The main feature of these systems is that their  $H_{\infty}$  norms are bounded to unity [15]. Let define

$$\delta_1(s) = e^{-sT} \tag{12}$$

$$\delta_1(s) = -\frac{1 - e^{-sT}}{sT} \tag{13}$$

such that

$$\left\|\delta_{k}\left(s\right)\right\|_{\infty} = \sup_{\operatorname{Re}(s)\geq 0} \left|\delta_{k}\left(s\right)\right| \leq 1 \quad , \quad k = 1, 2$$
(14)

## Small Gain Theorem

Let a linear system with transfer function G(s) to be stable and the nonlinear map F(y) be BIBO. Then, the closed loop system, shown in Figure 6, is stable if

$$\gamma(G)\gamma(F) < 1 \tag{15}$$

where  $\gamma(F)$  is the gain of the nonlinear map F(y) and  $\gamma(G) = \sup_{m} \|G(j\omega)\|$  [16].



Figure 6: A typical closed-loop system

Without lost of generality, the structure given in Figure 4 can be rearranged as in Figure 7, in which  $G(s) = G_m(s)\hat{G}_s(s)$ . Also, it is obvious that the stability of the proposed closed-loop model is the same as the stability of M(s), which has been shown in dashed rectangle in Figure 7

$$M(s) = \frac{\hat{F}_e}{F_h} = \frac{C_m(s)G(s)}{1 + C_m(s)G(s)e^{-2Ts}}$$
(16)

And the transfer function of the entire system is

$$M_{c}(s) = M(s)e^{-Ts} = \frac{C_{m}(s)G(s)}{1 + C_{m}(s)G(s)e^{-2Ts}}e^{-Ts}$$
(17)

Therefore, the local master controller must be designed to guarantee the stability of the closed-loop system  $M_c(s)$ , when the delay time in communication channel varies.



Figure 7: Equivalent control structure for Figure 2

#### 5. Stability Analysis

In this section, the stability of the proposed structure will be analyzed and conditions, which satisfy this stability, will be given. According to section 4.2, the stability of the proposed closed-loop structure is equivalent to the stability of M(s). In the followings, the stability theorem will be given, in which the stable scalar functions, as in (14), and the small gain theorem has been employed. Moreover, the uncertainty in the dynamics of the feedback system will be modeled with  $\delta_k$ .

Theorem:

Let a linear, time-invariant, and single-input-single-output control system be given as in Figure 7., and let G(s) be stable and the closed-system also be stable with no time delay (T = 0), then the closed-loop system M(s) is stable, according to the small gain theorem, if

$$\left| \frac{C_m(s)G(s)}{1 + C_m(s)G(s)} \right|_{s=i\omega} \le \frac{1}{2\omega T_{\max}} \quad \forall \omega, 0 \le T \le T_{\max}$$
(18)

Proof:

Let M(s) in Figure 7 be redrawn as in Figures 8a and then 8b. It is clear that the stability of structure in Figure 8b is the same as the stability of M(s).



Fig. 8: The equivalent structure of M(s) in Figure 7.

Now, let  $G_{uy}(s) = \frac{sC_m(s)G(s)}{1+C_m(s)G(s)}$ . Then, according to the small gain theorem, the closed-loop system is stable if

$$\gamma(G_{uy})\gamma(\tilde{\delta}) < 1 \tag{19}$$

Considering the property of stable scalar functions  $\|\delta(s)\|_{\infty} = \left\|\frac{1-e^{-2Ts}}{2sT}\right\|_{\infty} \le 1$  and assuming the worst case for delay time in communication channel  $T = T_{\text{max}}$ , we have

$$\gamma(\tilde{\delta}) < 2T_{\max} \rightarrow |\gamma(G_{uy})| \le \frac{1}{2T_{\max}}$$
(20)

Therefore,

$$\left|G_{uy}(j\omega)\right| = \left|\frac{\omega C_m(j\omega)G(j\omega)}{1 + C_m(j\omega)G(j\omega)}\right| \le \frac{1}{2T_{\max}}$$
(21)

which completes the proof.

## 6. Modelling of Teleoperation Systems

## 6.1 Slave Model

The Remote site has two parts: the slave manipulator, and the environment, where the task takes place. The slave used as the teleoperation system, is usually a robotic manipulator with several Degrees of Freedom (DoF). The dynamic Model of an n DoF robotic manipulator is usually given as [17]

$$\tau = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\mathbf{q},\dot{\mathbf{q}})$$
(22)

where  $\tau \in \Re^{n \times 1}$  is the torque produced by the actuators,  $\mathbf{M}(\mathbf{q}) \in \Re^{n \times n}$  is the inertia matrix,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \Re^{n \times 1}$  represents centrifugal and coriolis terms,  $\mathbf{G}(\mathbf{q}) \in \Re^{n \times 1}$  is the gravitational load, and  $\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}) \in \Re^{n \times 1}$  represents the frictional load. For the purpose of illustration, consider a single DOF with linear equations for the dynamics of the remote robot manipulator. Taking the interaction with the environment into account, yields

$$\tau - \tau_e = M_s \dot{q} + F_s \dot{q} \tag{23}$$

where  $F_s$  is the linear friction and  $\tau_e$  is the interaction torque between the manipulator end-effector and the environment.

## 6.3 Master Model

The master used in a teleoperation system is affected by the human force. The dynamics of a single-DOF master manipulator is

$$J_m \ddot{\theta} + b_m \dot{\theta} = \tau_m \tag{24}$$

where  $J_m$  and  $b_m$  are the manipulators inertia and damping coefficient. The force  $\tau_m$  applied to the Manipulator depends on the interaction with the human operator.

## 7. Simulations

In simulations, two mechanical arms have been used as the master and the slave systems

$$(J_m s^2 + b_m)x_m = F_m$$
$$(M_s s^2 + B_s)x_s = F_s$$

in which *B* and *b* are the friction coefficients, *M* is the mass, *J* is the moment of inertia, *x* is the displacement, and *F* is the force. Indices *m* and *s* are for the master and the slave systems, respectively. It should be noted that  $F_m$  ( $F_s$ ) is the applied force to the master (slave) arm according to the interactions between the environment and the control action. The system parameters have been given in table 1. The simulation results have been shown in Figures 9 and 10. Figure 9 shows a random and varying time delay in communication channel. In figure 10 exhibits the position/speed tracking and force tracking. It is obvious that in addition to the stability of the teleoperation system, the slave position follows the master position with good accuracy, such that the operator command, applied to the master, is tracked by the slave. In addition to that, force tracking on the master side is very satisfactory.

#### 8. Conclusion

To achieve transparency and stability for a teleoperation system with model uncertainty and time delay uncertainty in communication channel, a new control scheme was proposed in this paper. Two local controllers, one in the master side and in the slave side was design based on Compliance control and direct force-measurement force-reflection control method, such that the master controller guarantees the position tracking and the slave controller guarantees force tracking. The advantage of the proposed method is that one can use the classical control methods as well as modern intelligent control methods. In this paper, by using two classical and simple controllers (i.e., PI for position and force tracking) it was shown that the new control scheme is a practical choice for teleoperation systems with time varying delay in communication channel, because the stability of the closed-loop system is guaranteed.

Future works in this area will include considering model mismatch in teleoperation system and some analytical work and conditions for stability of the closed-loop system.

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Table 1: Model Parameters

Figure 9: Time Delay in Communication Channel



Figure 10: Transparency Response, (a) Position Tracking, (b) Force Tracking