ROBUST ADAPTIVE CONTROL OF NONLINEAR SYSTEMS USING NEURAL NETWORKS

S Mohammad-Hoseini[*], M Farrokhi[*,†] and A J Koshkouei[††]

[*]*Department of Electrical Engineering, Iran University of Science and Technology, Tehran 16846, IRAN*
[†] *Centre of Excellence for Power System Operation and Automation, Iran University of Science and Technology, Tehran 16846, IRAN*
E-mails: {sm_hoseini, farrokhi}@iust.ac.ir
[††] *Control Theory and Applications Centre, Coventry University, Coventry, UK*

Keywords: Output feedback, nonlinear systems, asymptotic stability, adaptive control, neural networks.

Abstract

This paper presents a direct adaptive output feedback control design method for uncertain non-affine nonlinear systems, which does not rely on state estimation. The approach is applicable to systems with unknown, but bounded dimensions and with known relative degree. A neural network is employed to approximate and adaptively make ineffective unknown plant nonlinearities. An adaptive control law for the weights in the hidden layer and the output layer of the neural network are also established so that the entire closed-loop system is stable in the sense of Lyapunov. Moreover, the tracking error is guaranteed to be uniformly and asymptotically stable, rather than uniformly ultimately bounded with the aid of an additional adaptive robustifying control part. The proposed control algorithm is relatively strightforward and no restrictive conditions on the design parameters for achieving the systems stability are required. The efficiency of the proposed scheme is shown through the simulation of a non-affine nonlinear system with unmodelled dynamics.

1 Introduction

Control system design for complex nonlinear systems has been widely studied in the last decade. Many remarkable results in this area have been reported, including feedback linearization techniques [1, 2] and backstepping design [3] for systems with unmatched uncertainties [4]. Most of these researches are conducted for certain systems in affine form. Several adaptive schemes have been developed in dealing with the problem of parametric uncertainties [5, 6] for affine nonlinear systems. However, there are some practical systems such as chemical reactions [7] in which the input variables cannot be expressed in an affine form. So using classic approaches such as feedback linearisation, the control of such systems may be difficult or impossible.

In recent years, several methods based on Neural Networks (NNs) have been presented to control nonlinear systems by removing the unknown nonlinear part of the system [7]-[13]. Most of these approaches have been proposed based upon the state feedback [9, 10] or output feedback [11]-[13]. In particular, because of approximation errors inherent in NNs, when the number of neurons is limited, most of these methods can guarantee only uniformly ultimately bounded (UUB) stability. To remove this obstacle and to compensate the approximation errors, a method has been widely used in which an extra robustifying input part is considered [8, 14]. In this method the gain is computed from the suitable information about an upper bound of the system uncertainties, which is normally unavailable and there is no direct method for obtaining it. Therefore, these methods yield an overestimate resulting from a conservative design. To overcome this problem, an adaptive robustifying control part based on states of system is introduced in [15]. In this approach, the system states should be available or be estimated, and the dimension of system must also be known a priori.

In this paper an adaptive robustifying control part which guarantees asymptotic stability of tracking error, is proposed. The overall proposed control law is based on output feedback control methods and estimates of the states are not required. Therefore, the plant dimension is not necessary to be known a priori and for designing the controller only the relative degree of system is required. Since the control law comprises of the stabiliser, adaptive and robustifying parts, the closed-loop system is robust against unmodelled dynamics and asymptotically stabilises the system. In addition, the method is applicable to a class of nonlinear systems with any relative degree. The method is based on strictly positive realness (SPR) condition of the closed-loop error dynamics and the Kalman-Yakobovich's lemma as well as NN techniques.

This paper is organised as follows: Section 2 describes the class of nonlinear systems to be controlled and the problem of the tracking error. The structure and approximation properties of the neural networks are addressed in Section 3. In Section 4, the stability of the closed-loop system is proved. An example which illustrates the effectiveness of the proposed controller is presented in Section 5. Conclusions are given in Section 6.

2 Problem statement

Consider the nonlinear system

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \tag{1}$$

where $x \in \Omega_x \subset R^n$ is the state vector on the compact set Ω_x as an operating region, $u \in \Omega_u \subset R$ is the input on the compact set Ω_u , and $y \in R$ is the output. The mapping $f : R^{n+1} \to R^n$ is an unknown smooth vector field and $h : R^n \to R$ is a smooth and known real function. Assume that the relative degree of the system (1) is $r \le n$. Under this assumption, there is a diffeomorphism transformation

$$\Phi(x) = [z,\eta] = \left[h(x), L_f h, \ldots, L_f^{r-1} h, \varphi_1(x), \ldots, \varphi_{n-r}(x) \right]$$

which transforms the system (1) into the following normal form with a new coordinate $\left[z,\eta\right] = \left[z_1,\ldots,z_r,\eta_{r+1},\ldots,\eta_n\right]$ [1]

$$\begin{cases} \dot{z}_i = z_{i+1}, & 1 \le i \le r-1 \\ \dot{z}_r = b_1\left(z,\eta,u\right) \\ \dot{\eta} = w\left(z,\eta\right) \\ y = z_1 \end{cases} \tag{2}$$

Assumption 1: *The zero dynamics of system (2), $\dot{\eta} = w\left(0,\eta\right)$ is exponentially stable. Moreover, the desired trajectory and its derivates are bounded such that the internal dynamics remain bounded.*

Assumption 2: *Assume that $b_u = \partial b_1\left(z,\eta,u\right)/\partial u \ne 0$. This condition implies that the smooth function b_u is strictly either positive or negative on the compact set*

$$U = \left\{ \left(z,\eta,u\right) \middle| \left(z,\eta\right) = \Phi\left(x\right), x \in \Omega_x ; u \in \Omega_u \right\}$$

From (2), the input-output relation of the system is

$$y^{(r)} = b_1(z,\eta,u) = b(x,u) \tag{3}$$

Define the pseudo-control

$$v = \hat{b}(y,u) \tag{4}$$

where $\hat{b}(y,u)$ is the available approximation of $b(\mathbf{x},u)$. One may consider $\hat{b}(y,u) = cu$, where c is a constant. However, it should be appropriately selected based upon the criteria which are presented in Section 3. Hence, the modelling error is

$$\Delta(\mathbf{x},u) = \hat{b}(y,u) - b(\mathbf{x},u) \tag{5}$$

Using (3), (4) and (5) $y^{(r)} = \hat{b}(y,u) + \Delta(x,u) = v + \Delta(x,u)$

Now, selecting the pseudo-control v as

$$v = y_d^{(r)} + u_L + u_{ad} - u_R \tag{6}$$

where $y_d^{(r)}$ is the r-derivative of the desired output y_d , u_L stabilises the closed-loop system, u_{ad} is the adaptive part and it cancels out the modelling error $\Delta(\mathbf{x},u)$ whilst the control part u_{R} is proposed to achieve robust asymptotic stability. The robust control u_{R} could be continuous or discontinuous. In particular, one may consider a sliding mode control since it is robust in the presence of uncertainties.

Define $e = y_d - y$. Then the closed-loop error dynamics of the system is

$$e^{(r)} = -u_L + \left(\Delta(x,u) - u_{ad}\right) + u_R \tag{7}$$

2.1 SPR of the error dynamic

In this section, a strictly positive realness (SPR) property of closed-loop error dynamic is studied. Assume that u_L is a suitable filter with the following structure

$$u_L = \frac{N_L}{D_L} e, \tag{8}$$

and filtered error signal $\tilde{e}$ is defined as

$$\tilde{e} = G_{\text{ad}}(s)e = \frac{N_{\text{ad}}}{D_{\text{ad}}} e \tag{9}$$

where $G_{\text{ad}}(s)$ is chosen so that $G_{\text{ad}}(0) \ne 0$. Substituting (8) and (9) in (7), the closed-loop tracking error is

$$\tilde{e}(s) = G(s)\left(\left(\Delta(x,u) - u_{\text{ad}}\right) + u_{\text{R}}\right)(s) \tag{10}$$

where

$$G(s) = \frac{D_L N_{\text{ad}}}{D_{\text{ad}}\left(s^r D_L + N_L\right)} \tag{11}$$

By applying Routh-Hurwitz stability criterion to (11), one can conclude that a necessary condition for stability of the closed-loop system (11) is that the degree of N_L (and hence D_L) should be at least $r-1$. Therefore, k is defined as

$$k = \deg(D_L) \ge \deg(N_L) \ge r-1 \tag{12}$$

In addition, to simplify the design procedure, D_{ad} and D_{L} are selected such that

$$\deg(D_{\text{ad}}) = \deg(D_L) \tag{13}$$

Hence, the relative degree of $G(s)$ is

$$\rho = k + r - \deg(N_{\text{ad}}) \tag{14}$$

where $\deg(N_{\text{ad}}) \le k$. Therefore, $\rho \ge r$. If $\rho > 1$ then G(s) is not a SPR function [16]. So there exits a suitable filter $T(s)$ with the following characteristic

$$\deg(N_{\text{ad}}) + \deg(T) = k + r - 1 \tag{15}$$

and the new filtered error dynamic is

$$\tilde{e}(s) = \bar{G}(s)T^{-1}(s)\left(\left(\Delta(x,u) - u_{ad}\right) + u_{sl}\right)(s) \tag{16}$$

By selecting N_{ad} and $T(s)$ appropriately, the auxiliary transfer function

$$\bar{G}(s) = G(s)T(s) = \frac{b_1 s^{p-1} + b_2 s^{p-2} + \ldots + b_p}{s^p + a_1 s^{p-1} + \ldots + a_p}, \quad p = 2k + r \tag{17}$$

has the SPR property. Therefore, the state space model of closed-loop error dynamic (16) can be represented as

$$\begin{aligned} \dot{\xi} &= A_{cl}\xi + b_{cl}\left[T^{-1}(s)\left(\left(\Delta(x,u) - u_{ad}\right) + u_R\right)\right] \\ \tilde{e} &= c_{cl}^T \xi \end{aligned} \tag{18}$$

Due to the SPR property of $\bar{G}(s)$ and according to the Kalman-Yakobovich lemma, for any positive-definite matrix Q , there is a positive-definite symmetric matrix P such that

$$A_{cl}^T P + P A_{cl} = -Q \tag{19}$$

and

$$P b_{cl} = c_{cl} \tag{20}$$

3 Neural network-based approximation

The modelling error $\Delta(x(t),u(t))$ affects the pseudo-control ν. So the adaptive part $u_{\text{ad}}(t)$ of the control ν is designed to cancel the unknown modelling approximation error $\Delta(\mathbf{x},u)$. So, there exists a fixed point problem as $u_{ad}(t)=\Delta\left(x(t),L(u_{ad}(t),u_L,u_R)\right)$ where L is a function, which can be directly found from (4) and (6). The conditions that guarantee the existence and uniqueness are [13]

$$\begin{aligned}&\operatorname{sgn}(\partial b/\partial u)=\operatorname{sgn}(\partial\hat{b}/\partial u)\\&\left|\partial\hat{b}/\partial u\right|>\left|\partial b/\partial u\right|/2>0\end{aligned}\tag{21}$$

In the following lemma, it is shown that the error $\Delta(x,u)$ can be approximated by a neural network. Moreover, it is proved that if any non-affine system satisfies conditions (21) then there is unnecessary to use $u_{\text{ad}}(t)$ as a feedback signal.

Lemma 1: *If conditions (21) are satisfied then the modelling error $\Delta(x,u)$ can be approximated by a single hidden layer Multilayer Perceptron (MLP) as $w^T\boldsymbol{\sigma}\left(V^T\boldsymbol{\zeta}\right)$ where w is a vector containing synaptic weights of the output layer, V is a matrix containing the weights for the hidden layer, respectively, and ζ is the input vector, which is equal to $\zeta=\left[1,\bar{y},\bar{u}_\alpha,\bar{u}_{\text{ad}}\right]^T$ where*

$$\begin{aligned}\bar{y}&=[y(t),...,y(t-T_d(n_1-1))]\\\bar{u}_\alpha&=[u_\alpha(t),...,u_\alpha(t-T_d(n_1-r-1))]\\\bar{u}_{\text{ad}}&=[u_{\text{ad}}(t-T_d),...,u_{\text{ad}}(t-T_d(n_1-r-1))]\end{aligned}\tag{22}$$

in which $u_\alpha=u_{\text{dc}}+u_{\text{R}}+y_d^{(r)}$.

Proof: Under the observability condition of the system (1), it is shown that the system states can be expressed as a function of $\bar{y}$ and $\bar{\nu}$ as

$$x=F\left(\bar{y},\bar{\nu}\right)\tag{23}$$

where $\bar{\nu}=[\nu(t),\nu(t-T_d),...,\nu(t-T_d(n_1-r-1))]^T$, $n_1\geq n$ [17]. In addition, (4) indicates that the control u is a function of y and ν. Thus

$$\Delta(x,u)=\Delta(x,y,\nu)=H(\bar{y},\bar{\nu})\tag{24}$$

In it is proved that adaptive part u_{ad} is a function of states and u_α [16]. So

$$u_{\text{ad}}=I(x,u_\alpha)\tag{25}$$

From (23) and (25)

$$u_{\text{ad}}(t)=I(\bar{y},u_{\text{ad}}(t),\bar{u}_\alpha,\bar{u}_{\text{ad}})\tag{26}$$

Assume that

$$\frac{\partial\left[u_{\text{ad}}(t)-I(\bar{y},u_{\text{ad}}(t),\bar{u}_\alpha,\bar{u}_{\text{ad}})\right]}{\partial u_{\text{ad}}}\neq 0$$

Then according to the implicit function theorem, the adaptive part of control law

$$u_{\text{ad}}(t)=K(\bar{y},\bar{u}_\alpha,\bar{u}_{\text{ad}})\tag{27}$$

is obtained. Finally, by substituting (27) in (24) yields

$$\Delta(x,y,\nu)=M(\bar{y},\bar{u}_\alpha,\bar{u}_{\text{ad}})$$

On the other hand, any sufficiently smooth function can be approximated on a compact set with an arbitrarily bounded error by a suitable large MLP [18]. Therefore, on the compact set Ω_ζ a set of ideal weights w^* and V^* exist such that

$$\Delta\left(x,y,\nu\right)=w^{*T}\sigma(V^{*T}\zeta)+\varepsilon\tag{28}$$

where $|\varepsilon|\leq\varepsilon_M$ and ε_M is an appropriate positive number. The ideal constant weights w^* and V^* are defined as

$$\left(w^*,V^*\right):=\arg\min_{(w,V)\in\Omega_w}\left\{\sup_{\zeta\in\Omega_\zeta}\left|w^T\sigma\left(V^T\zeta\right)-\Delta(x,y,\nu)\right|\right\}\tag{29}$$

in which $\Omega_w=\left\{(w,V)\,\middle|\,\|w\|_F\leq M_w,\|V\|_F\leq M_v\right\}$ and ε_M , M_w and M_v are positive numbers, and $\|\cdot\|_F$ denotes the Frobenius norm. However, in practice, the weights may be different from ideal ones, so an approximation error occurs, which can be calculated as in [8]

$$\begin{aligned}\Delta(x,y,\nu)-u_{ad}&=w^{*T}\sigma(V^{*T}\zeta)+\varepsilon-w^T\sigma(V^T\zeta)\\&=\tilde{w}\left(\sigma-\dot{\sigma}v^T\zeta\right)+w^T\dot{\sigma}\tilde{V}^T\zeta+\delta(t)\end{aligned}\tag{30}$$

where

$$\begin{aligned}&|\delta(t)|\leq c_0+c_1\left\|\tilde{Z}\right\|_F+c_2\left\|\tilde{Z}\right\|_F|\tilde{e}|\\&\tilde{w}=w^*-w\\&\tilde{V}=V^*-V\\&\tilde{Z}=Z^*-Z\end{aligned}\tag{31}$$

in which

$$Z=\begin{bmatrix}w&0\\0&v\end{bmatrix}$$

and $\dot{\sigma}\in R^{m\times m}$ is the derivative of σ with respect to the input signal of all neuron in the hidden layer of NN.

4 Stability performance

In this section the asymptotic stability of the error system is proved. Based on the results of Section 3, the system (18) is first converted into a new form. Then a lemma is presented which is needed for proof of the system stability. By substituting (30) in (18), the closed-loop error dynamic can be represented as

$$\begin{aligned}&\dot{\xi}=A_{cl}\xi+b_{cl}\left(T^{-1}(s)\tilde{w}^T\left(\sigma-\dot{\sigma}V^T\zeta\right)+T^{-1}w^T\dot{\sigma}\tilde{V}^T\zeta+\delta_f(t)+u_{Rf}\right)\\&\tilde{e}=c_{cl}^T\xi\end{aligned}$$

where

$$\begin{aligned}\delta_{\text{f}}&=T^{-1}(s)\,\delta\\u_{\text{Rf}}&=T^{-1}(s)u_{\text{R}}\end{aligned}$$

Define

$$\begin{aligned}\psi_1&=\sigma-\dot{\sigma}V^T\zeta\\\Psi_2&=\zeta w^T\dot{\sigma}\end{aligned}$$

Then the closed-loop error dynamic is

$$\begin{aligned}&\dot{\xi}=A_{cl}\xi+b_{cl}\left(T^{-1}(s)\tilde{w}^T\psi_1+tr\left(T^{-1}(s)\tilde{V}^T\Psi_2\right)+\delta_f(t)+u_{Rf}\right)\\&\tilde{e}=c_{cl}^T\xi\end{aligned}\tag{32}$$

When $\tilde{w}$ and $\tilde{V}$ are time-variant signals, the transfer function of the system (32) is not commutable. To remove this obstacle, the error parts are defined as

$$\begin{aligned}\delta_w &= T^{-1}(s)\tilde{w}^T\psi_1 - \tilde{w}^T T^{-1}(s)\psi_1 \\ \delta_V &= tr\left(T^{-1}(s)\tilde{V}^T\Psi_2 - \tilde{V}^T T^{-1}(s)\Psi_2\right)\end{aligned} \tag{33}$$

where

$$|\delta_w| \le c_3 \|\tilde{w}\|_F, \quad |\delta_V| \le c_4 \|\tilde{V}\|_F \tag{34}$$

with positive numbers c_3 and c_4. Substituting (33) in (32) yields

$$\begin{aligned}\dot{\xi} &= A_{cl}\xi + b_{cl}\left(\tilde{w}\psi_{1f} + tr\left(\tilde{V}^T\Psi_{2f}\right) + \delta_w + \delta_V + \delta_f + u_{Rf}\right) \\ \tilde{e} &= c_{cl}^T\xi\end{aligned} \tag{35}$$

where

$$\begin{aligned}\psi_{1f} &= T^{-1}(s)\left(\sigma - \dot{\sigma}V^T\zeta\right) \\ \Psi_{2f} &= T^{-1}(s)\left(\zeta w^T\dot{\sigma}\right)\end{aligned} \tag{36}$$

In order to show that the system is asymptotically stable via the proposed control, the following lemma is needed.

Lemma 2: *It can be shown that the following inequality holds*

$$\left|\delta_f + \delta_w + \delta_V\right| \le \varphi^* T^{-1}(s) g \tag{37}$$

where, $g = \left(1 + \sqrt{\|V\|_F^2 + \|w\|_F^2}\right)\left(1 + |\tilde{e}|\right) + \left(1 + \|V\|_F + \|w\|_F\right)$ *and* φ^* *is a constant.*

Proof: Assume that $\left|T^{-1}(s)\right| \le 1$. Using (31), (34) and

$$\left|\delta_f + \delta_w + \delta_V\right| \le \left|T^{-1}\delta(t)\right| + |\delta_w| + |\delta_V| \le |\delta(t)| + |\delta_w| + |\delta_V|$$

One can obtain

$$\begin{aligned}\left|\delta_f(t) + \delta_w + \delta_V\right| &\le c_0 + c_1\|Z^* - Z\|_F + c_2\|Z^* - Z\|_F|\tilde{e}| \\ &\quad + c_3\|w^* - w\|_F + c_4\|V^* - V\|_F \\ &\le c_0 + c_1\|Z^*\|_F + c_1\|Z\|_F + c_2\|Z^*\|_F|\tilde{e}| + c_2\|Z\|_F|\tilde{e}| \\ &\quad + c_3\|w^*\|_F + c_3\|w\|_F + c_4\|V^*\|_F + c_4\|V\|_F \\ &\le c_0 + c_1 M + c_1\|\mathbf{Z}\|_F + c_2 M|\tilde{e}| + c_2\|\mathbf{Z}\|_F|\tilde{e}| + c_3 M_w \\ &\quad + c_3\|\mathbf{w}\|_F + c_4 M_V + c_4\|\mathbf{V}\|_F \\ &\le \varphi_1\left(\left(1 + \|\mathbf{Z}\|_F\right)\left(1 + |\tilde{e}|\right) + \left(1 + \|\mathbf{V}\|_F + \|\mathbf{w}\|_F\right)\right) \\ &= \varphi_1\left(\left(1 + \sqrt{\|\mathbf{V}\|_F^2 + \|\mathbf{w}\|_F^2}\right)\left(1 + |\tilde{e}|\right) + \left(1 + \|\mathbf{V}\|_F + \|\mathbf{w}\|_F\right)\right)\end{aligned}$$

where

$$M = \sqrt{M_w^2 + M_v^2}$$

and

$$\varphi_1 = \max\left\{c_0 + c_1 M, c_1, c_2 M, c_2, c_3 M_w + c_4 M_V, c_3, c_4\right\}.$$

Assume that the highest frequency of signal s is ω_s and $T^{-1}(s)$ is a low pass filter. Then

$$\left|\delta_f + \delta_w + \delta_V\right| \le \frac{\varphi_1}{T^{-1}(j\omega_s)} T^{-1}(s) g$$

Therefore, φ^* may be selected as

$$\varphi^* = \frac{\varphi_1}{T^{-1}(j\omega_s)}$$

Theorem: Consider the discontinuous control

$$u_R = -g\varphi\,\mathrm{sgn}\left(\tilde{e}\right) \tag{38}$$

and select the adaptation laws for NN weights, and the gain of the robust part φ as

$$\begin{aligned}\dot{w} &= \gamma_w \tilde{e}\psi_{1f} \\ \dot{V} &= \gamma_v \tilde{e}\Psi_{2f} \\ \dot{\varphi} &= \gamma_\varphi |\tilde{e}|\left(T^{-1}g\right)\end{aligned} \tag{39}$$

Then the closed-loop tracking error is asymptotically stable and the weights of the neural network remain bounded.

Proof: Consider the Lyapunov function

$$W = \frac{1}{2}\xi^T P\xi + \frac{1}{2\gamma_w}\|\tilde{w}\|^2 + \frac{1}{2\gamma_v}\|\tilde{V}\|_F^2 + \frac{1}{2\gamma_\varphi}|\tilde{\varphi}|^2 \tag{40}$$

where P is the unique positive-definite symmetric solution (19) and $\tilde{\varphi} = \varphi^* - \varphi$. Assume that w^* and V^* are ideal constant weights defined as in (29). Then from (31) $\dot{w} = -\dot{\tilde{w}},\ \dot{W} = -\dot{\tilde{V}}$. Hence, the time-derivative of W is

$$\begin{aligned}\dot{W} &= -\frac{1}{2}q\|\xi\|^2 + \xi^T P b_{cl}\left[T^{-1}(s)\tilde{w}^T\psi_1 + tr\left(T^{-1}(s)\tilde{V}^T\Psi_2\right)\right. \\ &\quad \left. + u_{Rf} + \delta_f\right] - \frac{1}{\gamma_w}\tilde{w}^T\dot{w} - \frac{1}{\gamma_v}tr\left(\tilde{V}^T\dot{V}\right) - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi}\end{aligned}$$

Using (33)

$$\begin{aligned}\dot{W} &\le -\frac{1}{2}q\|\xi\|^2 + \xi^T P b_{cl}\left[\tilde{w}^T\psi_{1f} + tr\left(\tilde{V}^T\Psi_{2f}\right) + u_{Rf} + \delta_f\right. \\ &\quad \left. + \delta_w + \delta_V\right] - \frac{1}{\gamma_w}\tilde{w}^T\dot{w} - \frac{1}{\gamma_v}tr\left(\tilde{V}^T\dot{V}\right) - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi}\end{aligned} \tag{41}$$

From (18) and (20)

$$\tilde{e} = \xi^T P b_{cl} \tag{41a}$$

Lemma 2 implies

$$\left|\delta_f + \delta_w + \delta_V\right| \le \varphi^* T^{-1}(s)\, g \tag{41b}$$

Substituting (41a) and (41b) in (41) yields

$$\begin{aligned}\dot{W} &\le -\frac{1}{2}q\|\xi\|^2 + tr\left(\tilde{V}^T\left(\tilde{e}\Psi_{2f} - \frac{1}{\gamma_v}\dot{V}\right)\right) + \tilde{w}^T\left(\tilde{e}\psi_{1f} - \frac{1}{\gamma_w}\dot{w}\right) \\ &\quad + |\tilde{e}|T^{-1}(s)\varphi^* g + \tilde{e}u_{Rf} - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi}\end{aligned}$$

Using adaptation laws (39) the time-derivative of W satisfied the following inequality

$$\begin{aligned}\dot{W} &\le -\frac{1}{2}q\|\xi\|^2 + |\tilde{e}|T^{-1}\varphi^* g - \tilde{e}\left(T^{-1}\,\mathrm{sgn}(\tilde{e})\varphi g\right) + \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi} \\ &= -\frac{1}{2}q\|\xi\|^2 + |\tilde{e}|T^{-1}g\left(\varphi^* - \varphi\right) + \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi} = \\ &= -\frac{1}{2}q\|\xi\|^2 + \tilde{\varphi}\left(|\tilde{e}|T^{-1}g + \frac{1}{\gamma_\varphi}\dot{\varphi}\right) = -\frac{1}{2}q\|\xi\|^2\end{aligned} \tag{42}$$

Since W is a positive function and $\dot{W} \le 0$, so $\|\xi\|,\ \|\tilde{V}\|,\ \|\tilde{w}\|$ and $|\tilde{\varphi}|$ are bounded. In addition, from (29), V^* and w^* are bounded, therefore, according to (31), W and w must remain bounded. Moreover, by integrating (42)

$$\int_0^\infty \|\xi(t)\|^2 dt \le \frac{2}{q}\left(W(t)\big|_{t=0} - W(t)\big|_{t=\infty}\right) \tag{43}$$

Since, the right-hand side of (43) is bounded, then the Barbalet's lemma yields

$$\lim_{t\to\infty}\|\xi\|^2 = 0 \tag{44}$$

Since $\tilde{e} = \mathbf{c}_{\mathrm{cl}}^{\mathrm{T}}\boldsymbol{\xi}$,

$$\lim_{t\to\infty}\tilde{e}(t) = 0 \tag{45}$$

According to the final value theorem and using (9), it yields

$$\lim_{s\to 0} s\,\tilde{e}(s) = \lim_{s\to 0} s\,G_{ad}(s)e(s) = 0$$

Since $G_{ad}(0) \neq 0$ one can conclude

$$\lim_{s\to 0} s\,e(s) = 0$$

so

$$\lim_{t\to\infty} e(t) = 0 \tag{46}$$

Remark: *When a discontinuous control is applied to a system, a phenomenon, the so-called chattering appears. Many methods have been proposed to reduce chattering including continuous approximation of the discontinuous control. A continuous approximation of* $\operatorname{sgn}(\tilde{e})$ *in (38) is the saturation function*

$$\operatorname{sat}(\tilde{e}) = \begin{cases} \operatorname{sgn}(\tilde{e}) & \text{if } |\tilde{e}| \geq \varepsilon \\ \dfrac{\tilde{e}}{\varepsilon} & \text{otherwise} \end{cases} \tag{47}$$

Alternatively, one may consider the smoothing function $\tanh\left(\tilde{e}/\varepsilon\right)$ *or* $\dfrac{\tilde{e}}{|\tilde{e}|+\delta}$ *where* $\varepsilon > 0$ *and* $0 < \delta < 1$ *as an approximation of* $\operatorname{sgn}(\tilde{e})$.

Figure 1 shows the block diagram of the system with the proposed control. Note that if there is a finite time t_s such that $\tilde{e} = 0$ for all $t \geq t_s$, the system trajectories move on the sliding surface $\tilde{e} = 0$ and along this surface tending to an equilibrium point [19]. The control (38) guarantees the robustness of the method in the presence of disturbances or unmodelled dynamics provided that the gain φ is selected sufficiently large.

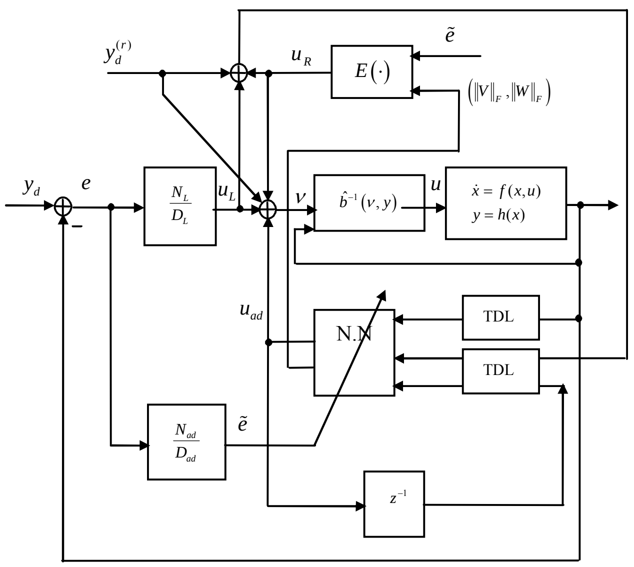

Figure 1: Block diagram of the proposed control method.

5 Example

The performance of the proposed controller is illustrated by considering the following non-affine nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_2 + x_1^2 x_2 + (2x_1x_2 + u)^3 \\ \dot{x}_3 = x_1 - 0.8x_3 \\ y = x_1 + x_3 \end{cases}$$

The relative degree of the system with output y *is* $r = 2$. In fact, the zero dynamic of the system is $\dot{x}_3 = -0.8x_3$ which is asymptotically stable. Therefore, in practice it is assumed that the system is modelled as a second order nonlinear plant model, whose realization consists of states x_1 and x_2 (state x_3 is omitted) the output is modelled as $y = x_1$. Hence, the system without unmodelled dynamic can be presented as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_2 + x_1^2 x_2 + (2x_1x_2 + u)^3 \\ y = x_1 \end{cases}$$

The second order compensator

$$\frac{N_L}{D_L} = \frac{18s^2 + 16s + 12}{s(s+7)}$$

is selected to stabilize the linear second order system $\ddot{y} = u_L$. Now, based on the assumptions on N_{ad} and D_{ad} in Section 2, the following filtered error signal $\tilde{e}$

$$\frac{N_{ad}}{D_{ad}} = 50\frac{s^2 + 6s + 6}{(s+10)(s+20)}$$

is considered. It is desired that the above filter has high bandwidths. Finally, $T(s) = 0.5s + 1$ is selected based on SPR property of $\bar{G}$.

The MLP has 20 neurons in hidden layer with tangent hyperbolic activation functions. The weights are initialised randomly with small numbers. The input to the NN for $n_1 = 4 \geq n$ is

$$\xi = [1, y(t), y(t-T_d), y(t-2T_d), y(t-3T_d), u_\alpha(t), u_\alpha(t-T_d), u_{ad}(t-T_d)]^T$$

with $T_d = 0.01\text{sec.}$. Also, the training constants are selected as $\gamma_W = \gamma_V = 0.8, \gamma_\varphi = 0.9$. And finally, to avoid chattering, $\tanh(\tilde{e}/0.4)$ is used instead of $\operatorname{sgn}(\tilde{e})$.

Simulation results have been depicted in Figures 2-7. First, the controlled system performance is evaluated without the unmodelled mode dynamics. Figure 2, compares the system response with three different control laws. First, simulations have been performed without the adaptive part u_{ad} and without the robustifying part u_R. Then, only the adaptive part has been added. And finally, both the adaptive part and the robustifying part have been used.

Figure 2, clearly demonstrates that the system response is very oscillatory and almost unstable. Then, for the second case, by using the adaptive part, these oscillations are eliminated (Figure 2). Moreover, by adding the robustifying part the desired tracking with asymptotic stability is achieved. Figures 3 and 4 show the action of the controllers and the norms of weights, respectively. They also show that the weights remain bounded.

Next, the effect of the unmodelled dynamics is examined and the simulation results are shown in Figure 5. In this case, the response without the adaptive part is unstable, whereas by using the additional control parts, the good tracking is achieved. In addition, these results demonstrate that the proposed control law can compensate the effect of unmodelled dynamics appropriately.

6 Conclusions

In this paper, a direct adaptive output feedback control method has been developed for uncertain non-affine nonlinear systems that do not rely on state estimation. Moreover, it has been shown that the use of an additional robustifying part of the control guarantees the uniform asymptotic stability of the tracking error system. Without this control part, only the uniform ultimately boundedness of the tracking error system is demonstrated. The proposed control algorithm is relatively simple and requires no restrictive conditions on the design constants for the stability. The efficiency of the proposed scheme has been shown using the simulation of a nonlinear system with unmodelled dynamics. The simulation results showed the effectiveness of the proposed control method as compared to linear and linear-adaptive controllers.

References

[1] Isidori, A., Nonlinear control system, 2nd ed. Berlin: Springer-Verlag, 1989.

[2] Esfandiari, F. and K. Khalil, H., Output feedback stabilization of fully linearisable system, *Int. J. Control*, **56**, 1007-1037, 1992.

[3] Krstic, M., Kanellakopoulos, I. and Kokotovic, P., Nonlinear and *a*daptive control design, New York: Wiley, 1995.

[4] Koshkouei A. J., Zinober A. S. I. and Burnham, K., Adaptive sliding mode backstepping control of nonlinear systems with unmatched uncertainty, *Asian Journal of Control*, **6**, 447-457, 2004.

[5] Marino, R. and Tomei, P., Nonlinear adaptive design: geometric, adaptive and robust, London: Prentice-Hall, 1995.

[6] Khalil, H., Adaptive output feedback control of nonlinear system represented by input-output models, *IEEE Trans. Automat. Control,* **41**, 177-188, 1996.

[7] Ge, S. S., Hang, C. and Zhang, T., Nonlinear adaptive control using neural network and its application to CSTR systems, *J. Process Control*, **9**, 313–323, 1998.

[8] Lewis, F., Yesildirek, A. and Liu, K., Multilayer neural-net robot controller with guaranteed tracking performance, *IEEE Trans. Neural Networks*, **7**, 1996.

[9] Lewis, F., Yesildirek, A. and Liu, K., Feedback linearization using neural networks, *Automatica*, **31**, 1659-1664, 1995.

[10] Ge, S.S., Hang, C. C. and Zhang, T., Adaptive neural network control of nonlinear system by state and output feedback, *IEEE, Trans. System Man. Cyber,* **29**, 1999.

[11] Ge, S. S. and Zhang, T., Neural network control of non-affine nonlinear with zero dynamics by state and output feedback, *IEEE, Trans. on Neural Networks*, **14**, 2003.

[12] Kim, B. S. and Calise, A. J., Nonlinear flight control using neural networks", *J. of Guidance, Control and Dynamics*, **20**, 1997.

[13] Hovakimyan , N., Nardi, F. and Calise, A. J., A novel error observer based adaptive output feedback approach for control of uncertain systems, *IEEE Trans. Automat. Control,* **47**, 2002.

[14] Polycarpou, M. U. and Mears, M. J., Stable adaptive tracking of uncertain systems using nonlinearly parameterized on-line approximators, *Int. J. Control*, **70**, 363–384, 1998.

[15] Park, J. H. and Huh, S. H., Direct adaptive controller for non-affine nonlinear system using self-structuring neural networks, *IEEE Trans. Neural Networks,* **16**, 2005.

[16] Narendra, K. S. and Annaswamy, A. M., Stable adaptive system, London: Prentice-Hall, 1989.

[17] Lavertsky, E. Calise, A. J., Hovakimyan, N., Upper bounds for approximation of continuous-time dynamics using delayed outputs and feedforward neural networks. IEEE *Trans. on Automat. Control*, **48**, 1606-1610, 2003.

[18] Hornik, K., Stinchombe, M. and White, H., Multilayer feed-forward networks are universal approximators, *Neural Networks*, **2**, 359-366, 1989.

[19] Koshkouei, A. J. and Zinober, A. S. I., Sliding mode controller-observer design for SISO linear systems, *Int. J. Systems Science*, **29**, 1363-1373, 1998.

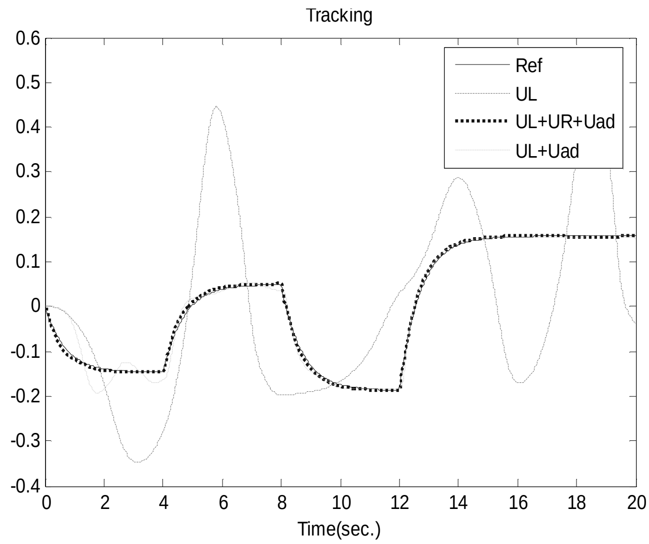

Figure 2: Tracking performance for the system without unmodelled dynamics.

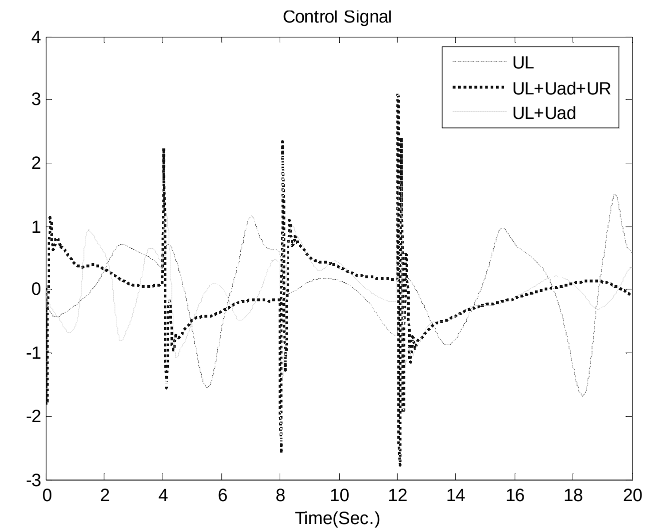

Figure 3: The action of controllers for the system without unmodelled dynamics.

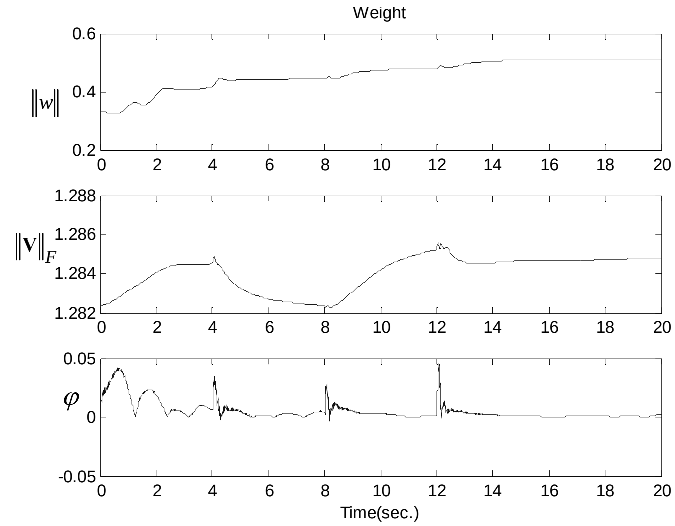

Figure 4: Norm of the weights for the system without unmodelled dynamics.

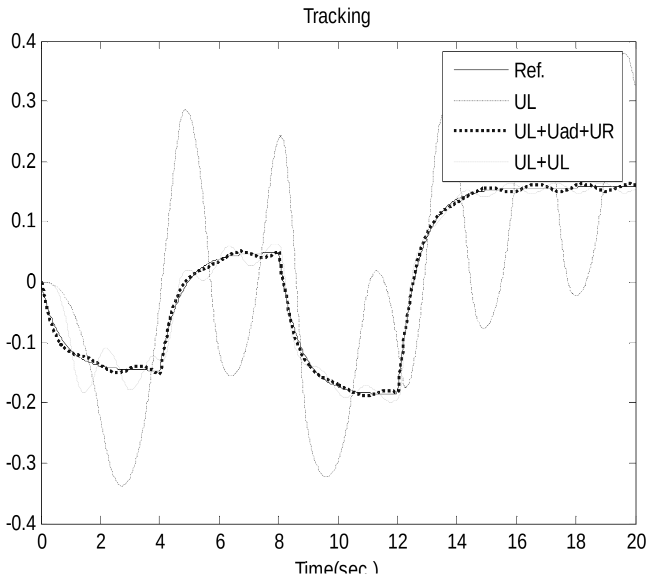

Figure 5: Tracking performance for the system with unmodelled dynamics.

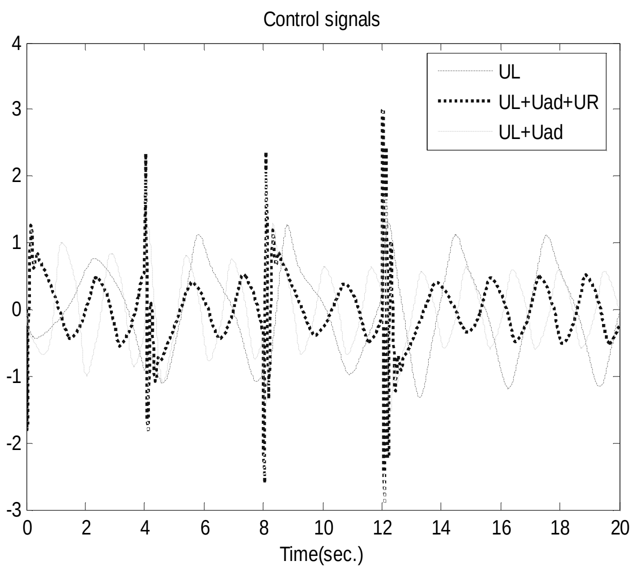

Figure 6: The action of controllers for system with unmodelled dynamics.

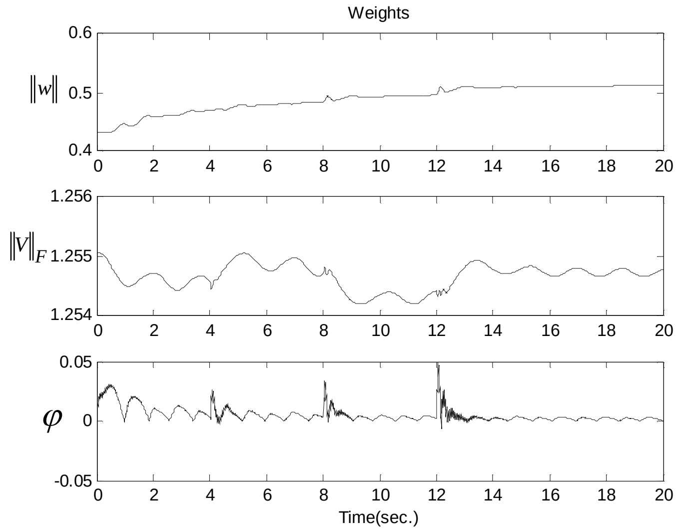

Figure 7: Norm of the weights for the system with unmodelled dynamics.