Target-Based Line-Of-Sight Stabilization in Periscopes

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***Abstract*— This paper presents a novel method for target-based line-of-sight stabilization in periscopes with respect to disturbances in the platform movements. The platform can be a submarine or a tank. The disturbances can cause unstable images taken by the periscope and give altered information from the target, which can be located on the sea surface, on the earth, or in the air. To overcome this problem, a stabilization method is required to generate three reference signals for servomotors to remove the unwanted motion incurred on the image sequences. The stabilization equations, proposed in this paper, map the three-dimensional input space to the three-dimensional output space. The input space consists of the measurable pitch and yaw movements of the platform, and the target distance which can be measured or estimated. The output space consists of three reference paths. Simulation results show good results for the proposed method.**

I. INTRODUCTION

Line-of-sight stabilization has been widely used by many researchers for different applications [1]. This problem can be categorized in three groups [2]. The first category consists of software approaches, which are mainly used to compensate small motions or disturbances in the image sequences. The second approach is the platform stabilization, in which the compensation is performed by moving the camera [3]. The third category, which is called the steering stabilization, uses a mirror to stabilize the image sequences. In this case, by controlling the mirror movements, the reflected image of targets on the camera is stabilized with respect to the plant motions [4].

One of the optical instruments that enable the observer to see his surroundings, while remaining under cover is the periscope. The stabilization procedure for this system usually falls within the third category, mentioned above. The structure of a periscope is like a gyro stabilized mirror systems [5], [6]. A common periscope structure has been depicted in Fig. 1.

Image sequences, taken from the target, are reflected by the mirror to the camera, and observed by the operator. The image sequences taken from the camera have unwanted motions, which are caused by the platform movements, yielding an unstable picture from the still-standing or moving targets. In this paper, it is assumed that θ_1 and θ_2 in Fig. 1 are the motions of the platform that can be measured by gyroscopes, and r in Fig. 2 is the distance from the periscope to the target, which can be measured or estimated.

Periscopes have different shapes, structures and applications [7], [8]. A usual periscope has 3 Degrees Of Freedom (DOF) (Fig. 1). These are θ_3, which is used for rotation compensate of image, in addition to changing the view field by the operator, θ_4 and θ_5 for changing the Line Of Sight (LOS) and stabilizing the image sequences. Since the operator must be able to rotate the periscope to any desired location, there is no limit on θ_3 axis, but the forth and the fifth axis have limits with respect to the periscope structure.

II. FORWARD KINEMATICS

The link coordinate frames of a periscope are shown in Fig. 3, where *d* is the length of periscope. The first three frames are coincided and can be expressed using the basic rotation matrices [9], [10]. The fourth and the fifth coordinate frames are coincided too. The composite transformation matrices from the coordinate zero to the coordinate three can be written as

$$
{}^{0}\mathbf{T}_3 = {}^{0}\mathbf{T}_1\,{}^{1}\mathbf{T}_2\,{}^{2}\mathbf{T}_3 =
\begin{bmatrix} 0 & 0 & 1 & 0 \\ -s_1 & -c_1 & 0 & 0 \\ c_1 & -s_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)
$$

$$
= \begin{bmatrix} c_2c_3 & -c_2s_3 & s_2 & 0 \\ s_1s_2c_3+c_1s_3 & -s_1s_2s_3+c_1c_3 & -s_1c_2 & 0 \\ -c_1s_2c_3+s_1s_3 & c_1s_2s_3+s_1c_3 & c_1c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

where $s_1 \equiv \sin(\theta_1)$, $c_1 \equiv \cos(\theta_1)$, ${}^{0}\mathbf{T}_1$ is the homogeneous transformation matrix from the first coordinate frame to the zero (base) coordinate frame, and so on. Moreover, the transformation matrix from the third coordinate to the fifth coordinate is equal to

$$
{}^{3}\mathbf{T}_5 = {}^{3}\mathbf{T}_4\,{}^{4}\mathbf{T}_5 =
\begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_4 & -c_4 & 0 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c_4c_5 & -c_4s_5 & s_4 & 0 \\ s_5 & c_5 & 0 & 0 \\ -s_4c_5 & s_4s_5 & c_4 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)
$$

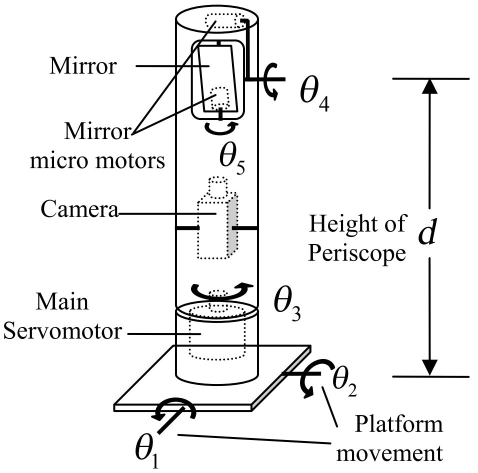

Fig. 1. Structure of a periscope

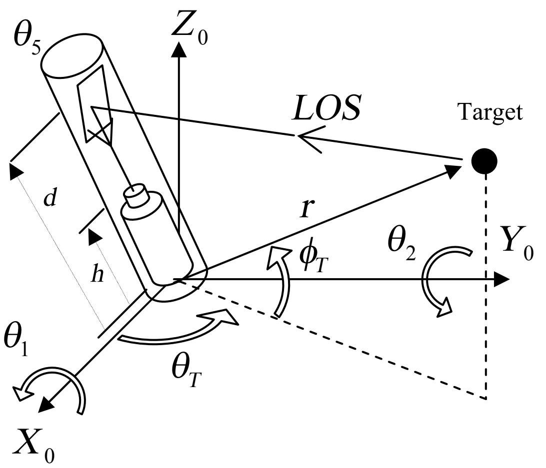

Fig. 2. View plane and the real and the desired LOS.

The forward kinematics is given by

$$ {}^0\mathbf{T}_5 = {}^0\mathbf{T}_3 \, {}^3\mathbf{T}_5 \tag{3}$$

where ${}^0\mathbf{T}_5$ is the transformation matrix from the coordinate five to the reference coordinate.

III. LINE OF SIGHT STABILIZATION

The goal of target-based image stabilization in a periscope is to stabilize the line of sight (LOS) to a standing or moving target (Fig. 2). Hence, it is necessary to derive the equations for the LOS. The operator specifies the target deviations in base coordinate frame by determining θ_T and ϕ_T. The target distance r can be measured by a range finder or estimated by the operator. In case of tracking target, these parameters should be estimated by an appropriate filter [11] and an image processing procedure. The LOS of periscope should focus on the target, regardless of any movements on the X axis (θ_1) and/or Y axis (θ_2).
The target vector in reference coordinate frame is (Fig. 2)

$$ {}^0\mathbf{V}_T = \begin{bmatrix} r\,\mathrm{c}_\phi \mathrm{c}_\theta & r\,\mathrm{c}_\phi \mathrm{s}_\theta & r\,\mathrm{s}_\phi & 1 \end{bmatrix}^T . \tag{4}$$

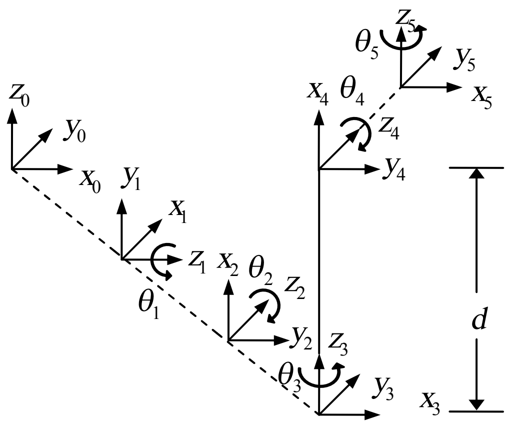

Fig. 3. Link coordinate frames of the periscope.

where $\mathrm{c}_\varphi = \cos(\varphi_T)$, $\mathrm{c}_\theta = \cos(\theta_T)$ and so on. The coordinate of camera in reference frame is

$$\begin{aligned} {}^0\mathbf{V}_C &= {}^0\mathbf{T}_3 \cdot \begin{bmatrix} 0 & 0 & h & 1 \end{bmatrix}^T \\ &= \begin{bmatrix} h\mathrm{s}_2 & -h\mathrm{s}_1\mathrm{c}_2 & h\mathrm{c}_1\mathrm{c}_2 & 1 \end{bmatrix}^T , \end{aligned} \tag{5}$$

where h is the position of camera (Fig. 2). This vector should be transform to the fifth coordinate frame, which is the end coordinate

$$ {}^5\mathbf{V}_C = {}^0\mathbf{T}_5^T . {}^0\mathbf{V}_C = \begin{bmatrix} w\mathrm{c}_5\mathrm{s}_4 & w\mathrm{s}_5\mathrm{s}_4 & w\mathrm{c}_4 & 1 \end{bmatrix}^T , \tag{6}$$

where $w = (d-h)$ and ${}^5\mathbf{V}_C$ is the camera optical axis in the fifth coordinate frame.

The actual LOS can be found by calculating the camera optical axis reflectance on the mirror. The perpendicular line to the mirror is vector $\mathbf{x}_5$ (Fig. 3), and every vector reflected from the mirror should have positive x. According to (6), ${}^5\mathbf{V}_C$ has positive x with respect to the following constraints:

$$\begin{cases} 0^\circ < \theta_4 < 90^\circ \\ -90^\circ < \theta_5 < 90^\circ \\ \mathrm{w} > 0 \end{cases} \tag{7}$$

Constraints (7) yield $wc_5s_4 > 0$, which gives positive values for x.

Finding the reflecting vector on the mirror with respect to the Cartesian coordinates is straight forward. Suppose the mirror is on the *y-z* plane and the normal vector to this plane is *x*. Reflection of a vector from the *y-z* plane can be calculated by inversing the *y* and the *z* elements of that vector. So, the LOS can be represented as follows (Fig. 5):

$$ {}^5LOS = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . {}^5\mathbf{V}_C = \begin{bmatrix} w\mathrm{c}_5\mathrm{s}_4 \\ -w\mathrm{s}_5\mathrm{s}_4 \\ -w\mathrm{c}_4 \\ 1 \end{bmatrix} = \begin{bmatrix} x_L \\ y_L \\ z_L \\ 1 \end{bmatrix} \tag{8}$$

Now, the target vector (4) should be transferred to the fifth coordinate frame.

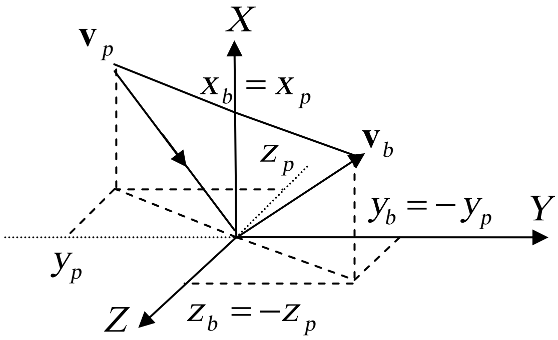

Fig. 4. Finding the reflectance on mirror

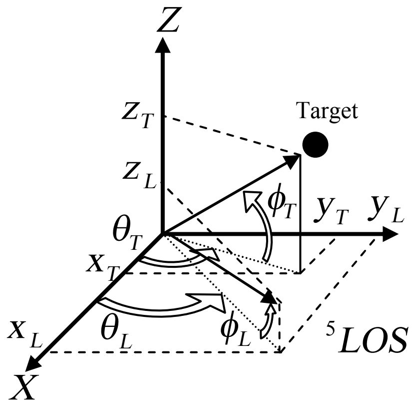

Fig. 5. Desired and real line of sight.

$$
{}^5\mathbf{V}_T = {}^0\mathbf{T}_5^T . {}^0\mathbf{V}_T = \begin{bmatrix} x_T & y_T & z_T & 1 \end{bmatrix}^T =
$$

$$
\begin{bmatrix}
\begin{array}{l}
(\text{-c}_2\text{s}_3\text{s}_5\text{-c}_5\text{s}_2\text{s}_4\text{+c}_5\text{c}_2\text{c}_3\text{c}_4)\text{rc}_\phi\text{c}_\theta\text{+(s}_5\text{c}_1\text{c}_3\text{-s}_5\text{s}_2\text{s}_1\text{s}_3 \\
\text{+c}_2\text{c}_5\text{s}_4\text{s}_1\text{+c}_4\text{c}_5\text{c}_1\text{s}_3\text{+s}_2\text{c}_4\text{c}_5\text{s}_1\text{c}_3)\text{rc}_\phi s_\theta\text{+(s}_5\text{s}_1\text{c}_3 \\
\text{+s}_5\text{s}_2\text{c}_1\text{s}_3\text{-c}_2\text{c}_5\text{s}_4\text{c}_1\text{+c}_4\text{c}_5\text{s}_1\text{s}_3\text{-s}_2\text{c}_4\text{c}_5\text{c}_1\text{c}_3)\text{rs}_\phi\text{+dc}_5\text{s}_4 \\
\\
(\text{-s}_5\text{c}_2\text{c}_3\text{c}_4\text{+s}_5\text{s}_2\text{s}_4\text{-c}_2\text{s}_3\text{c}_5)\text{rc}_\phi\text{c}_\theta\text{+(-s}_5\text{c}_4\text{s}_1\text{s}_2\text{c}_3\text{-s}_5\text{c}_4\text{c}_1\text{s}_3 \\
\text{-s}_5\text{s}_1\text{c}_2\text{s}_4\text{-c}_5\text{s}_1\text{s}_2\text{s}_3\text{+c}_5\text{c}_1\text{c}_3)\text{rc}_\phi s_\theta\text{+(s}_5\text{c}_4\text{c}_1\text{s}_2\text{c}_3\text{-s}_5\text{c}_4\text{s}_1\text{s}_3 \\
\text{+s}_5\text{c}_1\text{c}_2\text{s}_4\text{+c}_5\text{c}_1\text{s}_2\text{s}_3\text{+c}_5\text{s}_1\text{c}_3)\text{rs}_\phi\text{-ds}_5\text{s}_4 \\
\\
(\text{c}_2\text{c}_3\text{s}_4\text{+s}_2\text{c}_4)\text{rc}_\phi\text{c}_\theta\text{+(s}_4\text{s}_1\text{s}_2\text{c}_3\text{+s}_4\text{c}_1\text{s}_3\text{-s}_1\text{c}_2\text{c}_4)\text{rc}_\phi s_\theta \\
\text{+(-s}_4\text{c}_1\text{s}_2\text{c}_3\text{+s}_4\text{s}_1\text{s}_3\text{+c}_1\text{c}_2\text{c}_4)\text{rs}_\phi\text{-dc}_4 \\
\\
\qquad\qquad\qquad 1
\end{array}
\end{bmatrix} \tag{9}
$$

According to Fig. 5, to align ^5LOS on $^5\mathbf{V}_T$, the following conditions must be met:

$$
\begin{cases}
\theta_L = \tan^{-1}\left(\dfrac{y_L}{x_L}\right) = \theta_T \\
\phi_L = \tan^{-1}\left(\dfrac{z_L}{\sqrt{x_L + y_L}}\right) = \phi_T
\end{cases} \tag{10}
$$

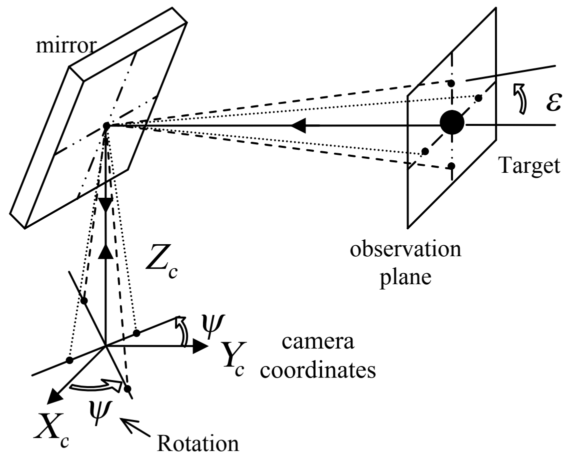

Fig. 6. Image rotation with respect to the camera coordinates.

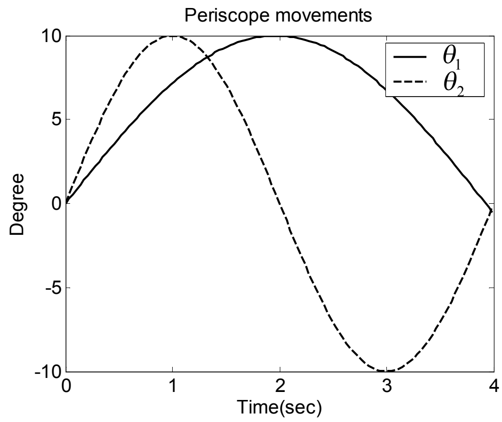

Fig. 7. θ_1 and θ_2

Another important issue that may not be obvious in the first place, is the image rotation. That is, when ^5LOS is aligned with $^5\mathbf{V}_T$, the image might have been rotated with respect to the camera axis. Fig. 6 shows how the rotation can be calculated. To do this, consider a vector, which is deviated by ε on the *x-y* plane from $^0\mathbf{V}_T$

$$
{}^0\mathbf{V} = \begin{bmatrix} r\,\text{c}_\beta\text{c}_\theta & \text{rc}_\beta\text{s}_\theta & \text{rs}_\beta & 1 \end{bmatrix}^T, \tag{11}
$$

where $c_\beta = \cos(\varphi_T + \varepsilon)$ and $s_\beta = \sin(\varphi_T + \varepsilon)$. Transforming this vector to the fifth coordinates yields

$$
{}^5\mathbf{V} = {}^0\mathbf{T}_5^T . {}^0\mathbf{V}. \tag{12}
$$

And its reflectance on the mirror is

$$
{}^5\mathbf{V}_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . {}^5\mathbf{V} = \begin{bmatrix} x_R \\ y_R \\ z_R \\ 1 \end{bmatrix} \tag{13}
$$

And finally, ${}^{5}\mathbf{V}_R$ should be transformed to the third coordinate frames in order to calculate the amount of rotation

$$ {}^{3}\mathbf{V}_R = {}^{3}\mathbf{T}_5 . {}^{5}\mathbf{V}_R = \begin{bmatrix} x_R & y_R & z_R & 1 \end{bmatrix}^T \tag{14}$$

To remove the rotation ψ from the image, y_R should be zero according to Fig. 6. Hence, the equations for image stabilization are

$$\left| \begin{array}{l}
-s_4 w\,(2dc_5 s_5 s_4 + 2rc_\varphi c_\theta c_2 s_3 c_5^2 + 2rc_5 c_\varphi s_\theta s_5 s_1 c_2 s_4 \\
-2rc_5 s_\varphi s_5 c_4 c_1 s_2 c_3 + 2rc_5 s_\varphi s_5 c_4 s_1 s_3 - 2rc_5 s_\varphi s_5 c_1 c_2 s_4 \\
+ 2rc_\varphi s_\theta c_5^2 s_1 s_2 s_3 - 2rc_\varphi s_\theta c_5^2 c_1 c_3 - 2rs_\varphi c_5^2 c_1 s_2 s_3 \\
- 2rs_\varphi c_5^2 s_1 c_3 + 2rc_5 c_\varphi c_\theta s_5 c_2 c_3 c_4 - 2r c_5 c_\varphi c_\theta s_5 s_2 s_4 \\
\ 2rc_5 c_\varphi s_\theta s_5 c_4 s_1 s_2 c_3 + 2rc_5 c_\varphi s_\theta s_5 c_4 c_1 s_3 - rc_\varphi c_\theta c_2 s_3 \\
+ rc_\varphi s_\theta c_1 c_3 - rc_\varphi s_\theta s_2 s_1 s_3 + r s_\varphi s_1 c_3 + r s_\varphi s_2 c_1 s_3) = 0 \\
\\
c_4^2 w^2\,(((-c_2 s_3 s_5 - c_5 s_2 s_4 + c_5 c_3 c_2 c_4)\, rc_\varphi c_\theta + (s_5 c_1 c_3 \\
- s_5 s_2 s_1 s_3 + c_2 c_5 s_4 s_1 + s_3 c_5 c_1 c_4 + s_2 c_4 c_5 s_1 c_3)\, r c_\varphi s_\theta \\
+ (s_5 s_1 c_3 + s_5 s_2 c_1 s_3 - c_2 c_5 s_4 c_1 + c_4 c_5 s_1 s_3 - s_2 c_4 c_5 c_1 c_3) rs_\varphi \\
+ d c_5 s_4)^2 + ((-s_5 c_2 c_3 c_4 + s_5 s_2 s_4 - c_2 s_3 c_5) rc_\varphi c_\theta \\
+ (-s_5 c_4 s_1 s_2 c_3 - s_5 c_4 c_1 s_3 - s_5 s_1 c_2 s_4 - c_5 s_1 s_2 s_3 \\
+ c_5 c_1 c_3)\, r c_\varphi s_\theta + (s_5 c_4 c_1 s_2 c_3 - s_5 c_4 s_1 s_3 + s_5 c_1 c_2 s_4 \\
+ c_5 c_1 s_2 s_3 + c_5 s_1 c_3) rs_\varphi - ds_5 s_4)^2) - ((c_2 c_3 s_4 + s_2 c_4) rc_\varphi c_\theta \\
+ (s_4 s_1 s_2 c_3 + s_4 c_1 s_3 - s_1 c_2 c_4)\, r c_\varphi s_\theta + (-s_4 c_1 s_2 c_3 + s_4 s_1 s_3 \\
\qquad\qquad + c_1 c_2 c_4) rs_\varphi - dc_4)^2 (c_5^2 s_4^2 w^2 + s_5^2 s_4^2 w^2) = 0
\end{array} \right.$$

$$\left| \begin{array}{l}
-r c_\beta c_\theta c_2 s_3 + 2rc_\beta c_\theta c_2 s_3 c_5^2 + rc_\beta s_\theta c_1 c_3 - 2rc_\beta s_\theta c_5^2 c_1 c_3 \\
- r c_\beta s_\theta s_2 s_1 s_3 + 2rc_\beta s_\theta c_5^2 s_1 s_2 s_3 + rs_\beta s_1 c_3 - 2r s_\beta c_5^2 s_1 c_3 \\
+ r s_\beta s_2 c_1 s_3 - 2rs_\beta c_5^2 c_1 s_2 s_3 - 2 r s_5 c_\beta c_\theta s_4 s_2 c_5 \\
-2rs_5 s_\beta c_2 c_5 s_4 c_1 + 2rs_5 c_\beta c_\theta c_5 c_2 c_3 c_4 + 2rs_5 c_\beta c_\theta c_2 c_5 s_4 s_1 \\
+ 2rs_5 c_\beta s_\theta c_5 c_4 c_1 s_3 + 2rs_5 c_\beta s_\theta s_2 c_4 c_5 s_1 c_3 + 2dc_5 s_5 s_4 \\
\qquad\qquad + 2rs_5 s_\beta c_4 c_5 s_1 s_3 - 2rs_5 s_\beta s_2 c_4 c_5 c_1 c_3 = 0
\end{array} \right.$$

$$\tag{15}$$

IV. Simulation Results

Equation (8) represents the actual LOS of periscope in the fifth coordinate system. The LOS in the reference coordinate frame is

$$ {}^{0}LOS = {}^{0}\mathbf{T}_5 . {}^{5}LOS \tag{16}$$

This equation is used for simulations. Two sinusoidal disturbances, as in Fig. 7, are used to simulate movements of the platform (angles θ_1 and θ_2). These disturbances cause unwanted motions in the image sequences. In order to solve the image stabilization problem, three nonlinear equations, presented in (15), must be solved numerically. In this paper, the Newton method has been employed for solving these equations [12], [13]. Since equations (15) must be solved simultaneously at every step of simulations, and there is need for initial values in the Newton method, the solutions at every step are used as the initial guess for the next step. Figs. 8 and 9 show θ_T, ϕ_T and the target trajectory, respectively.

Fig. 10 shows the three-reference path for three servomotors in periscope. Figs. 11 shows head of the periscope movements in Cartesian coordinate frames (i.e. q_1 and q_2). Figs. 12 and 13 show real target trajectory and the ${}^{0}LOS$ track in no compensated case and in the compensated case, respectively. As the simulation results show, the proposed method has stabilized the image sequences to an acceptable level. Noting that for the case of no compensation, the target will be lost especially when it is close to the periscope.

V. Conclusion

In this paper presents a new method for target-based line-of-sight stabilization in periscopes with respect to disturbances in the platform movements was presented. The stabilization method generates three reference signals for servomotors to remove the unwanted motion incurred on the image sequences due to the platform movements. The platform can be a submarine, a tank, or similar vehicles. The stabilization equations map the three-dimensional input space to the three-dimensional output space. The input space consists of the measurable pitch and yaw movements of the platform, and the target distance which can be measured or estimated. The output space consists of three reference paths. Simulations were performed on a target with movements and changing distance from the periscope. The results were satisfactory in tracking the moving target and at the same time rejecting the platform disturbances, yielding an stable image to the operator.

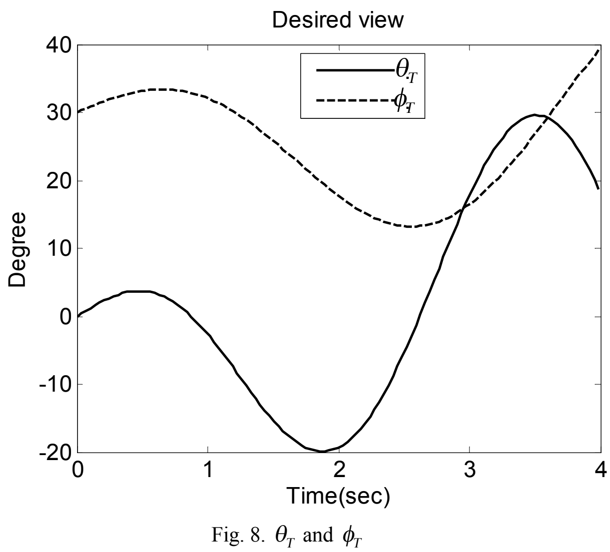

Fig. 8. θ_T and ϕ_T

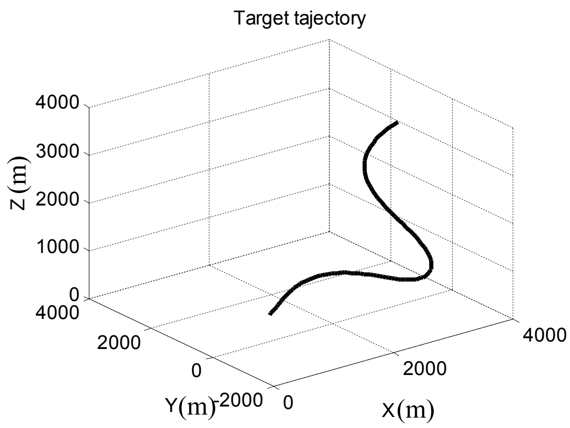

Fig. 9. Target trajectory

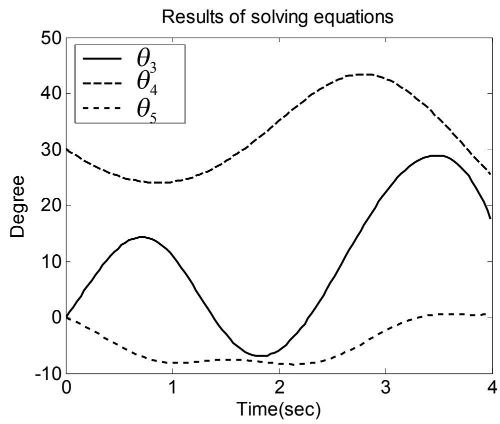

Fig. 10. Three-reference path for three servomotors

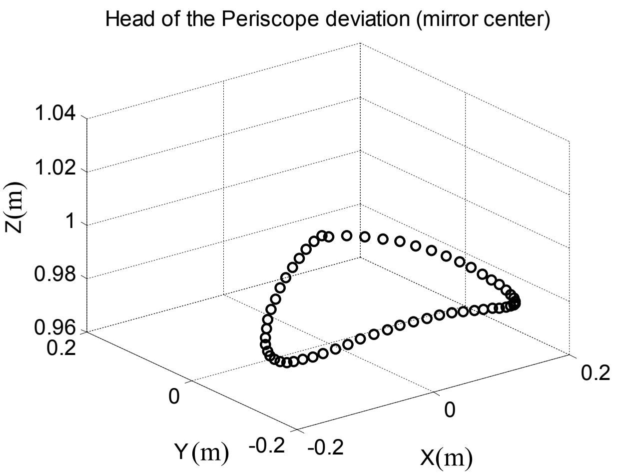

Fig. 11. Head of the periscope movements along θ_1 and θ_2

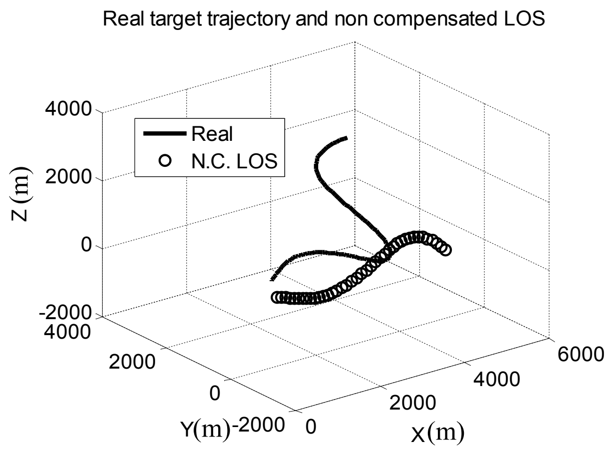

Fig. 12. Real target trajectory and ^{0}LOS track in no compensated case

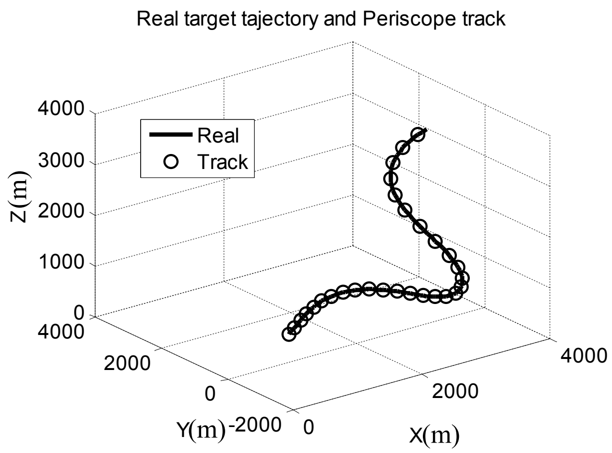

Fig. 13. . Real target trajectory and ^{0}LOS track in compensated case

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