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ADAPTIVE OUTPUT FEEDBACK CONTROL OF NONLINEAR NON-MINIMUM PHASE SYSTEMS USING NEURAL NETWORKS

This paper presents an adaptive output feedback control method for nonlinear non-minimum phase systems. The control is designed based on neural networks and robust control techniques. The inputs to the neural network are the tapped delayed values of the system input-output signals. The adaptation law for the neural network weights is obtained using the Lyapunov's direct method. In addition, since the controller depends on the output signal, the controller is always accessible and the state estimation is not required. The effectiveness of the proposed scheme will be shown through simulations for a nonlinear non-minimum phase system.

1. INTRODUCTION

Output feedback control of nonlinear systems is a challenging problem in control theory. This problem has been an active research area for many years. Several researchers have recently proposed fundamental methods in this area. These methods include using geometric techniques [7]; adaptive observers and output feedback controllers for system in output feedback form[13]; high gain observers [10]; backstepping algorithms for systems with parametric uncertainty [11]; and combining backstepping with small gain theorem [8]. The aim of all these research efforts is to develop systematic design methods for controlling systems in the presence of structured uncertainties in the form of parameters variations and unstructured uncertainties such as unmodeled dynamics. Recently, some results for output feedback control based on Neural Networks (NNs) have been presented, which can be applied to a wide class of systems with structured and unstructured uncertainties. Remarkably, these results include, methods for uncertain systems based on high gain observer [1, 2, 15], and using adaptive error observer [5].

In these methods, it is assumed that the zero dynamics of the system is globally asymptotically stable or input-to-state stable (ISS). Isidori has presented a solution for robust semi-global stabilization based on auxiliary constructions and with relaxed conditions [6]. Karagiannis *et al.* have proposed a method for global output feedback stabilization by designing an observer and using classical backstepping and small-gain techniques [9]. In this method, they have considered the stabilization problem for systems where nonlinearities depend only on the output. A neuro-adaptive output feedback control for non-minimum phase nonlinear systems using a high order error observer is proposed in [4].

An adaptive control method based on output feedback for minimum phase non-affine nonlinear systems has been proposed in [3]. The main advantages of this method are the semi-global asymptotic stability of the closed-loop system and its robustness to model uncertainties

This paper presents an adaptive output feedback control method for stabilization of observable and stabilizable nonlinear non-minimum phase systems. In this method, the linear model of a system is

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first derived. This model represents the non-minimum phase zeros of the nonlinear system with desired accuracy. In fact, there is a conic sector bound on the modelling error of non-minimum phase zeros. Then a linear controller is designed to satisfy the performance requirements in the absence of the modelling errors. Next, the linear controller is augmented with a neuro-adaptive element and a robustifying term, which are used to cancel out the modelling uncertainties. The NN operates over a tapped delay line of memory units, comprised of the system input/output signals. Moreover, the adaptation law for the NN weights depends only on the output tracking error with the aid of strictly positive realness (SPR) of the augmented tracking error dynamic.

This paper is organized as follows: Section 2 describes the class of nonlinear systems to be controlled and defines the problem of tracking error. The controller design procedure and approximation properties of the NN are addressed in section 3. In Section 4, the stability of the closed-loop system is analytically proved. An example, which illustrates the effectiveness of the proposed controller, is presented in Section 5. Conclusions are given in section 6.

2. PROBLEM STATEMENT

Consider the nonlinear SISO system

$$\begin{cases} \dot{z}_i = z_{i+1} & 1 \le i \le r-1 \\ \dot{z}_r = f\left(\mathbf{z}, \boldsymbol{\eta}, u\right) \\ \dot{\boldsymbol{\eta}} = \mathbf{v}\left(\mathbf{z}, \boldsymbol{\eta}\right) \\ y = z_1, \end{cases} \tag{1}$$

where r is the relative degree, $\boldsymbol{\eta} \in \Omega_\eta \subset R^{n-r}$ is the state vector associated with the internal dynamics, $\mathbf{z} = \left[z_1, \cdots, z_r\right]^T \in \Omega_z \subset R^r$, Ω_η and Ω_z are the compact sets of operating regions, and $u \in R$ and $y \in R$ are the input and the output of the system, respectively. The mappings $f : R^{n+1} \to R$ and $\mathbf{v} : R^n \to R^{n-r}$ are partially known and Lipschitz continuous functions with initial conditions $f\left(\mathbf{0}, \mathbf{0}, 0\right) = 0$ and $\mathbf{v}\left(\mathbf{0}, \mathbf{0}\right) = \mathbf{0}$. Note that the stability assumption on the system zero dynamics is not required.

Assumption 1. Assume that $f_u = \partial f\left(\mathbf{z}, \boldsymbol{\eta}, u\right) / \partial u \ne 0$. This condition implies that the smooth function f_u is strictly either positive or negative on the compact set

$$U = \left\{\left(\mathbf{z}, \boldsymbol{\eta}, u\right) \middle| \mathbf{z} \in \Omega_z,\ \boldsymbol{\eta} \in \Omega_\eta,\ u \in R\right\}.$$

It is also assumed that the system output $y(t)$ is measurable and it is desired the output tracks a reference signal y_d with a bounded error trajectory.

3. CONTROLLER DESIGN

3.1. CONSTRUCTION OF ERROR DYNAMIC

Using Taylor expansion, the system (1) around its equilibrium at the origin can be represented as

$$\begin{cases} \dot{z}_i = z_{i+1} & 1 \le i \le r-1 \\ \dot{z}_r = \mathbf{m}^T \mathbf{z} + \mathbf{n}^T \boldsymbol{\eta} + b\, u_L + b\left(\Delta\left(\mathbf{z}, \boldsymbol{\eta}, u\right) - u_{ad} - u_R\right) \\ \dot{\boldsymbol{\eta}} = \mathbf{F}\boldsymbol{\eta} + \mathbf{G}\mathbf{z} + \boldsymbol{\Delta}_{\boldsymbol{\eta}}\left(\mathbf{z}, \boldsymbol{\eta}\right) \\ y = z_1. \end{cases} \tag{2}$$

where the control signal in (1) is introduced as

$$u = u_L - u_{ad} - u_R \tag{3}$$

in which u_L , u_{ad} and u_R are the linear, the adaptive and the robustifying terms, respectively. The adaptive term is considered to compensate the unknown term $\Delta(\mathbf{z}, \boldsymbol{\eta}, u)$. Moreover, **m**, **n**, **F**, **G**, are coefficient matrices with appropriate dimensions, and $\boldsymbol{\Delta}_{\boldsymbol{\eta}}(\mathbf{z}, \boldsymbol{\eta})$ denotes the zero dynamic modelling error.

Assumption 2. The modelling error of the internal dynamics is bounded with a conic sector bound as

$$\left\|\boldsymbol{\Delta}_{\boldsymbol{\eta}}(\mathbf{z}, \boldsymbol{\eta})\right\| \le c_1 + c_2 \left\|\mathbf{z}\right\| + c_3 \left\|\boldsymbol{\eta}\right\| \tag{4}$$

where c_i (i=1,2,3) are known positive constants.

The linear model of the system (2) is

$$\begin{cases} \dot{z}_i^l = z_{i+1}^l & 1 \le i \le r-1 \\ \dot{z}_r^l = \mathbf{m}^T \mathbf{z}^l + \mathbf{n}^T \boldsymbol{\eta}^l + b u_L \\ \dot{\boldsymbol{\eta}}^l = \mathbf{F}\boldsymbol{\eta}^l + \mathbf{G}\mathbf{z}^l \\ y_l = z_1^l \end{cases} \tag{5}$$

Assumption 3. There is a tracking linear controller for the linear dynamic (5), which satisfies the performance requirements.

This linear controller can be represented in the state-space form as

$$\begin{aligned} \dot{\mathbf{x}}_c &= \mathbf{A}_c \mathbf{x}_c + \mathbf{b}_c (y_d - y) \\ u_L &= \mathbf{c}_c \mathbf{x}_c + d_c (y_d - y). \end{aligned} \tag{6}$$

The linear model (5), when regulated by (6), defines a closed-loop reference model as

$$\begin{aligned} \dot{\mathbf{x}}_l &= \mathbf{A}_{cl} \mathbf{x}_l + \mathbf{b}_{cl} y_d \\ y_l &= \mathbf{c}_{cl} \mathbf{x}_l, \end{aligned} \tag{7}$$

where $\mathbf{x}_l^T = [\mathbf{z}_l^T, \boldsymbol{\eta}_l^T, \mathbf{x}_{cl}^T]$ and

$$\mathbf{A}_{cl} = \begin{bmatrix} \mathbf{M} - \mathbf{b} d_c \mathbf{c} & \mathbf{N} & \mathbf{b}\mathbf{c}_c \\ \mathbf{G} & \mathbf{F} & 0 \\ -\mathbf{b}_c \mathbf{c} & 0 & \mathbf{A}_c \end{bmatrix}, \quad \mathbf{b}_{cl} = \begin{pmatrix} \mathbf{b} d_c \\ 0 \\ \mathbf{b}_c \end{pmatrix}, \quad \mathbf{c}_{cl} = \begin{pmatrix} \mathbf{c} & 0 & 0 \end{pmatrix},$$

$$\mathbf{M} = \left[\begin{array}{cc} \mathbf{0}_{(r-1)\times 1} & \mathbf{I}_{(r-1)\times(r-1)} \\ \hline \multicolumn{2}{c}{\mathbf{m}^T} \end{array} \right], \quad \mathbf{N} = \begin{bmatrix} \mathbf{0}_{r\times(n-r)} \\ \mathbf{n}^T \end{bmatrix}, \quad \mathbf{c} = [1 \quad \mathbf{0}_{1\times(r-1)}], \ \mathbf{b} = \begin{bmatrix} \mathbf{0}_{1\times(r-1)} & b \end{bmatrix}^{\mathrm{T}},$$

$\mathbf{x}_{cl}$ denotes the state vector of controller (6) when applied to linear model (5). In addition, the nonlinear system (2) under regulation of (6) can be written as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}_{cl} \mathbf{x} + \mathbf{b}_{cl} y_d + \mathbf{g}\left(\Delta - u_{ad} - u_R\right) + \mathbf{H}\boldsymbol{\Delta}_{\eta} \\ y &= \mathbf{c}_{cl} \mathbf{x}, \end{aligned} \tag{8}$$

where

$$\mathbf{g} = \begin{bmatrix} \mathbf{b}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{0}_{r\times(n-r)} \\ \mathbf{I}_{(n-r)\times(n-r)} \\ \mathbf{0}_{n_c \times(n-r)} \end{bmatrix}.$$

The objective is to design u_{ad} and u_R such that the output y tracks the reference signal y_d .The error dynamics for the output signal $e = y_l - y$ is derived in this section and in Section 3.2. The ultimately boundedness of the error signal is concluded using the Lyapunov's direct method. This ensures that y tracks y_d with an error bound.

Using (7) and (8) and with the following definition of error vector

$$\mathbf{E} = \mathbf{x}_l - \mathbf{x}, \tag{9}$$

error dynamics can be represented as

$$\begin{aligned} \dot{\mathbf{E}} &= A_{cl}\mathbf{E} + \mathbf{g}\left(u_{ad} - \Delta + u_R\right) - \mathbf{H}\boldsymbol{\Delta}_\eta \\ e &= \mathbf{c}_{cl}\mathbf{E}. \end{aligned} \tag{10}$$

Using (9), the upper bound of the modelling error, defined in (4), can be represented as

$$\left\|\boldsymbol{\Delta}_{\boldsymbol{\eta}}(\mathbf{z},\boldsymbol{\eta})\right\| \le c_1 + \alpha_1\left(\|\mathbf{E}\| + \|\mathbf{x}_l\|\right) \tag{11}$$

Since the closed-loop reference model in (7) is stable, it is always possible to find a positive constant c_4 that satisfies $\|\mathbf{x}_l\| \le c_4$. Then, substituting in (11) yields

$$\left\|\boldsymbol{\Delta}_{\boldsymbol{\eta}}(\mathbf{z},\boldsymbol{\eta})\right\| \le \alpha_0 + \alpha_1\|\mathbf{E}\|, \qquad \alpha_0 = c_1 + \alpha_1 c_4 \tag{12}$$

3.2. CONSTRUCTION OF SPR ERROR DYNAMIC

In this section, a strictly positive realness property of the closed-loop error dynamic is studied. As it will be shown in the next section, in order for the NN adaptation law to be dependent only on the system output, the transfer function of the closed-loop error dynamic should be Strictly Positive Real (SPR). Because of the non-minimum phase properties of the system, the closed-loop error dynamic (10) cannot be SPR [14]. To achieve an SPR error dynamic, an augmented error is introduce as

$$\begin{aligned} \dot{\mathbf{E}} &= \mathbf{A}_{cl}\mathbf{E} + \mathbf{g}\left(u_{ad} - \Delta + u_R\right) - \mathbf{H}\boldsymbol{\Delta}_\eta \\ e_{ag} &= \mathbf{c}_{ag}\mathbf{E}, \end{aligned} \tag{13}$$

where $\mathbf{c}_{ag} = \mathbf{c}_{cl} + \mathbf{c}_a$. The Kalman-Yakobuvich lemma is used to design $\mathbf{c}_a$.

SPR of the augmented error dynamic assures the existence of a matrix $\mathbf{P} = \mathbf{P}^T > 0$, which satisfies

$$\mathbf{A}_{cl}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{cl} = -\mathbf{Q} \tag{14}$$

and

$$\mathbf{c}_{ag}^T = \mathbf{P}\mathbf{g} \tag{15}$$

where $\mathbf{Q} = \mathbf{Q}^T > 0$. Using (14) and (15) and substituting $\mathbf{A}_{cl}$ and $\mathbf{g}$ from (7) and (8) into (14) and (15), $\mathbf{c}_{ag}$ is determined.

Thus, the SPR augmented error can be derived as

$$e_{ag} = e + G_a(s)\left(u_{ad} - \Delta + u_R\right) + \mathbf{H}_a(s)\boldsymbol{\Delta}_{\boldsymbol{\eta}} \tag{16}$$

where

$$G_a(s) = \mathbf{c}_a\left(s\mathbf{I} - \mathbf{A}_{cl}\right)^{-1}\mathbf{g} \tag{17}$$

$$\begin{aligned} \mathbf{H}_a(s) &= \left[\mathbf{0}_{1\times r}, \mathrm{h}_1, \cdots, \mathrm{h}_{n-r}, \mathbf{0}_{1\times n_c}\right] \\ &= -\mathbf{c}_a\left(s\mathbf{I} - \mathbf{A}_{cl}\right)^{-1}\mathbf{H} \end{aligned} \tag{18}$$

Assumption 4. It is assumed that the smallest eigenvalue Q_m of $\mathbf{Q}$ in (14) satisfies the condition $Q_m > 4 + 2\mu\alpha_1\|\mathbf{H}_a\| + 2\alpha_1\|\mathbf{PH}\|$, where $\mu > 0$ is the robustifying control gain and α_1 is a positive number.

3.3.1. NEURAL NETWORK-BASED APPROXIMATION

The adaptive part u_{ad} of the control law is designed to cancel out the modelling error $\Delta(\mathbf{z},\boldsymbol{\eta},u) = \Delta(\mathbf{z},\boldsymbol{\eta},u_L - u_{ad} - u_R)$. Hence, there exists a fixed-point problem as

$$u_{ad}(t) = \Delta(\mathbf{z}, \boldsymbol{\eta}, u_L - u_{ad} - u_R) \tag{19}$$

The following assumption provides conditions that guarantee the existence and uniqueness of a solution for u_{ad}.

Assumption 5. The map $u_{ad} \to \Delta$ is a contraction over the entire input domain. This means, the following inequality should be satisfied

$$\left| \frac{\partial \Delta}{\partial u_{ad}} \right| < 1 \tag{20}$$

Substituting (1), (2) and (3) into (20), yields

$$\begin{aligned} \left| \frac{\partial \Delta}{\partial u_{ad}} \right| &= \left| \frac{1}{b} \frac{\partial \left(f\,(\mathbf{z}, \boldsymbol{\eta}, u) - \mathbf{m}^T \mathbf{z} - \mathbf{n}^T \boldsymbol{\eta} - b\,u \right)}{\partial u} \frac{\partial u}{\partial u_{ad}} \right| \\ &= \left| 1 - \frac{1}{b} \frac{\partial f}{\partial u} \right| < 1 \end{aligned} \tag{21}$$

Condition (21) is equivalent to the following two conditions

$$\begin{aligned} &\operatorname{sgn}(b) = \operatorname{sgn}(\partial f / \partial u) \\ &|b| > |\partial f / \partial u| / 2 > 0 \end{aligned} \tag{22}$$

In the following lemma, it is shown that the modelling error $\Delta(\mathbf{z}, \boldsymbol{\eta}, u)$ can be approximated on compact set by a linear parameterized neural network, based on input-output data. Moreover, it is proved that if any non-affine system satisfies conditions (22), then it is unnecessary to use $u_{ad}(t)$ as an input signal to the NN. Therefore, it is possible to employ the static NN rather than the recurrent NN to approximate $u_{ad}(t)$.

Lemma: If conditions (22) are satisfied, then, modelling error $\Delta(\mathbf{z}, \boldsymbol{\eta}, u)$ can be approximated by a Radial Bases Function Network (RBFN) as $\mathbf{w}^T \boldsymbol{\Phi}(\boldsymbol{\zeta})$, where $\mathbf{w} \in R^{m+1}$ is a vector containing synaptic weights and $\boldsymbol{\Phi}(\cdot) \in R^m$ is a vector of nonlinear functions as $\phi_i(\boldsymbol{\zeta}) = \exp\left(-\|\boldsymbol{\zeta} - \boldsymbol{\zeta}_{ci}\|^2 / \sigma_i^2\right)$, $|\phi_i(\boldsymbol{\zeta})| \le 1$, and $\boldsymbol{\zeta} \in R^N$ is the input vector, which is equal to $\boldsymbol{\zeta} = \left[1 \quad \bar{\mathbf{y}} \quad \bar{\mathbf{u}}_\alpha \quad \bar{\mathbf{u}}_{ad}\right]^T$, where

$$\begin{aligned} &\bar{\mathbf{y}} = \left[y(t) \quad \cdots \quad y(t - T_d(n_1 - 1)) \right], \quad \bar{\mathbf{u}}_\alpha = \left[u_\alpha(t) \quad \cdots \quad u_\alpha(t - T_d(n_1 - r - 1)) \right] \\ &\bar{\mathbf{u}}_{ad} = \left[u_{ad}(t - T_d) \quad \cdots \quad u_{ad}(t - T_d(n_1 - r - 1)) \right] \end{aligned} \tag{23}$$

in which $u_\alpha = u + u_{ad} = u_L - u_R$ and T_d is the sampling time.

Proof: Under the observability condition of the system (1), it has been shown in [12] that the continuous-time dynamic $\Delta(\mathbf{z}, \boldsymbol{\eta}, u)$ can be approximated using the delayed inputs and outputs as

$$\Delta(\mathbf{z}, \boldsymbol{\eta}, u) = \mathrm{F}(\bar{\mathbf{y}}, \bar{\mathbf{u}}) + \varepsilon_1, \tag{24}$$

where

$$\bar{\mathbf{u}} = \left[u(t) \quad u(t - T_d) \quad \cdots \quad u(t - T_d(n_1 - r - 1)) \right], \quad n_1 \ge n$$

and $|\varepsilon_1| \le \varepsilon_{1M}$, in which ε_{1M} is directly proportional to the sampling time interval T_d. Hence, ε_1 can be ignored by selecting T_d sufficiently small. Moreover, assumption 5 guarantees the existence and uniqueness of a solution for u_{ad} satisfying the following equation

$$M(\mathbf{z}, \boldsymbol{\eta}, u, u_{ad}) = \Delta(\mathbf{z}, \boldsymbol{\eta}, u) - u_{ad}(t) = 0 \tag{25}$$

Differentiating M with respect to u_{ad}, similar to (21), yields

$$\frac{\partial}{\partial u_{ad}} M(\mathbf{z},\boldsymbol{\eta},u,u_{ad}) = \frac{\partial}{\partial u_{ad}}\left(\Delta(\mathbf{z},\boldsymbol{\eta},u) - u_{ad}\right) = \frac{\partial \Delta}{\partial u_{ad}} - 1 = -\frac{1}{b}\frac{\partial f}{\partial u} \tag{26}$$

which is nonzero by Assumption 1. Substituting (24) into (25) implies

$$\Delta(\mathbf{z},\boldsymbol{\eta},u) - u_{ad}(t) = F_1\left(\bar{\mathbf{y}},\bar{\mathbf{u}}_{\alpha} + \bar{\mathbf{u}}_{ad}, u_{ad}(t)\right) - u_{ad}(t) = 0 \tag{27}$$

where $F_1\left(\bar{\mathbf{y}},\bar{\mathbf{u}}_{\alpha} + \bar{\mathbf{u}}_{ad}, u_{ad}(t)\right) = F\left(\bar{\mathbf{y}},\bar{\mathbf{u}}\right)$.

Thus, from (26) and according to the implicit function theorem, there exists a unique solution for u_{ad} as

$$u_{\mathrm{ad}}(t) = \Gamma(\bar{\mathbf{y}},\bar{\mathbf{u}}_{\alpha},\bar{\mathbf{u}}_{ad}) \tag{28}$$

From (28) and (27)

$$\Delta(\mathbf{z},\boldsymbol{\eta},u) = \Gamma(\boldsymbol{\zeta}) \tag{29}$$

On the other hand, any sufficiently smooth function can be approximated on a compact set with an arbitrarily bounded error by a suitable large RBFN. Therefore, on the compact set $\Omega \subset U$, there exists a set of ideal weights $\mathbf{w}^*$ such that

$$\Delta(\mathbf{z},\boldsymbol{\eta},u) = \mathbf{w}^{*T}\boldsymbol{\Phi}(\boldsymbol{\zeta}) + \delta \tag{30}$$

where and $|\delta| \le \varepsilon_M$, in which ε_M depends on the network architecture. The ideal constant weights $\mathbf{w}^*$ is defined as

$$\mathbf{w}^* := \arg \min_{\mathbf{w}\in\Omega_w}\left\{\sup_{\zeta\in\Omega_\zeta}\left|\mathbf{w}^T\boldsymbol{\Phi}(\zeta) - \Gamma(\zeta)\right|\right\} \tag{31}$$

where $\Omega_w = \left\{w \,\middle|\, \|w\| \le M_w\right\}$, in which M_w is a positive constant. However, in practice, the weights of neural network may be different from the ideal weights. Hence, an approximation error occurs, which satisfies the following equality:

$$u_{ad} - \Delta(\mathbf{z},\boldsymbol{\eta},u) = \mathbf{w}^T\boldsymbol{\Phi}(\boldsymbol{\zeta}) - \mathbf{w}^{*T}\boldsymbol{\Phi}(\boldsymbol{\zeta}) - \delta = \tilde{\mathbf{w}}\boldsymbol{\Phi}(\boldsymbol{\zeta}) + \varepsilon \tag{32}$$

where

$$|\varepsilon| \le \varepsilon_M, \quad \tilde{\mathbf{w}} = \mathbf{w} - \mathbf{w}^* \tag{33}$$

□

4. STABILITY ANALYSIS

In this section, the ultimately bounded stability of the error dynamics is shown based on the results of Section 3. First, the augmented error dynamics (13) is used for the stability proof. Then, it is replaced with the actual error dynamics (16). The control input u_R is designed to compensate the effect of the difference between the augmented error and the actual error. Finally, the uniformly ultimately boundedness of the actual error is proved.

Theorem: Consider the control u_R as

$$u_R = -\mu e, \tag{34}$$

where, $\mu = \lambda / \|G_a\|_\infty$ with $0 < \lambda < 1$ and the adaptation law for NN weights as

$$\dot{\mathbf{w}} = -\gamma_w\left(\boldsymbol{\Phi}e + \boldsymbol{\Phi}_f u_R + \boldsymbol{\Phi}u_{Rf} + \kappa\mathbf{w}\right) \tag{35}$$

Then, the errors $\mathbf{E}$ and $\tilde{\mathbf{w}}$ in the closed-loop system are uniformly ultimately bounded.

Proof:

By substituting (32) into (13), the closed-loop augmented error dynamic can be represented as

$$\begin{aligned}\dot{\mathbf{E}} &= \mathbf{A}_{cl}\mathbf{E} + \mathbf{g}\left(\tilde{\mathbf{w}}^T\boldsymbol{\Phi} + \varepsilon + u_R\right) - \mathbf{H}\boldsymbol{\Delta}_\eta \\ e_{ag} &= \mathbf{c}_{ag}\mathbf{E}.\end{aligned} \tag{36}$$

Define the Lyapunov function

$$L=\frac{1}{2}\mathbf{E}^T\mathbf{PE}+\frac{1}{2\gamma_{\mathbf{w}}}\|\tilde{\mathbf{w}}\|^2, \tag{37}$$

where matrix $\mathbf{P}$ is the unique positive-definite symmetric solution of (14). Moreover, assume that $\mathbf{w}^*$ is the ideal constant weight defined in (31). Then, from (33) $\dot{\mathbf{w}}=\dot{\tilde{\mathbf{w}}}$. Using (36), the time-derivative of L becomes

$$\dot{L}=-\frac{1}{2}\mathbf{E}^{\mathrm{T}}\mathbf{QE}+\mathbf{E}^T\mathbf{Pg}\left(\tilde{\mathbf{w}}^T\mathbf{\Phi}+\varepsilon+u_R\right)-\mathbf{E}^T\mathbf{PH}\mathbf{\Delta}_{\boldsymbol{\eta}}+\frac{1}{\gamma_w}\tilde{\mathbf{w}}^T\dot{\mathbf{w}}$$

From (13), (15) and (16)

$$\mathbf{E}^T\mathbf{Pg}=e_{ag}=e+G_a(s)\left(u_{ad}-\Delta+u_R\right)+\mathbf{H}_a(s)\mathbf{\Delta}_{\boldsymbol{\eta}} \tag{38}$$

Now, (32) and (38), yields

$$\dot{L}=-\frac{1}{2}\mathbf{E}^{\mathrm{T}}\mathbf{QE}+\left(e+\tilde{\mathbf{w}}^T\mathbf{\Phi}_f+\delta_w+\varepsilon_f+u_{Rf}+\mathbf{H}_a(s)\mathbf{\Delta}_{\boldsymbol{\eta}}\right)\left(\tilde{\mathbf{w}}^T\mathbf{\Phi}+\varepsilon+u_R\right)-\mathbf{E}^T\mathbf{PH}\mathbf{\Delta}_{\boldsymbol{\eta}}+\frac{1}{\gamma_w}\tilde{\mathbf{w}}^T\dot{\mathbf{w}}$$

where

$$\delta_w=G_a(s)\tilde{\mathbf{w}}^T\mathbf{\Phi}-\tilde{\mathbf{w}}^TG_a(s)\mathbf{\Phi},\quad \varepsilon_f=G_a(s)\varepsilon,\quad \mathbf{\Phi}_f=G_a(s)\mathbf{\Phi},\qquad u_{Rf}=-\mu G_a(s)e$$

for witch the following bound can be defined

$$|\delta_{\mathbf{w}}|\le c_0\|\tilde{\mathbf{w}}\|,\quad \|\mathbf{\Phi}_f\|\le\|G_a\|_\infty,\quad |\varepsilon_f|\le\varepsilon_M\|G_a\|_\infty \tag{39}$$

Then

$$\begin{aligned}\dot{L}\le&-\frac{1}{2}\mathbf{E}^{\mathrm{T}}\mathbf{QE}+\tilde{\mathbf{w}}^T\left(e\mathbf{\Phi}+\mathbf{\Phi}_fu_R+\mathbf{\Phi}u_{Rf}+\kappa\mathbf{w}+\frac{1}{\gamma_w}\dot{\mathbf{w}}\right)+\varepsilon\left(e+\tilde{\mathbf{w}}^T\mathbf{\Phi}_f+\varepsilon_f+\delta_{\mathbf{w}}+u_{Rf}+\mathbf{H}_a(s)\Delta_{\boldsymbol{\eta}}\right)\\&+\tilde{\mathbf{w}}^T\mathbf{\Phi}\left(\tilde{\mathbf{w}}^T\mathbf{\Phi}_f+\varepsilon_f+\delta_{\mathbf{w}}+\mathbf{H}_a(s)\Delta_{\boldsymbol{\eta}}\right)-\kappa\tilde{\mathbf{w}}^T\left(\tilde{\mathbf{w}}+\mathbf{w}^*\right)+u_R\left(e+u_{Rf}+\delta\mathbf{w}+\varepsilon_f+\mathbf{H}_a(s)\Delta_{\boldsymbol{\eta}}\right)-\mathbf{E}^T\mathbf{PH}\mathbf{\Delta}_{\boldsymbol{\eta}}\end{aligned}$$

The adaptation law (35) and upper bounds in (12) and (39) yields

$$\begin{aligned}\dot{L}\le&-\frac{1}{2}Q_m\|\mathbf{E}\|^2+\varepsilon_M\left[|e|+\|G_a\|_\infty\|\tilde{\mathbf{w}}\|+\|G_a\|_\infty\varepsilon_M+c_0\|\tilde{\mathbf{w}}\|+\mu\|G_a\|_\infty|e|+\|\mathbf{H}_a\|_\infty\left(\alpha_0+\alpha_1\|\mathbf{E}\|\right)\right]\\&+\|G_a\|_\infty\|\tilde{\mathbf{w}}\|^2-\kappa\|\tilde{\mathbf{w}}\|^2+\kappa M_{\mathbf{w}}\|\tilde{\mathbf{w}}\|+\|\tilde{\mathbf{w}}\|\left(\|G_a\|_\infty\varepsilon_M+c_0\|\tilde{\mathbf{w}}\|+\|\mathbf{H}_a\|_\infty\left(\alpha_0+\alpha_1\|\mathbf{E}\|\right)\right)+\mu^2\|G_a\|_\infty|e|^2-\mu|e|^2\\&+\mu\|G_a\|_\infty\varepsilon_M|e|+\mu c_0\|\tilde{\mathbf{w}}\||e|+\mu|e|\|\mathbf{H}_a\|_\infty\left(\alpha_0+\alpha_1\|\mathbf{E}\|\right)+\|\mathbf{E}\|\|\mathbf{PH}\|\left(\alpha_0+\alpha_1\|\mathbf{E}\|\right)\end{aligned}$$

From (10), $|e|\le\|\mathbf{E}\|$. Then

$$\begin{aligned}\dot{L}\le&-\left(\frac{1}{2}Q_m-\mu\alpha_1\|\mathbf{H}_a\|_\infty-\alpha_1\|\mathbf{PH}\|\right)\|\mathbf{E}\|^2-\left(\kappa-\|G_a\|_\infty-c_0\right)\|\tilde{\mathbf{w}}\|^2+\left(\alpha_1\|\mathbf{H}_a\|_\infty+\mu c_0\right)\|\tilde{\mathbf{w}}\|\|\mathbf{E}\|\\&+\left(c_0\varepsilon_M+2\varepsilon_M\|G_a\|_\infty+\kappa M_{\mathbf{w}}+\alpha_0\|\mathbf{H}_a\|_\infty\right)\|\tilde{\mathbf{w}}\|+\left(\alpha_0\|\mathbf{PH}\|+\varepsilon_M\alpha_1\|\mathbf{H}_a\|_\infty\right)\|\mathbf{E}\|+\varepsilon_M^2\|G_a\|_\infty\\&+\alpha_0\varepsilon_M\|\mathbf{H}_a\|_\infty-\left(\mu-\mu^2\|G_a\|_\infty\right)|e|^2+\left(2\mu\|G_a\|_\infty\varepsilon_M+\varepsilon_M+\alpha_0\mu\|\mathbf{H}_a\|_\infty\right)|e|\end{aligned} \tag{40}$$

Select $\mu=\lambda/\|G_a\|_\infty$ with $0<\lambda<1$, which ensures that $\left(\mu-\mu^2\|G_a\|_\infty\right)>0$. Hence by substituting μ and completion of square terms in (40), gives

$$\begin{aligned}\dot{L}\le&-\left(\frac{1}{2}Q_m-\mu\alpha_1\|\mathbf{H}_a\|_\infty-\alpha_1\|\mathbf{PH}\|-2\right)\|\mathbf{E}\|^2-\left(\kappa-\|G_a\|_\infty-c_0-\frac{1}{4}\left(\alpha_1\|\mathbf{H}_a\|_\infty+\mu c_0\right)^2-1\right)\|\tilde{\mathbf{w}}\|^2\\&+\frac{1}{4}\left(\alpha_0\|\mathbf{PH}\|+\varepsilon_M\alpha_1\|\mathbf{H}_a\|_\infty\right)^2+\varepsilon_M^2\|G_a\|_\infty+\alpha_0\varepsilon_M\|\mathbf{H}_a\|_\infty+\frac{1}{4}\left(c_0\varepsilon_M+2\varepsilon_M\|G_a\|_\infty+\kappa M_{\mathbf{w}}+\alpha_0\|\mathbf{H}_a\|_\infty\right)^2\\&+\frac{1}{4}\left(\lambda-\lambda^2\right)^{-1}\|G_a\|_\infty\left((2\lambda+1)\varepsilon_M+\alpha_0\mu\|\mathbf{H}_a\|_\infty\right)^2\end{aligned} \tag{41}$$

Let $\kappa>1+\|G_a\|_\infty+c_0+\frac{1}{4}\left(\alpha_1\|\mathbf{H}_a\|_\infty+\mu c_0\right)^2$. Thus, $\dot{L}$ is negative on either of the following conditions:

$$\|\mathbf{E}\|>R_E,\qquad \|\tilde{\mathbf{w}}\|>R_w, \tag{42}$$

where

$$R_E = \sqrt{\frac{K}{\left(\frac{1}{2}Q_m - \mu\alpha_1\|\mathbf{H}_a\|_\infty - \alpha_1\|\mathbf{PH}\| - 2\right)}}, \qquad R_W = \sqrt{\frac{K}{\left(\kappa - \|G_a\|_\infty - c_0 - \frac{1}{4}\left(\alpha_1\|\mathbf{H}_a\|_\infty + \mu c_0\right)^2 - 1\right)}}$$

in which K is

$$K = \frac{1}{4}\left(\alpha_0\|\mathbf{PH}\| + \varepsilon_M\alpha_1\|\mathbf{H}_a\|_\infty\right)^2 + \varepsilon_M^2\|G_a\|_\infty + \alpha_0\varepsilon_M\|\mathbf{H}_a\|_\infty + \frac{1}{4}\left(c_0\varepsilon_M + 2\varepsilon_M\|G_a\|_\infty + \kappa M_{\mathbf{w}} + \alpha_0\|\mathbf{H}_a\|_\infty\right)^2$$
$$+ \frac{1}{4}\left(\lambda - \lambda^2\right)^{-1}\|G_a\|_\infty\left((2\lambda+1)\varepsilon_M + \alpha_0\mu\|\mathbf{H}_a\|_\infty\right)^2$$

To prove the ultimate boundedness, a set of initial conditions for the error variables should be defined and it should be ensured that this set comprises of the origin and it is invariant.

Let Ω be a compact set in which NN can approximate the modelling error and Ω_r be the largest hypersphere in the error space $\boldsymbol{\xi} = [\mathbf{E}, \tilde{\mathbf{w}}]$, defined as

$$\Omega_r = \left\{\boldsymbol{\xi} \,\middle|\, \|\boldsymbol{\xi}\| \le r\right\} \tag{43}$$

such that for every $\boldsymbol{\xi} \in \Omega_r$ there exists $(\mathbf{z}, \boldsymbol{\eta}, u) \in \Omega$. From (42) one can imply that $\dot{L}$ is negative outside the compact set

$$\Omega_\gamma = \left\{\boldsymbol{\xi} \in \Omega_r \,\middle|\, \|\boldsymbol{\xi}\| \le \gamma\right\}$$

where $\gamma = \max(R_E, R_w)$. Consider the Lyapunov function (37), which can be rewritten as $L = \boldsymbol{\xi}^T \mathbf{S} \boldsymbol{\xi}$ where

$$\mathbf{S} = \frac{1}{2}\begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \gamma_w \mathbf{I}_{(m+1)\times(m+1)} \end{bmatrix}$$

Let Γ be the maximum value of the Lyapunov function L on the boundary of Ω_γ, $\Gamma = S_M\gamma^2$, and α be its minimum value on the boundary of $\Omega_r : \alpha = S_m r^2$, where S_M and S_m are maximum and minimum eigenvalues of $\mathbf{S}$, respectively. Define the sets

$$\Omega_\Gamma = \left\{\boldsymbol{\xi} \,\middle|\, L \le \Gamma\right\},\ \Omega_\alpha = \left\{\boldsymbol{\xi} \in \Omega_r \,\middle|\, L \le \alpha\right\}.$$

If $\Gamma < \alpha$ or equivalently

$$r > \sqrt{S_M / S_m}\,\gamma \tag{44}$$

then $\Omega_\Gamma \subset \Omega_\alpha$. This ensures that if the initial error $\boldsymbol{\xi}(0) \in \Omega_\alpha$, then the error trajectory $\boldsymbol{\xi}$ is ultimately bounded with the ultimate bound γ. □

Remark 1. Note that according to (42) the bounds on the errors depends on the NN reconstruction error ε, the unmatched modelling error $\boldsymbol{\Delta}_\eta$ and $\|G_a\|_\infty$. Smaller bound on tracking error can be achieved by selecting a suitable network with sufficient neurons, and also by selecting Q_m sufficiently large, without proportionally increasing **P**, requires the increment of the compensator gain in (6). Moreover, for a fixed Q_m, the increment of the compensator gain yields a smaller $\|G_a\|_\infty$, but unfortunately increasing the compensator gain leads to peaking phenomenon [15]. However, the proof of the theorem is valid as long as Assumptions 1-5 and condition (44) hold.

Remark 2. The stability results are semi-global in the sense that they are local with respect to the NN approximation domain Ω. If the NN globally approximates the modelling error over the entire space R^{n+1} then the global stability is obtained.

Remark 3. When some of the internal dynamics are exponentially stable, they can be ignored in controller design, but their effects should be considered in the stability analysis.

5. SIMULATION EXAMPLE

The performance of the proposed controller is illustrated by considering the following non-minimum phase non-affine system

$$\begin{cases} \dot{z} = -1.8z - \eta_1 + 1.1u - \left(0.1z\,\eta_1 - u\right)^3 \\ \dot{\eta}_1 = \eta_1 + 2z + 0.8\eta_2 - 0.2z^2 \\ \dot{\eta}_2 = -3\eta_2 + 1.5z \\ y = z \end{cases}$$

The zero dynamics of the system are

$$\begin{cases} \dot{\eta}_1 = \eta_1 + 0.8\eta_2 \\ \dot{\eta}_2 = -3\eta_2 \end{cases}$$

where η_1 and η_2 are the unstable and exponentially stable states, respectively. Moreover, it is assumed that the reference signal and its derivates are defined such that the internal dynamic η_2 is input-to-state stable. Hence, in the design procedure, η_2 is neglected. Hence, linear system can be written as

$$\begin{cases} \dot{z} = -2z - \eta_1 + u \\ \dot{\eta}_1 = \eta_1 + 2z \\ y = z \end{cases}$$

and the modelling errors are

$$\Delta\left(z, \boldsymbol{\eta}, u\right) = 0.2z + 0.1u - \left(0.1z\,\eta_1 - u\right)^3, \quad \Delta_{\boldsymbol{\eta}}\left(z, \boldsymbol{\eta}\right) = 0.8\eta_2 - 0.2z^2$$

Note that Assumption 2 is satisfied

$$\frac{\partial \mathrm{f}(z, \eta, u)}{\partial u} = 1.1 + 3(0.1z\,\eta_2 + u)^2 > 0$$

The dynamic compensator (6) is an LQG controller based on the linear model, which is designed as

$$\frac{N_c}{D_c} = -\frac{523s + 529}{s^2 + 318s + 1277}$$

To construct the augmented error, the following filter is designed when $\mathbf{Q} = 10\mathbf{I}$

$$G_a(s) = \frac{97.95s^3 + 31300s^2 + 103500s + 92570}{s^4 + 319.6s^3 + 1073s^2 + 1271s + 529.2}$$

The NN is of RBF type and has five neurons in the hidden layer. The weights are initialised randomly with small numbers. The centres of the Gaussian functions $\boldsymbol{\zeta}_{ci}$ $(i = 1, \ldots, 5)$ are randomly selected over the possible values of vector $\boldsymbol{\zeta}$, and variance is $\sigma_i^2 = 1$ for all neurons. The input vector to the NN for $n_1 = 4 > n$ is

$$\boldsymbol{\xi} = [1, y(t), y(t - T_d), y(t - 2T_d), y(t - 3T_d), u_\alpha(t), u_\alpha(t - T_d), u_{ad}(t - T_d)]^T$$

with $T_d = 10$ msec. Moreover, the learning rate is selected as $\gamma_{\mathbf{w}} = 0.2$ and $\lambda = 0.95$. The reference signal y_d is a square signal, which has been passed through a first order filter with transfer function $0.2/(s + 0.2)$.

Simulation results are presented in Figures 1 and 2. Figure 1 shows the system response when the linear controller is applied. It is clear that the system response is not satisfactory, because of the nonlinear behaviour of the system. By adding the proposed control terms, the response is improved and a relatively suitable tracking performance with bounded error is achieved. Figure 2.a demonstrates the internal dynamics, which are stable in the presence of the modelling error. The control signals are shown in Figure 2.b. The norm of the weight vector is depicted in Figure 2.c, which shows that weights remain bounded. Finally, Figure 2.d demonstrates the approximate cancellation of matched modelling error Δ by u_{ad} in the absence of unmatched modelling error $\boldsymbol{\Delta}_\eta$.

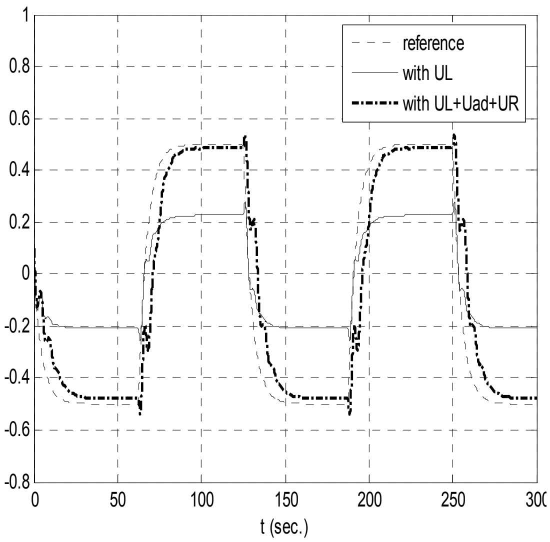

Fig.1. Tracking performance of the linear and the proposed controller.

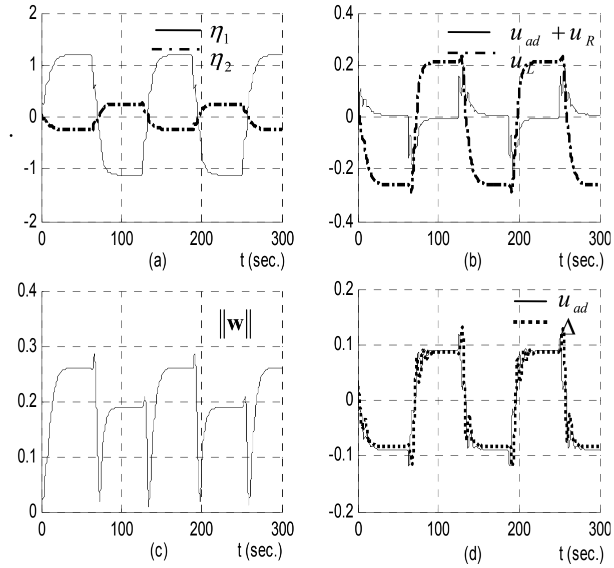

Fig.2. (a) Internal dynamics; (b) control signals; (c) norm of weights; (d) matched modelling error compensation in the absence of unmatched modelling error

6. CONCLUSIONS

A direct adaptive output feedback control method has been proposed for non-minimum phase nonlinear systems. The proposed method introduces an augmented error signal, which is strictly positive real. Since the control depends on the output signal, the state estimation is not required. It has been shown that the tracking error and the NN weights are uniformly ultimately bounded. The control design is based on the definitions of some parameters and functions. However, the method is not very sensitive to these constants and functions.

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