ADAPTIVE STABILIZATION OF NON-MINIMUM PHASE NONLINEAR SYSTEMS USING NEURAL NETWORKS

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Abstract: This paper, presents a direct adaptive control design method for uncertain nonlinear non-minimum phase systems. First, an appropriate reference signal is designed such that the internal dynamic subsystem is input-to-state practical stable. Then an output feedback control, which does not rely on the state estimation, is designed such that the output of system asymptotically tracks this reference signal. This controller is comprised of a dynamic linear controller, an adaptive neural network and a discontinuous robustifying term. Stability of the overall system is guaranteed using the small gain theorem. The effectiveness of the proposed scheme is shown in simulations using a non-minimum phase non-affine system.

Keywords: Nonlinear systems; Adaptive control; Output feedback; Non-minimum phase systems; Neural networks

1. INTRODUCTION

Control system design for complex nonlinear systems has been widely studied in the last decade. Many remarkable results in this area have been reported, including feedback linearization techniques (Isidori, 1989; Esfandiari and Khalil, 1992), backstepping design (Krstic *et al.,* 1995), and systems with unmatched uncertainties (Koshkouei *et al.,* 2004).

For uncertain systems, some researchers (Slotine and Li, 1991; Levant, 2003, 2005) have developed sliding-mode control approaches. In these methods, the controller gains are computed using the upper bound information on the system uncertainties, which is normally unavailable and there is no direct method to obtain them. Therefore, these methods can yield overestimation resulting from a conservative design. To overcome this problem, several adaptive schemes have been developed for affine nonlinear systems in order to deal with the problem of parametric uncertainties (Marino and Tomei, 1995; Khalil, 1996). During the last decade, adaptive methods based on Neural Networks (NNs) have been developed to control uncertain nonlinear systems by removing the unknown nonlinear part of the system. Most of these approaches have been proposed for affine systems (Lewis *et al.,* 1995, 1996) and some of them consider non-affine systems based on the state feedback (Kim and Calise, 1997) or based on the output feedback (Ge *et al.,* 1999; Ge and Zhang, 2003; Hovakimyan *et al.,* 2002). A common assumption in the aforementioned methods is that the zero dynamics are globally asymptotically stable or input-to-state stable (ISS). Isidori (2000) has presented a solution for robust semi-global stabilization of non-minimum phase systems based on auxiliary constructions. Karagiannis *et al.* (2005) have proposed a method for global output feedback stabilization by designing an observer and using classical backstepping and small-gain techniques. In this method, they have considered the stabilization problem for systems where nonlinearities depend only on the output. Chen and Chen (2003) have proposed a state feedback method for stabilization of uncertain non-minimum phase affine systems based on linearization of the internal dynamics and using an enhanced radial-basis-function NN.

In this paper, a direct adaptive control method is developed for non-minimum phase non-affine nonlinear systems. The system dynamic is considered as two subsystems. The internal dynamics, which is called the η-subsystem and other dynamics, referred to as the z-subsystem. First, it will be shown that the η-subsystem is Input-to-State Practical Stable (ISPS) (Jiang, 1999), with input tracking error e, by designing a suitable reference signal y_d which is applied as the input to the η-subsystem. Then, the asymptotic stability of the tracking error dynamic in the z-subsystem is guaranteed by using a combined output feedback control. This controller contains a linear controller, which is augmented with a neuro-adaptive term and an adaptive robustifying term. These adaptive terms are used to cancel out the modeling uncertainties. The NN operates over a tapped delay line of memory units, comprised of the system input/output signals. Moreover, the adaptation law for the NN weights depends only on the output tracking error with the aid of strictly positive realness (SPR) of the tracking error dynamic. In particular, because of approximation errors inherent in NNs, when the number of neurons is limited or initialization of weights are not suitable, most of

these methods can guarantee only uniformly ultimately bounded (UUB) stability. To remove this obstacle and to compensate the reconstruction errors, a method has been widely used in which an extra non-adaptive robustifying input term is considered based on information about boundary conditions of system uncertainties, which may be unavailable. This approach may results in a conservative design (Lewis *et al.*1996, Polycarpou and Mears 1998, Seshagiri and Khalil 2000). To overcome this problem, an adaptive robustifying control term, which guarantees the robustness of the system against approximation error of NN and assures the asymptotic stability of tracking error, is proposed in this paper.
The stability of the interconnection of two subsystems is proved by using small gain theorem.

The rest of this paper is organized as follows: Section 2 describes the class of nonlinear systems to be controlled and the problem of the tracking error. In Section 3, ISPS of internal dynamics, with tracking error as input, is studied. The controller design procedure and approximation properties of the NNs are addressed in Section 4. In Section 5, stability of the closed-loop system is presented. Simulation results are given in Section 6, followed by conclusions in Section 7.

2. PROBLEM STATEMENT

Consider the nonlinear SISO system in normal form

$$\begin{cases} \dot{z}_i = z_{i+1} & 1 \le i \le r-1 \\ \dot{z}_r = b(\mathbf{z}, \boldsymbol{\eta}, u) \\ \dot{\boldsymbol{\eta}} = \mathbf{F}\boldsymbol{\eta} + \mathbf{g} z_1 + \boldsymbol{\Delta}_\eta(z_1, \boldsymbol{\eta}) \\ y = z_1, \end{cases} \tag{1}$$

where r is the relative degree, $\boldsymbol{\eta} \in \Omega_\eta \subset R^{n-r}$ is the state vector associated with the internal dynamics, which can be unstable, i.e. the system may be non-minimum phase. $\mathbf{z} = [z_1, \cdots, z_r]^T \in \Omega_z \subset R^r$, Ω_η and Ω_z are the compact sets of operating regions, and $u \in R$ and $y \in R$ are the input and output of the system, respectively. Moreover, $b: R^{n+1} \to R$ is an uncertain mapping, and $\mathbf{F}$ and $\mathbf{g}$ are such that they represents non-minimum phase zeros of the nonlinear system up to a tolerable accuracy. This means that the modelling error satisfies a conic sector bound as follows.

Assumption 1. *The pair* (F, g) *is stabilizable and the modeling error of internal dynamics* $\boldsymbol{\Delta}_\eta$ *is bounded with a conic sector bound as*

$$\|\boldsymbol{\Delta}_{\boldsymbol{\eta}}(z_1, \boldsymbol{\eta})\| \le c_0 + c_1\|z_1\| + c_2\|\boldsymbol{\eta}\| \tag{2}$$

such that $c_1 < \|\mathbf{g}\|$.

Assumption 2. *Assume that* $b_u = \partial b(\mathbf{z}, \boldsymbol{\eta}, u)/\partial u \ne 0$ *is strictly either positive or negative on the compact set*

$$U = \{(\mathbf{z}, \boldsymbol{\eta}, u) | \mathbf{z} \in \Omega_z,\ \boldsymbol{\eta} \in \Omega_\eta,\ u \in R\}.$$

Defining the error signal $e = y_d - y$, the system (1) can be described as the following subsystems

$$\Sigma_e: \begin{cases} \dot{e}_i = e_{i+1} & 1 \le i \le r-1 \\ \dot{e}_r = y_d^{(r)} - b(\mathbf{z}, \boldsymbol{\eta}, u) \end{cases} \tag{3}$$

$$\Sigma_\eta: \{\dot{\boldsymbol{\eta}} = \mathbf{F}\boldsymbol{\eta} + \mathbf{g}\, y_d - \mathbf{g}e + \boldsymbol{\Delta}_\eta(z_1, \boldsymbol{\eta}) \tag{4}$$

The controller design procedure is considered in two parts. First, $y_d = y_d(\boldsymbol{\eta})$ is designed such that $\boldsymbol{\eta}$-subsystem Σ_η becomes ISPS with input *e*. Then, a combined adaptive output-feedback control law that utilizes the available measurement $y(t)$, will be used to obtain the system output tracking for the trajectory y_d, which is assumed to be *r*-times differentiable.

3. INPUT-TO-STATE STABILITY OF THE $\boldsymbol{\eta}$-SUBSYSTEM

Considering the internal subsystem (4), $y_d(\boldsymbol{\eta})$ is introduced as

$$y_d(\boldsymbol{\eta}) = \mathbf{k}\boldsymbol{\eta} + \mathrm{v}(\boldsymbol{\eta}) \tag{5}$$

Then, the closed loop form of Σ_η is

$$\dot{\boldsymbol{\eta}} = (\mathbf{F} + \mathbf{g}\mathbf{k})\boldsymbol{\eta} - \mathbf{g}e + \mathbf{g}\,\mathrm{v}(\boldsymbol{\eta}) + \boldsymbol{\Delta}_\eta(z_1, \boldsymbol{\eta}) \tag{6}$$

Assumption 1 ensures the existence of gain vector **k**, such that $\mathbf{F} + \mathbf{g}\mathbf{k}$ is Hurwitz. This in turn assures existence of a matrix $\mathbf{P}_1 = \mathbf{P}_1^T > 0$, which satisfies

$$(\mathbf{F} + \mathbf{g}\mathbf{k})^T \mathbf{P}_1 + \mathbf{P}_1(\mathbf{F} + \mathbf{g}\mathbf{k}) = -\mathbf{Q}_1 \tag{7}$$

where $\mathbf{Q}_1 = \mathbf{Q}_1^T > 0$.

Using (5), the upper bound of the modeling error, introduced in (2), can be represented as

$$\|\boldsymbol{\Delta}_{\boldsymbol{\eta}}(z_1, \boldsymbol{\eta})\| \le c_0 + \beta_1\|\boldsymbol{\eta}\| + \beta_2\|\mathbf{g}\,\mathrm{v}\| + c_1|e| \tag{8}$$

where $\beta_1 = c_2 + c_1\|\mathbf{k}\|$ and $\beta_2 = c_1/\|\mathbf{g}\|$.

Theorem 1: *Consider the control* $\mathrm{v}(\boldsymbol{\eta})$ *as*

$$\mathrm{v}(\boldsymbol{\eta}) = -k_c\left(\mathbf{g}^T\mathbf{g}\right)^{-1}\mathbf{g}^T \frac{1}{1-\beta_2}\frac{\mathbf{w}}{\underline{\sigma}(\mathbf{P}_1)} \tag{9}$$

where $k_c > \beta_1$, $\mathbf{w}^T = \boldsymbol{\eta}^T\mathbf{P}_1$ *and* $\underline{\sigma}$ *denotes the minimum singular value. Then, the* η*-subsystem is ISPS with input e.*

Proof:
Let define a Lyapunov function as

$$L_1 = \frac{1}{2}\boldsymbol{\eta}^T\mathbf{P}_1\boldsymbol{\eta} \tag{10}$$

where matrix $\mathbf{P}_1$ is the unique positive-definite symmetric solution of (7). Using (6), (8) and (9) the time-derivative of L_1 becomes

$$\dot{L}_1 = \frac{\partial L_1}{\partial \eta}(\mathbf{F} + \mathbf{g}\mathbf{k})\eta - \frac{\partial L_1}{\partial \eta}\mathbf{g}e + \frac{\partial L_1}{\partial \eta}\left[\mathbf{g}\,\mathrm{v}(\boldsymbol{\eta}) + \boldsymbol{\Delta}_\eta(\boldsymbol{\eta}, y)\right]$$
$$\le -Q_{1m}\|\boldsymbol{\eta}\|^2 - \frac{k_c\|\mathbf{w}\|^2}{\underline{\sigma}(\mathbf{P}_1)} + \beta_1\|\boldsymbol{\eta}\|\|\mathbf{w}\| + c_0\|\mathbf{w}\| + (c_1 + \|\mathbf{g}\|)|e|\|\mathbf{w}\|$$

where Q_{1m} is the minimum eigenvalue of Q_1. Using $\underline{\sigma}(\mathbf{P}_1)\|\boldsymbol{\eta}\| \le \|\mathbf{w}\| \le \bar{\sigma}(\mathbf{P}_1)\|\boldsymbol{\eta}\|$ it gives

$$\begin{aligned} \dot{L}_1 &\le -Q_{1m}\|\boldsymbol{\eta}\|^2 - (k_c - \beta_1)\|\boldsymbol{\eta}\|\|\mathbf{w}\| + c_0\|\mathbf{w}\| + (c_1 + \|\mathbf{g}\|)|e|\|\mathbf{w}\| \\ &\le -\left(Q_{1m} + \underline{\sigma}(\mathbf{P}_1)(k_c - \beta_1)\right)\|\boldsymbol{\eta}\|^2 + c_0\bar{\sigma}(\mathbf{P}_1)\|\boldsymbol{\eta}\| \\ &\quad + (c_1 + \|\mathbf{g}\|)\bar{\sigma}(\mathbf{P}_1)\|\boldsymbol{\eta}\||e| \end{aligned} \tag{11}$$

Completion of square terms in (11) yields

$$\begin{aligned} \dot{L}_1 &\le -\left(Q_{1m} + \underline{\sigma}(\mathbf{P}_1)(k_c - \beta_1) - 2\right)\|\boldsymbol{\eta}\|^2 \\ &\quad + \frac{\left((c_1 + \|\mathbf{g}\|)\bar{\sigma}(\mathbf{P}_1)\right)^2}{4}|e|^2 + \frac{1}{4}\left(c_0\bar{\sigma}(\mathbf{P}_1)\right)^2 \end{aligned} \tag{12}$$

It is always possible to select k_c large enough such that

$$Q_{1m} + \underline{\sigma}(\mathbf{P}_1)(k_c - \beta_1) - 2 > 0 \tag{13}$$

Thus L_1 is an ISPS-Lyapunov function for subsystem Σ_η, which means this system is ISPS with input e (Jiang, 1999). I.e., there exist a continuous positive definite function α, a κ-function χ and a constant $c \geq 0$ such that

$$\dot{L}_1 \leq -\alpha\left(\|\boldsymbol{\eta}\|\right) \qquad \forall \|\boldsymbol{\eta}\| \geq \chi\left(|e|\right)+c \tag{14}$$

4. CONTROLLER DESIGN

4.1. Feedback Linearization

By introducing the following transformation:

$$v = \hat{b}(y,u) \tag{15}$$

feedback linearization is performed. In (15), v is referred to as a pseudo control signal and $\hat{b}(y,u)$ is a good approximation of the $b(\mathbf{x},u)$. Hence, the modeling error is

$$\Delta(\mathbf{z},\boldsymbol{\eta},u) = \hat{\mathrm{b}}(y,u) - \mathrm{b}(\mathbf{z},\boldsymbol{\eta},u) \tag{16}$$

Using (3), (15) and (16) the error dynamic can be expressed as

$$e^{(r)} = y_d^{(r)} - \hat{\mathrm{b}}(y,u) + \Delta(\mathbf{z},\boldsymbol{\eta},u) = y_d^{(r)} - v + \Delta(\mathbf{z},\boldsymbol{\eta},u) \tag{17}$$

This equation represents the dynamic relation of r integrations between the pseudo control v and the system output y, where the modeling error $\Delta(\mathbf{z},\boldsymbol{\eta},u)$ acts as a disturbance signal. The pseudo control is selected to have the form

$$v = y_d^{(r)} + u_L + u_{ad} - u_R \tag{18}$$

Now, substituting (18) into (17), the close-loop error dynamic can be presented as

$$e^{(r)} = -u_L + \left(\Delta - u_{ad}\right) + u_R \tag{19}$$

where u_L is the output of a stabilizing linear dynamic compensator for the linear dynamics in (19) when $\Delta = u_{ad}$, and $u_R = 0$. u_{ad} is the adaptive part of the control signal designed to approximately cancel out $\Delta(\mathbf{z},\boldsymbol{\eta},u)$ whilst the control part u_R is proposed to achieve robust asymptotic stability. The robustifying term u_R could be continuous or discontinuous. For instance, one may use the sliding-mode control since it is robust in the presence of uncertainties.

Note that the model approximation function $\hat{b}(y,u)$ should be invertible with respect to u, allowing the actual control input to be computed by

$$u = \hat{b}^{-1}(y,v)$$

As it was mentioned before, u_{ad} is designed to cancel the unknown modeling error $\Delta(\mathbf{z},\boldsymbol{\eta},\hat{b}^{-1}(y,v))$, where Δ depends on $u_{ad}(t)$ through v. Therefore, there exists a fixed-point problem as

$$u_{ad}(t) = \Delta\left(\mathbf{x}(t), \hat{b}^{-1}(y, y_d^{(r)} + u_L + u_{ad} - u_R)\right) \tag{20}$$

The following assumption provides conditions that guarantee the existence and the uniqueness of a solution for u_{ad}.

Assumption 3. *The map $u_{ad} \to \Delta$ is a contraction over the entire input domain. This means, the following inequality should be satisfied:*

$$\left|\partial\Delta \big/ \partial u_{ad}\right| < 1 \tag{21}$$

Substituting (15), (16) and (18) into (21) implies

$$\left|\frac{\partial\Delta}{\partial u_{ad}}\right| = \left|\frac{\partial(\hat{b}-b)}{\partial u}\frac{\partial u}{\partial v}\frac{\partial v}{\partial u_{ad}}\right| = \left|1 - \frac{\partial b/\partial u}{\partial\hat{b}/\partial u}\right| < 1 \tag{22}$$

Inequality (22) is equivalent to the following conditions:

$$\begin{gathered}\operatorname{sgn}(\partial b/\partial u) = \operatorname{sgn}(\partial\hat{b}/\partial u) \\ \left|\partial\hat{b}/\partial u\right| > \left|\partial b/\partial u\right|/2 > 0\end{gathered} \tag{23}$$

Hence, $\hat{b}(y,u)$ should be selected such that it satisfies conditions (23).

4.2. Construction of SPR Error Dynamic

In this section, the strictly positive realness (SPR) property of the closed-loop error dynamic is studied. Assume that u_L is constructed as

$$u_L = \frac{N_L}{D_L}e \tag{24}$$

and the filtered error signal $\tilde{e}$ is defined as

$$\tilde{e} = G_{ad}(s)e = \frac{N_{ad}}{D_{ad}}e \tag{25}$$

where $G_{ad}(s)$ is selected such that $G_{ad}(0) \neq 0$. This signal is constructed to ensure a realizable error signal, which is used to adapt the NN weights.

Using (19) and based on the compensators defined in (24) and (25), the closed-loop transfer function of the system can be written as

$$\tilde{e}(s) = G(s)\left(\left(\Delta(\mathbf{x},u) - u_{ad}\right) + u_R\right)(s) \tag{26}$$

where

$$G(s) = \frac{D_L N_{ad}}{D_{ad}\left(s^r D_L + N_L\right)} \tag{27}$$

Analyzing the denominator of (27), the Routh-Hurwitz stability criterion implies that a necessary condition for the closed-loop system stability is that the polynomial $s^r D_L + N_L$ is complete, i.e. all of the polynomial coefficients should be nonzero. Therefore, the numerator degree of the compensator N_L (and hence D_L) should be at least $r-1$.

In addition, to simplify the design procedure, D_{ad} and D_L are selected with the same degree. Hence, the relative degree of $G(s)$ is

$$\rho = \deg(D_L) + r - \deg(N_{ad}) \tag{28}$$

where $\deg(N_{ad}) \leq \deg(D_L)$. Therefore, $\rho \geq r$. As it will be shown in the next section, for the NN adaptation rules to be realizable (i.e. dependent only on the available data), the transfer function $G(s)$ must be strictly positive real (SPR). However, the relative degree of $G(s)$ is at least r. When the relative degree of $G(s)$ is one, it can be made SPR by a proper construction of $N_{ad}(s)$. If $\rho > 1$, $G(s)$ cannot be SPR (Narendra and Annaswamy 1989). To achieve SPR when $\rho > 1$, a stable low pass filter $T(s)$ is introduced such that

$$\deg(N_{ad}) + \deg(T) = \deg(D_L) + r - 1 \tag{29}$$

Thus, the new filtered error dynamic is

$$\tilde{e}(s) = \bar{G}(s)T^{-1}(s)\left(\left(\Delta(\mathbf{x},u) - u_{\mathrm{ad}}\right) + u_R\right)(s) \tag{30}$$

where $\bar{G}(s)$ can be represented as

$$\bar{G}(s) = G(s)T(s) = \frac{b_1 s^{p-1} + b_2 s^{p-2} + \ldots + b_p}{s^p + a_1 s^{p-1} + \ldots + a_p} \tag{31}$$

with $p = 2\deg(D_L) + r$.

Since $\bar{G}(s)$ is a stable transfer function, its zeros (roots of N_{ad} and $T(s)$) can be easily placed to make it SPR. Moreover it is important to note that $T(s)$ should be designed such that the step response of $T^{-1}(s)$ has no overshoot and $\left|T^{-1}(s)\right| \le 1$. This is a mild constrain that is used in stability proof. Hence, the state space model of the closed-loop error dynamic (31) can be represented as

$$\begin{aligned} \dot{\xi} &= \mathbf{A}_{cl}\xi + \mathbf{b}_{cl}\left[T^{-1}(s)\left(\left(\Delta(\mathbf{x},u) - u_{ad}\right) + u_R\right)\right] \\ \tilde{e} &= \mathbf{c}_{cl}^T \xi \end{aligned} \quad (32)$$

According to the Kalman-Yakobovich lemma, the SPR of $\bar{G}(s)$ assures the existence of a matrix $\mathbf{P} = \mathbf{P}^T > 0$ which satisfies

$$\mathbf{A}_{cl}^T \mathbf{P} + \mathbf{P}\mathbf{A}_{cl} = -\mathbf{Q} \quad (33)$$

and

$$\mathbf{P}\mathbf{b}_{cl} = \mathbf{c}_{cl} \quad (34)$$

where $\mathbf{Q} = \mathbf{Q}^T > 0$

4.3. Neural Network-Based Approximation

It can be shown that under the observability condition of system (1), a Multi Layer Perceptron (MLP) neural network can approximate the modelling error $\Delta(\mathbf{z},\boldsymbol{\eta},u)$, based on the input-output data only, as (Lavertsky, *et al.*, 2003)

$$\Delta\left(\mathbf{z},\boldsymbol{\eta},u\right) = \mathbf{w}^{*T}\boldsymbol{\sigma}(\mathbf{V}^{*T}\boldsymbol{\zeta}) + \varepsilon_1 \quad (35)$$

where $\left|\varepsilon_1\right| \le \varepsilon_M$ (ε_M depends on the network architecture), $\boldsymbol{\zeta} = \begin{bmatrix}1 & \bar{\mathbf{y}} & \bar{\mathbf{u}}\end{bmatrix}^T$, in which

$$\begin{aligned} \bar{\mathbf{y}} &= \left[y(t) \ \cdots \ y(t - T_d(n_1 - 1))\right], \\ \bar{\mathbf{u}} &= \left[u(t) \ \cdots \ u(t - T_d(n_1 - r - 1))\right] \end{aligned}$$

$\boldsymbol{\sigma}(\cdot) \in R^m$ is a vector function containing the nonlinear functions in the neurons of the hidden layer. The ideal constant weights $\mathbf{w}^*$ and $\mathbf{V}^*$ are

$$\left(\mathbf{w}^*,\mathbf{V}^*\right) := \arg \min_{(\mathbf{w},\mathbf{V})\in\Omega_w} \left\{ \sup_{\zeta\in\Omega_\zeta} \left|\mathbf{w}^T\boldsymbol{\sigma}\left(\mathbf{V}^T\boldsymbol{\zeta}\right) - \Delta\right| \right\} \quad (36)$$

where $\Omega_{\mathrm{w}} = \left\{\left(\mathbf{w},\mathbf{V}\right) \middle| \left\|\mathbf{w}\right\|_{\mathrm{F}} \le M_{\mathrm{w}}, \left\|\mathbf{V}\right\|_{\mathrm{F}} \le M_{\mathrm{V}}\right\}$, M_{w} and M_{V} are positive numbers, and $\left\|\cdot\right\|_{\mathrm{F}}$ denotes the Forbenius norm. Therefore, it is possible to use a MLP to construct u_{ad} and to cancel out Δ as

$$u_{ad} = \mathbf{w}^T\boldsymbol{\sigma}(\mathbf{V}^T\boldsymbol{\zeta}) \quad (37)$$

However, in practice, the weights of the neural network may be different from the ideal ones in (36). Hence, an approximation error occurs.

Lemma 1: *The approximation error, which arises from the difference between (35) and the output of the NN (37) satisfies the following equality:*

$$\Delta(\mathbf{x},u) - u_{ad} = \tilde{\mathbf{w}}\left(\boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}\mathbf{V}^T\boldsymbol{\zeta}\right) + \mathbf{w}^T\dot{\boldsymbol{\sigma}}\tilde{\mathbf{V}}^T\boldsymbol{\zeta} + \delta(t) \quad (38)$$

where

$$\begin{aligned} \left|\delta(t)\right| &\le c_0 + c_1\left\|\tilde{\mathbf{w}}\right\|\left\|\boldsymbol{\zeta}\right\| + c_2\left\|\tilde{\mathbf{V}}\right\|_{\mathrm{F}}\left\|\boldsymbol{\zeta}\right\| \\ \tilde{\mathbf{w}} &= \mathbf{w}^* - \mathbf{w}, \quad \tilde{\mathbf{V}} = \mathbf{V}^* - \mathbf{V} \end{aligned} \quad (39)$$

$\dot{\boldsymbol{\sigma}} \in R^{m\times m}$ *is the derivative of* $\boldsymbol{\sigma}$ *with respect to the input signals of all neurons in the hidden layer of the NN, and* c_i $(i = 0,1,2)$ *are positive known constants.*

Proof: See (Hoseini, *et al*. 2006).

5. STABILITY ANALYSIS

In this section, first, asymptotic stability of the tracking error is proved, followed by stability of the overall system using the small gain theorem.

Substituting (38) in (32), the closed-loop error dynamic can be represented as

$$\dot{\boldsymbol{\xi}} = \mathbf{A}_{cl}\boldsymbol{\xi} + \mathbf{b}_{cl}\left(T^{-1}\tilde{\mathbf{w}}^T\left(\boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}\mathbf{V}^T\boldsymbol{\zeta}\right) + T^{-1}\mathbf{w}^T\dot{\boldsymbol{\sigma}}\tilde{\mathbf{V}}^T\boldsymbol{\zeta} + \delta_f(t) + u_{Rf}\right)$$

where $\delta_f(t) = T^{-1}(s)\delta(t)$ and $u_{Rf}(t) = T^{-1}(s)u_R(t)$.

Now, let define $\boldsymbol{\psi} = \boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}\mathbf{V}^T\boldsymbol{\zeta}$ and $\boldsymbol{\Psi} = \boldsymbol{\zeta}\,\mathbf{w}^T\dot{\boldsymbol{\sigma}}$, and consider the discontinues control signal

$$u_R = -g\varphi\,\mathrm{sgn}\left(\tilde{e}\right) \quad (40)$$

where φ is an adaptive gain and g is a function of the NN weights and input vector $\boldsymbol{\xi}$.

Using the equality $\mathbf{w}^T\dot{\boldsymbol{\sigma}}\tilde{\mathbf{V}}^T\boldsymbol{\zeta} = \mathrm{tr}(\tilde{\mathbf{V}}^T\boldsymbol{\zeta}\mathbf{w}^T\dot{\boldsymbol{\sigma}})$, the closed-loop error dynamic results as

$$\begin{aligned} \dot{\boldsymbol{\xi}} = \mathbf{A}_{cl}\boldsymbol{\xi} + \mathbf{b}_{cl}\Big(&T^{-1}(s)\tilde{\mathbf{w}}^T\boldsymbol{\psi} + \mathrm{tr}\left(T^{-1}(s)\tilde{\mathbf{V}}^T\boldsymbol{\Psi}\right) \\ &+ \delta_f(t) - T^{-1}(s)g\varphi\,\mathrm{sgn}\left(\tilde{e}\right)\Big) \end{aligned} \quad (41)$$

Here, $\tilde{\mathbf{V}}$, $\tilde{\mathbf{w}}$ and $\varphi\,\mathrm{sgn}\left(\tilde{e}\right)$ are time varying signals. Hence, the transfer function operator in (41) is not commutable. Now, consider the following error terms

$$\begin{aligned} \delta_w &= T^{-1}(s)\tilde{\mathbf{w}}^T\boldsymbol{\psi} - \tilde{\mathbf{w}}^T T^{-1}(s)\boldsymbol{\psi} \\ \delta_V &= \mathrm{tr}\left(T^{-1}(s)\tilde{\mathbf{V}}^T\boldsymbol{\Psi}\right) - \mathrm{tr}\left(\tilde{\mathbf{V}}^T T^{-1}(s)\boldsymbol{\Psi}\right) \\ \delta_\varphi &= T^{-1}(s)g\varphi\,\mathrm{sgn}(\tilde{e}) - \varphi\,\mathrm{sgn}(\tilde{e})T^{-1}(s)g \end{aligned} \quad (42)$$

for which the following bounds can be defined

$$\left|\delta_{\mathrm{w}}\right| \le c_3\left\|\tilde{\mathbf{w}}\right\|, \quad \left|\delta_{\mathrm{V}}\right| \le c_4\left\|\tilde{\mathbf{V}}\right\|_{\mathrm{F}}, \quad \left|\delta_\varphi\right| \le c_5\left|\varphi\right| \quad (43)$$

where c_3, c_4 and c_5 are positive numbers. Substituting (42) into (41) yields

$$\begin{aligned} \dot{\boldsymbol{\xi}} = \mathbf{A}_{cl}\boldsymbol{\xi} + \mathbf{b}_{cl}\Big(&\tilde{\mathbf{w}}\boldsymbol{\psi}_f + \mathrm{tr}\left(\tilde{\mathbf{V}}^T\boldsymbol{\Psi}_f\right) \\ &- \varphi\,\mathrm{sgn}(\tilde{e})T^{-1}g + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} + \delta_f - \delta_\varphi\Big) \end{aligned} \quad (44)$$

where $\boldsymbol{\psi}_f = T^{-1}(s)\,\boldsymbol{\psi}$ and $\boldsymbol{\Psi}_f = T^{-1}(s)\,\boldsymbol{\Psi}$.

In order to show that the error dynamics are asymptotically stable in the proposed control method, the following lemma is needed.

Lemma 2: *The following inequality holds:*

$$\left|\delta_f + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} - \delta_\varphi\right| \le \varphi^* T^{-1}(s)g \quad (45)$$

where, $g = \left(2 + \left\|\mathbf{V}\right\|_{\mathrm{F}} + \left\|\mathbf{w}\right\|\right)\left(1 + \left\|\boldsymbol{\zeta}\right\|\right) + \left|\varphi\right|$ *and* φ^* *is a positive constant.*

Proof: Let $\left|T^{-1}(s)\right| \le 1$. Using (39) and (43) and after omitting some intermediate steps

$$\begin{aligned} \left|\delta_f + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} - \delta_\varphi\right| &\le c_0 + c_1\left\|\mathbf{w}^* - \mathbf{w}\right\|\left|\boldsymbol{\zeta}\right| + c_2\left\|\mathbf{V}^* - \mathbf{V}\right\|_F\left|\boldsymbol{\zeta}\right| \\ &\quad + c_3\left\|\mathbf{w}^* - \mathbf{w}\right\| + c_4\left\|\mathbf{V}^* - \mathbf{V}\right\|_{\mathrm{F}} + c_5\left|\varphi\right| \\ &\le \varphi_1\left(\left(2 + \left\|\mathbf{V}\right\|_{\mathrm{F}} + \left\|\mathbf{w}\right\|\right)\left(1 + \left\|\boldsymbol{\zeta}\right\|\right) + \left|\varphi\right|\right) \end{aligned}$$

where $\varphi_1 = \max\left\{c_0, c_1 M_{\mathrm{w}}, c_1, c_2 M_{\mathrm{V}}, c_2, c_3 M_{\mathrm{w}} + c_4 M_{\mathrm{V}}, c_3, c_4, c_5\right\}$.

Considering that g is a positive signal and low pass filter $T^{-1}(s)$ is designed such that its step response has no overshoot, so $T^{-1}(s)g$ remains positive. By suitable initialization of filter states, it is easy to find $0 < \lambda < 1$ such that $g \le {T^{-1}(s)g}/{\lambda}$. Consequently

$$\left|\delta_f + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} - \delta_\varphi\right| \le {\varphi_1}/{\lambda}\, T^{-1}(s)g \quad (46)$$

Therefore, φ^* may be selected as

$$\varphi^* = \varphi_1 / \lambda \qquad \square$$

Theorem 2: *Consider the discontinuous control (44) and select the adaptation laws for NN weights and the gain of the robustifying term* φ *as*

$$\dot{\mathbf{w}} = \gamma_{\mathrm{w}} \tilde{e} \boldsymbol{\psi}_f, \ \dot{\mathbf{V}} = \gamma_{\mathrm{V}} \tilde{e} \boldsymbol{\Psi}_f, \ \dot{\varphi} = \gamma_\varphi |\tilde{e}| \left(T^{-1} g\right) \quad (47)$$

Then, the closed-loop tracking error is asymptotically stable and the weights of the NN remain bounded.

Proof: Let define a Lyapunov function as

$$L_2 = \frac{1}{2}\boldsymbol{\xi}^T \mathbf{P} \boldsymbol{\xi} + \frac{1}{2\gamma_{\mathrm{w}}}\|\tilde{\mathbf{w}}\|^2 + \frac{1}{2\gamma_{\mathrm{V}}}\|\tilde{\mathbf{V}}\|_{\mathrm{F}}^2 + \frac{1}{2\gamma_\varphi}|\tilde{\varphi}|^2 \quad (48)$$

where $\mathbf{P}$ is the unique positive-definite symmetric solution of (33) and $\tilde{\varphi} = \varphi^* - \varphi$. Moreover, assume that $\mathbf{w}^*$ and $\mathbf{V}^*$ are ideal constant weights defined in (36); then, from (39) $\dot{\mathbf{w}} = -\dot{\tilde{\mathbf{w}}}$, $\dot{\mathbf{V}} = -\dot{\tilde{\mathbf{V}}}$. Using (44), the time-derivative of L becomes

$$\begin{aligned}\dot{L}_2 \le & -\frac{1}{2} Q_m \|\boldsymbol{\xi}\|^2 + \boldsymbol{\xi}^T \mathbf{P} \mathbf{b}_{cl} \big[\tilde{\mathbf{w}}^T \boldsymbol{\psi}_f + \mathrm{tr}\left(\tilde{\mathbf{V}}^T \boldsymbol{\Psi}_f\right) - \varphi \,\mathrm{sgn}(\tilde{e}) T^{-1} g \\ & + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} + \delta_f - \delta_\varphi \big] - \frac{1}{\gamma_{\mathrm{w}}} \tilde{\mathbf{w}}^T \dot{\mathbf{w}} - \frac{1}{\gamma_{\mathrm{V}}} \mathrm{tr}\left(\tilde{\mathbf{V}}^T \dot{\mathbf{V}}\right) - \frac{1}{\gamma_\varphi} \tilde{\varphi}\dot{\varphi}\end{aligned}$$

where Q_m is the smallest eigenvalue of $\mathbf{Q}$. Substituting $\tilde{e} = \boldsymbol{\xi}^T \mathbf{P} \mathbf{b}_{cl}$ from (32) and (34) and using Lemma 2

$$\begin{aligned}\dot{L}_2 \le & -\frac{1}{2} Q_m \|\boldsymbol{\xi}\|^2 + \tilde{\mathbf{w}}^T \left(\tilde{e}\, \boldsymbol{\psi}_f - \frac{1}{\gamma_{\mathrm{w}}} \dot{\mathbf{w}}\right) + \mathrm{tr}\left(\tilde{\mathbf{V}}^T \left(\tilde{e}\, \boldsymbol{\Psi}_f - \frac{1}{\gamma_{\mathrm{V}}} \dot{\mathbf{V}}\right)\right) \\ & - \tilde{e} \varphi \,\mathrm{sgn}(\tilde{e}) T^{-1}(s) g + |\tilde{e}| \varphi^* T^{-1}(s) g - \tilde{\varphi}\dot{\varphi} / \gamma_\varphi\end{aligned}$$

Using the adaptation laws in (47), it yields

$$\begin{aligned}\dot{L}_2 & \le -\frac{1}{2} Q_m \|\boldsymbol{\xi}\|^2 + |\tilde{e}| \varphi^* T^{-1} g - |\tilde{e}| \varphi T^{-1} g - \frac{1}{\gamma_\varphi} \tilde{\varphi}\dot{\varphi} \\ & = -\frac{1}{2} Q_m \|\boldsymbol{\xi}\|^2 + \left(|\tilde{e}| T^{-1} g - \frac{1}{\gamma_\varphi} \dot{\varphi}\right) \tilde{\varphi} \qquad (49) \\ & = -\frac{1}{2} Q_m \|\boldsymbol{\xi}\|^2 \le -\frac{1}{2} \frac{Q_m}{\|\mathbf{c}_{cl}\|^2} |\tilde{e}|^2 \le -\frac{1}{2} \frac{Q_m}{G_{\inf}^2 \|\mathbf{c}_{cl}\|^2} |e|^2\end{aligned}$$

where $G_{\inf} = \inf_{\omega} |G_{ad}(j\omega)|$.

Since L is a positive function and $\dot{L} \le 0$, then one can conclude that $\|\boldsymbol{\xi}\|$, $\|\tilde{\mathbf{V}}\|$, $\|\tilde{\mathbf{w}}\|$ and $|\tilde{\varphi}|$ are bounded. In addition, from (36), $\mathbf{V}^*$ and $\mathbf{w}^*$ are bounded; therefore, according to (39), $\mathbf{V}$ and $\mathbf{w}$ remain bounded. Moreover, by integrating (49)

$$\int_0^\infty \|\boldsymbol{\xi}(t)\|^2 dt \le \frac{2}{Q_m}\left(L_2(t)\big|_{t=0} - L_2(t)\big|_{t=\infty}\right). \quad (50)$$

Since, the right-hand side of (50) is bounded, then, according to the Barbalet's lemma

$$\lim_{t\to\infty} \|\boldsymbol{\xi}\|^2 = 0$$

Since $\tilde{e} = \mathbf{c}_{cl}^T \boldsymbol{\xi}$, then $\lim_{t\to\infty} \tilde{e}(t) = 0$, and according to the final value theorem and using (14), it gives

$$\lim_{s\to 0} s\, \tilde{e}(s) = \lim_{s\to 0} s\, G_{\mathrm{ad}}(s) e(s) = 0 .$$

Since $G_{\mathrm{ad}}(0) \ne 0$, one can conclude that $\lim_{s\to 0} s\, e(s) = 0$, and hence

$$\lim_{t\to\infty} e(t) = 0 \quad (51)$$

which concludes the proof. $\square$

Now, the overall stability of the system is proved in the following theorem.

Theorem 3: *Consider the following systems:*

$$\Sigma_{x_1} : \dot{x}_1 = f_1(x_1, x_2, \delta) \quad (52)$$

$$\Sigma_{x_2} : \dot{x}_2 = f_1(x_1, x_2, \delta) \quad (53)$$

where Σ_{x_1} *and* Σ_{x_2} *are ISS with inputs* (x_2, δ) *and* (x_1, δ), *respectively. I.e. there exist Lyapunov functions* $V_1(x_1)$ *and* $V_2(x_2)$ *such that*

$$\dot{V}_1 < -k_{11}\|x_1\|^2 + \gamma_{11}\|x_2\|^2 + \gamma_{12}\|\delta\|^2 \quad (54)$$

$$\dot{V}_2 < -k_{21}\|x_2\|^2 + \gamma_{21}\|x_1\|^2 + \gamma_{22}\|\delta\|^2 \quad (55)$$

Then, the interconnected systems (52) and (53), depicted in Fig. 1., is ISS with input δ *if there exist constants* $0 < \varepsilon_1 < 1$ *and* $0 < \varepsilon_2 < 1$ *such that*

$$\frac{\gamma_{11}}{(1-\varepsilon_1)k_{11}} \frac{\gamma_{21}}{(1-\varepsilon_2)k_{21}} < 1 \quad (56)$$

Proof: (Karagiannis, et al., 2005)

Remark: By comparing (12) and (49) with (54) and (55) it can be shown that $\gamma_{21} = 0$, so the small gain condition (56) is always satisfied and the interconnected systems (3) and (4) are ISPS. Moreover, since $\delta = 0$, the overall system is uniformly ultimately bounded. Note from (12) that smaller bound on error can be achieved by selecting large compensator gains $\mathbf{k}$ and k_c; but unfortunately increasing the compensator gain leads to peaking phenomenon (Seshagiri and Khalil, 2000). In addition if Σ_η is ISS (i.e. $c_0 = 0$) then the overall system is asymptotically stable.

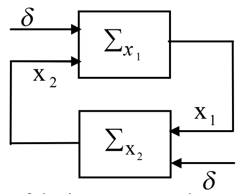

Fig. 1. Block diagram of the interconnected systems (52) and (53).

6. SIMULATION EXAMPLE

The performance of the proposed controller is illustrated by considering the following non-minimum non-affine nonlinear system

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = z_2 + \eta + u - e^{-z_1^2 u} \\ \dot{\eta} = z_1 + 0.8\eta + 0.1 z_1^2 + \eta^2 \\ y = z_1 \end{cases}$$

The relative degree of the system with output y is 2. In fact, the zero dynamic of the system is $\dot{\eta} = 0.8\eta + \eta^2$, which is unstable.

Note that assumption 2 is satisfied; that is

$$\frac{\partial b(x,u)}{\partial u} = 1 + z_1^2 e^{-z_1^2 u} > 0 \quad \forall \mathbf{z} \in \Omega_{\mathbf{z}}, \eta \in \Omega_\eta$$

and best available approximation of $b(\mathbf{x}, u)$ is selected as $v = \hat{b}(y, u) = u$. The second order compensator

$$\frac{N_L}{D_L} = \frac{18 s^2 + 16 s + 12}{s(s+7)}$$

is selected to stabilize the linear second-order system $\ddot{e} = -u_L$. Based on the assumptions on N_{ad} and D_{ad} in Section 4, the following filter is used to construct the error signal $\tilde{e}$

$$\frac{N_{\text{ad}}}{D_{\text{ad}}} = 50\frac{s^2+6s+6}{(s+10)(s+20)}.$$

It is desirable that the above filter possess high bandwidths. Finally, $T(s) = 0.5s + 1$ is selected based on SPR property of $\bar{G}$. The NN is of MLP type and has 20 neurons in one hidden layer with tangent hyperbolic activation functions. The weights are initialised randomly with small numbers. The input to the NN for $n_1 = 4 \geq n$ is

$$\xi = [1, y(t), y(t-T_d), y(t-2T_d), y(t-3T_d), u(t), u(t-T_d)]^T$$

with $T_d = 10$ msec. The NN is trained using the adaptation laws, given in (47), with learning rates $\gamma_{\mathbf{W}} = \gamma_{\mathbf{V}} = \gamma_{\varphi} = 0.1$. The reference signal y_d is designed using k =-1.5 and $k_c = -4$. And finally, to avoid chattering, $\tanh(\tilde{e}/0.9)$ is used instead of $\operatorname{sgn}(\tilde{e})$. Simulation results are presented in Fig. 2. The output y, the internal dynamic η and the control signal are shown for three different input laws. Figs. 2a, 2b and 2c are for the cases where u_L, $u_L + u_{ad}$ and $u_L + u_{ad} - u_R$ are inputs to the system, respectively. Fig 2a shows that when only the linear controller is applied, the system response is very oscillatory and unstable because of nonlinear behaviour of the system. When the adaptive control term u_{ad} is added to the control law, the states of system are still oscillatory, but stable (Fig. 2b). By adding the robustifying term u_{R}, all the states of system are stable with good transient response and small steady-state error. The norms of weights of the NN are depicted in Figure 2d. It shows that the weights remain bounded.

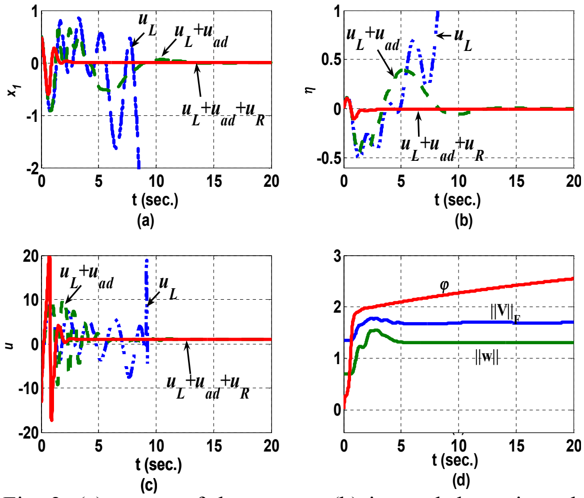

Fig. 2. (a) output of the system, (b) internal dynamic and (c) control signal. The dash-dot (blue) line, the dotted line (black) and the solid line (red) are for the control laws u_L, $u_L + u_{ad}$ and $u_L + u_{ad} - u_R$, respectively. (d) norm of the weights of the NN.

7. CONCLUSION

In this paper, a direct adaptive output-feedback control method was developed for non-minimum phase nonlinear systems. The main feature of the proposed method is that it does not need estimation of the external dynamics. The system dynamic was considered as two subsystems. The η-subsystem, which includes the internal dynamics, was stabilized input-to-state practical with input *e* using a suitable reference signal. The asymptotic stability of the error dynamic was guaranteed by using a combined output-feedback control method. The overall system stability was shown using the small gain theorem. The effectiveness of the proposed scheme was demonstrated using a non-minimum phase nonlinear system.

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