OUTPUT-FEEDBACK STABILIZATION OF NONLINEAR NON-MINIMUM PHASE SYSTEMS USING NEURAL NETWORK

Saeed M. Hoseini[1] *Mohamad Farrokhi*[1,2]

[1] Department of Electrical Engineering,
[2] Center of Excellence for Power System Automation and Operation,
Iran University of Science and Technology
Tehran 16846-13114, IRAN

sm_hoseini @iust.ac.ir farrokhi@iust.ac.ir

ABSTRACT

This paper presents an adaptive output-feedback stabilization method for non-affine nonlinear non-minimum phase systems using neural networks. The proposed controller is comprised of a linear, a neuro-adaptive, and an adaptive robustifying control term. The learning rules for adaptive gains, including weights of the neural network, are derived using the Lyapunov's direct method. These adaptation laws employ a suitable output of a linear observer of system dynamics that is realizable. The effectiveness of the proposed scheme will be shown in simulations for the benchmark Translation Oscillator Rotational Actuator (TORA) problem.

1. INTRODUCTION

Output-feedback control of nonlinear systems is a challenging problem in control theory. This problem has been an active research area for many years. Several researchers have recently proposed fundamental methods in this area. These methods include using geometric techniques [1], adaptive observers and output-feedback controllers for system in output-feedback form [2], high gain observers [3], and backstepping method for systems with parametric uncertainties [4]. The aim of all these research efforts is to develop a systematic design method for controlling systems in the presence of structured uncertainties in the form of parameters variations and unstructured uncertainties such as unmodeled dynamics. Recently, some results, based on the output-feedback control method, using Neural Networks (NNs), have been presented. These methods can be applied to a wide class of systems with structured and unstructured uncertainties. Remarkably, these results include methods for uncertain systems based on high-gain observer [5], [6], using adaptive error observer [7], and constructing an SPR error signal using the Kalman-Yakobovic lemma [8].

A common assumption in the aforementioned methods is that the zero dynamics are globally asymptotically stable or input-to-state stable (ISS). In other words, the system is assumed to be minimum phase. Recently, some papers have dealt with output-feedback stabilization for non-minimum phase systems. Isidori has presented a solution for robust semi-global output-feedback stabilization of non-minimum phase systems based on auxiliary constructions using a high-gain observer [9]. Karagiannis *et al.* have proposed a method for global output-feedback stabilization using the classical backstepping and the small-gain techniques [10]. A design method for semi-global stabilization of a class of non-minimum phase nonlinear systems that can be transformed to the global normal form as well as to the form of linear observer error dynamic is presented by Ding [11]. These methods can only be applied to the systems, where the nonlinearities or the high frequency gains depend only on the output of the system. By using the approximation ability of NNs, theses restrictions on the model of the system are relaxed; the local and non-local stabilization methods for uncertain non-minimum phase systems with unstructured uncertainties are presented in [12], [13] and [14]. However, these methods are based on the state feedback.

This paper presents an adaptive output-feedback stabilization method for observable and stabilizable nonlinear non-affine non-minimum phase systems. Only an approximate linear model of the nonlinear system is required in the design procedure. This linear system should present the non-minimum phase zeros of the nonlinear system with desired accuracy. In fact, there is a conic sector bound on the modelling error of the non-minimum phase zeros that is referred to as the unmatched uncertainty. Hence, the proposed approach can be applied to uncertain systems, which have partially known Lipschitz continuous functions in their arguments. Moreover, in this method, the dynamics of the system won't be restricted only to the output of the system.

In the design proedure, first, a static linear controller is proposed that stabilises the linear part of dynamics. Then, this linear controller is augmented with a neuro-adaptive term, which is used to approximate the matched uncertainty. The NN operates over a tapped-delay line of memory units comprised of the system input/output signals. Also an adaptive robustifying control term is added to the control law to compensate the NN approximation error. In addition, a suitable linear observer is design such that the combined control law depends only on the output of the system.

This paper is organized as follows: Section 2 describes the class of nonlinear systems to be stabilised and defines the problem of stabilization. The procedure for the controller and observer design and approximation properties of the NN are addressed in Section 3. In Section 4, the stability of the closed-loop system is analytically proved. The simulation example, which illustrates the effectiveness of the proposed controller, is presented in Section 5. Conclusions are given in Section 6.

2. PROBLEM STATEMENT

Consider the nonlinear SISO system in the following normal form with the coordinates $\left[\mathbf{z}^T, \boldsymbol{\eta}^T\right] = \left[z_1, ..., z_r, \eta_{r+1}, ..., \eta_n\right]$ [1]:

$$\begin{cases} \dot{z}_i = z_{i+1} & 1 \le i \le r-1 \\ \dot{z}_r = f\left(\mathbf{z}, \boldsymbol{\eta}, u\right) \\ \dot{\boldsymbol{\eta}} = \mathbf{v}\left(\mathbf{z}, \boldsymbol{\eta}\right) \\ y = z_1, \end{cases} \tag{1}$$

where r is the relative degree, $\boldsymbol{\eta} \in \Omega_\eta \subset R^{n-r}$ is the state vector associated with the internal dynamics, $\mathbf{z} = \left[z_1, \cdots, z_r\right]^T \in \Omega_z \subset R^r$, Ω_η and Ω_z are the compact sets of the operating regions, and $u \in R$ and y are the input and the output of the system, respectively. The mappings $f: R^{n+1} \to R$ and $\mathbf{v}: R^n \to R^{n-r}$ are partially known Lipschitz continuous functions of their arguments with the initial conditions $f\left(\mathbf{0}, \mathbf{0}, 0\right) = 0$ and $\mathbf{v}\left(\mathbf{0}, \mathbf{0}\right) = \mathbf{0}$. Note that the system (1) can be non-minimum phase; hence, there is no need for any assumption on the stability of the zero dynamics.

Assumption 1. *Assume that $f_u = \partial f\left(\mathbf{z}, \boldsymbol{\eta}, u\right) / \partial u \neq 0$. This condition implies that the smooth function f_u is strictly either positive or negative on the compact set*

$$U = \left\{\left(\mathbf{z}, \boldsymbol{\eta}, u\right) \middle| \mathbf{z} \in \Omega_z,\ \boldsymbol{\eta} \in \Omega_\eta,\ u \in R\right\}.$$

It is also assumed that only the output of system y is measurable. The goal is to design a combined controller such that it stabilizes all state variables including internal dynamics of the system. The various features of the proposed control design scheme are presented in the next section.

3. CONTROLLER DESIGN

3.1. Linearization

Using the Taylor expansion method, the system (1) can be expressed around its equilibrium point at the origin as

$$\begin{cases} \dot{z}_i = z_{i+1} & 1 \le i \le r-1 \\ \dot{z}_r = \mathbf{m}^T \mathbf{z} + \mathbf{n}^T \boldsymbol{\eta} + b\left(\Delta\left(\mathbf{z}, \boldsymbol{\eta}, u\right) + u\right) \\ \dot{\boldsymbol{\eta}} = \mathbf{F}\boldsymbol{\eta} + \mathbf{G}\mathbf{z} + \boldsymbol{\Delta}_{\boldsymbol{\eta}}\left(\mathbf{z}, \boldsymbol{\eta}\right) \\ y = z_1, \end{cases} \tag{2}$$

where $\mathbf{m}$ and $\mathbf{n}$ are coefficient vectors and $\mathbf{F}$ and $\mathbf{G}$ are matrices with appropriate dimensions. In addition, $\Delta\left(\mathbf{z}, \boldsymbol{\eta}, u\right)$ is the matched uncertainty and $\boldsymbol{\Delta}_{\boldsymbol{\eta}}(\mathbf{z}, \boldsymbol{\eta})$ denotes the vector of zero-dynamic modelling error or the unmatched uncertainties.

Assumption 2. The unmatched uncertainties are bounded with a conic sector bound as

$$\left\|\boldsymbol{\Delta}_{\boldsymbol{\eta}}(\mathbf{z}, \boldsymbol{\eta})\right\| \le c_0 + c_1 \left\|\mathbf{z}\right\| + c_2 \left\|\boldsymbol{\eta}\right\|, \tag{3}$$

where $c_i \quad (i = 0, 1, 2)$ are known positive constants.

Let us define $\boldsymbol{\xi} = \left[\mathbf{z}^T, \boldsymbol{\eta}^T\right]^T$ and introduce the combined control law as

$$u = u_L - u_{ad} - u_R, \tag{4}$$

Then, the system (2) can be described as

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= \mathbf{A}\boldsymbol{\xi} + \mathbf{b}u_L + \mathbf{b}\left(\Delta - u_{ad} - u_R\right) + \mathbf{H}\boldsymbol{\Delta}_{\boldsymbol{\eta}} \\ y &= \mathbf{c}\boldsymbol{\xi} \end{aligned} \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{M} & \mathbf{N} \\ \mathbf{G} & \mathbf{F} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \mathbf{0}_{1\times(r-1)} & b & \mathbf{0} \end{bmatrix}^T, \mathbf{H} = \begin{bmatrix} \mathbf{0}_{(n-r)\times r} & \mathbf{I}_{(n-r)\times(n-r)} \end{bmatrix}^T$$

$$\mathbf{c} = \begin{bmatrix} 1 & \mathbf{0}_{1\times(n-1)} \end{bmatrix}, \mathbf{M} = \begin{bmatrix} \mathbf{0}_{(r-1)\times 1} & \mathbf{I}_{(r-1)\times(r-1)} \\ \hline \multicolumn{2}{c}{\mathbf{m}^T} \end{bmatrix}, \mathbf{N} = \begin{bmatrix} \mathbf{0}_{(r-1)\times(n-r)} \\ \mathbf{n}^T \end{bmatrix}.$$

Note that, since the system is non-minimum phase, some eigenvalues of $\mathbf{A}$ are positive. First, the linear controller u_L is designed to stabilize the linear part of the system.

3.2. Linear Controller Design

Let $\mathbf{P}_1 = \mathbf{P}_1^T > 0$ be the solution of the following Riccati equation for some $\mathbf{Q}_1 = \mathbf{Q}_1^T > 0$ **:**

$$\mathbf{P}_1\mathbf{A} + \mathbf{A}^T\mathbf{P}_1 + \mathbf{Q}_1 - 2\mathbf{P}_1\mathbf{b}\mathbf{b}^T\mathbf{P}_1 = 0. \tag{6}$$

The linear control law is proposed as

$$u_L = -\rho = -K_c\hat{\boldsymbol{\xi}}, \tag{7}$$

where $\hat{\boldsymbol{\xi}}$ denotes the estimation of $\boldsymbol{\xi}$ and the vector gain K_c is derived as

$$K_c^T = \mathbf{P}_1\mathbf{b}. \tag{8}$$

Substituting (8) in (6) gives

$$\left(\mathbf{A} - \mathbf{b}K_c\right)^T \mathbf{P}_1 + \mathbf{P}_1\left(\mathbf{A} - \mathbf{b}K_c\right) + \mathbf{Q}_1 = 0. \tag{9}$$

Equation (9) ensures that $\mathbf{A} - \mathbf{b}K_c$ is a stable matrix. From (5) it can be easily concluded that u_L stabilizes the system in the absence of nonlinearities.

3.3. Neural Network-Based Adaptive Controller

The adaptive part of the control law in (4) u_{ad} is designed to approximate $\Delta(\mathbf{z}, \boldsymbol{\eta}, u)$. Hence, there exists a fixed-point problem as

$$u_{ad}(t) = \Delta(\mathbf{z}, \boldsymbol{\eta}, u_L - u_R - u_{ad}), \tag{10}$$

The following assumption provides conditions, which guarantee the existence and uniqueness of a solution for u_{ad} [7].

Assumption 3. *The map $u_{ad} \to \Delta$ is a contraction over the entire input domain. This means, the following inequality should be satisfied*

$$\left|\frac{\partial \Delta}{\partial u_{ad}}\right| < 1. \tag{11}$$

Substituting (1), (2) and (4) into (11), yields

$$\begin{aligned} \left|\frac{\partial \Delta}{\partial u_{ad}}\right| &= \left|\frac{1}{b}\frac{\partial\left(f\left(\mathbf{z}, \boldsymbol{\eta}, u\right) - \mathbf{m}^T\mathbf{z} - \mathbf{n}^T\boldsymbol{\eta} - bu\right)}{\partial u}\frac{\partial u}{\partial u_{ad}}\right| \\ &= \left|1 - \frac{1}{b}\frac{\partial f}{\partial u}\right| < 1 \end{aligned} \tag{12}$$

It can be easily verified that condition (12) is equivalent to the following two conditions

$$\operatorname{sgn}(b) = \operatorname{sgn}(\partial f / \partial u), \quad b > 0.5\,\partial f / \partial u. \tag{13}$$

Under the observability condition of system in (1), it has been shown by Lavertsky et al. that the continuous-time dynamic $\Delta(\mathbf{z}, \boldsymbol{\eta}, u)$ can be approximated using the delayed version of inputs and outputs as [15]

$$\Delta(\mathbf{z}, \boldsymbol{\eta}, u) = \Gamma\left(\boldsymbol{\zeta}\right) + \varepsilon_0, \tag{14}$$

where $\boldsymbol{\zeta} = \begin{bmatrix} 1 & \bar{\mathbf{y}} & \bar{\mathbf{u}} \end{bmatrix}^T \in R^N$ and

$$\begin{aligned} \bar{\mathbf{u}} &= \left[u(t) \quad u(t - T_d) \quad \cdots \quad u(t - T_d(n_1 - r - 1))\right], \quad n_1 \ge n \\ \bar{\mathbf{y}} &= \left[y(t) \quad \cdots \quad y(t - T_d(n_1 - 1))\right] \end{aligned}$$

and ε_0 is directly proportional to the sampling time interval T_d. Hence, ε_0 can be ignored by selecting T_d sufficiently small.

On the other hand, any sufficiently smooth function can be approximated on a compact set with an arbitrarily bounded error by a suitable large MLP [16]. Therefore, a set of ideal weights $\mathbf{w}^*$ and $\mathbf{V}^*$ on the compact set Ω_ζ exists such that

$$\Delta\left(\mathbf{z},\boldsymbol{\eta},u\right)=\mathbf{w}^{*T}\boldsymbol{\sigma}(\mathbf{V}^{*T}\zeta)+\varepsilon_1\,, \tag{15}$$

where $\mathbf{w}^* \in R^m$ is a vector containing synaptic weights of the output layer, $\mathbf{V}^* \in R^{N\times m}$ is a matrix containing the weights for the hidden layer, $\boldsymbol{\sigma}=\left[\sigma_1 \cdots \sigma_m\right]^T$ is a vector function containing the nonlinear functions of the neurons in the hidden layer, and $\left|\varepsilon_1\right| \le \varepsilon_{1M}$, in which ε_{1M} depends on the network architecture. The ideal constant weights $\mathbf{w}^*$ and $\mathbf{V}^*$ are defined as

$$\left(\mathbf{w}^*,\mathbf{V}^*\right):=\arg\min_{(\mathbf{w},\mathbf{V})\in\Omega_{\mathbf{w}}}\left\{\sup_{\zeta\in\Omega_\zeta}\left|\mathbf{w}^T\boldsymbol{\sigma}\left(\mathbf{V}^T\zeta\right)-\Gamma(\zeta)\right|\right\}, \tag{16}$$

where $\Omega_{\mathbf{w}}=\left\{\left(\mathbf{w},\mathbf{V}\right)\middle|\left\|\mathbf{w}\right\|_{\mathrm{F}}\le M_{\mathbf{w}},\left\|\mathbf{V}\right\|_{\mathrm{F}}\le M_{\mathbf{V}}\right\}$, $M_{\mathbf{w}}$ and $M_{\mathbf{V}}$ are positive numbers, and $\left\|\cdot\right\|_{\mathrm{F}}$ denotes the Frobenius norm. Since Δ can be approximated with an MLP NN, hence, an MLP is employed to construct the adaptive control term as

$$u_{ad}=\mathbf{w}^T\boldsymbol{\sigma}\left(\mathbf{V}^T\zeta\right). \tag{17}$$

In practice, the weights of this neural network may be different from the ideal ones, defined in (16). Therefore, an approximation error exists.

Lemma 1. *The approximation error, which arises from the difference between (15) and (17), satisfies the following equality:*

$$\Delta(\mathbf{z},\boldsymbol{\eta},u)-u_{ad}=\tilde{\mathbf{w}}^T\left(\boldsymbol{\sigma}-\dot{\boldsymbol{\sigma}}\mathbf{V}^T\zeta\right)+\mathrm{tr}\left(\tilde{\mathbf{V}}^T\zeta\mathbf{w}^T\dot{\boldsymbol{\sigma}}\right)+\delta(t)\,, \tag{18}$$

where

$$\begin{aligned}&\left|\delta(t)\right|\le d_0+d_1\left\|\tilde{\mathbf{w}}\right\|\left\|\zeta\right\|+d_2\left\|\tilde{\mathbf{V}}\right\|_{\mathrm{F}}\left\|\zeta\right\|,\\&\tilde{\mathbf{w}}=\mathbf{w}^*-\mathbf{w},\quad \tilde{\mathbf{V}}=\mathbf{V}^*-\mathbf{V},\end{aligned} \tag{19}$$

and $\dot{\boldsymbol{\sigma}}\in R^{m\times m}$ *is the derivative of* $\boldsymbol{\sigma}$ *with respect to the input signals of all neurons in the hidden layer of NN, and* d_i $(i=0,1,2)$ *are positive constants.*

Proof : See [8] and [13].

The adaptation rules for the weights of neuro-adaptive control term u_{ad}, defined in (17), is proposed as

$$\begin{aligned}\dot{\mathbf{w}}&=\gamma_{\mathbf{w}}\left(\rho\left(\boldsymbol{\sigma}-\dot{\boldsymbol{\sigma}}\mathbf{V}^T\zeta\right)-k_w\mathbf{w}\right)\\\dot{\mathbf{V}}&=\gamma_{\mathbf{V}}\left(\rho\zeta\mathbf{w}^T\dot{\boldsymbol{\sigma}}-k_V\mathbf{V}\right)\end{aligned} \tag{20}$$

where ρ is introduce in (8), $\gamma_{\mathbf{w}}$ and $\gamma_{\mathbf{V}}$ are learning constants, and k_w and k_V are σ -modification gains [17, 18].

Using (15)-(17) and the fact that $\left|\sigma_i\left(\cdot\right)\right|\le 1$, the following conservative upper bound of the approximation error can be calculated

$$\begin{aligned}\left|\Delta(\mathbf{x},u)-u_{ad}\right|&=\left|\mathbf{w}^*\boldsymbol{\sigma}\left(\mathbf{V}^{*T}\zeta\right)+\varepsilon_1-\mathbf{w}\boldsymbol{\sigma}\left(\mathbf{V}^T\zeta\right)\right|\\&\le 2mM_{\mathbf{w}}+\varepsilon_{1M}\,,\end{aligned} \tag{21}$$

3.4. The Adaptive Robustifying Control Term

The neuro-adaptive control term u_{ad}, with adaptation rules, given in (20), cannot provide exact solution to the matched uncertainty and there still exists an approximation error $\delta(t)$. In order to compensate for this error, an adaptive robustifying term u_R is proposed. Using (16) and (19) the upper bound of the approximation error can be calculated as

$$\begin{aligned}\left|\delta\right|&\le d_0+d_1\left\|\mathbf{w}^*-\mathbf{w}\right\|\left|\zeta\right|+d_2\left\|\mathbf{V}^*-\mathbf{V}\right\|_F\left|\zeta\right|\\&\le d_0+d_1M_{\mathbf{w}}\left|\zeta\right|+d_1\left\|\mathbf{w}\right\|\left|\zeta\right|+d_2M_{\mathbf{V}}\left\|\zeta\right\|+d_2\left\|\mathbf{V}\right\|_{\mathrm{F}}\left\|\zeta\right\|\\&\le\varphi^*\left(1+\left|\zeta\right|\left(1+\left\|\mathbf{V}\right\|_{\mathrm{F}}+\left\|\mathbf{w}\right\|\right)\right)=\varphi^*\chi\end{aligned} \tag{22}$$

where $\varphi^*=\max\left\{d_0,d_1,d_2,d_1M_{\mathbf{w}},d_2M_{\mathbf{V}}\right\}$ and

$$\chi\equiv 1+\left|\zeta\right|\left(1+\left\|\mathbf{V}\right\|_{\mathrm{F}}+\left\|\mathbf{w}\right\|\right)$$

Hence, $\delta(t)$ can be limited to a multiplication of the known function χ and an unknown gain φ^*. The following adaptive robustifying term is introduced:

$$u_R=-\chi\,\varphi\,sign\left(\rho\right), \tag{23}$$

with the following adaptation rule:

$$\dot{\varphi}=\gamma_\varphi\chi\left|\rho\right|, \tag{24}$$

where γ_φ is the learning constant and φ denotes estimation of the unknown gain φ^*.

Note that, because of the universal approximation property of NNs, the approximation error $\delta(t)$ is bounded, so it is always possible to find a positive constant U_M such that

$$\left|u_R\right|\le U_M \tag{25}$$

3.5. Observer Design

For realization of weight adaptation rules, given in (20) and (24), (i.e. dependent only on the available data), a linear state estimator is proposed as

$$\dot{\hat{\boldsymbol{\xi}}}=\mathbf{A}\hat{\boldsymbol{\xi}}+\mathbf{b}u_L+K_o\left(y-\mathbf{c}\hat{\boldsymbol{\xi}}\right), \tag{26}$$

where K_o is selected such that $\mathbf{A}-K_o\mathbf{c}$ is stable. The stability of $\mathbf{A}-K_o\mathbf{c}$ assures existence of the solution $\mathbf{P}_2=\mathbf{P}_2^T>0$ for the following Riccati equation for some $\mathbf{Q}_2=\mathbf{Q}_2^T>0$:

$$\begin{aligned}\mathbf{P}_2\left(\mathbf{A}-K_o\mathbf{c}\right)+\left(\mathbf{A}-K_o\mathbf{c}\right)^T\mathbf{P}_2=&-\mathbf{Q}_2-2K_c^TK_c\\&-\mathbf{c}^TK_o{}^T\mathbf{P}_1\mathbf{Q}_1^{-1}\mathbf{P}_1K_o\mathbf{c}\end{aligned} \tag{27}$$

Let the nonlinear system (5) be equipped with the observer (26), and define

$$\mathbf{E}=\left[\boldsymbol{\xi}^T,\tilde{\boldsymbol{\xi}}^T\right]^T \tag{28}$$

where $\tilde{\boldsymbol{\xi}}=\hat{\boldsymbol{\xi}}-\boldsymbol{\xi}$. Then, the augmented system dynamic can be described as

$$\dot{\mathbf{E}} = \underbrace{\begin{bmatrix} \mathbf{A}-\mathbf{b}K_c & -\mathbf{b}K_c \\ 0 & \mathbf{A}_{cl}-K_o\mathbf{c} \end{bmatrix}}_{\mathbf{A}_0}\mathbf{E} + \underbrace{\begin{bmatrix} \mathbf{b} \\ 0 \end{bmatrix}}_{\mathbf{b}_0}\left(u_L + K_c\hat{\boldsymbol{\xi}} + \beta\right) - \underbrace{\begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix}}_{\mathbf{b}_1}\beta + \underbrace{\begin{bmatrix} \mathbf{H} \\ -\mathbf{H} \end{bmatrix}}_{\mathbf{H}_1}\boldsymbol{\Delta}_{\boldsymbol{\eta}} \tag{29}$$

where $\beta = \Delta - u_{ad} - u_R$.

The available output signal, defined in (7), can be represented as

$$\rho = K_c\hat{\boldsymbol{\xi}} = \underbrace{\begin{bmatrix} K_c & K_c \end{bmatrix}}_{\mathbf{c}_0}\mathbf{E}. \tag{30}$$

Moreover, using (3) and (28) the following upper bound can be represented as

$$\left\|\boldsymbol{\Delta}_{\boldsymbol{\eta}}\right\| \le \alpha_0 + \alpha_1\left\|\mathbf{E}\right\|, \tag{31}$$

where $\alpha_0 = c_0$ and $\alpha_1 = c_1 + c_2$

4. STABILITY ANALYSIS

Theorem: *Consider the linear controller (7), the neuro-adaptive control u_{ad} in (17) with the adaptation rules (20) and the robustifing control term u_R in (23). Then, the error signals $\mathbf{E}$, $\tilde{\mathbf{w}}$, and $\tilde{\mathbf{V}}$ in the closed-loop system (29) are uniformly ultimately bounded.*

Proof: Define the Lyapunov function as

$$L = \frac{1}{2}\left(\hat{\boldsymbol{\xi}}^T\mathbf{P}_1\hat{\boldsymbol{\xi}} + \tilde{\boldsymbol{\xi}}^T\mathbf{P}_2\tilde{\boldsymbol{\xi}}\right) + \frac{1}{2\gamma_{\mathbf{w}}}\|\tilde{\mathbf{w}}\|^2 + \frac{1}{2\gamma_V}\|\tilde{\mathbf{V}}\|_F^2 + \frac{1}{2\gamma_\varphi}|\tilde{\varphi}|^2.$$

Using (28), this Lyapunov function can be represented as

$$L = \frac{1}{2}\mathbf{E}^T\underbrace{\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_1 \\ \mathbf{P}_1 & \mathbf{P}_1+\mathbf{P}_2 \end{bmatrix}}_{\mathbf{P}}\mathbf{E} + \frac{1}{2\gamma_{\mathbf{w}}}\|\tilde{\mathbf{w}}\|^2 + \frac{1}{2\gamma_V}\|\tilde{\mathbf{V}}\|_F^2 + \frac{1}{2\gamma_\varphi}|\tilde{\varphi}|^2 \tag{32}$$

Define $\tilde{\varphi} = \varphi^* - \varphi$, where φ^* is the ideal gain of their corresponding estimated value φ, respectively, and $\mathbf{w}^*$ and $\mathbf{V}^*$ are the ideal constant weights, defined in (16). Then, from (19) $\dot{\mathbf{w}} = -\dot{\tilde{\mathbf{w}}}$ and $\dot{\mathbf{V}} = -\dot{\tilde{\mathbf{V}}}$. Using (29) and (32), the time-derivative of L becomes

$$\begin{aligned}\dot{L} &= -\frac{1}{2}\mathbf{E}^{\mathbf{T}}\left(\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_1 \\ \mathbf{P}_1 & \mathbf{P}_1+\mathbf{P}_2 \end{bmatrix}\begin{bmatrix} \mathbf{A}-\mathbf{b}K_c & -\mathbf{b}K_c \\ 0 & \mathbf{A}-K_o\mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{A}-\mathbf{b}K_c & -\mathbf{b}K_c \\ 0 & \mathbf{A}-K_o\mathbf{c} \end{bmatrix}^T\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_1 \\ \mathbf{P}_1 & \mathbf{P}_1+\mathbf{P}_2 \end{bmatrix}\right)\mathbf{E} \\ &+ \mathbf{E}^T\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_1 \\ \mathbf{P}_1 & \mathbf{P}_1+\mathbf{P}_2 \end{bmatrix}\mathbf{b}_0\left(\beta + u_L + K_c\hat{\boldsymbol{\xi}}\right) - \mathbf{E}^T\mathbf{P}\mathbf{b}_1\beta \\ &+ \mathbf{E}^T\mathbf{P}\mathbf{H}_1\boldsymbol{\Delta}_{\boldsymbol{\eta}} - \frac{1}{\gamma_w}\tilde{\mathbf{w}}^T\dot{\mathbf{w}} - \frac{1}{\gamma_V}\operatorname{tr}\left(\tilde{\mathbf{V}}^T\dot{\mathbf{V}}\right) - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi}\end{aligned}$$

Using (8) and (30), it yields

$$\mathbf{E}^T\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_1 \\ \mathbf{P}_1 & \mathbf{P}_1+\mathbf{P}_2 \end{bmatrix}\mathbf{b}_0 = \mathbf{E}^T\begin{bmatrix} \mathbf{P}_1\mathbf{b} \\ \mathbf{P}_1\mathbf{b} \end{bmatrix} = \mathbf{E}^T\begin{bmatrix} K_c^T \\ K_c^T \end{bmatrix} = \rho \tag{33}$$

Substituting from (33) and (7) and using (6) and (27) it gives

$$\begin{aligned}\dot{L} &= -\frac{1}{2}\mathbf{E}^{\mathbf{T}}\underbrace{\begin{bmatrix} \mathbf{Q}_1 & \left(\mathbf{Q}_1 + \mathbf{P}_1K_o\mathbf{c}\right) \\ \left(\mathbf{Q}_1 + \mathbf{c}^TK_o^T\mathbf{P}_1\right) & \mathbf{Q}_2 + \left(\mathbf{Q}_1 + \mathbf{c}^TK_o^T\mathbf{P}_1\right)\mathbf{Q}_1^{-1}\left(\mathbf{Q}_1 + \mathbf{P}_1K_o\mathbf{c}\right) \end{bmatrix}}_{\mathbf{Q}}\mathbf{E} \\ &+ \rho\,\beta - \mathbf{E}^T\mathbf{P}\mathbf{b}_1\beta + \mathbf{E}^T\mathbf{P}\mathbf{H}_1\boldsymbol{\Delta}_{\boldsymbol{\eta}} - \frac{1}{\gamma_w}\tilde{\mathbf{w}}^T\dot{\mathbf{w}} - \frac{1}{\gamma_V}\operatorname{tr}\left(\tilde{\mathbf{V}}^T\dot{\mathbf{V}}\right) - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi}\end{aligned}$$

since $\mathbf{Q}_1 = \mathbf{Q}_1^T > 0$ and $\mathbf{Q}_2 = \mathbf{Q}_2^T > 0$, it is easy to conclude that the matrix $\mathbf{Q}$ is positive definite. Substituting $\beta = \tilde{\mathbf{w}}^T\left(\boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}\mathbf{V}^T\boldsymbol{\zeta}\right) + \operatorname{tr}\left(\tilde{\mathbf{V}}^T\boldsymbol{\zeta}\mathbf{w}^T\dot{\boldsymbol{\sigma}}\right) + \delta - u_R$ from (18) and (29), results in

$$\begin{aligned}\dot{L} &= -\frac{1}{2}\mathbf{E}^{\mathbf{T}}\mathbf{Q}\mathbf{E} + \rho\left(\tilde{\mathbf{w}}^T\left(\boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}\mathbf{V}^T\boldsymbol{\zeta}\right) + \operatorname{tr}\left(\tilde{\mathbf{V}}^T\boldsymbol{\zeta}\mathbf{w}^T\dot{\boldsymbol{\sigma}}\right)\right) - \frac{1}{\gamma_w}\tilde{\mathbf{w}}^T\dot{\mathbf{w}} \\ &- \frac{1}{\gamma_V}\operatorname{tr}\left(\tilde{\mathbf{V}}^T\dot{\mathbf{V}}\right) + \rho\left(\delta - u_R\right) - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi} - \mathbf{E}^T\mathbf{P}\mathbf{b}_1\beta + \mathbf{E}^T\mathbf{P}\mathbf{H}_1\boldsymbol{\Delta}_{\boldsymbol{\eta}}\end{aligned}$$

Using the bounds (22) and (31), and the robustifying control (23) yields

$$\begin{aligned}\dot{L} &\le -\frac{1}{2}Q_m\|\mathbf{E}\|^2 + \tilde{\mathbf{w}}^T\left(\rho\,\boldsymbol{\psi} - k_1\mathbf{w} - \frac{1}{\gamma_w}\dot{\mathbf{w}}\right) + k_1\tilde{\mathbf{w}}^T\mathbf{w} + k_2\operatorname{tr}\left(\tilde{\mathbf{V}}^T\mathbf{V}\right) \\ &+ \operatorname{tr}\left(\tilde{\mathbf{V}}^T\left(\rho\boldsymbol{\Psi} - k_2\mathbf{V} - \frac{1}{\gamma_V}\dot{\mathbf{V}}\right)\right) + |\rho|\left(\varphi^* - \varphi\right)\chi - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi} \\ &+ \|\mathbf{E}\|\|\mathbf{P}\mathbf{b}_1\|\beta_M + \|\mathbf{E}\|\|\mathbf{P}\mathbf{H}_1\|\left(\alpha_0 + \alpha_1\|\mathbf{E}\|\right)\end{aligned} \tag{34}$$

where from (21) and (25) $\beta_M := 2mM_w + \varepsilon_{1M} + U_M$ and Q_m denotes the smallest eigenvalue of $\mathbf{Q}$. By applying the adaptation rules (20), it gives

$$\begin{aligned}\dot{L} &\le -\frac{1}{2}Q_m\|\mathbf{E}\|^2 - k_1\|\tilde{\mathbf{w}}\|^2 + k_1M_w\|\tilde{\mathbf{w}}\| - k_2\|\tilde{\mathbf{V}}\|^2 + k_2M_V\|\tilde{\mathbf{V}}\| \\ &+ \tilde{\varphi}\left(|\rho|\chi - \frac{1}{\gamma_\varphi}\dot{\varphi}\right) + \|\mathbf{E}\|\|\mathbf{P}\mathbf{b}_1\|\beta_M + \|\mathbf{E}\|\|\mathbf{P}\mathbf{H}_1\|\left(\alpha_0 + \alpha_1\|\mathbf{E}\|\right)\end{aligned}$$

Using the learning rules (24) and completing the square terms, yields

$$\dot{L} \le -A_E\|\mathbf{E}\|^2 - \left(k_1 - 1\right)\|\tilde{\mathbf{w}}\|^2 - \left(k_2 - 1\right)\|\tilde{\mathbf{V}}\|^2 + R \tag{35}$$

where

$$A_E = \left(\frac{1}{2}Q_m - \alpha_1\|\mathbf{P}\mathbf{H}_1\| - 1\right)$$

$$R = \frac{\left(k_1M_w\right)^2}{4} + \frac{\left(k_2M_V\right)^2}{4} + \left(\frac{\alpha_0\|\mathbf{P}\mathbf{H}_1\| + \beta_M\|\mathbf{P}\mathbf{b}_1\|}{2}\right)^2$$

Let α_1 be such that the matrix $\mathbf{Q}$ can be find to ensure

$$Q_m > 2\alpha_1\|\mathbf{P}\mathbf{H}_1\| + 2, \tag{36}$$

and let $k_2 > 1$ and $k_1 > 1$, and define the following compact sets around the origin:

$$\Omega = \left\{\left(\mathbf{E}, \tilde{\mathbf{w}}, \tilde{\mathbf{V}}\right) \middle| A_E\|\mathbf{E}\|^2 + A_w\|\tilde{\mathbf{w}}\|^2 + A_V\|\tilde{\mathbf{V}}\|^2 \le R\right\}$$

$$\Omega_E = \left\{\mathbf{E} \middle| \|\mathbf{E}\| \le \sqrt{R/A_E}\right\}$$

Equation (35) shows that $\dot{L} < 0$ once the errors are outside the compact set Ω. Hence, according to the standard Lyapunov theorem extension [19], the error trajectories $\mathbf{E}$, $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{V}}$ are ultimatelybounded.□

Remark1: From (35) it can be seen that $\dot{L}$ is strictly negative as long as $\mathbf{E}$ is outside the compact set Ω_E .Therefore, there exists a constant time T such that for $t > T$, the error $\mathbf{E}$ converge to Ω_E [5]. This means that $\|\boldsymbol{\xi}\| \le \sqrt{R/A_E} = \varepsilon_E$ and consequently $\|\mathbf{z}\| \le \varepsilon_E$ and $\|\boldsymbol{\eta}\| \le \varepsilon_E$. Figure 1 shows the block diagram of the proposed control method.

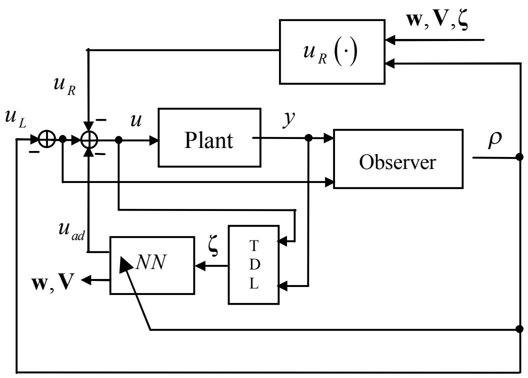

Figure 1. *Block diagram of the proposed control method.*

Remark 2. As Eq. (35) shows, the unmatched uncertainties, and the NN reconstruction error embodied in the constants α_0 and β_M increase the error bound. Note that, since u_{ad} and u_R are designed to cancel out Δ, the upper bound β_M defined in (34) is very conservative, and in practice the real bound would be much smaller.

5. SIMULATION EXAMPLES

The proposed controller in this paper is applied to stabilize the TORA system [10], [14]. This system, depicted in Figure 2, is described by the following equations:

$$(M + m)\ddot{x} + m\,l(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = -k\,x$$
$$(J + m\,l^2)\ddot{\theta} + m\,l\cos\theta\,\ddot{x} = \tau,$$

where θ is the angle of rotation, x is the translational displacement, and τ is the control torque. The positive constants *k*, *l*, *J*, *M*, and *m* denote the spring stiffness, the radius of rotation, the moment of inertia, the mass of the cart, and the eccentric mass, respectively. Define the states and the input variables as

$$\eta_1 = x + \frac{m\,l}{M + m}\sin\theta, \quad \eta_2 = \dot{x} + \frac{m\,l}{M + m}\dot{\theta}\cos\theta,$$
$$z_1 = \theta, \qquad z_2 = \dot{\theta}, \qquad u = \tau.$$

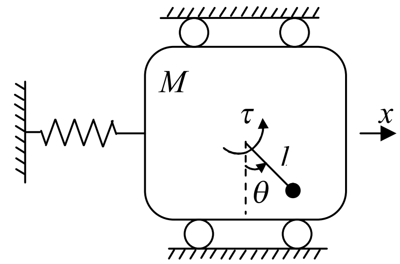

Figure 2. *A translational oscillator with a rotational actuator*

In these coordinates, the system can be described by a set of equations in the form of (2) as

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = k\,\alpha_1\phi^{-1}(z_1)\cos z_1\eta_1 - \alpha_1^2\alpha_2\phi^{-1}(z_1)\sin z_1\cos z_1 \\ \qquad - m^2 l^2\phi^{-1}(z_1)z_2^2\sin z_1\cos z_1 + (M + m)\phi^{-1}(z_1)\,u \\ \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = -\alpha_2\eta_1 + \alpha_3\sin z_1, \end{cases}$$

where

$$\phi(z_1) = (M + m)(J + m\,l^2) - m^2 l^2\cos^2 z_1,$$
$$\alpha_1 = m\,l, \quad \alpha_2 = \frac{k}{M + m} \quad \text{and} \quad \alpha_3 = \frac{k\,m\,l}{(M + m)^2}.$$

The zero dynamics of this system are

$$\begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = -\alpha_2\eta_1. \end{cases}$$

It is easy to check that these zero dynamics are unstable, so the system is non-minimum phase [14]. The linearized version of the TORA system is

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\alpha_1^2\alpha_2\phi^{-1}(0)z_1 + k\,\alpha_1\phi^{-1}(0)\eta_1 + (M + m)\phi^{-1}(0)\,u \\ \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = -\alpha_2\eta_1 + \alpha_3 z_1 \end{cases}$$

Hence, the matched and the unmatched uncertainties can be represented as

$$\Delta(\boldsymbol{\eta}, \mathbf{z}, u) = \frac{1}{(M + m)\phi^{-1}(0)}\Big\{\Big[(M + m)\big(\phi^{-1}(z_1) - \phi^{-1}(0)\big)\Big]u$$
$$+ k\,\alpha_1\big[\phi^{-1}(z_1)\cos z_1 - \phi^{-1}(0)\big]\eta_1 - m^2 l^2\phi^{-1}(z_1)z_2^2\sin z_1\cos z_1$$
$$-\alpha_1^2\alpha_2\big[\phi^{-1}(z_1)\sin z_1\cos z_1 - \phi^{-1}(0)z_1\big]\Big\}$$
$$\Delta_{\boldsymbol{\eta}}(\boldsymbol{\eta}, z_1) = \alpha_3(\sin z_1 - z_1)$$

Note that Assumption 1 is satisfied; that is

$$\frac{\partial\,\mathrm{f}(z, \eta, u)}{\partial u} = (M + m)\phi^{-1}(z_1) > 0$$

Moreover, it is easy to show that $\phi(0) \le \phi(z_1)$ so the following inequality, which verified Assumption 3, is always satisfied:

$$(M + m)\phi^{-1}(0) \ge 0.5(M + m)\phi^{-1}(z_1)$$

The NN is of MLP type and has five neurons in the hidden layer with tangent hyperbolic activation functions. The weights are initialised randomly with small numbers. The input vector to the NN for $n_1 = 4 \ge n$ is

$$\boldsymbol{\xi} = [1, y(t), y(t - T_d), y(t - 2T_d), y(t - 3T_d), u(t), u(t - T_d)]^T$$

with $T_d = 10$ msec . Also, the learning constants are selected as $\gamma_{\mathbf{w}} = \gamma_{\mathbf{V}} = 0.2$, $\gamma_\varphi = 1$ and $k_1 = k_2 = 5$

In addition, the closed-loop system is simulated using the following parameters:

$$J = 0.0002175 \text{ kg/m}^2,\ M = 1.3608 \text{ kg},\ m = 0.096 \text{ kg},$$
$$l = 0.0592 \text{ m},\ k = 186.3 \text{ N/m}$$

The initial states are

$$\eta_1(0) = 0.025\,,\ \eta_2(0) = z_1(0) = z_2(0) = 0\,.$$

For the sake of comparison, the simulations are carried out using the same parameters and the same initial conditions as in the reference [10], where the remarkable back-stepping approach is employed. Simulation results show that the response of the closed-loop system using the proposed controller is nearly the same as the response of the back-stepping control method (Figure 3). The unmatched uncertainty cancellation, norm of NN's weights, and estimation errors are shown in Figure 4.

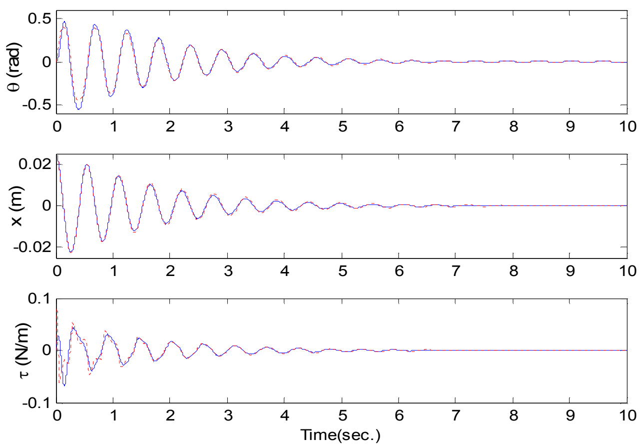

Figure 3. Response of the TORA system. Dotted line: back-stepping based controller, solid line: the proposed controller.

-

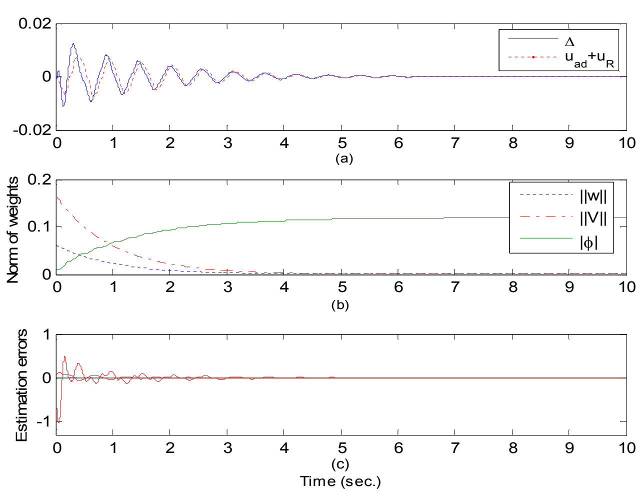

Figure 4. Response of the TORA system. (a) matched uncertainty cancellation (b) norm of weigth (c) errors of estimation $\tilde{\xi}$

6. CONCLUSIONS

A direct adaptive output-feedback stabilization method for nonlinear non-minimum phase systems was proposed in this paper. The proposed method relays on state estimation. The approach can be applied to uncertain non-affine nonlinear systems, from which a linear approximation can be derived. The ultimate boundedness of all states including internal dynamics and the NN weights was shown using the Lyapunov direct method. Simulation results, performed on the TORA system, showed good performance as compared to the back-stepping control method.

REFERENCES

[1] Isidori, A., *Nonlinear Control Systems.* Berlin: Springer-Verlag, 1995.

[2] Marino, R., and Tomei, P., *Nonlinear Adaptive Design: Geometric, Adaptive and Robust,* Prentice-Hall, London, 1995.

[3] Khalil, H., *Nonlinear Systems.* Englewood Cliffs, NJ: Prentice-Hall, 2002.

[4] Krstic, M., Kanellakopoulos, I., and Kokotovic, P., *Nonlinear and Adaptive Control Design,* John Wiley & Sons, New York, 1995.

[5] Ge, S. S. and Zhang, T., "Neural network control of non-affine nonlinear with zero dynamics by state and output feedback," *IEEE Trans. Neural Networks*, *Vol. 14, pp. 900-918, 2003.*

[6] Seshagiri, S., and Khalil, H., "Output feedback control of nonlinear systems using RBF neural networks," *IEEE Trans. Neural Nets., Vol. 11, pp. 69-79, 2000.*

[7] Hovakimyan, N., Nardi, F., and Calise, A. J., "A novel error observer based adaptive output feedback approach for control of uncertain systems," *IEEE Trans. Automat. Contr., Vol. 47, pp. 1310 – 1314, 2002.*

[8] Hoseini, S. M., Farrokhi, M., Koshkouei, A. J., "Robust adaptive control of uncertain nonlinear systems using neural networks", *Int. J. Control., in Press.*

[9] Isidori, A., "A tool for semiglobal stabilization of uncertain non-minimum phase nonlinear systems via output feedback," *IEEE Trans. Automat. Contr., Vol. 45, pp. 1817-1827, 2000.*

[10] Karagiannis, D., Jiang, Z. P., Ortega, R., and Astolfi, A.," Output-feedback stabilization of a class of uncertain non-minimum phase nonlinear systems," *Automatic*, *Vol. 41, pp. 1609-1615, 2005.*

[11] Ding, Z.,"Semi global stabilization of a class of non-minimum phase nonlinear output-feedback system," *IEE Proc. Contr. Theory and Appl., Vol. 152, pp. 460-464, 2005.*

[12] Chen, S. C. and Chen, W. L., "Output regulation of nonlinear uncertain system with non-minimum phase via enhances RBFN controller," *IEEE Trans. Sys. Man Cyber. PART A, Vol. 33, 265-270, 2003.*

[13] Hoseini, S. M., and Farrokhi, M., "Adaptive stabilization of non-minimum phase nonlinear systems using neural networks," *IFAC Workshop on Adaptation and Learning in Control and Signal Processing. Saint Petersburg, Russia, 2007.*

[14] Lee, C. H., "Stabilization of nonlinear nonminimum phase system: adaptive parallel approach using recurrent fuzzy neural network," *IEEE Trans. on System Man and Cybernetics Part B. Vol. 34, 1075-1088, 2004.*

[15] Lavertsky, E., Calise, A. J. and Hovakimyan, N., "Upper bounds for approximation of continuous-time dynamicsusing delayed outputs and feedforward neural networks," *IEEE Trans. Automat. Contr., Vol. 48, pp.1606-1610, 2003.*

[16] Hornik, K., Stinchombe, M. and White, H.,"Multilayer feed-forward networks are universal approximatros," *Neural Networks,Vol. 2, 359-366, 1989.*

[17] Ioannou, P. A. and Kokotovic, P. V., *Adaptive Systems with Reduced Models*, Springer-Verlag, New York, 1983.

[18] Lewis, F., Yesildirek, A., and Liu, K., "Multilayer neural-net robot controller with guaranteed tracking performance," *IEEE Trans. Neural Nets., Vol. 7 pp., 1996.*

[19] Narendra, K. S., and Annaswamy, A. M., "A new adaptive law for robust adaptation without persistence excitation," *IEEE Trans. Automat. Contr., vol. 32, pp. 134-145, 1987.*