Delay-Independent Robust Absolute Stability Criteria of Uncertain Lur'e Systems with Multiple Time-Delays

Ali Kazemy
Department of Electrical Engineering
Iran University of Science and Technology
Tehran, Iran
ali_kazemy@ee.iust.ac.ir

Mohammad Farrokhi
Department of Electrical Engineering
Iran University of Science and Technology
Tehran, Iran
farrokhi@iust.ac.ir

Abstract— **The delay-independent robust absolute stability of uncertain Lur'e systems with multiple time-delays for both the time-varying and time-invariant sector bounded nonlinearities is considered in this paper. Based on the Lyapunov-Krasovskii stability theory and the linear matrix inequality (LMI) approach, some delay-independent sufficient conditions for the robust absolute stability are derived and are expressed as the feasibility problem of a certain LMI system. Finally, some examples are given to illustrate the proposed results.**

Keywords- Lur'e system; Multiple time-delay, Robust stability; Delay-independent stability; LMI.

I. Introduction

It is well known that many nonlinear control systems can be represented as feedback connection of a linear dynamical system and a nonlinear element, where the nonlinear element satisfies certain sector constraints [1]. Based on these classes of nonlinear systems, the notion of absolute stability was introduced by Lur'e [2]; since then, the problem of the absolute stability of Lur'e system has been widely studied for several decades [3--6].

Time-delays appear in many real engineering systems. When time-delays appear in a dynamic system, behavior analysis of such systems becomes more complex. The existence of time-delays is often a source of instability and performance degradation [7]. The existing stability criteria for time-delay systems can be classified into two types: the delay-independent and the delay-dependent [7]. Delay-independent conditions are useful for the systems that are stable for any value of time-delays, whereas delay-dependent conditions provide stability for systems that are stable for limited time-delays.

For the case of time-delayed Lur'e systems without uncertainty, some remarkable results have been developed in literatures. In [8], the problem of delay-independent absolute stability has been considered. Delay-dependent absolute stability conditions of Lur'e systems with multiple time delay and nonlinearities have been developed in [9--11]. In addition, some researchers focus on Lur'e systems with time-varying delays [12, 13].

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Recently, practical considerations such as model uncertainties and time delays are considered for stability analysis of Lur'e systems [14--19].

This paper discusses the problem of delay-independent absolute stability of uncertain Lur'e systems with multiple time-delays. Based on the Lyapunov-Krasovskii stability theory and the Linear Matrix Inequality (LMI) approach, some delay-independent sufficient conditions for the robust absolute stability are derived and expressed as the feasibility problem of certain LMI systems. Both cases with the time-varying and time-invariant nonlinearities are considered. Finally, some examples are given to validate the results.

Notations. Through this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices. $\mathbf{P}>0$ means that $\mathbf{P}$ is a real positive definite symmetric matrix. $\mathbb{C}[-h,0]$ denotes space of continuous functions defined on $[-h,0]$, and $\mathbf{I}$ is the Identity matrix with appropriate dimensions. $\operatorname{diag}\{W_1, \dots, W_m\}$ refers to a real matrix with diagonal elements $W_1, \dots, W$. $\mathbf{A}^T$ denotes the transpose of the real matrix $\mathbf{A}$. Symmetric terms in a symmetric matrix are denoted by $*$.

II. Problem Formulation and Preliminaries

Consider the uncertain Lur'e system with multiple time-delays as

$$\begin{cases} \dot{\mathbf{x}}(t)=\bar{\mathbf{A}}\mathbf{x}(t)+\sum_{i=1}^{m}\bar{\mathbf{B}}_i\mathbf{x}(t-h_i)+\bar{\mathbf{D}}\boldsymbol{\omega}(t), \\ \mathbf{z}(t)=\mathbf{M}\mathbf{x}(t)+\sum_{i=1}^{m}\mathbf{N}_i\mathbf{x}(t-h_i), \\ \boldsymbol{\omega}(t)=-\boldsymbol{\varphi}(t,\mathbf{z}(t)), \\ \mathbf{x}(t)=\boldsymbol{\phi}(t), \qquad \forall t\in[-\max_{1\le i\le m}\{h_i\},0], \end{cases} \tag{1}$$

where $\mathbf{x}(t)\in\mathbb{R}^n$ denotes the state vector, $\boldsymbol{\omega}(t)\in\mathbb{R}^p$ is the input, $\mathbf{z}(t)\in\mathbb{R}^p$ is the output, and $\boldsymbol{\phi}(t)\in\mathbb{C}([h,0],\mathbb{R}^n)$ is a continuous vector-valued initial function; $h_i\ge 0$ $(i=1,\dots,m)$ are time-delays, and $\mathbf{M}$ and $\mathbf{N}_i$ $(i=1,\dots,m)\in\mathbb{R}^{p\times n}$ are known real constant matrices; $\bar{\mathbf{A}}$, $\bar{\mathbf{B}}_i$ $(i=1,\dots,m)\in\mathbb{R}^{n\times n}$, and $\bar{\mathbf{D}}\in\mathbb{R}^{n\times p}$ are time-varying matrices with the following structures:

$$\bar{\mathbf{A}} = \mathbf{A} + \Delta\mathbf{A}(t), \bar{\mathbf{D}} = \mathbf{D} + \Delta\mathbf{D}(t),$$
$$\bar{\mathbf{B}}_i = \mathbf{B}_i + \Delta\mathbf{B}_i(t), (i = 1, \ldots, m) \tag{2}$$

where $\mathbf{A}$, $\mathbf{B}_i$ $(i = 1, \cdots, m)$, and $\mathbf{D}$ are known real constant matrices. $\Delta\mathbf{A}(t)$, $\Delta\mathbf{B}_i(t)$ $(i = 1, \cdots, m)$, and $\Delta\mathbf{D}(t)$ are norm bounded parameter uncertainties and are assumed to be of the form

$$\left[\Delta\mathbf{A}(t), \Delta\mathbf{B}_1(t), \cdots, \Delta\mathbf{B}_m(t), \Delta\mathbf{D}(t)\right] = \mathbf{LF}(t)\left[\mathbf{E}, \mathbf{E}_1, \ldots, \mathbf{E}_m, \mathbf{H}\right], \tag{3}$$

where $\mathbf{L}$, $\mathbf{E}$, $\mathbf{E}_i$ $(i = 1, \ldots, m)$, and $\mathbf{H}$ are known real constant matrices with appropriate dimensions and $\mathbf{F}(t) \in \mathbb{R}^{q \times k}$ is the unknown time-varying real matrix satisfying

$$\mathbf{F}^T(t)\mathbf{F}(t) \le \mathbf{I}. \tag{4}$$

The nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t)) \in \mathbb{R}^p$ in (1) is piecewise continuous in t, globally Lipschitz in $\mathbf{z}(t)$, $\boldsymbol{\varphi}(t, 0) = 0$, and satisfies the following sector condition for any $t \ge 0$ and $\mathbf{z}(t) \in \mathbb{R}^p$:

$$\left[\boldsymbol{\varphi}(t, \mathbf{z}(t)) - \mathbf{K}_1\mathbf{z}(t)\right]^T \left[\boldsymbol{\varphi}(t, \mathbf{z}(t)) - \mathbf{K}_2\mathbf{z}(t)\right] \le 0. \tag{5}$$

Such a nonlinear function is said to belong to the sector bound $[\mathbf{K}_1, \mathbf{K}_2]$.

Definition 1. The nonlinear delay system (1) is said to be robustly absolutely stable in the sector $[\mathbf{K}_1, \mathbf{K}_2]$ if it is globally uniformly asymptotically stable for any nonlinear function $\boldsymbol{\varphi}(\mathbf{z}(t))$ satisfying $\boldsymbol{\varphi}(0) = 0$ and (5) for all admissible uncertainties [18].

Lemma 1. [20] For given matrices $\boldsymbol{\Psi} = \boldsymbol{\Psi}^T$, $\mathbf{U}$, and $\mathbf{V}$ with appropriate dimensions, inequality

$$\boldsymbol{\Psi} + \mathbf{UF}(t)\mathbf{V} + \mathbf{V}^T\mathbf{F}^T(t)\mathbf{U}^T < 0, \tag{6}$$

holds for all $\mathbf{F}^T(t)\mathbf{F}(t) \le \mathbf{I}$ if and only if there exists $\delta > 0$ such that

$$\boldsymbol{\Psi} + \delta^{-1}\mathbf{U}\mathbf{U}^T + \delta\mathbf{V}^T\mathbf{V} < 0. \tag{7}$$

Lemma 2. (Schur complement [21]) Let the symmetric matrix $\mathbf{M}$ be partitioned as

$$\mathbf{M} = \begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix},$$

with $\mathbf{X}$ and $\mathbf{Z}$ being symmetric matrices. Then, $\mathbf{M} > 0$ if and only if

$$\begin{cases} \mathbf{Z} > 0, \\ \mathbf{X} - \mathbf{Y}\mathbf{Z}^{-1}\mathbf{Y}^T > 0. \end{cases} \tag{8}$$

Based on these lemmas, the following section will show the main results, proposed in this paper.

III. Main Results

For the first step, consider the case that the nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t))$ belongs to the sector $[0, \mathbf{K}]$. Hence, it must satisfy

$$\boldsymbol{\varphi}^T(t, \mathbf{z}(t))\left[\boldsymbol{\varphi}(t, \mathbf{z}(t)) - \mathbf{Kz}(t)\right] \le 0, \tag{9}$$

where $\mathbf{K} > 0$.

Theorem 1. The nonlinear delay system (1) with the nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t))$ satisfying (9) and $\boldsymbol{\varphi}(t, 0) = 0$, is robustly absolutely stable if there exist scalars $\gamma > 0$, $\delta > 0$, and symmetric matrices $\mathbf{P} > 0$, $\mathbf{Q}_i > 0$ $(i = 1, \ldots, m)$ such that the following LMI holds:

$$\begin{bmatrix} \Gamma & \mathbf{PB}_1 + \delta\mathbf{E}^T\mathbf{E}_1 & \mathbf{PB}_2 + \delta\mathbf{E}^T\mathbf{E}_2 & \cdots & \mathbf{PB}_m + \delta\mathbf{E}^T\mathbf{E}_m & \Psi & \mathbf{PL} \\ * & -\mathbf{Q}_1 + \delta\mathbf{E}_1^T\mathbf{E}_1 & \delta\mathbf{E}_1^T\mathbf{E}_2 & \cdots & \delta\mathbf{E}_1^T\mathbf{E}_m & \Psi_1 & 0 \\ * & * & -\mathbf{Q}_2 + \delta\mathbf{E}_2^T\mathbf{E}_2 & \cdots & \delta\mathbf{E}_2^T\mathbf{E}_m & \Psi_2 & 0 \\ \vdots & & \vdots & \ddots & \vdots & \vdots & 0 \\ * & * & * & \cdots & -\mathbf{Q}_m + \delta\mathbf{E}_m^T\mathbf{E}_m & \Psi_m & 0 \\ * & * & * & & * & \mho & 0 \\ * & * & * & & * & * & -\delta\mathbf{I} \end{bmatrix} < 0 \tag{10}$$

where

$$\Gamma = \mathbf{A}^T\mathbf{P} + \mathbf{PA} + \sum_{i=1}^{m}(\mathbf{Q}_i) + \delta\mathbf{E}^T\mathbf{E},$$

$$\Psi = \mathbf{PD} - \gamma\mathbf{M}^T\mathbf{K}^T + \delta\mathbf{E}^T\mathbf{H}, \mho = -2\gamma\mathbf{I} + \delta\mathbf{H}^T\mathbf{H}.$$

$$\Psi_i = -\gamma\mathbf{N}_i^T\mathbf{K}^T + \delta\mathbf{E}_i^T\mathbf{H}, \ (i = 1, \ldots, m).$$

Proof. Let select the Lyapunov-Krasovskii functional candidate as

$$V(\mathbf{x}_t) = \mathbf{x}^T(t)\mathbf{Px}(t) + \sum_{j=1}^{m}\int_{t-h_j}^{t}\mathbf{x}^T(s)\mathbf{Q}_j\mathbf{x}(s)ds. \tag{11}$$

where $\mathbf{x}_t$ is defined as $\mathbf{x}_t = \mathbf{x}(t + \theta)$, $\theta \in [-\max_{1 \le i \le m}\{h_i\}, 0]$. Taking the time derivative of $V(\mathbf{x}_t)$ yields

$$\dot{V}(\mathbf{x}_t) = 2\mathbf{x}^T(t)\mathbf{P}\dot{\mathbf{x}}(t) + \mathbf{x}^T(t)\left(\sum_{j=1}^{m}\mathbf{Q}_j\right)\mathbf{x}(t) - \sum_{j=1}^{m}\left[\mathbf{x}^T(t - h_j)\mathbf{Q}_j\mathbf{x}(t - h_j)\right]. \tag{12}$$

Using $\boldsymbol{\omega}(t) = -\boldsymbol{\varphi}(t, \mathbf{z}(t))$ as in (1), the sector condition (9) can be written as

$$-\boldsymbol{\omega}^T(t)\left[\boldsymbol{\omega}(t) + \mathbf{Kz}(t)\right] \ge 0 \tag{13}$$

where $\mathbf{z}(t)$ and $\boldsymbol{\omega}(t)$ are defined in (1). Hence, $\dot{V}(\mathbf{x}_t)$ can be expressed as

$$\dot{V}(\mathbf{x}_t) \le 2\mathbf{x}^T(t)\mathbf{P}\dot{\mathbf{x}}(t) + \mathbf{x}^T(t)\left(\sum_{j=1}^{m}\mathbf{Q}_j\right)\mathbf{x}(t)$$
$$-\sum_{j=1}^{m}\left[\mathbf{x}^T(t-h_j)\mathbf{Q}_j\mathbf{x}(t-h_j)\right] \qquad (14)$$
$$-2\gamma\boldsymbol{\omega}^T(t)\left[\boldsymbol{\omega}(t)+\mathbf{K}\mathbf{z}(t)\right],$$

where γ is the same as in (10). Substituting $\dot{\mathbf{x}}(t)$ from (1) into (14) and considering (11)--(14), it is straightforward to show that

$$\dot{V}(\mathbf{x}_t) \le \boldsymbol{\xi}^T(t)\,\boldsymbol{\Xi}\,\boldsymbol{\xi}(t), \qquad (15)$$

where

$$\boldsymbol{\Xi} = \begin{bmatrix} \hat{\Gamma} & \mathbf{P}\bar{\mathbf{B}}_1 & \mathbf{P}\bar{\mathbf{B}}_2 & \cdots & \mathbf{P}\bar{\mathbf{B}}_m & \mathbf{P}\bar{\mathbf{D}}-\gamma\mathbf{M}^T\mathbf{K}^T \\ * & -\mathbf{Q}_1 & 0 & \cdots & 0 & -\gamma\mathbf{N}_1^T\mathbf{K}^T \\ * & * & -\mathbf{Q}_2 & \cdots & 0 & -\gamma\mathbf{N}_2^T\mathbf{K}^T \\ * & * & * & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -\mathbf{Q}_m & -\gamma\mathbf{N}_m^T\mathbf{K}^T \\ * & * & * & & * & -2\gamma\mathbf{I} \end{bmatrix}, \qquad (16)$$

in which

$$\hat{\Gamma} = \bar{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\bar{\mathbf{A}} + \sum_{i=1}^{m}\mathbf{Q}_i$$

$$\boldsymbol{\xi}(t) = \begin{bmatrix}\mathbf{x}^T(t) & \boldsymbol{\xi}_1^T(t) & \omega^T(t)\end{bmatrix}^T,$$

$$\boldsymbol{\xi}_1(t) = \begin{bmatrix}\mathbf{x}^T(t-h_1) & \cdots & \mathbf{x}^T(t-h_m)\end{bmatrix}^T.$$

If it can be shows that $\boldsymbol{\Xi} < 0$ in (15), then $\dot{V}(\mathbf{x}_t) < 0$ and by Definition 1 and the Lyapunov-Krasovskii theorem [22], the considered nonlinear delayed system in (1) is robustly absolutely stable. But matrix $\boldsymbol{\Xi}$ is not in the form of an LMI and should be transformed. This Matrix can be rewritten as

$$\begin{bmatrix} \underset{\sim}{\Gamma} & \mathbf{P}\mathbf{B}_1 & \mathbf{P}\mathbf{B}_2 & \cdots & \mathbf{P}\mathbf{B}_m & \mathbf{P}\mathbf{D}-\gamma\mathbf{M}^T\mathbf{K}^T \\ * & -\mathbf{Q}_1 & 0 & \cdots & 0 & -\gamma\mathbf{N}_1^T\mathbf{K}^T \\ * & * & -\mathbf{Q}_2 & \cdots & 0 & -\gamma\mathbf{N}_2^T\mathbf{K}^T \\ * & * & * & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -\mathbf{Q}_m & -\gamma\mathbf{N}_m^T\mathbf{K}^T \\ * & * & * & & * & -2\gamma\mathbf{I} \end{bmatrix} \qquad (17)$$
$$+\mathbf{U}\mathbf{F}(t)\mathbf{V} + \mathbf{V}^T\mathbf{F}^T(t)\mathbf{U}^T < 0$$

where

$$\underset{\sim}{\Gamma} = \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \sum_{i=1}^{m}\mathbf{Q}_i$$

$$\mathbf{U} = \begin{bmatrix}\mathbf{L}^T\mathbf{P} & \underbrace{0 \ \cdots \ 0}_{m} & 0\end{bmatrix}^T$$

$$\mathbf{V} = \begin{bmatrix}\mathbf{E} & \mathbf{E}_1 & \cdots & \mathbf{E}_m & \mathbf{H}\end{bmatrix}^T.$$

Using Lemma 1 and inequality (4), (17) becomes

$$\begin{bmatrix} \Gamma & \mathbf{P}\mathbf{B}_1+\delta\mathbf{E}^T\mathbf{E}_1 & \mathbf{P}\mathbf{B}_2+\delta\mathbf{E}^T\mathbf{E}_2 & \cdots & \mathbf{P}\mathbf{B}_m+\delta\mathbf{E}^T\mathbf{E}_m & \mathbf{P}\mathbf{D}-\gamma\mathbf{M}^T\mathbf{K}^T \\ * & -\mathbf{Q}_1+\delta\mathbf{E}_1^T\mathbf{E}_1 & \delta\mathbf{E}_1^T\mathbf{E}_2 & \cdots & \delta\mathbf{E}_1^T\mathbf{E}_m & -\gamma\mathbf{N}_1^T\mathbf{K}^T \\ * & * & -\mathbf{Q}_2+\delta\mathbf{E}_2^T\mathbf{E}_2 & \cdots & \delta\mathbf{E}_2^T\mathbf{E}_m & -\gamma\mathbf{N}_2^T\mathbf{K}^T \\ * & * & * & \ddots & \vdots & \vdots \\ * & * & * & \cdots & -\mathbf{Q}_m+\delta\mathbf{E}_m^T\mathbf{E}_m & -\gamma\mathbf{N}_m^T\mathbf{K}^T \\ * & * & * & & * & -2\gamma\mathbf{I} \end{bmatrix} \qquad (18)$$
$$+\delta^{-1}\mathbf{U}\mathbf{U}^T < 0$$

Using Lemma 2, (18) can be transformed into (10). This completes the proof. □

Remark 1. The matrix inequality (10) is linear in the unknown parameters $\mathbf{P} > 0$, $\mathbf{Q}_i > 0 \ (i = 1, \ldots, m)$, and $\gamma > 0$, $\delta > 0$. Therefore, it can be solved using available softwares such MATLAB LMI Toolbox.

Remark 2. Since condition (10) is independent of any delay, stability of the system (1) will be guaranteed for all values of $h_i > 0 \ (i = 1, \ldots, m)$.

Remark 3. Since condition (10) is independent of $\mathbf{F}(t)$, robust absolute stability of the system (1) with the nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t))$ satisfying (9) is guaranteed for all admissible $\mathbf{F}(t)$ satisfying (4).

Next, the problem of robust absolute stability analysis of the nonlinear delayed system (1), with the nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t))$ in the sector $[\mathbf{K}_1, \mathbf{K}_2]$ is considered.

Theorem 2. The nonlinear delay system (1) with the nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t))$ satisfying (5) and $\boldsymbol{\varphi}(t, 0) = 0$ is robustly absolutely stable if there exist scalars $\gamma > 0$, $\delta > 0$, and symmetric matrices $\mathbf{P} > 0$, $\mathbf{Q}_i > 0 \ (i = 1, \ldots, m)$ such that the following LMI holds:

$$\begin{bmatrix} \tilde{\Gamma} & \mathbf{P}\tilde{\mathbf{B}}_1+\delta\tilde{\mathbf{E}}^T\tilde{\mathbf{E}}_1 & \mathbf{P}\tilde{\mathbf{B}}_2+\delta\tilde{\mathbf{E}}^T\tilde{\mathbf{E}}_2 & \cdots & \mathbf{P}\tilde{\mathbf{B}}_m+\delta\tilde{\mathbf{E}}^T\tilde{\mathbf{E}}_m & \tilde{\Psi} & \mathbf{P}\mathbf{L} \\ * & -\mathbf{Q}_1+\delta\tilde{\mathbf{E}}_1^T\tilde{\mathbf{E}}_1 & \delta\tilde{\mathbf{E}}_1^T\tilde{\mathbf{E}}_2 & \cdots & \delta\tilde{\mathbf{E}}_1^T\tilde{\mathbf{E}}_m & \tilde{\Psi}_1 & 0 \\ * & * & -\mathbf{Q}_2+\delta\tilde{\mathbf{E}}_2^T\tilde{\mathbf{E}}_2 & \cdots & \delta\tilde{\mathbf{E}}_2^T\tilde{\mathbf{E}}_m & \tilde{\Psi}_2 & 0 \\ \vdots & & \vdots & \ddots & \vdots & \vdots & 0 \\ * & * & * & \cdots & -\mathbf{Q}_m+\delta\tilde{\mathbf{E}}_m^T\tilde{\mathbf{E}}_m & \tilde{\Psi}_m & 0 \\ * & * & * & & * & \mho & 0 \\ * & * & * & & * & * & -\delta\mathbf{I} \end{bmatrix} < 0 \qquad (19)$$

where

$$\tilde{\Gamma} = \tilde{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}} + \sum_{i=1}^{m}\mathbf{Q}_i + \delta\tilde{\mathbf{E}}^T\tilde{\mathbf{E}},$$
$$\tilde{\Psi} = \mathbf{P}\mathbf{D} - \gamma\mathbf{M}^T\tilde{\mathbf{K}}^T + \delta\tilde{\mathbf{E}}^T\mathbf{H}, \ \tilde{\Psi}_i = -\gamma\mathbf{N}_i^T\tilde{\mathbf{K}}^T + \delta\tilde{\mathbf{E}}_i^T\mathbf{H},$$
$$\tilde{\mathbf{B}}_i = \mathbf{B}_i - \mathbf{D}\mathbf{K}_1\mathbf{N}_i, \ \tilde{\mathbf{E}}_i = \mathbf{E}_i - \mathbf{H}\mathbf{K}_1\mathbf{N}_i, \ (i = 1, \ldots, m),$$
$$\tilde{\mathbf{K}} = \mathbf{K}_2 - \mathbf{K}_1, \ \tilde{\mathbf{A}} = \mathbf{A} - \mathbf{D}\mathbf{K}_1\mathbf{M}, \ \tilde{\mathbf{E}} = \mathbf{E} - \mathbf{H}\mathbf{K}_1\mathbf{M},$$

and $\mho$ is the same as in (10).

Proof: By applying the loop transformation suggested in [1], (1) can be transformed into

$$\begin{cases} \dot{\mathbf{x}}(t) = \left(\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{M}\right)\mathbf{x}(t) + \sum_{i=1}^{m}\left(\bar{\mathbf{B}}_i - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{N}_i\right)\mathbf{x}(t-h_i) + \bar{\mathbf{D}}\tilde{\boldsymbol{\omega}}(t), \\ \mathbf{z}(t) = \mathbf{M}\mathbf{x}(t) + \sum_{i=1}^{m}\mathbf{N}_i\mathbf{x}(t-h_i), \\ \tilde{\boldsymbol{\omega}}(t) = -\tilde{\boldsymbol{\varphi}}(t,\mathbf{z}(t)), \\ \mathbf{x}(t) = \boldsymbol{\phi}(t), \qquad \forall t \in \left[-\max_{1\le i\le m}\{h_i\}, 0\right], \end{cases} \tag{20}$$

where the nonlinear function $\tilde{\boldsymbol{\varphi}}(t,\mathbf{z}(t))$ satisfies

$$\tilde{\boldsymbol{\varphi}}^T\left(t,\mathbf{z}(t)\right)\left[\tilde{\boldsymbol{\varphi}}\left(t,\mathbf{z}(t)\right) - \tilde{\mathbf{K}}\mathbf{z}(t)\right] \le 0, \tag{21}$$

for any $t > 0$. Noting (2) and (3), it can be written

$$\begin{aligned} \bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{M} &= \mathbf{A} + \mathbf{L}\mathbf{F}(t)\mathbf{E} - \left(\mathbf{D} + \mathbf{L}\mathbf{F}(t)\mathbf{H}\right)\mathbf{K}_1\mathbf{M} \\ &= \left(\mathbf{A} - \mathbf{D}\mathbf{K}_1\mathbf{M}\right) + \mathbf{L}\mathbf{F}(t)\left(\mathbf{E} - \mathbf{H}\mathbf{K}_1\mathbf{M}\right) = \tilde{\mathbf{A}} + \mathbf{L}\mathbf{F}(t)\tilde{\mathbf{E}}, \end{aligned} \tag{22}$$

$$\begin{aligned} \bar{\mathbf{B}}_i - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{N}_i &= \left(\mathbf{B}_i + \mathbf{L}\mathbf{F}(t)\mathbf{E}_i\right) - \left(\mathbf{D} + \mathbf{L}\mathbf{F}(t)\mathbf{H}\right)\mathbf{K}_1\mathbf{N}_i \\ &= \left(\mathbf{B}_i - \mathbf{D}\mathbf{K}_1\mathbf{N}_i\right) + \mathbf{L}\mathbf{F}(t)\left(\mathbf{E}_i - \mathbf{H}\mathbf{K}_1\mathbf{N}_i\right) = \tilde{\mathbf{B}}_i + \mathbf{L}\mathbf{F}(t)\tilde{\mathbf{E}}_i. \end{aligned} \tag{23}$$

Hence, Theorem 1 can be applied to the transformed system (20) with new matrices (22) and (23). This completes the proof. □

In the sequel, it is assumed that the nonlinear function in (1) is time-invariant and decentralized. Therefore, $\boldsymbol{\omega}(t)$ in (1) will be $\boldsymbol{\omega}(t) = -\boldsymbol{\varphi}\left(\mathbf{z}(t)\right)$. In this case, the nonlinear function

$$\boldsymbol{\varphi}\left(\mathbf{z}(t)\right) = \left[\varphi_1\left(\mathbf{z}_1(t)\right) \quad \varphi_2\left(\mathbf{z}_2(t)\right) \quad \cdots \quad \varphi_p\left(\mathbf{z}_p(t)\right)\right] \tag{24}$$

satisfies

$$\alpha_i z_i^2(t) \le z_i(t)\varphi_i\left(z_i(t)\right) \le \beta_i z_i^2(t), \quad (i = 1,\cdots,p), \tag{25}$$

for any $t \ge 0$ and $\beta_i \ge \alpha_i > 0$ $(i = 1, \ldots, p)$. The following theorem shows the delay-independent robust absolute stability result of this system.

Theorem 3. The nonlinear delay system (1) with the nonlinear function $\boldsymbol{\varphi}(\mathbf{z}(t))$ satisfying (25) is robustly absolutely stable if there exist scalars $\gamma > 0$, $\delta > 0$, and symmetric matrices $\mathbf{P} > 0$, $\mathbf{Q}_i > 0$, $\mathbf{T}_i > 0$ $(i = 1, \ldots, m)$, diagonal matrices $\mathbf{Y} = \mathrm{diag}\{y_1, y_2, \ldots, y_p\} > 0$ and $\mathbf{C} = \mathrm{diag}\{c_1, c_2, \ldots, c_p\} > 0$ such that the following LMI holds:

$$\begin{bmatrix} \Gamma & \Pi_1 & \Pi_2 & \cdots & \Pi_m & \Psi & 0 & \mathbf{A}^T\Phi & \mathbf{PL} \\ & \Sigma_1 & \theta_{12} & & \theta_{1m} & \Psi_1 & 0 & \mathbf{B}_1^T\Phi & 0 \\ & & \Sigma_2 & \cdots & \theta_{2m} & \Psi_2 & 0 & \mathbf{B}_2^T\Phi & 0 \\ & & & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & \Sigma_m & \Psi_m & 0 & \mathbf{B}_m^T\Phi & 0 \\ & & & & & \mho & -\mathbf{C}\tilde{\mathbf{N}} & \mathbf{D}^T\Phi & -\mathbf{CML} \\ & & & & & & -\tilde{\mathbf{T}} & 0 & 0 \\ & & & & & & & -\Phi & \Phi\mathbf{L} \\ & & & & & & & & -\delta\mathbf{I} \end{bmatrix} < 0 \tag{26}$$

where

$$\Gamma = \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \sum_{i=1}^{m}\mathbf{Q}_i - 2\mathbf{M}^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{M} + \delta\mathbf{E}^T\mathbf{E},$$

$$\Pi_i = \mathbf{P}\mathbf{B}_i - 2\mathbf{M}^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{N}_i + \delta\mathbf{E}^T\mathbf{E}_i, \ \theta_{ij} = -\mathbf{N}_i^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{N}_j + \delta\mathbf{E}_i^T\mathbf{E}_j,$$

$$\Phi = \sum_{i=1}^{m}\left(\mathbf{T}_i\right), \ \Sigma_i = -\mathbf{Q}_i - 2\mathbf{N}_i^T\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{N}_i + \delta\mathbf{E}_i^T\mathbf{E}_i,$$

$$\Psi_i = -\mathbf{B}_i^T\mathbf{M}^T\mathbf{C} - \mathbf{N}_i^T\mathbf{Y}\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\right) + \delta\mathbf{E}_i^T\mathbf{H},$$

$$\left(i \ne j \text{ and } i, j = 1,\cdots,m\right),$$

$$\Psi = \mathbf{P}\mathbf{D} - \mathbf{A}^T\mathbf{M}^T\mathbf{C} - \mathbf{M}^T\mathbf{Y}\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\right) + \delta\mathbf{E}^T\mathbf{H},$$

$$\mho = -\mathbf{C}\mathbf{M}\mathbf{D} - \mathbf{D}^T\mathbf{M}^T\mathbf{C} - 2\mathbf{Y} + \delta\mathbf{H}^T\mathbf{H},$$

$$\tilde{\mathbf{T}} = \mathrm{diag}\{-\mathbf{T}_1, -\mathbf{T}_2, \cdots, -\mathbf{T}_m\}, \ \tilde{\mathbf{N}} = \left[\mathbf{N}_1 \quad \mathbf{N}_2 \quad \cdots \quad \mathbf{N}_m\right],$$

$$\boldsymbol{\alpha} = \mathrm{diag}\{\alpha_1, \alpha_2, \cdots, \alpha_p\}, \ \boldsymbol{\beta} = \mathrm{diag}\{\beta_1, \beta_2, \cdots, \beta_p\}.$$

Proof: Let select the Lyapunov-Krasovskii functional candidate as

$$\hat{V}(\mathbf{x}_t) = V(\mathbf{x}_t) + \sum_{j=1}^{m}\int_{t-h_j}^{t}\dot{\mathbf{x}}^T(s)\mathbf{T}_j\dot{\mathbf{x}}(s)ds + 2\sum_{j=1}^{p}s_j\int_{0}^{z_i(t)}\varphi_i(s)ds \tag{27}$$

where $V(\mathbf{x}_t)$ is defined in (11). Taking the time derivative of $\hat{V}(\mathbf{x}_t)$ yields

$$\begin{aligned} \dot{\hat{V}}(\mathbf{x}_t) &= \dot{V}(\mathbf{x}_t) + \dot{\mathbf{x}}^T(t)\left(\sum_{j=1}^{m}\mathbf{T}_j\right)\dot{\mathbf{x}}(t) \\ &\quad - \sum_{j=1}^{m}\dot{\mathbf{x}}^T\left(t-h_j\right)\mathbf{T}_j\dot{\mathbf{x}}\left(t-h_j\right) + 2\varphi^T\left(\mathbf{z}(t)\right)\mathbf{C}\dot{\mathbf{z}}(t). \end{aligned} \tag{28}$$

Considering that $\mathbf{Y} > 0$, the sector condition (25) can be written as

$$2y_i\left[\varphi_i\left(z_i(t)\right) - \alpha_i z_i(t)\right]\left[\varphi_i\left(z_i(t)\right) - \beta_i z_i(t)\right] \le 0,$$

for $(i = 1, \ldots, p)$. That is

$$\begin{aligned} &2y_i\varphi_i^2\left(z_i(t)\right) - 2y_i\left(\alpha_i + \beta_i\right)\varphi_i\left(z_i(t)\right)z_i(t) \\ &\qquad + 2y_i\alpha_i\beta_i z_i^2(t) \le 0, \quad (i = 1,\cdots,p). \end{aligned}$$

Therefore,

$$-2\boldsymbol{\omega}^T(t)\mathbf{Y}\boldsymbol{\omega}(t) - 2\boldsymbol{\omega}^T(t)\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\right)\mathbf{Y}\mathbf{z}(t) - 2\mathbf{z}^T(t)\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{z}(t) \ge 0, \tag{29}$$

where $\mathbf{z}(t)$ and $\boldsymbol{\omega}(t)$ are defined in (1) and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\mathbf{Y}$ are defined in (26). Hence, $\dot{\hat{V}}(\mathbf{x}_t)$ can be expressed as

$$\begin{aligned} \dot{\hat{V}}(\mathbf{x}_t) &\le \dot{V}(\mathbf{x}_t) + \dot{\mathbf{x}}^T(t)\left(\sum_{j=1}^{m}\mathbf{T}_j\right)\dot{\mathbf{x}}(t) - \sum_{j=1}^{m}\dot{\mathbf{x}}^T\left(t-h_j\right)\mathbf{T}_j\dot{\mathbf{x}}\left(t-h_j\right) \\ &\quad + 2\varphi^T\left(\mathbf{z}(t)\right)\mathbf{C}\dot{\mathbf{z}}(t) - 2\boldsymbol{\omega}^T(t)\mathbf{Y}\boldsymbol{\omega}(t) \\ &\quad - 2\boldsymbol{\omega}^T(t)\left(\boldsymbol{\alpha} + \boldsymbol{\beta}\right)\mathbf{Y}\mathbf{z}(t) - 2\mathbf{z}^T(t)\boldsymbol{\alpha}\boldsymbol{\beta}\mathbf{Y}\mathbf{z}(t). \end{aligned} \tag{30}$$

Substituting $\dot{\mathbf{x}}(t)$ from (1) into (30) and taking the time derivative of $\mathbf{z}(t)$ in (1) and considering (12), it is straightforward to show that

$$\dot{\hat{V}}\left(\mathbf{x}_t\right) \le \hat{\boldsymbol{\xi}}^T(t)\,\hat{\boldsymbol{\Xi}}\,\hat{\boldsymbol{\xi}}(t), \tag{31}$$

where

$$\hat{\boldsymbol{\xi}}(t) = \left[\boldsymbol{\xi}^T(t) \quad \dot{\mathbf{x}}^T(t-h_1) \quad \cdots \quad \dot{\mathbf{x}}^T(t-h_m)\right]^T,$$

$\xi(t)$ is defined in (15), and

$$\hat{\boldsymbol{\Xi}} = \begin{bmatrix} \hat{\Gamma} & \hat{\prod}_1 & \hat{\prod}_2 & \cdots & \hat{\prod}_m & \hat{\Psi} & 0 \\ * & \hat{\sum}_1 & \hat{\theta}_{12} & & \hat{\theta}_{1m} & \hat{\Psi}_1 & 0 \\ * & * & \hat{\sum}_2 & \cdots & \hat{\theta}_{2m} & \hat{\Psi}_2 & 0 \\ * & * & * & \ddots & \vdots & & \vdots \\ * & * & * & * & \hat{\sum}_m & \hat{\Psi}_m & 0 \\ * & * & * & * & * & \hat{\mho} & -\mathbf{C}\tilde{\mathbf{N}} \\ * & * & * & * & * & * & -\tilde{\mathbf{T}} \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{A}}^T \\ \bar{\mathbf{B}}_1^T \\ \bar{\mathbf{B}}_2^T \\ \vdots \\ \bar{\mathbf{B}}_m^T \\ \bar{\mathbf{D}}^T \\ 0 \end{bmatrix} \Phi \begin{bmatrix} \bar{\mathbf{A}}^T \\ \bar{\mathbf{B}}_1^T \\ \bar{\mathbf{B}}_2^T \\ \vdots \\ \bar{\mathbf{B}}_m^T \\ \bar{\mathbf{D}}^T \\ 0 \end{bmatrix}^T, \tag{32}$$

where

$$\hat{\Gamma} = \bar{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\bar{\mathbf{A}} + \sum_{i=1}^{m}\left(\mathbf{Q}_i\right) - 2\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YM},$$

$$\hat{\prod}_i = \mathbf{P}\bar{\mathbf{B}}_i - 2\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YN}_i, \ \hat{\theta}_{ij} = -\mathbf{N}_i^T\boldsymbol{\alpha\beta}\mathbf{YN}_j,$$

$$\hat{\sum}_i = -\mathbf{Q}_i - 2\mathbf{N}_i^T\boldsymbol{\alpha\beta}\mathbf{YN}_i, \ \hat{\Psi}_i = -\bar{\mathbf{B}}_i^T\mathbf{M}^T\mathbf{C} - \mathbf{N}_i^T\mathbf{Y}(\boldsymbol{\alpha}+\boldsymbol{\beta}),$$

$$(i \ne j \text{ and } i, j = 1,\cdots,m), \ \hat{\Psi} = \mathbf{P}\bar{\mathbf{D}} - \bar{\mathbf{A}}^T\mathbf{M}^T\mathbf{C} - \mathbf{M}^T\mathbf{Y}(\boldsymbol{\alpha}+\boldsymbol{\beta}),$$

$$\hat{\mho} = -\mathbf{CM}\bar{\mathbf{D}} - \bar{\mathbf{D}}^T\mathbf{M}^T\mathbf{C} - 2\mathbf{Y}.$$

Using Lemma 2, (32) can be transformed into

$$\begin{bmatrix} \hat{\Gamma} & \hat{\prod}_1 & \hat{\prod}_2 & \cdots & \hat{\prod}_m & \hat{\Psi} & 0 & \bar{\mathbf{A}}^T\Phi \\ * & \hat{\sum}_1 & \hat{\theta}_{12} & & \hat{\theta}_{1m} & \hat{\Psi}_1 & 0 & \bar{\mathbf{B}}_1^T\Phi \\ * & * & \hat{\sum}_2 & \cdots & \hat{\theta}_{2m} & \hat{\Psi}_2 & 0 & \bar{\mathbf{B}}_2^T\Phi \\ * & * & * & \ddots & \vdots & & & \vdots \\ * & * & * & * & \hat{\sum}_m & \hat{\Psi}_m & 0 & \bar{\mathbf{B}}_m^T\Phi \\ * & * & * & * & * & \hat{\mho} & -\mathbf{C}\tilde{\mathbf{N}} & \bar{\mathbf{D}}^T\Phi \\ * & * & * & * & * & * & -\tilde{\mathbf{T}} & 0 \\ * & * & * & * & * & * & * & -\Phi \end{bmatrix} < 0 \tag{33}$$

Hence,

$$\begin{bmatrix} \underset{\sim}{\Gamma} & \underset{\sim}{\prod}_1 & \underset{\sim}{\prod}_2 & \cdots & \underset{\sim}{\prod}_m & \underset{\sim}{\Psi} & 0 & \mathbf{A}^T\Phi \\ * & \hat{\sum}_1 & \hat{\theta}_{12} & & \hat{\theta}_{1m} & \underset{\sim}{\Psi}_1 & 0 & \mathbf{B}_1^T\Phi \\ * & * & \hat{\sum}_2 & \cdots & \hat{\theta}_{2m} & \underset{\sim}{\Psi}_2 & 0 & \mathbf{B}_2^T\Phi \\ * & * & * & \ddots & \vdots & & & \vdots \\ * & * & * & * & \hat{\sum}_m & \underset{\sim}{\Psi}_m & 0 & \mathbf{B}_m^T\Phi \\ * & * & * & * & * & \underset{\sim}{\mho} & -\mathbf{C}\tilde{\mathbf{N}} & \mathbf{D}^T\Phi \\ * & * & * & * & * & * & -\tilde{\mathbf{T}} & 0 \\ * & * & * & * & * & * & * & -\Phi \end{bmatrix} \tag{34}$$

$$+\mathbf{UF}(t)\mathbf{V} + \mathbf{V}^T\mathbf{F}^T(t)\mathbf{U}^T < 0$$

where

$$\underset{\sim}{\Gamma} = \mathbf{A}^T\mathbf{P} + \mathbf{PA} + \sum_{i=1}^{m}\left(\mathbf{Q}_i\right) - 2\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YM},$$

$$\underset{\sim}{\prod}_i = \mathbf{PB}_i - 2\mathbf{M}^T\boldsymbol{\alpha\beta}\mathbf{YN}_i, \ \underset{\sim}{\Psi}_i = -\mathbf{B}_i^T\mathbf{M}^T\mathbf{C} - \mathbf{N}_i^T\mathbf{Y}(\boldsymbol{\alpha}+\boldsymbol{\beta}),$$

$$(i = 1,\cdots,m), \ \underset{\sim}{\Psi} = \mathbf{PD} - \mathbf{A}^T\mathbf{M}^T\mathbf{C} - \mathbf{M}^T\mathbf{Y}(\boldsymbol{\alpha}+\boldsymbol{\beta}),$$

$$\underset{\sim}{\mho} = -\mathbf{CMD} - \mathbf{D}^T\mathbf{M}^T\mathbf{C} - 2\mathbf{Y},$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{L}^T\mathbf{P} & \underbrace{0 \ \cdots \ 0}_{m} & -\mathbf{L}^T\mathbf{M}^T\mathbf{C} & 0 & \mathbf{L}^T\Phi \end{bmatrix}^T$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{E} & \mathbf{E}_1 & \cdots & \mathbf{E}_m & \mathbf{H} & 0 & 0 \end{bmatrix}^T.$$

Similar to the methods introduced in (17) and (18) and by using Lemmas 1 and 2, it is straightforward to show that (34) can be transformed into (26). This completes the proof. □

IV. Numerical Example

To demonstrate the applicability of the present results, the following examples are provided.

Example 1. Consider the system described by (1) with single time delay and the following parameters:

$$\mathbf{A} = \begin{bmatrix} -0.5 & 0 \\ 1 & -1 \end{bmatrix}, \ \mathbf{B}_1 = \begin{bmatrix} -0.2 & 0.5 \\ 0.3 & -1 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.3 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.8 \end{bmatrix}, \ \mathbf{N} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \mathbf{K} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \ \mathbf{L} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

$$\mathbf{E} = \mathbf{E}_1 = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.03 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Theorem 1 guarantees stability of this system for any value of the time delay. The solution given by the LMI control toolbox is

$$\mathbf{P} = \begin{bmatrix} 2.260 & 0.589 \\ 0.589 & 1.514 \end{bmatrix}, \ \mathbf{Q} = \begin{bmatrix} 0.286 & -0.340 \\ -0.340 & 1.323 \end{bmatrix},$$

$$\delta = 72.529, \ \gamma = 1.9795.$$

Example 2. In this example, consider the system described by (1) with single time delay and the following parameters:

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}, \ \mathbf{B}_1 = \begin{bmatrix} -0.2 & 0.5 \\ 0.3 & -1 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} -0.3 & 0 \\ 0 & -0.3 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 0.2 & 0.3 \\ -0.1 & 1 \end{bmatrix}, \ \mathbf{N} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \mathbf{K} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$\mathbf{L} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{E} = \mathbf{E}_1 = \begin{bmatrix} 0.05 & 0.3 \\ 0.2 & 0.05 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

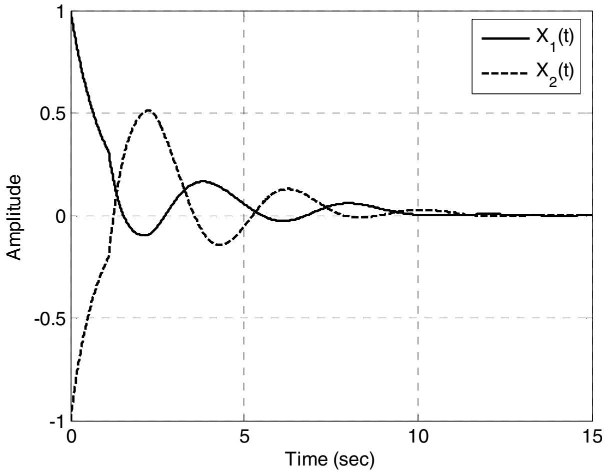

Fig. 1. State trajectories of the system in Example 2.

In this case, Theorem 1 cannot provide stability of the system. However, Theorem 3 guarantees stability of this system. The solution given by Theorem 3 is

$$\mathbf{P}=\begin{bmatrix}14.637 & 6.225\\ 6.225 & 6.506\end{bmatrix},\ \mathbf{Q}=\begin{bmatrix}3.541 & 0.111\\ 0.111 & 4.303\end{bmatrix},$$

$$\mathbf{T}=\begin{bmatrix}3.981 & 1.430\\ 1.430 & 1.375\end{bmatrix},\ \mathbf{Y}=\begin{bmatrix}32.33 & 0\\ 0 & 6.304\end{bmatrix},$$

$$\mathbf{S}=\begin{bmatrix}5.837 & 0\\ 0 & 0.768\end{bmatrix},\ \delta=30.272\,.$$

For the case of

$$\mathbf{F}(t)=\begin{bmatrix}0.5\sin(t) & 0\\ 0 & 0.8\cos(t)\end{bmatrix},\ h_1=1.1$$

states trajectories of the system are shown in Fig. 1.

V. Conclusion

This paper provided some conditions for delay-independent robust absolute stability for uncertain Lur'e systems with multiple time-delays and sector-bounded nonlinearity. Moreover, both cases with the time-varying and time-invariant nonlinearities were considered. The conditions were based on the Lyapunov-Krasovskii stability theory and were expressed as linear matrix inequalities. Simulation examples showed effectiveness of the proposed method.

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