Robust Nonlinear Model Predictive Trajectory Free Control of Biped Robots Based on Nonlinear Disturbance Observer

Mohsen Parsa Department of Electrical Engineering Iran University of Science and Technology Tehran, IRAN mparsa@ee.iust.ac.ir

*Abstract***— This paper employs nonlinear disturbance observer for robust Nonlinear Model Predictive Control (NMPC) of biped robots. The NMPC is used in order to imitate some properties of human walking, which is optimal and uses some basic goals and constraints, yielding safe and stable walking. Since there may be some uncertainties in the dynamics or parameters variations in the biped model, the controller robustness is also considered. However, the NMPC is a model based controller; this characteristic reduces the effectiveness of the NMPC based controlling. In order to overcome this shortcoming of the NMPC, the nonlinear disturbance observer (NDO) will be used to robustify the proposed controller against dynamic uncertainties in the biped robot and rejecting external disturbances. Simulation results reveal better performance of the nonlineardisturbance-observer-based NMPC as compared to the previously reported NMPC controllers.**

Keywords- biped robots, nonlinear model predictive control, nonlinear disturbance observer.

I. INTRODUCTION

The development of legged locomotion systems has recently received an increased attention due to their higher mobility than conventional wheeled vehicles. Legs are adapted to cluttered environments allowing the machine to stride over obstacles and limiting the damages to the environment thanks to their small support surface. An important branch of the legged robots are the biped robots, which are based on humanoriented facilities. The biped robots are expected to imitate human behaviors and locomotion abilities, e.g. getting up and down stairs and ladders, passing uneven and rough grounds. Some of these demands, which are not achievable by the wheeled robots, emphasize more on the use of the biped robots. These new demands together with the new concepts in the biped robots field (i.e. the stable walking and the biped robot balance) demand applying new and well adapted motion control approaches. As much as these methods are inspired by human walking algorithms, the expectations of the biped robot would be satisfied more [1].

The ordinary motion control methods in robotics are comprised of two phases: 1) the motion planning phase [4] and 2) the trajectory following phase. In the biped robots, the motion planning (i.e. the gait generation) phase may be

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Mohammad Farrokhi Department of Electrical Engineering Iran University of Science and Technology Tehran, IRAN farrokhi@iust.ac.ir

performed off-line or on-line [2]. The offline gait generation cannot adapt to the environment changes like obstacles, which can reduce the robot's abilities to walk. There are different methods for the on-line gait generation that can adapt to the environment. An on-line adaptive optimal gait pattern would facilitate best the biped robot motion control. Although consuming more efforts to reduce the error of tracking is the goal of lots of control problems, perfect joint trajectory tracking is not necessary in the biped motion control since the biped robot may have normal and acceptable walk even if there are some errors in the trajectory tracking of the joints. Thus, ordinary robot motion planning methods may not fit well to the biped robots.

The Human walking approach is based on optimal algorithms, which use some goals and constraints to displace the body or the Center of Mass (CoM) from one point to another, while considering and predicting the environment changes, in order to decide adaptively to accomplish safe and without falling walk [2]. A suitable way of imitating this behavior for motion control of the biped robot is to state the problem as a Non-linear Model Based Predictive Control (NMPC) [3], [4], [5]. With an appropriate objective function, while considering the state and the control signal constraints plus the physical constraints, it is possible to combine the gait pattern generation phase with the control phase and allowing the NMPC to decide about both the gait pattern and the control signals. In this approach, there are no trajectories to follow. Instead, the control signals are generated by the NMPC directly in such a way that the biped robot is able to walk. In addition to the advantages of the on-line gait generation, this method considers the biped dynamics, constraints of the control signals, the present and future of the biped states, and the physical constraints in the robot to execute more optimal and practical walking.

Practically, there may be some uncertainties in the biped robot parameters and/or parameter variations in the biped dynamics, or even some unmodeled dynamics in the biped robot model. Since the NMPC is a model based controller, this characteristic would reduce the NMPC efficiencies while there are differences between the biped real dynamics and the model used in the NMPC. In this case, it is possible that the NMPC control loop becomes unstable or the biped balance fails. Thus,

some remedies must be taken to improve the NMPC robustness. In this paper, a novel approach based on the nonlinear disturbance observer (NDO) is suggested to increase the NMPC robustness against unmodeled dynamics and/or external disturbances [8], [9]. In other words, the NDO observes the dynamic changes and disturbances and tries to compensate for them. In biped robots, there are several sources of disturbances. These disturbances can be divided into two categories: 1) the internal disturbances and 2) the external disturbances. The internal disturbances contain unmodeled dynamics, which can be due to the flexibility in links or joints, and uncertainties, which can be due to unknown parameters and/or parameter variations. The external disturbances include the external forces, friction in joints, and torque ripple of the actuators. In this paper, using the NDO, the additive internal and external disturbances will be compensated for; hence, providing robustness to NMPC.

This paper is organized as follows. Section II presents dynamics for the 5-link planar biped robot in the single support phase and the impact effect. Section III provides the proposed NMPC strategy by defining an appropriate objective function and the constraints. In Section IV, the NDO is added to the control loop. Section V shows simulation results. Section VI concludes this paper.

II. BYPED ROBOT DYNAMIC

In this paper, the control of a planar biped robot with five links is considered. This biped robot contains a torso and two identical lower limbs with each limb having a thigh and a shank. Moreover, the biped has two hip joints, two knee joints, and two ankles at tips of lower limbs. There is an actuator located at each joint; in the biped model, all joints are considered friction free rotating in the sagittal plane. In addition, in this model, feet have no mass. This assumption simplifies the biped model while does not reduce much of the biped dynamics [12]. Although the dynamics of the feet is neglected, it is assumed that the biped can apply torque at the ankles. Each gait consists of two successive dynamics: 1) The single support phase (SSP) and 2) The impact event, which happens in an infinitesimal period of time as swing limb collides with the ground and joint velocities are subject to a sudden jump resulting from this event. The double support phase (DSP) span is assumed too short and has been omitted. The friction between the feet and the ground is assumed sufficient to prevent slippage during walking [12].

A. Single Support Phase

The biped locomotion with the single-foot support can be considered as an open-loop kinematic chain model [18]. The dynamic equations to describe the biped robot can be derived using the standard procedure of Lagrangian formulation as

-

$$
\mathbf{D}(\theta)\ddot{\theta} + \mathbf{H}(\theta, \dot{\theta})\dot{\theta} + \mathbf{G}(\theta) = \mathbf{T}
$$
 (1)

where $\mathbf{D}(\theta)$ is 5×5 positive definite and symmetric inertia matrix and $H(\theta)$ is 5×5 Coriolis matrix; $G(\theta)$, θ , $\dot{\theta}$, $\ddot{\theta}$ and **T** are 5×1 vectors of gravity terms, generalized coordinates, velocities, acceleration and torques, respectively [12].

Fig. 1: The planar five link biped robot model [12].

B. Impact Effect

At the end of the SSP, the tip of the swing limb contacts the ground surface with an impact. The joint velocities will be subject to a sudden jump resulting from this impact event. The vertical velocity of the tip of the swing limb becomes zero immediately after the impact due to the ground collision

$$
\dot{\boldsymbol{\theta}}_{\text{impact}}^{+} = \dot{\boldsymbol{\theta}}^{-} + \mathbf{D}^{-1} \mathbf{J}^{T} \left[\mathbf{J} \mathbf{D}^{-1} \mathbf{J}^{T} \right]^{-1} \left(-\mathbf{J} \dot{\boldsymbol{\theta}}^{-} \right)
$$
(2)

where $\dot{\theta}_{\text{impact}}^+$ and $\dot{\theta}^-$ are 5×1 vectors of generalized velocities immediately after and before the impact, respectively [12].

III. NMPC APPROACH

The ordinary motion planning methods in robotics are comprised of two phases: 1) the motion planning phase [4] and 2) the trajectory tracking phase. However, the human walking strategy is not based on following a planned or desired trajectory. Perfect joint trajectory tracking is not necessary during the walking gait. The human walking strategy is based on defining some goals and constraints, which can be briefly stated as follows:

- Moving the body or the Centre of Mass (CoM) from one point to another with almost constant speed.
- Walking in such a way to reduce the energy consumption.
- Satisfying physical constraints and the environmental constraint like the obstacles avoidance.
- Maintaining the balance during walk.

This problem statement can bring new vision in the biped robot motion control. Bearing in mind the previous goals and constraints, the NMPC, which is based on minimizing an objective function with some constraints, can be a good selection. The NMPC, in addition to be optimal, can handle non-linear multivariable non-minimum phase systems like the biped robots. The main part of the NMPC is defining the objective function. After defining the objective function, some equality and inequality constraints, due to the states and the control signals and the physical constraints, have to be added. Optimizing the objective function while satisfying the constraints, forces the robot to become stable and perform a

desired walk. The objective function and the constraints are defined in the next two sections.

A. Objective Function

In SSP, the rear foot has to swing and land in front of the stance foot to make new supporting area. As soon as the front foot tip lands on the ground, a sudden jump in the velocity of the joints happens, which is called the impact event. Then, the stance foot and the swing foot exchange their roles and the exswing foot would become the stance foot. The walking cycle is continued by repeating this cycle. During the SSP, the CoM ground projection has to remain in the stance foot ground contact area (Fig. 2) in order to maintain the biped static stability. In order to have a constant transitional speed with optimum energy consumption, the following objective function is proposed:

$$
\min_{\mathbf{U} \in \mathbb{R}^{5n_c}} J_{\text{SSP}} = \mathbf{W}_1 \sum_{j=0}^{n_c-1} \mathbf{T}(t + j \Delta t)^T \mathbf{T}(t + j \Delta t) + \n+ \mathbf{W}_2 \sum_{i=1}^{n_p} \left| y_e(t + i \Delta t) - y_{e\xi} \right| + \mathbf{W}_3 \sum_{i=1}^{n_p} \left| \dot{x}_{\text{COM}}(t + i \Delta t) - \dot{x}_{\text{COM}}^d \right|
$$
\n(3)

where n_p is the prediction horizon, n_c is the control horizon, Δt is the sampling time, $\dot{x}_{CoM}(t + i \Delta t)$ is the CoM ground projection speed of the biped at the i^{th} prediction, \dot{x}_{COM}^d is the desired CoM horizontal speed, $T(t + j\Delta t)$ is the exerted torque to the biped robot, and $v_e(t + i \Delta t)$ is the vertical position of the swing foot tip at the ith prediction. The SSP objective function comprises of two parts: 1) the swing foot behind the stance foot $\xi = 1$ and 2) the swing foot at the front of the stance foot $\xi = 2$; y_{e1} and y_{e2} are the desired maximum and minimum of y_e ; w_1 , w_2 and w_3 are the weights, which determine the importance of each term in the objective function; these weights may affect the walking behavior of the biped robot.

B. Constarints

In order to obtain a practical and normal walk, a set of equality and inequality constraints will be established in this section. Some constraints help the NMPC to force the biped robot to walk.

- 1) The joint constraints: $q_{i, \min} \leq q_i \leq q_{i, \max}$, where $q_i = \theta_{i-1} - \theta_i$, $(i = 1,...,5)$ and $\theta_0 = 0$. In order to prevent singularity in the Jacobian matrix of the robot, constraints on the angle of joints two and five have to be defined in such a way that $q_2 \neq 0$ and $q_5 \neq 0$.
- 2) During walking, the CoM ground projection of the biped has to move only forward, i.e. $\dot{x}_{CoM} \ge 0$.
- 3) The hip level constraint guarantees the biped to maintain its erected posture during the locomotion; that is $h_{\min} \leq h_{\min} \leq h_{\max}$, where h_{\min} is the vertical position of the tip of link 2 (Fig. 1).
- 4) The torso has to maintain almost the upright position during the whole cycle $\alpha_{\min} \leq \theta_3 \leq \alpha_{\max}$.

- 5) In order to guarantee the static stability, the CoM ground projection has to remain in the support area. This constraint can be given as: $x_{min}^{SSP} \le x_{CoM} \le x_{max}^{SSP}$.
- 6) The swing foot has to remain above the ground level at all times $y_e \geq 0$.
- 7) The height of the swing foot is limited $y_e \le h_m$.
- 8) The swing foot must maintain a minimum velocity $2\dot{x}_{\text{CoM}} \sin(\pi y_e/2h_m) \leq \dot{x}_e$ where \dot{x}_e is the horizontal velocity of the swing foot tip. This constraint synchronizes the swing foot with the CoM horizontal speed. It also adapts the swing foot velocity to its height and helps a smoother touch of the swing foot on the ground. Hence, it can reduce the effect of the impact.
- 9) During SSP, the tip position of the swing foot follows almost a parabolic trajectory. Using this concept, the following constraints are proposed:
	- *a*) While the swing foot is behind the stance foot, the swing foot height increases $\dot{y}_e \ge 0$.
	- *b*) As soon as the swing foot passes the stance foot, the swing foot height decreases until it touches the ground $\dot{y}_e \leq 0$.
- 10) There are constraints on the joint torques $T_{\min} \leq T \leq T_{\max}$.

IV. NONLINEAR DISTURBANCE OBSERVER

The Disturbance Observer (DO) is a robust compensator proposed by Ohnishi in 1987 [18]. In many control loops, the external disturbances can be modeled by additive signal added to the control signal. In some systems, it may also be possible to model the parameter variation of the plant by additive input disturbance signals. Knowing the disturbance value, it can be rejected by adding an appropriate canceling signal that have the disturbance magnitude but is in the opposite sign. Since the external disturbance value is unknown, it should be estimated. This is the main idea behind developing a disturbance observer: First, the DO estimates the equivalent disturbance and then the estimated disturbance is fed back as a cancellation signal and makes the whole system to behave like the nominal system (Fig. 3). In order to estimate the external disturbance, the DO compares the control input signal applied to the system with the virtual control input signal applied to the nominal model. The virtual control input signal is derived using the output response of the inverse of the nominal model.

The linear and nonlinear DOs have been proposed in literatures [9]. The Linear DO (LDO) is able to cancel disturbances in a specified frequency range; thus LDO may not be realized without using a low-pass filter. Moreover, optimizing the filter to achieve the required performance is the main challenge in LDOs. Since the disturbance estimation in LDO is based on calculating the inverse of the nominal system model, another problem in using LDO is that the system has to be minimum-phase. The LDO based controllers are usually designed according to the linear control theory, even if the plant is strongly nonlinear. Thus using LDO for nonlinear systems, the stability analysis may not be valid. Since the biped robot is a highly nonlinear and coupled system, the validity of using linear analysis may be doubtful. Therefore, in this paper the Nonlinear DO (NDO) is used. The NDO contains nonlinear dynamics and its stability can be guaranteed by Lyapunov based approaches [9], [13], and [14].

According to Eq. (1), the SSP dynamics, considering nominal parameters and the external and the internal disturbances, can be written as:

$$
\mathbf{D}_{n}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{H}_{n}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{G}_{n}(\boldsymbol{\theta}) = \mathbf{T} + \mathbf{T}_{ext}^{dist} + \mathbf{T}_{int}^{dist}
$$
(4)

where \mathbf{D}_n , \mathbf{H}_n and \mathbf{G}_n are the nominal values of \mathbf{D}_{act} , \mathbf{H}_{act} and \mathbf{G}_{act} , respectively. $\mathbf{T}_{\text{ext}}^{\text{dist}}$ is the external disturbance vector and contains torques due to the unknown load(s), the external force(s), the friction force(s) and the torque ripples. T_{int}^{dist} is the unmodeled dynamics and the difference between the nominal and actual **D** , **H** and **G** due to parameter changes and may be written as

$$
T_{int}^{dist} = -\delta D\ddot{\theta} - \delta H\dot{\theta} - \delta G \tag{5}
$$

where δD , δH , and δG are the additive unmodeled parts of the inertia, Coriolis, and gravity matrices, respectively. By satisfying the following two properties, Nikoobin and Haghighi have shown sufficient conditions for asymptotic stability of the NDO [9].

Property 1: $\mathbf{D}_{n \times n}(\theta)$ is symmetric positive definite, and bounded below and above, i.e., $\exists \alpha \ge \beta > 0$ such that $\beta I_n \leq D(\theta) \leq \alpha I_n$, $\forall \theta \in \mathbb{R}^n$ where I_n is the $n \times n$ identity matrix.

Property 2: The torque vector **T** is bounded, thus the angular velocity vector $\dot{\theta}$ lies in a known bounded set. That means $\dot{\boldsymbol{\theta}} \in \boldsymbol{\Omega}_{\dot{\boldsymbol{\theta}}} \coloneqq \{\dot{\boldsymbol{\theta}} : \left\|\dot{\boldsymbol{\theta}}\right\| \leq \dot{\boldsymbol{\theta}}_{\text{max}}\}$

Fig. 3: The NDO structure [9].

$$
c \geq \left(\frac{n(n-1)}{2}\right) \times \max\left\{\begin{vmatrix} X_1^1 \dot{\boldsymbol{\theta}}_{\text{max2}} \Big|, \cdots, \begin{vmatrix} X_{n-1}^1 \dot{\boldsymbol{\theta}}_{\text{max2},n} \Big|, \\ \cdots, \begin{vmatrix} X_{n-2}^2 \dot{\boldsymbol{\theta}}_{\text{max3},n} \Big| \cdots, \begin{vmatrix} X_{n-1}^{n-1} \end{vmatrix} \dot{\boldsymbol{\theta}}_{\text{max}} \end{vmatrix} \right\} \tag{6}
$$

where c is the observer gain and X_i^i are the constant parameters, which depend on the masses and the length of the biped links, and *n* is the number of links [9].

V. SIMULATION RESULTS

The parameters of the biped robot, which have been taken from [10] and [11], are given in Table I. Moreover, the following values are used in the NMPC simulation:

$$
n_p = 5
$$
, $n_c = 3$, $\Delta t = 0.02$ s, $\dot{x}_{COM}^d = 0.3$ m.s⁻¹,
\n $y_{e1} = 0.05$ m, $y_{e2} = 0$ m,
\n $w_1 = 10^{-3}$, $w_2 = 100$, $w_3 = 500$

The foot length is $F_L = 0.15$ m. In Table II, the maximum and minimum of the NMPC constraints have been listed.

According to Eq. (6), the observer would be globally asymptotically stable, if parameter c is bigger than 960. Since bigger values of *c* accelerate convergence of the observer error, it is selected larger (2000 in this case) in order to have better convergence rate. The optimization problem is solved using the *fmincon* function in the MATLAB optimization toolbox dedicated to the minimization of a constrained nonlinear multivariable function. The *fmincon* is based on the sequential quadratic programming (SQP) algorithm. The SQP is an iterative technique in which the objective is replaced by a quadratic approximation and the constraints by linear approximations. Simulations are performed using Intel T7500 Core2 Duo 2.2Mhz processor with 1Gbyte of RAM. The objective functions in Eq. (3) with constraints given before are minimized using the *fmincon* function. The input sequence

$$
\mathbf{U} = [\mathbf{T}(t)^{\mathrm{T}} \ \mathbf{T}(t + \Delta t)^{\mathrm{T}} \cdots \mathbf{T}(t + (\mathbf{n}_{\mathrm{c}} - 1)\Delta t)^{\mathrm{T}}]^{\mathrm{T}}
$$

is the solution of the optimization problem. The first element of this vector, i.e. $T_c(t) = T(t)$ is applied to the joints of the biped robot as control signals.

Simulations are carried out for six steps walk on a flat ground, with and without the NDO. During the first two steps, no disturbance is exerted. During steps three and four, the following Coulomb and viscous frictions are exerted as the external disturbance [15]:

$$
T_{ext,i}^{dist} = K_{i1} sign(\dot{\theta}_i) + K_{i2} \dot{\theta}_i \qquad i = 1,...,5
$$
 (7)

where $K_{i1} = 50$ N.m and $K_{i2} = 6$ N.ms.rad⁻¹ are the Coulomb and viscous friction coefficients, respectively. During steps five and six, internal disturbances are also applied to the biped robot by 50% increase to the mass and inertia of all five links of the robot. Fig. 4 illustrates the hip and the swing foot tip vertical and horizontal positions during these six steps. Joint torques and the estimated disturbances are shown in Fig. 5 and Fig. 6, respectively. Finally, the hip and the swing foot tip vertical and horizontal positions, while the NDO is not used, are illustrated in Fig. 7.

Fig. 4 shows that during the first two steps, the swing foot has almost a parabolic trajectory and the hip height changes are limited. This means that the biped has smooth and normal walking. Because the DSP has been omitted in this paper, the step length must decrease in order for the CoM ground projection to have a safe supporting area during the foot switching. Here, the step length is almost 14 cm. Fig. 4 illustrates that the biped robot has almost 25 cm/s constant transitional speed while human normal speed is approximately 125 cm/s. Thus, the biped robot walking speed is almost one fifth of the normal speed. This is mainly due to the static walking and cancellation of the DSP.

Fig. 4 shows that in the presence of disturbances, while using the NDO, the biped robot continues its normal walking the same way as the first two steps. Fig. 7 illustrates the simulation results of references [10] and [11] (i.e., the same situation as in Fig. 4 but without the NDO). As this figure shows, the biped robot acts unmorally and becomes unstable.

Fig. 4: The hip and the swing foot tip vertical and horizontal positions with NDO

VI. CONCLUSION

In this paper, by defining a suitable objective function and appropriate constraints, the gait generation phase was discarded and included in the control phase using the NMPC. In contrast to the previous papers, which had fixed step length, here, the step length is not fixed and the NMPC can change it to optimize the energy consumption and stability. The NMPC is a model based controller and since there may be some uncertainties in the biped robot parameters or parameters variations in the biped dynamics the NMPC efficiency may reduce and/or it may become unstable. In order to overcome this problem, in a new method, the NMPC was robustified using the nonlinear disturbance observer. The NDO improved robustness of the motion controller in presence of the biped robot parameter variations and the unmodeled dynamics. Simulation results show that the proposed method has the ability to reject external disturbances as well.

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