Robust Absolute Stability Criteria for Uncertain Lur'e Systems with Multiple Time-delays

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Abstract **— This paper deals with the problem of robust absolute stability of uncertain multiple time-delay Lur'e systems with sector-bounded nonlinearity. The nonlinearities are assumed to be time-varying. Based on the Lyapunov-Krasovskii stability theory and linear matrix inequalities (LMIs) approach, some delay-dependent sufficient conditions for the robust absolute stability are derived and are expressed as the feasibility problem of a certain LMI system. Finally, some examples are given to illustrate the proposed results.**

Key Words**:** Lur'e system, Absolute stability, Time delay, Delay-dependent criteria, Linear matrix inequality, Lyapunov-Krasovskii stability.

I. INTRODUCTION

Many nonlinear control systems can be represented as feedback connection of a linear dynamical system and a nonlinear element, where the nonlinear element satisfies certain sector constraints [1]. Based on these classes of nonlinear systems, the notion of absolute stability was introduced by Lur'e in [2]. After that, the problem of the absolute stability of Lur'e system has been widely studied for several decades [3-6].

The existence of time-delays is often a source of instability and performance degradation. In addition, when time delays appear in a dynamic system, analysis of such systems becomes more complex [7]. The existing stability criteria for time delay-systems can be classified into two categories: delay-independent and delay-dependent [7]. Delay-independent conditions are useful for the systems that are stable for any value of time-delays. However, it is difficult to find such systems. Delay-dependent conditions are less conservative than delay-independent ones, especially when the size of the delay is small. In most of practical problems, the size of the time delay or the maximum value of that is known. Hence, the delay-dependent conditions are more practical. For the case of time-delayed Lur'e systems without uncertainty, some remarkable results have been developed in literatures. The problem of delay-independent absolute stability is considered in [8]. Delay-dependent absolute stability conditions of Lur'e systems with multiple time delay and nonlinearities have developed in [9-11]. In addition, some researchers focus on Lur'e systems with time-varying delay [12, 13].

Recently, practical considerations such as model uncertainties and time delays are considered for stability analysis of Lur'e systems [14-19]. To the best of our knowledge, delay-dependent absolute stability of uncertain Lur'e systems with multiple time-delays has not been fully investigated [20, 21].

This paper discusses the problem of delay-dependent absolute stability of uncertain Lur'e systems with multiple time-delays. Based on the Lyapunov-Krasovskii stability theory and Linear Matrix Inequalities (LMIs) approach, some delay-dependent sufficient conditions for the robust absolute stability will be derived and expressed as the feasibility problem of certain LMI systems. Finally, some examples are given to validate the results.

Notations. Through this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices. $\mathbf{P}>0$ means that $\mathbf{P}$ is a real positive definite symmetric matrix. $\mathbb{C}[-h,0]$ denotes the space of continuous functions defined on $[-h,0]$, and $\mathbf{I}$ is the identity matrix with appropriate dimensions. $\operatorname{diag}\{\mathbf{w}_1,\cdots,\mathbf{w}_m\}$ refers to a real matrix with diagonal elements $\mathbf{w}_1,\cdots,\mathbf{w}_m$. $\mathbf{A}^T$ denotes the transpose of real matrix $\mathbf{A}$. Symmetric terms in a symmetric matrix are denoted by $*$.

II. PROBLEM FORMULATION

Consider the uncertain Lur'e system with multiple time-delays as

$$\begin{cases}\dot{\mathbf{x}}(t)=\overline{\mathbf{A}}\mathbf{x}(t)+\sum_{i=1}^{m}\overline{\mathbf{B}}_i\mathbf{x}(t-h_i)+\overline{\mathbf{D}}\boldsymbol{\omega}(t),\\ \mathbf{z}(t)=\mathbf{M}\mathbf{x}(t)+\sum_{i=1}^{m}\mathbf{N}_i\mathbf{x}(t-h_i),\\ \boldsymbol{\omega}(t)=-\boldsymbol{\varphi}(t,\mathbf{z}(t)),\\ \mathbf{x}(t)=\boldsymbol{\phi}(t),\qquad \forall t\in[-\max\limits_{1\le i\le m}\{h_i\},0],\end{cases}\tag{1}$$

where $\mathbf{x}(t)\in\mathbb{R}^n$ denotes the state vector, $\boldsymbol{\omega}(t)\in\mathbb{R}^p$ is the input, $\mathbf{z}(t)\in\mathbb{R}^p$ is the output and $\phi(t)\in\mathbb{C}([h,0],\mathbb{R}^n)$ is a continuous vector-valued initial function. $h_i\ge 0$ $(i=1,\cdots,m)$ are time delays, and $\mathbf{M}$ and $\mathbf{N}_i \in \mathbb{R}^{p\times n}$ $(i=1,\cdots,m)$ are known real constant matrices. $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}_i$ $(i=1,\cdots,m) \in \mathbb{R}^{n\times n}$, and $\bar{\mathbf{D}} \in \mathbb{R}^{n\times p}$ are time-varying matrices with the following structures:

$$\begin{aligned} &\bar{\mathbf{A}} = \mathbf{A} + \Delta\mathbf{A}(t), \bar{\mathbf{D}} = \mathbf{D} + \Delta\mathbf{D}(t), \\ &\bar{\mathbf{B}}_i = \mathbf{B}_i + \Delta\mathbf{B}_i(t), (i=1,\cdots,m) \end{aligned} \tag{2}$$

where $\mathbf{A}$, $\mathbf{B}_i$ $(i=1,\cdots,m)$, and $\mathbf{D}$ are known real constant matrices. $\Delta\mathbf{A}(t)$, $\Delta\mathbf{B}_i(t)$ $(i=1,\cdots,m)$, and $\Delta\mathbf{D}(t)$ are norm bounded parameter uncertainties and are assumed to be of the form

$$\begin{aligned} &[\Delta\mathbf{A}(t), \Delta\mathbf{B}_1(t), \cdots, \Delta\mathbf{B}_m(t), \Delta\mathbf{D}(t)] \\ &\quad = \mathbf{LF}(t)[\mathbf{E}, \mathbf{E}_1, \cdots, \mathbf{E}_m, \mathbf{H}], \end{aligned} \tag{3}$$

where $\mathbf{L}$, $\mathbf{E}$, $\mathbf{E}_i$ $(i=1,\cdots,m)$, and $\mathbf{H}$ are known real constant matrices with appropriate dimensions and $\mathbf{F}(t) \in \mathbb{R}^{q\times k}$ is the unknown time-varying real matrix satisfying

$$\mathbf{F}^T(t)\mathbf{F}(t) \le \mathbf{I}. \tag{4}$$

The nonlinear function $\boldsymbol{\varphi}(t, \mathbf{z}(t)) \in \mathbb{R}^p$ is piecewise continuous in t, globally Lipschitz in $\mathbf{z}(t)$, $\boldsymbol{\varphi}(t,0)=0$, and satisfies the following sector condition for any $t \ge 0$ and $\mathbf{z}(t) \in \mathbb{R}^p$:

$$\boldsymbol{\varphi}(t,\mathbf{z}(t))[\boldsymbol{\varphi}(t,\mathbf{z}(t)) - \mathbf{Kz}(t)] \le 0. \tag{5}$$

Such a nonlinear function $\boldsymbol{\varphi}(t,\mathbf{z}(t))$ is said to belong to the sector $[0, \mathbf{K}]$.

Definition 1. The nonlinear delay system (1) is said to be robustly absolutely stable in the sector $[0,\mathbf{K}]$ if it is globally uniformly asymptotically stable for any nonlinear function $\boldsymbol{\varphi}(t,\mathbf{z}(t))$ satisfying $\boldsymbol{\varphi}(t,0)=0$ and (5) and for all admissible uncertainties [18].

Lemma 1. (Jensen inequality [22]) For any constant matrix $\mathbf{R} \in \mathbb{R}^{n\times n}$, scalar $h>0$ and a vector function $\mathbf{x}(t) \in \mathbb{C}([h,0],\mathbb{R}^n)$, such that the integrations concerned are well defined; then,

$$\begin{aligned} &-h\int_{t-h}^{t} \mathbf{x}^T(s)\mathbf{R}\mathbf{x}(s)\,ds \le -\left(\int_{t-h}^{t}\mathbf{x}^T(s)\,ds\right)\mathbf{R}\left(\int_{t-h}^{t}\mathbf{x}(s)\,ds\right) \\ &-h\int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\mathbf{R}\dot{\mathbf{x}}(s)\,ds \le -\left(\int_{t-h}^{t}\dot{\mathbf{x}}^T(s)\,ds\right)\mathbf{R}\left(\int_{t-h}^{t}\dot{\mathbf{x}}(s)\,ds\right) \end{aligned} \tag{6}$$

Lemma 2. ([23]) For given matrices $\boldsymbol{\Psi} = \boldsymbol{\Psi}^T$, $\mathbf{U}$ and $\mathbf{V}$ with appropriate dimensions, the following inequality

$$\boldsymbol{\Psi} + \mathbf{UF}(t)\mathbf{V} + \mathbf{V}^T\mathbf{F}^T(t)\mathbf{U}^T < 0,$$

holds for all $\mathbf{F}(t)^T\mathbf{F}(t) \le \mathbf{I}$ if and only if there exists $\varepsilon > 0$ such that

$$\boldsymbol{\Psi} + \varepsilon^{-1}\mathbf{U}\mathbf{U}^T + \varepsilon\mathbf{V}^T\mathbf{V} < 0. \tag{7}$$

Lemma 3. (Schur complement [24]) Let the symmetric matrix $\mathbf{M}$ be partitioned as

$$\mathbf{M} = \begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix},$$

where $\mathbf{X}$ and $\mathbf{Z}$ are symmetric matrices. Then, $\mathbf{M} > 0$ if and only if

$$\begin{cases} \mathbf{Z} > 0, \\ \mathbf{X} - \mathbf{YZ}^{-1}\mathbf{Y}^T > 0. \end{cases} \tag{8}$$

In the following section, the main results will be given based on the above lemmas.

III. MAIN RESULTS

Theorem 1. The nonlinear delay system (1) with the nonlinear function $\boldsymbol{\varphi}(t,\mathbf{z}(t))$ satisfying (5) and $\boldsymbol{\varphi}(t,0)=0$, is robustly absolutely stable if there exist scalars $\gamma > 0$, $\delta > 0$, and symmetric matrices $\mathbf{P} > 0$, $\mathbf{Q}_i > 0$, $\mathbf{R}_i > 0$, $\mathbf{W}_i > 0$, $\mathbf{S}_i > 0$, $(i=1,\cdots,m)$ such that the following LMI holds:

$$\begin{bmatrix} \Gamma & \Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_m & \Psi & \Omega & \mathbf{A}^T\Phi & \mathbf{PL} \\ * & \Sigma_1 & \delta\mathbf{E}_1^T\mathbf{E}_2 & \delta\mathbf{E}_1^T\mathbf{E}_3 & & \delta\mathbf{E}_1^T\mathbf{E}_m & \Psi_1 & & \mathbf{B}_1^T\Phi & 0 \\ * & * & \Sigma_2 & \delta\mathbf{E}_2^T\mathbf{E}_3 & \cdots & \delta\mathbf{E}_2^T\mathbf{E}_m & \Psi_2 & & \mathbf{B}_2^T\Phi & 0 \\ * & * & * & \Sigma_3 & \cdots & \delta\mathbf{E}_3^T\mathbf{E}_1 & \Psi_3 & \tilde{\mathbf{W}} & \mathbf{B}_3^T\Phi & 0 \\ \vdots & & & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots \\ * & * & * & * & \cdots & \Sigma_m & \Psi_m & & \mathbf{B}_m^T\Phi & 0 \\ * & * & * & * & & * & \mho & 0 & \mathbf{D}^T\Phi & 0 \\ * & * & * & * & & * & * & \tilde{\mathbf{R}} & 0 & 0 \\ * & * & * & * & & * & * & * & -\Phi & \Phi\mathbf{L} \\ * & * & * & * & \cdots & * & * & * & * & -\delta\mathbf{I} \end{bmatrix} < 0 \tag{9}$$

where

$$\Gamma = \mathbf{A}^T\mathbf{P} + \mathbf{PA} + \sum_{i=1}^{m}\left(\mathbf{Q}_i + h_i\mathbf{R}_i - \mathbf{S}_i\right) + \delta\mathbf{E}^T\mathbf{E},$$

$$\Psi = \mathbf{PD} - \gamma\mathbf{M}^T\mathbf{K}^T + \delta\mathbf{E}^T\mathbf{H},$$

$$\Omega = [\mathbf{W}_1 \quad \mathbf{W}_2 \quad \cdots \quad \mathbf{W}_m],$$

$$\tilde{\mathbf{W}} = \mathrm{diag}\{-\mathbf{W}_1, -\mathbf{W}_2, \cdots, -\mathbf{W}_m\}, \Phi = \sum_{i=1}^{m} h_i^2\mathbf{S}_i,$$

$$\tilde{\mathbf{R}} = \mathrm{diag}\{-1/h_1.\mathbf{R}_1, -1/h_2.\mathbf{R}_2, \cdots, -1/h_m.\mathbf{R}_m\},$$

$$\mho = -2\gamma\mathbf{I} + \delta\mathbf{H}^T\mathbf{H}, \ \Pi_i = \mathbf{PB}_i + \mathbf{S}_i + \delta\mathbf{E}^T\mathbf{E}_i,$$

$$\Sigma_i = -\mathbf{Q}_i - \mathbf{S}_i + \delta\mathbf{E}_i^T\mathbf{E}_i, \ \Psi_i = -\gamma\mathbf{N}_i^T\mathbf{K}^T + \delta\mathbf{E}_i^T\mathbf{H},$$

$$(i=1,\cdots,m).$$

Proof. Let select the Lyapunov-Krasovskii functional candidate as

$$V(\mathbf{x}_t) = V_1(\mathbf{x}) + V_2(\mathbf{x}_t) + V_3(\mathbf{x}_t) + V_4(\mathbf{x}_t) + V_5(\mathbf{x}_t) \quad (10)$$

where

$$V_1(\mathbf{x}) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t),$$

$$V_2(\mathbf{x}_t) = \sum_{j=1}^{m}\int_{t-h_j}^{t} \mathbf{x}^T(s)\mathbf{Q}_j\mathbf{x}(s)\,ds,$$

$$V_3(\mathbf{x}_t) = \sum_{j=1}^{m}\int_{-h_j}^{0}\int_{t+\beta}^{t} \mathbf{x}^T(s)\mathbf{R}_j\mathbf{x}(s)\,ds d\beta,$$

$$V_4(\mathbf{x}_t) = \sum_{j=1}^{m}\left(\int_{t-h_j}^{t} \mathbf{x}^T(s)\,ds\right)\mathbf{W}_j\left(\int_{t-h_j}^{t} \mathbf{x}(s)\,ds\right),$$

$$V_5(\mathbf{x}_t) = \sum_{j=1}^{m} h_j\int_{-h_j}^{0}\int_{t+\beta}^{t} \dot{\mathbf{x}}^T(s)\mathbf{S}_j\dot{\mathbf{x}}(s)\,ds d\beta,$$

where $\mathbf{x}_t$ is defined as $\mathbf{x}_t = \mathbf{x}(t+\theta)$, $\theta \in [-\max_{1\le i\le m}\{h_i\}, 0]$. Taking the derivative of $V(\mathbf{x}_t)$ with respect to t yields

$$\begin{aligned}\dot{V}_1(\mathbf{x}) &= 2\mathbf{x}^T(t)\mathbf{P}\dot{\mathbf{x}}(t) = \mathbf{x}^T(t)\left(\bar{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\bar{\mathbf{A}}\right)\mathbf{x}(t) \\ &\quad + 2\mathbf{x}^T(t)\mathbf{P}\sum_{i=1}^{m}\bar{\mathbf{B}}_i\mathbf{x}(t-h_i) + 2\mathbf{x}^T(t)\mathbf{P}\mathbf{D}\omega(t),\end{aligned} \quad (11)$$

$$\begin{aligned}\dot{V}_2(\mathbf{x}_t) &= \mathbf{x}^T(t)\left(\sum_{j=1}^{m}\mathbf{Q}_j\right)\mathbf{x}(t) \\ &\quad - \sum_{j=1}^{m}\left(\mathbf{x}^T(t-h_j)\mathbf{Q}_j\mathbf{x}(t-h_j)\right)\end{aligned} \quad (12)$$

$$\begin{aligned}\dot{V}_3(\mathbf{x}_t) &= \mathbf{x}^T(t)\left(\sum_{j=1}^{m}h_j\mathbf{R}_j\right)\mathbf{x}(t) \\ &\quad - \sum_{j=1}^{m}\int_{t-h_j}^{t}\mathbf{x}^T(s)\mathbf{R}_j\mathbf{x}(s)\,ds\end{aligned} \quad (13)$$

Using Lemma 1, it gives

$$\begin{aligned}\dot{V}_3(\mathbf{x}_t) &\le \mathbf{x}^T(t)\left(\sum_{j=1}^{m}h_j\mathbf{R}_j\right)\mathbf{x}(t) \\ &\quad - \sum_{j=1}^{m}\left(\int_{t-hj}^{t}\mathbf{x}^T(s)\,ds\right)\frac{1}{h_j}\mathbf{R}_j\left(\int_{t-hj}^{t}\mathbf{x}(s)\,ds\right)\end{aligned} \quad (14)$$

$$\dot{V}_4(\mathbf{x}_t) = 2\sum_{j=1}^{m}\left(\mathbf{x}^T(t) - \mathbf{x}^T(t-h_j)\right)\mathbf{W}_j\left(\int_{t-hj}^{t}\mathbf{x}(s)\,ds\right) \quad (15)$$

$$\dot{V}_5(\mathbf{x}_t) = \sum_{j=1}^{m}h_j^2\dot{\mathbf{x}}^T(t)\mathbf{S}_j\dot{\mathbf{x}}(t) - \sum_{j=1}^{m}\int_{t-hj}^{t}\dot{\mathbf{x}}^T(s)h_j\mathbf{S}_j\dot{\mathbf{x}}(s)\,ds$$

Using the Leibniz-Newton formula

$$\int_{t-hj}^{t}\dot{\mathbf{x}}(s)\,ds = \mathbf{x}(t) - \mathbf{x}(t-h_j),$$

It yields

$$\begin{aligned}\dot{V}_5(\mathbf{x}_t) &\le \sum_{j=1}^{m}h_j^2\dot{\mathbf{x}}^T(t)\mathbf{S}_j\dot{\mathbf{x}}(t) \\ &\quad - \sum_{j=1}^{m}\left(\mathbf{x}^T(t) - \mathbf{x}^T(t-h_j)\right)\mathbf{S}_j\left(\mathbf{x}(t) - \mathbf{x}(t-h_j)\right).\end{aligned} \quad (16)$$

Using $\boldsymbol{\omega}(t) = -\boldsymbol{\varphi}(t, \mathbf{z}(t))$ as in (1), the sector condition (5) can be written as

$$-\boldsymbol{\omega}^T(t)\left[\boldsymbol{\omega}(t) + \mathbf{K}\mathbf{z}(t)\right] \ge 0 \quad (17)$$

where $\mathbf{z}(t)$ and $\boldsymbol{\omega}(t)$ are defined in (1). Hence, $\dot{V}(\mathbf{x}_t)$ can be expressed by

$$\begin{aligned}\dot{V}(\mathbf{x}_t) &\le \dot{V}_1(\mathbf{x}_t) + \dot{V}_2(\mathbf{x}_t) + \dot{V}_3(\mathbf{x}_t) + \dot{V}_4(\mathbf{x}_t) \\ &\quad + \dot{V}_5(\mathbf{x}_t) - 2\gamma\boldsymbol{\omega}^T(t)\left[\boldsymbol{\omega}(t) + \mathbf{K}\mathbf{z}(t)\right],\end{aligned} \quad (18)$$

where γ is the same as in Theorem 1. By substituting $\dot{\mathbf{x}}(t)$ from (1) into the (18) and considering (11)-(18), it is easy to show that

$$\dot{V}(\mathbf{x}_t) \le \xi^T(t)\boldsymbol{\Xi}\xi(t), \quad (19)$$

where

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\eta} & \hat{\Pi}_1 & \hat{\Pi}_2 & \cdots & \hat{\Pi}_m & \hat{\Psi} & \Omega \\ * & \hat{\Sigma}_1 & \bar{\mathbf{B}}_1^T\Phi\bar{\mathbf{B}}_2 & \cdots & \bar{\mathbf{B}}_1^T\Phi\bar{\mathbf{B}}_m & \hat{\Psi}_1 & \\ * & * & \hat{\Sigma}_2 & \cdots & \bar{\mathbf{B}}_2^T\Phi\bar{\mathbf{B}}_m & \hat{\Psi}_2 & \tilde{\mathbf{W}} \\ \vdots & & \vdots & \ddots & \vdots & \vdots & \\ * & * & * & \cdots & \hat{\Sigma}_m & \hat{\Psi}_m & \\ * & * & * & * & * & \hat{\mho} & 0 \\ * & * & * & * & * & * & \tilde{\mathbf{R}} \end{bmatrix} \quad (20)$$

in which

$$\boldsymbol{\eta} = \bar{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\bar{\mathbf{A}} + \sum\nolimits_{i=1}^{m}\left(\mathbf{Q}_i + h_i\mathbf{R}_i - \mathbf{S}_i\right) + \bar{\mathbf{A}}^T\Phi\bar{\mathbf{A}},$$

$$\hat{\Psi} = \mathbf{P}\bar{\mathbf{D}} + \bar{\mathbf{A}}^T\Phi\bar{\mathbf{D}} - \gamma\mathbf{M}^T\mathbf{K}^T, \quad \hat{\mho} = \bar{\mathbf{D}}^T\Phi\bar{\mathbf{D}} - 2\gamma\mathbf{I},$$

$$\hat{\Psi}_i = \bar{\mathbf{B}}_i^T\Phi\bar{\mathbf{D}} - \gamma\mathbf{N}_i^T\mathbf{K}^T, \quad \hat{\Pi}_i = \mathbf{P}\bar{\mathbf{B}}_i + \bar{\mathbf{A}}^T\Phi\bar{\mathbf{B}}_i + \mathbf{S}_i,$$

$$\hat{\Sigma}_i = \bar{\mathbf{B}}_i^T\Phi\bar{\mathbf{B}}_i - \mathbf{Q}_i - \mathbf{S}_i, (i = 1, \cdots, m)$$

$$\xi(t) = \begin{bmatrix}\mathbf{x}^T(t) & \xi_1^T(t) & \omega^T(t) & \xi_2^T(t)\end{bmatrix}^T$$

$$\xi_1(t) = \begin{bmatrix}\mathbf{x}^T(t-h_1) & \cdots & \mathbf{x}^T(t-h_m)\end{bmatrix}^T$$

$$\xi_1(t) = \begin{bmatrix}\int_{t-h_1}^{t}x^T(s)\,ds & \cdots & \int_{t-h_m}^{t}x^T(s)\,ds\end{bmatrix}^T$$

and Ω, $\tilde{\mathbf{W}}$, $\tilde{\mathbf{R}}$ are defined in (9).

If it can be shows that $\boldsymbol{\Xi} < 0$ in (19), then $\dot{V}(\mathbf{x}_t) < 0$ and by definition 1 and the Lyapunov-Krasovskii theorem [25], the considered nonlinear delayed system is robustly absolutely stable. But the matrix Ξ is not an LMI and should be transformed into an LMI. Matrix Ξ can be rewritten as

$$\begin{bmatrix} \hat{\boldsymbol{\eta}} & \mathbf{P}\bar{\mathbf{B}}_1+\mathbf{S}_1 & \mathbf{P}\bar{\mathbf{B}}_2+\mathbf{S}_2 & \cdots & \mathbf{P}\bar{\mathbf{B}}_m+\mathbf{S}_m & \mathbf{P}\bar{\mathbf{D}}-\gamma\mathbf{M}^T\mathbf{K}^T & \Omega \\ * & -\mathbf{Q}_1-\mathbf{S}_1 & 0 & \cdots & 0 & -\gamma\mathbf{N}_1^T\mathbf{K}^T & \\ * & * & -\mathbf{Q}_2-\mathbf{S}_2 & \cdots & 0 & -\gamma\mathbf{N}_2^T\mathbf{K}^T & \tilde{\mathbf{W}} \\ \vdots & & \vdots & \ddots & \vdots & \vdots & \\ * & * & * & \cdots & -\mathbf{Q}_m-\mathbf{S}_m & -\gamma\mathbf{N}_m^T\mathbf{K}^T & \\ * & * & * & * & * & -2\gamma\mathbf{I} & 0 \\ * & * & * & * & * & * & \tilde{\mathbf{R}} \end{bmatrix}$$

$$+\begin{bmatrix} \bar{\mathbf{A}}^T \\ \bar{\mathbf{B}}_1^T \\ \bar{\mathbf{B}}_2^T \\ \vdots \\ \bar{\mathbf{B}}_m^T \\ \bar{\mathbf{D}}^T \\ 0 \end{bmatrix} \Phi \begin{bmatrix} \bar{\mathbf{A}}^T \\ \bar{\mathbf{B}}_1^T \\ \bar{\mathbf{B}}_2^T \\ \vdots \\ \bar{\mathbf{B}}_m^T \\ \bar{\mathbf{D}}^T \\ 0 \end{bmatrix}^T < 0, \qquad (21)$$

where Φ is defined in (9) and

$$\hat{\boldsymbol{\eta}} = \bar{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\bar{\mathbf{A}} + \sum\nolimits_{i=1}^{m}\left(\mathbf{Q}_i + h_i\mathbf{R}_i - \mathbf{S}_i\right).$$

Using Lemma 3, it is easy to show that (21) with can be transformed into

$$\begin{bmatrix} \hat{\boldsymbol{\eta}} & \mathbf{P}\bar{\mathbf{B}}_1+\mathbf{S}_1 & \mathbf{P}\bar{\mathbf{B}}_2+\mathbf{S}_2 & \cdots & \mathbf{P}\bar{\mathbf{B}}_m+\mathbf{S}_m & \mathbf{P}\bar{\mathbf{D}}-\gamma\mathbf{M}^T\mathbf{K}^T & \Omega & \bar{\mathbf{A}}^T\Phi \\ * & -\mathbf{Q}_1-\mathbf{S}_1 & 0 & \cdots & 0 & -\gamma\mathbf{N}_1^T\mathbf{K}^T & & \bar{\mathbf{B}}_1^T\Phi \\ * & * & -\mathbf{Q}_2-\mathbf{S}_2 & \cdots & 0 & -\gamma\mathbf{N}_2^T\mathbf{K}^T & \tilde{\mathbf{W}} & \bar{\mathbf{B}}_2^T\Phi \\ * & * & * & \ddots & \vdots & \vdots & & \vdots \\ * & * & * & * & -\mathbf{Q}_m-\mathbf{S}_m & -\gamma\mathbf{N}_m^T\mathbf{K}^T & & \bar{\mathbf{B}}_m^T\Phi \\ * & * & * & * & * & -2\gamma\mathbf{I} & 0 & \bar{\mathbf{D}}^T\Phi \\ * & * & * & * & * & * & \tilde{\mathbf{R}} & 0 \\ * & * & * & * & * & * & * & -\Phi \end{bmatrix} < 0 \qquad (22)$$

Noting (2) and (3), (22) can be written as

$$\begin{bmatrix} \boldsymbol{\eta} & \mathbf{P}\mathbf{B}_1+\mathbf{S}_1 & \mathbf{P}\mathbf{B}_2+\mathbf{S}_2 & \cdots & \mathbf{P}\mathbf{B}_m+\mathbf{S}_m & \mathbf{P}\mathbf{D}-\gamma\mathbf{M}^T\mathbf{K}^T & \Omega & \mathbf{A}^T\Phi \\ * & -\mathbf{Q}_1-\mathbf{S}_1 & 0 & \cdots & 0 & -\gamma\mathbf{N}_1^T\mathbf{K}^T & & \mathbf{B}_1^T\Phi \\ * & * & -\mathbf{Q}_2-\mathbf{S}_2 & \cdots & 0 & -\gamma\mathbf{N}_2^T\mathbf{K}^T & \tilde{\mathbf{W}} & \mathbf{B}_2^T\Phi \\ * & * & * & \ddots & \vdots & \vdots & & \vdots \\ * & * & * & * & -\mathbf{Q}_m-\mathbf{S}_m & -\gamma\mathbf{N}_m^T\mathbf{K}^T & & \mathbf{B}_m^T\Phi \\ * & * & * & * & * & -2\gamma\mathbf{I} & 0 & \mathbf{D}^T\Phi \\ * & * & * & * & * & * & \tilde{\mathbf{R}} & 0 \\ * & * & * & * & * & * & * & -\Phi \end{bmatrix}$$

$$+\mathbf{U}\mathbf{F}(t)\mathbf{V} + \mathbf{V}^T\mathbf{F}^T(t)\mathbf{U}^T < 0, \qquad (23)$$

where

$$\boldsymbol{\eta} = \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \sum\nolimits_{i=1}^{m}\left(\mathbf{Q}_i + h_i\mathbf{R}_i - \mathbf{S}_i\right),$$

$$\mathbf{U} = [\mathbf{L}^T\mathbf{P} \quad \underbrace{0 \quad 0 \quad \cdots \quad 0}_{m} \quad 0 \quad 0 \quad \mathbf{L}^T\boldsymbol{\Phi}]^T,$$

$$\mathbf{V} = \left[\mathbf{E} \quad \mathbf{E}_1 \quad \mathbf{E}_2 \quad \cdots \quad \mathbf{E}_m \quad \mathbf{H} \quad 0 \quad 0\right].$$

Using Lemma 2 and inequality (4), (23) is equivalent to

$$\begin{bmatrix} \Gamma & \Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_m & \Psi & \Omega & \mathbf{A}^T\Phi \\ * & \Sigma_1 & \delta\mathbf{E}_1^T\mathbf{E}_2 & \delta\mathbf{E}_1^T\mathbf{E}_3 & & \delta\mathbf{E}_1^T\mathbf{E}_m & \Psi_1 & & \mathbf{B}_1^T\Phi \\ * & * & \Sigma_2 & \delta\mathbf{E}_2^T\mathbf{E}_3 & \cdots & \delta\mathbf{E}_2^T\mathbf{E}_m & \Psi_2 & & \mathbf{B}_2^T\Phi 0 \\ * & * & * & \Sigma_3 & \cdots & \delta\mathbf{E}_3^T\mathbf{E}_1 & \Psi_3 & \tilde{\mathbf{W}} & \mathbf{B}_3^T\Phi \\ \vdots & * & * & \vdots & \ddots & \vdots & \vdots & & \vdots \\ * & * & * & * & \cdots & \Sigma_m & \Psi_m & & \mathbf{B}_m^T\Phi \\ * & * & * & * & * & * & \mho & 0 & \mathbf{D}^T\Phi \\ * & * & * & * & * & * & * & \tilde{\mathbf{R}} & 0 \\ * & * & * & * & * & * & * & * & -\Phi \end{bmatrix}$$

$$+\delta^{-1}\mathbf{U}\mathbf{U}^T < 0, \qquad (24)$$

where the elements are defined in (9).
Again, using Lemma 3, (24) can be transformed into (9). This completes the proof. □

Next, the problem of robust absolute stability analysis of the nonlinear delayed system (1), with the nonlinear function $\boldsymbol{\varphi}(t,\mathbf{z}(t))$ in a sector $[\mathbf{K}_1, \mathbf{K}_2]$ is considered.

***Theorem* 2.** The nonlinear delay system (1) with the nonlinear function $\boldsymbol{\varphi}(t,\mathbf{z}(t))$, satisfying (5) and $\boldsymbol{\varphi}(t,0)=0$, is robustly absolutely stable if there exist scalars $\gamma > 0$, $\delta > 0$, and symmetric matrices $\mathbf{P} > 0$, $\mathbf{Q}_i > 0$, $\mathbf{R}_i > 0$, $\mathbf{W}_i > 0$, $\mathbf{S}_i > 0$, $(i = 1,\cdots,m)$ such that the following LMI holds:

$$\begin{bmatrix} \tilde{\Gamma} & \tilde{\Pi}_1 & \tilde{\Pi}_2 & \tilde{\Pi}_3 & \cdots & \tilde{\Pi}_m & \tilde{\Psi} & \Omega & \tilde{\mathbf{A}}^T\Phi & \mathbf{PL} \\ * & \tilde{\Sigma}_1 & \delta\tilde{\mathbf{E}}_1^T\tilde{\mathbf{E}}_2 & \delta\tilde{\mathbf{E}}_1^T\tilde{\mathbf{E}}_3 & & \delta\tilde{\mathbf{E}}_1^T\tilde{\mathbf{E}}_m & \tilde{\Psi}_1 & & \tilde{\mathbf{B}}_1^T\Phi & 0 \\ * & * & \tilde{\Sigma}_2 & \delta\tilde{\mathbf{E}}_2^T\tilde{\mathbf{E}}_3 & \cdots & \delta\tilde{\mathbf{E}}_2^T\tilde{\mathbf{E}}_m & \tilde{\Psi}_2 & & \tilde{\mathbf{B}}_2^T\Phi & 0 \\ * & * & * & \tilde{\Sigma}_3 & \cdots & \delta\tilde{\mathbf{E}}_3^T\tilde{\mathbf{E}}_1 & \tilde{\Psi}_3 & \tilde{\mathbf{W}} & \tilde{\mathbf{B}}_3^T\Phi & 0 \\ \vdots & & & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots \\ * & * & * & * & \cdots & \tilde{\Sigma}_m & \tilde{\Psi}_m & & \tilde{\mathbf{B}}_m^T\Phi & 0 \\ * & * & * & * & & * & \mho & 0 & \mathbf{D}^T\Phi & 0 \\ * & * & * & * & & * & * & \tilde{\mathbf{R}} & 0 & 0 \\ * & * & * & * & & * & * & * & -\Phi & \Phi\mathbf{L} \\ * & * & * & * & \cdots & * & * & * & * & -\delta\mathbf{I} \end{bmatrix} < 0 \qquad (25)$$

where

$$\tilde{\Gamma} = \tilde{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}} + \sum\nolimits_{i=1}^{m}\left(\mathbf{Q}_i + h_i\mathbf{R}_i - \mathbf{S}_i\right) + \delta\tilde{\mathbf{E}}^T\tilde{\mathbf{E}},$$

$$\tilde{\Psi} = \mathbf{P}\mathbf{D} - \gamma\mathbf{M}^T\tilde{\mathbf{K}}^T + \delta\tilde{\mathbf{E}}^T\mathbf{H}, \tilde{\Pi}_i = \mathbf{P}\tilde{\mathbf{B}}_i + \mathbf{S}_i + \delta\tilde{\mathbf{E}}^T\tilde{\mathbf{E}}_i,$$

$$\tilde{\Sigma}_i = -\mathbf{Q}_i - \mathbf{S}_i + \delta\tilde{\mathbf{E}}_i{}^T\tilde{\mathbf{E}}_i, \ \tilde{\Psi}_i = -\gamma\mathbf{N}_i^T\tilde{\mathbf{K}}^T + \delta\tilde{\mathbf{E}}_i^T\mathbf{H},$$

$$\tilde{\mathbf{K}} = \mathbf{K}_2 - \mathbf{K}_1, \ \tilde{\mathbf{A}} = \mathbf{A} - \mathbf{D}\mathbf{K}_1\mathbf{M}, \ \tilde{\mathbf{E}} = \mathbf{E} - \mathbf{H}\mathbf{K}_1\mathbf{M},$$

$\tilde{\mathbf{B}}_i = \mathbf{B}_i - \mathbf{DK}_1\mathbf{N}_i$, $\tilde{\mathbf{E}}_i = \mathbf{E}_i - \mathbf{HK}_1\mathbf{N}_i$, $(i = 1, \cdots, m)$, and Ω, $\tilde{\mathbf{W}}$, Φ, $\tilde{\mathbf{R}}$, $\mho$ are defined in (9).

Proof: By applying the loop transformation suggested in [1], (1) can be transformed into

$$\begin{cases} \dot{\mathbf{x}}(t) = \left(\bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{M}\right)\mathbf{x}(t) + \sum_{i=1}^{m}\left(\bar{\mathbf{B}}_i - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{N}_i\right)\mathbf{x}(t - h_i) + \bar{\mathbf{D}}\tilde{\boldsymbol{\omega}}(t), \\ \mathbf{z}(t) = \mathbf{Mx}(t) + \sum_{i=1}^{m}\mathbf{N}_i\mathbf{x}(t - h_i), \\ \tilde{\boldsymbol{\omega}}(t) = -\tilde{\boldsymbol{\varphi}}(t, \mathbf{z}(t)), \\ \mathbf{x}(t) = \phi(t), \quad \forall t \in [-\max_{1 \le i \le m}\{h_i\}, 0], \end{cases} \tag{26}$$

where the nonlinear function $\tilde{\boldsymbol{\varphi}}(t, \mathbf{z}(t))$ satisfies

$$\tilde{\boldsymbol{\varphi}}^T(t, \mathbf{z}(t))[\tilde{\boldsymbol{\varphi}}(t, \mathbf{z}(t)) - \tilde{\mathbf{K}}\mathbf{z}(t)] \le 0, \tag{27}$$

for any $t > 0$.
Noting (2) and (3), it yields

$$\begin{aligned} \bar{\mathbf{A}} - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{M} &= \mathbf{A} + \mathbf{LF}(t)\mathbf{E} - \left(\mathbf{D} + \mathbf{LF}(t)\mathbf{H}\right)\mathbf{K}_1\mathbf{M} \\ &= \left(\mathbf{A} - \mathbf{DK}_1\mathbf{M}\right) + \mathbf{LF}(t)\left(\mathbf{E} - \mathbf{HK}_1\mathbf{M}\right) = \tilde{\mathbf{A}} + \mathbf{LF}(t)\tilde{\mathbf{E}}, \end{aligned} \tag{28}$$

$$\begin{aligned} \bar{\mathbf{B}}_i - \bar{\mathbf{D}}\mathbf{K}_1\mathbf{N}_i &= \left(\mathbf{B}_i + \mathbf{LF}(t)\mathbf{E}_i\right) - \left(\mathbf{D} + \mathbf{LF}(t)\mathbf{H}\right)\mathbf{K}_1\mathbf{N}_i \\ &= \left(\mathbf{B}_i - \mathbf{DK}_1\mathbf{N}_i\right) + \mathbf{LF}(t)\left(\mathbf{E}_i - \mathbf{HK}_1\mathbf{N}_i\right) = \tilde{\mathbf{B}}_i + \mathbf{LF}(t)\tilde{\mathbf{E}}_i, \end{aligned} \tag{29}$$

for $(i = 1, \cdots, m)$.

Hence, Theorem 1 can be applied to the transformed system (26) with new matrices (28) and (29). This completes the proof. □

IV. ILLUSTRATIVE EXAMPLES

To demonstrate the applicability of the presented results and to compare them with the previously reported results, the following examples are considered.

Example 1. Consider the system described by (1) with single time delay and the following parameters [26]:

$$\mathbf{A} = \begin{bmatrix} -2 & -1 \\ 0.5 & 0.2 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 0.5 & 1 \\ -0.1 & -0.8 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{N} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \mathbf{K}_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$\mathbf{K}_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{L} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{E} = \mathbf{E}_1 = \mathbf{H} = \mathbf{L}.$$

A comparison of the delay-dependent conditions between this article and different methods in [26] is given in Table 1. It is obvious that the maximum allowable delay $\bar{h}$ obtained by using Theorem 2 in this paper, is larger than those obtained in [26]. Hence, the theorems presented in this paper are less conservative as compared to those in [26].

Example 2: Consider the system described by (1) with two time delays and the following parameters:

$$\mathbf{A} = \begin{bmatrix} -1.2 & 0 \\ 0.8 & -1 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} -1 & 0.6 \\ -0.6 & -1 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} -0.5 & 0.4 \\ -1 & -1 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{N}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_1 = \mathbf{N}, \mathbf{N}_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \end{bmatrix},$$

$$\mathbf{L} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \mathbf{E}_1 = \mathbf{H} = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.03 \end{bmatrix},$$

$$\mathbf{E} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}, \mathbf{F}(t) = \begin{bmatrix} 0.5\sin(t) & 0 \\ 0 & 0.8\cos(t) \end{bmatrix},$$

$$\boldsymbol{\varphi}(\mathbf{z}(t)) = \begin{bmatrix} \tanh(z_1(t)) \\ 1.5\tanh(z_2(t)) \end{bmatrix}.$$

For various time delays, the maximum allowable value for h_1 and h_2 are calculated and shown in Fig. 1. Theorem 1 guarantees stability of the system in region A. For the case of $h_1 = 0.43$ and $h_2 = 0.63$, states trajectories of the system are shown in Figure 2.

V. CONCLUSION

This paper provided some conditions for delay-dependent robust absolute stability for uncertain Lur'e systems with multiple time-delays and sector-bounded nonlinearity. These conditions are based on the Lyapunov-Krasovskii stability theory and expressed as linear matrix inequalities. Finally, examples showed that the proposed theorems in this article are less conservative as compared to the recently published papers with single time delay. The main advantage of this paper is for systems with multiple time delay that was not performed in previously reported papers.

Table 1: Comparison of the delay-dependent conditions for Example 1

Methods	Maximum allowable delay $\bar{h}$
Corollary 8 in Han [26]	2.0239
Proposition 11 in Han [26]	2.0263
Theorem 2 in this article	2.1288

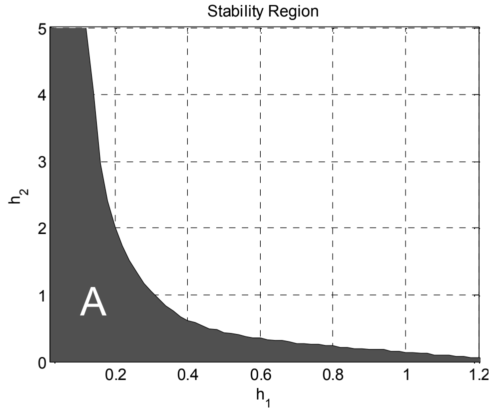

Figure 1. Stability region for various time delays in Example 2.

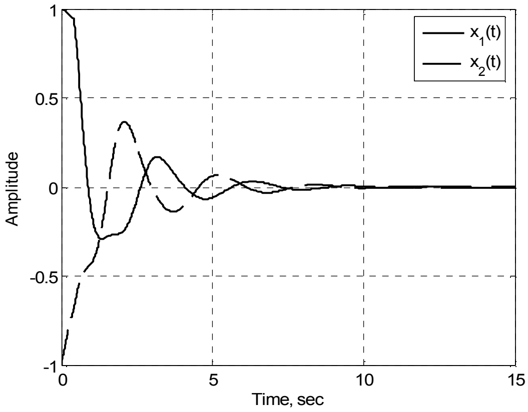

Figure 2. State trajectories of the system in Example 2.

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