Trajectory Free NMPC Based Control of 5-link Biped Robots with Adaptive Foot Positioning

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Abstract— **This paper is concerned with Nonlinear Model Predictive Motion Control (NMPC) of 5-link planar biped robots. In spite of different motion control methods like computed torque, sliding mode, and feedback linearization, which try to track previously designed gait pattern, the robot walking control, in this paper, is not based on any desired trajectory; rather it is trajectory free. Using the NMPC controller with appropriate definition of the cost function, there is no need for the gait generation phase. To this end, a new cost function, which is based on the concept of the human walking, will be proposed. Moreover, using this cost definition, the NMPC changes the robot gait length adaptively to optimize the gait length and to maintain the robot balance in the presence of external disturbances. Simulation results show that the proposed method provides better disturbance rejection as compared to the previous approaches on trajectory free methods using the NMPC.**

Keywords: Biped robots, MPC, Trajectory free, Adaptive foot positioning, Optimal control

I. INTRODUCTION

During the last few years, legged or walking robots have gained increasing interests among researchers. One of the important reasons for exploring legged robots is their mobility to move in difficult terrains, where existing robots like wheeled robots cannot go. Wheeled robots excel in prepared surfaces such as rails and roads, but they perform poorly in places where the terrain is irregular. On the other hand, biped robots have the ability to walk in rough and discontinuous terrains; therefore, they are expected to be more adaptive and robust to environment changes than wheeled robots. Recently, different methods have been proposed for the motion control of biped robots. This control is usually addressed in two steps: 1) the walking pattern or the gait generation [4], 2) the control of the robot along the reference trajectory. The gait generation phase may be performed offline or online [2]. The offline gait generation cannot adapt to environment changes like obstacles, which can deteriorate the biped advantages. There are different methods for the online gait generation that can adapt itself to its environment. However, an optimal and adaptive gait pattern would better facilitate the robot motion control [3]. Given the gait pattern, the controller must follow the joints trajectories and guarantees the stability while considering robot constraints like maximum allowable torque and unilateral constraints. In addition to these demands, the optimality is also important and should be considered. Although reducing the tracking error is the goal of almost all control problems, it may not be the case for biped robots because these robots may have normal and acceptable walking even if there are some errors in the joints trajectory tracking. This is mainly due to the fact that human walking approach is based on an optimal algorithm which uses some basic goals and constraints to displace the body (the center of mass) safely from one point to another, while considering and predicting the environmental changes. A suitable way of imitating this behaviour for motion control of the biped robot is to state the biped motion control problem as a Non-linear Model-based Predictive Control (NMPC) problem [5], [6], [7]. With an appropriate objective function, while considering the state and control signal constraints plus physical constraints, it's possible to combine the walking pattern generation phase with the control phase and allowing the NMPC to decide about the gait pattern and the control signals. In this approach, there is no trajectory to follow. The control signals are generated directly by the NMPC in such a way that the biped robot walks safely. In addition to advantages of the on-line gait generation, the biped dynamics, constraints of the control signals, the present and the future states of the biped, and physical constraints on the robot are considered in the proposed method to execute an optimal and practical walk.

Disturbance attenuation in the motion control of biped robots is more complex than the ordinary control problems because the external disturbances (e.g. pushing the biped robot) may endanger the biped static stability. In the previous NMPC based trajectory free control of the biped robots, the external disturbance rejection has not been considered. In order to maintain the static balance, the ground projection of the biped CoM has to remain within the supporting area. If the external disturbance is less than a certain level, then the NMPC can compensate for it and maintain the static balance of the biped. However, if the external disturbance is greater than a specific value, it may lead to the biped instability. Thus, some stability measures must be developed to maintain the robot stability even against large external disturbances. Imitating the human reaction to encounter with strong external disturbances, the step length has to change in order to provide a safe supporting area to cover the CoM ground projection [13]. In The proposed method, the NMPC decides about the gait length; hence, the controller can improve robustness of the walk in the presence of external disturbances. The adaptive foot positioning using linear MPC has been used before for biped robots [11], [12]. However, these papers use some simplifications like neglecting the inertia. Although this assumption helps to achieve faster algorithms but it deviates from the real robot dynamics. Moreover, these methods just suggest the CoM ground projection path and do not generate joint trajectories directly. This paper presents a new cost function for the Nonlinear MPC (NMPC) that is based on the human walking concept. In addition, this cost function provides an optimal adaptive gait length for better balancing the robot and rejecting disturbances such as external forces.

This paper is organized as follows. Section II presents dynamics for the 5-link planar biped robot in single-support and double-support phases. Section III provides the proposed NMPC strategy by defining an appropriate objective function and constraints. Section IV shows simulation results. Section V concludes this paper.

II. The Biped Robot Dynamic

In this paper, the control of a planar biped robot with five links is considered. The biped contains a torso and two identical lower limbs with each limb having a thigh and a shank. Moreover, the biped has two hip joints, two knee joints, and two ankles at tips of lower limbs. There is an actuator located at each joint; all joints are considered friction free rotating in the sagittal plane. It is assumed that feet have no mass. This simplification does not reduce that much efficiency of the biped dynamics [8]. Although the dynamics of the feet are neglected, it is assumed that the biped can apply torque at the ankles. Each gait consists of two successive dynamics: 1) the single support phase (SSP), where a stance limb is in contact with the ground and the other limb swings from rear to front, 2) the double support phase (DSP), where both limbs are on the ground while the body can slightly move forward. The impact happens in an infinitesimal period of time as the swing limb collides with the ground and joint velocities are subject to a sudden jump resulting from this impact event. During the DSP, a torque is applied to the leading ankle whereas the rear ankle does not possess a torque but can rotate through the knee torque and the effect of gravity. The friction between the feet and the ground is assumed sufficient to prevent slippage during walking [8].

A. Single Support Phase

The biped locomotion with single foot support can be considered as an open-loop kinematic chain model [18]. The dynamic equations to describe biped SSP can be derived using the standard procedure of Lagrangian formulation as:

$$\mathbf{D}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{G}(\boldsymbol{\theta}) = \mathbf{T} \tag{1}$$

where $\mathbf{D}(\boldsymbol{\theta})$ is a 5×5 positive definite and symmetric matrix of inertia, $\mathbf{H}(\boldsymbol{\theta})$ is a 5×5 matrix related to the centrifugal and Coriolis terms, $\mathbf{G}(\boldsymbol{\theta})$ is a 5×1 vector of gravity terms, $\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}}, \mathbf{T}$ are 5×1 vectors of generalized coordinates, velocities, accelerations, and torques, respectively [8].

B. Double Support Phase

The DSP begins with the front limb touching the ground and ends with the rear limb taking off the ground. As both of the contact points between the lower limbs and the ground are fixed during the DSP, there exists a set of holonomic constraints as

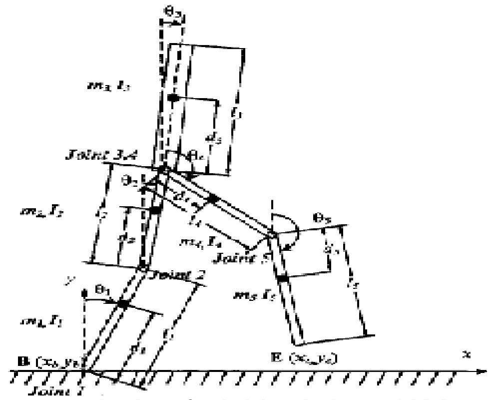


Fig. 1 The planar five link biped robot model [8].

$$\boldsymbol{\Phi}(\boldsymbol{\theta}) = \begin{pmatrix} x_e - x_b - L \\ y_e - y_b \end{pmatrix} = 0 \tag{2}$$

where L is the step length and x_b and x_e are the stance foot and the swing tip position, respectively. Hence, the Lagrangian equation of motion during the DSP is

$$\mathbf{D}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{G}(\boldsymbol{\theta}) = \mathbf{J}^{\mathrm{T}}(\boldsymbol{\theta})\boldsymbol{\lambda} + \mathbf{T} \tag{3}$$

where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers and $\mathbf{J} = \partial\boldsymbol{\Phi}/\partial\boldsymbol{\theta}$ is the 2×5 Jacobian matrix. As a dynamic system under holonomic constraints, a set of independent and generalized coordinates can be found to formulate the dynamic equations, which describe the constraint system without using the terms of constraint forces [17]. Let the independent and generalized coordinate be $\mathbf{p} = (x_h \quad y_h \quad \theta_3)^{\mathrm{T}}$ where $(x_h \quad y_h)$ is the hip position. With this new coordinates, Eq. (3) can be written as

$$\ddot{\mathbf{p}}(\boldsymbol{\theta}) = \mathbf{B}\,\mathbf{p}(\boldsymbol{\theta}) + \mathbf{C}(\mathbf{T} - \mathbf{N}) \tag{4}$$

where $\mathbf{B}$ is a 3×3 matrix, $\mathbf{C}$ is a 3×5 matrix and $\mathbf{N}$ is a 5×1 vector [8].

C. Impact Effect

At the end of the SSP, the tip of the swing limb contacts the ground surface with an impact. The joint velocities are subject to a sudden jump resulting from this impact event. The vertical velocity of the tip of the swing limb becomes zero, immediately after the impact due to the ground collision

$$\dot{\boldsymbol{\theta}}^{+}_{\text{impact}} = \dot{\boldsymbol{\theta}}^{-} + \mathbf{D}^{-1}\mathbf{J}^{\mathbf{T}}\left[\mathbf{J}\mathbf{D}^{-1}\mathbf{J}^{\mathbf{T}}\right]^{-1}\left(-\mathbf{J}\dot{\boldsymbol{\theta}}^{-}\right) \tag{5}$$

where $\dot{\boldsymbol{\theta}}^{+}_{\text{impact}}$ and $\dot{\boldsymbol{\theta}}^{-}$ are 5×1 vectors of generalized velocities immediately after and before the impact, respectively [8].

III. NMPC Control Approach

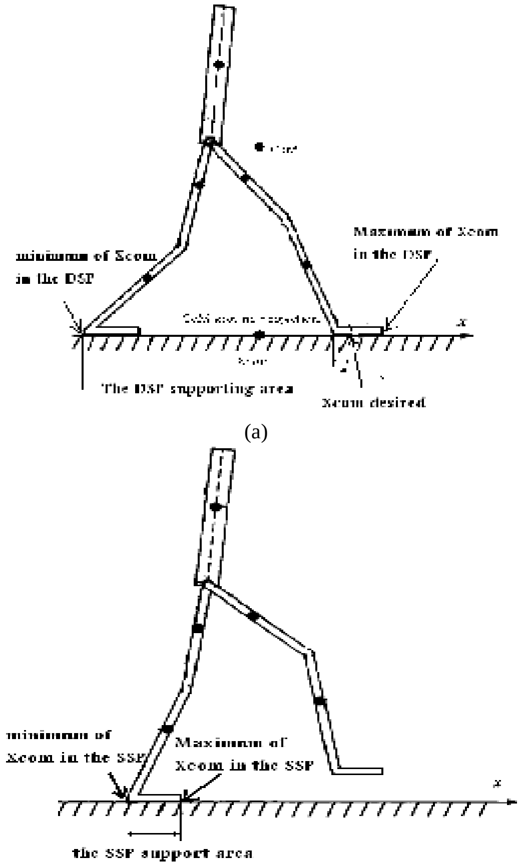
In general, motion planning in robotics can be separated in two phases: 1) the motion planning and 2) the trajectory following. However, human walking strategy is not based on following a planned or desired trajectory. Perfect joint trajectory tracking is not necessary during the walking gait. The human walking strategy is based on defining some basic goals and constraints like:

- Moving the body CoM from one point to another with almost constant speed.
- Walking in such a way to reduce the energy consumption.
- Satisfying the physical and environmental constraints, like obstacles avoidance.
- Maintaining the balance during walking.

This problem statement can bring new visions to the biped robot motion control. Having the previous goals and constraints, the nonlinear model predictive control (NMPC), which is based on minimizing an objective function subjected to some constraints, can be a good choice. In addition to be an optimal control method, the NMPC can handle nonlinear multivariable non-minimum phase systems like the biped robots. The more the motion control of the biped mimics closer the human walking scheme, the more adaptive, robust and optimum the robot walking would be. The NMPC consists of two parts: a nonlinear model and an optimizer, which requires an objective function with some constraints. Unlike previous reports on using the NMPC for biped robots [9], [10], which only try to minimize the energy consumption, the idea in this paper, in addition to minimizing the energy consumption, is to incorporate the strategy of the human walking strategy; i.e. pushing the CoM forward and providing new supporting area. In the CoM stability, walking can be classified into two different categories: 1) the static walk 2) the dynamic walk. In the static walk, the CoM ground projection never exits the supporting area. However, in the dynamic walk, the CoM ground projection can exit momentarily and then going back to the stable region. In this paper, the static walk is considered [14], [15]. In order to have a stable walking, the ground projection of CoM has to remain in the supporting area. As mentioned before, each step comprises of two phases: the single support phase and the double support phase. The supporting area changes in each phase. In the SSP, the supporting area is the contact area of the stance foot, while in the DSP the supporting area expands to the area between two feet (Fig. 2). If walking starts in the DSP, the CoM ground projection must move from its initial position, which is in DSP supporting area, until it enters the supporting area of the SSP, which is the contact area of the front foot. As soon as the ground projection of the CoM enters the SSP area, it's safe to switch to the SSP, where the biped must provide a new supporting area; this is accomplished by moving forward the limb and bringing it to the front of the stance foot. As the swinging foot lands on the ground, the SSP is finished and the DSP starts. At this moment, the CoM ground projection, which is in the rear foot contact area, moves to the front foot contact area. This cycle is repeated in the next step. Now, the main task is to incorporate the human walking strategy into the object function of the NMPC.

A. The Objective Function

Because of the existence of two phases in one step (i.e. the DSP and the SSP) there would be two different objective functions for these phases. In the DSP, the distance between the CoM ground projection and the minimum of the horizontal position of CoM in the SSP (Fig. 2) must be considered in the objective function. In the SSP, the rear foot has to swing and land in front of the stance foot to make a new supporting area. In previous NMPC papers [9], [10], the Step Length (SL) was considered fixed or may not be changed directly by the NMPC. In this paper, a desired SL is not defined implicitly. In other words, the SL is not fixed and may be changed by the NMPC in order to minimize the energy consumption and provide more stability to the robot. Here, the following objective functions are defined for the DSP and the SSP, respectively:



(a)

(b)

Fig. 2 (a) The DSP and (b) the SSP supporting area and their margins

$$\min_{\mathbf{U}\in\mathbf{R}^{5n_c}} J_{\mathrm{SSP}} = \mathrm{w}_{1V}^{\mathrm{SSP}}\sum_{j=0}^{n_c-1}\mathbf{T}(t+j\,\Delta t)^{\mathrm{T}}\mathbf{T}(t+j\,\Delta t) + + \mathrm{w}_{2\varsigma}^{\mathrm{SSP}}\sum_{i=1}^{n_p} y_e(t+i\,\Delta t) + \mathrm{w}_{3V}^{\mathrm{SSP}}\sum_{i=1}^{n_p}\left|\dot{x}_{\mathrm{CoM}}(t+i\,\Delta t)-\dot{x}_{\mathrm{CoM}}^{\mathrm{d}}\right| \tag{6}$$

$$\min_{\mathbf{U}\in\mathbf{R}^{5n_c}} J_{\mathrm{SSP}} = w_{1V}^{SSP}\sum_{j=0}^{n_c-1}\mathbf{T}(t+j\,\Delta t)^{\mathrm{T}}\mathbf{T}(t+j\,\Delta t) + +w_{2V}^{SSP}\sum_{i=1}^{n_p} y_e(t+i\,\Delta t) + w_{3V}^{SSP}\sum_{i=1}^{n_p}\left|\dot{x}_{\mathrm{CoM}}(t+i\,\Delta t)-\dot{x}_{\mathrm{CoM}}^{\mathrm{d}}\right| \tag{7}$$

where n_p is the prediction horizon, n_c is the control horizon, Δt is the sampling time, $x_{\mathrm{CoM}}(t+i\,\Delta t)$, $\dot{x}_{\mathrm{CoM}}(t+i\,\Delta t)$ are the horizontal position and the speed of the biped CoM ground projection at i^{th} prediction, respectively, $x_{CoM}^{d} = x_b + \Delta x$ is the minimum of CoM ground projection in the SSP that equals x_b, the horizontal position of the stance foot, plus Δx to guarantee more stability at the start of the SSP. $\dot{x}_{CoM}^{d}$ is the desired horizontal velocity of the CoM, $\mathbf{T}(t+j\Delta t)$ is the exerted torque to the joints, $y_e(t+i\,\Delta t)$ is the vertical tip position of the swing foot at the i^{th} prediction; The SSP objective function comprises two parts 1) the swing foot before stance foot ($V=1$) and 2) the swing foot after the stance foot ($V=2$) Eq.7. Changing the step length affects the walking speed and the stability. w_1^{DSP}, w_2^{DSP} And w_3^{DSP} are the weights of the DSP objective function, likewise, w_{11}^{SSP}, w_{21}^{SSP}, w_{31}^{SSP}, w_{12}^{SSP}, w_{22}^{SSP} and w_{32}^{SSP} are the weights of the SSP objective function. In order to obtain adaptive gait length, w_{22}^{SSP} is defined as

$$w_{22}^{SSP} = h\,\exp\left(\frac{F_L + s}{F_L + x_b + s - x_{CoM}(t)}\right) \quad (8)$$

where F_L is the foot length and s is a positive constant to prevent w_{22}^{SSP} to become infinite when $x_{CoM}(t)$ reaches to its maximum allowable position in the SSP. For $x_{CoM}(t) = x_b$, w_{22}^{SSP} is equal to h but as $x_{CoM}(t)$ approaches its maximum allowable position in the SSP, the biped stability reduces and w_{22}^{SSP} increases and forces the swing foot to land on the ground to supply a new supporting area. Adjusting h and s can change the step length. When there are no external disturbing forces, the step length is fixed. But as soon as the external force occurs, the controller pushes $x_{CoM}(t)$ to its maximum allowable position in the SSP. Hence, w_{22}^{SSP} rapidly increases and forces the swing foot to land sooner; in this way, the step length decreases in order to maintain stability for the biped.

B. The Constraints

Some constraints must be defined in order to obtain a normal and human-like walk.

i) The DSP Constraints

1) Joint constraints: $q_{i,\min} \le q_i \le q_{i,\max}$ where $q_i = q_{i-1} - q_i, (i:1,2,...5)$ and $q_0 = 0$. In order to prevent singularity in the Jacobian matrix of the robot, constraints on the joint angle two and five have to be selected in such a way that never q_2 and q_5 become zero.

2) During walking, the CoM ground projection of the biped robot should move only forward; that is $\dot{x}_{CoM} \ge 0$.

3) The hip level constraint guarantees the biped to maintain its erected posture during the locomotion $h_{\min} \le h_{hip} \le h_{\max}$, where h_{hip} is the vertical position of the tip of link 2 (Fig.1).

4) The Torso should maintain almost the upright position during the whole cycle $a_{\min} \le q_3 \le a_{\max}$.

5) In order to guarantee static stability of the biped robot, the CoM ground projection has to remain in the support area. This constraint can be given as $x_{\min}^{DSP} \le x_{CoM} \le x_{\max}^{DSP}$.

6) In the DSP, both feet must remain on the ground. The stance foot is always on the ground. In order to keep the other foot on the ground we must have $y_e = 0$.

ii) The SSP constraints: The first five constraints for the SSP are the same as the DSP constraints. The other constraints are as follows:

6) The swing foot is always in higher positions than the ground level $y_e \ge 0$.

7) The height of the swing foot is limited $y_e \le h_m$.

8) The swing foot must maintain a minimum velocity $2\dot{x}_{CoM}\sin(\frac{\pi}{2}\frac{y_e}{h_m}) \le \dot{x}_e$ where $\dot{x}_e$ is the horizontal velocity of the swing foot tip. This constraint synchronizes the swing foot with the CoM horizontal speed. It also adapts the swing foot velocity to its height and helps a smoother touch of the swing foot on the ground. Hence, it can reduce the effect of the impact.

9) During the SSP, the tip position of the swing foot follows almost a parabolic trajectory. Using this concept, the following constraints are suggested:

a) While the swing foot is behind the stance foot, its height increases $\dot{y}_e \ge 0$.

b) As soon as the swing foot passes the stance foot, its height decreases till it touches the ground $\dot{y}_e \le 0$.

iii) Joint Torque Constraints: $\mathrm{T}_{\min} \le \mathbf{T} \le \mathrm{T}_{\max}$.

IV. Simulation Results

The same parameters of the biped robot that have been used in [9], [10], are given in Table 1. The following values have been used in the NMPC: $\mathrm{n_p} = 5$, $\mathrm{n_c} = 3$, $\Delta t = 0.02s$, $\dot{x}_{CoM}^{d} = 0.3\mathrm{m.s}^{-1}$. $w_1^{DSP} = 0.001$, $w_2^{DSP} = 1000$, $w_3^{DSP} = 100$, $w_{11}^{SSP} = 0.001$, $w_{21}^{SSP} = 0$, $w_{31}^{SSP} = 100$, $w_{12}^{SSP} = 0.001$, $w_{32}^{SSP} = 100$, $F_L = 0.15m$, $h = 500$, $s = 0.05m$. Table 2 shows the maximum and the minimum of the MPC controller constraints. The optimization problem is solved using the *fmincon* function in the MATLAB Optimization Toolbox dedicated to the minimization of a constrained nonlinear multivariable function. *fmincon* is based on the sequential quadratic programming (SQP) algorithm. SQP is an iterative technique in which the objective is replaced by a quadratic approximation and the constraints by linear approximations. Simulations are performed using Intel T7500 Core2 Duo 2.2Mhz processor with 1Gbyte of RAM. The objective functions in Eqs. (6) and (7) subject to constraints are minimized using the *fmincon* function. The input sequence $\mathbf{U} = [\mathbf{T}(t)^{\mathrm{T}}\ \mathbf{T}(t+\Delta t)^{\mathrm{T}}\ \mathbf{L}\ \mathbf{T}(t+(\mathrm{n_c}-1)\Delta t)^{\mathrm{T}}]^{\mathrm{T}}$ is the solution of the optimization problem. The first element of this vector, i.e. $\mathbf{T_c}(t) = \mathbf{T}(t)$, is applied to the biped robot as control signals. Simulations are carried out for two-step walk on the flat ground. In the first step, there are no external disturbing forces. At $t = 1.46\mathrm{s}$, a pushing force for a time span of $t = 0.2\mathrm{s}$ is exerted to the biped robot. This external force is modelled with $\mathrm{T_d} = 400\mathrm{N.m}$ applied to the joint one (i.e. the supporting ankle). Fig. 3 illustrates the vertical and the horizontal position of the hip and the tip of the swing foot. The joints torque is shown in Fig. 4 and Fig. 5, respectively. The vertical and the horizontal position of the CoM in the fixed foot length mode are illustrated in Fig. 6.

Table I

Physical parameters of the robot [9], [10]

Link No.	l_i **Length(m)**	m_i **Mass(kg)**	I_i **Inertia(kgm^2)**	d_i **Center of Mass(m)**
1	0.41	5.93	0.69	0.258
2	0.41	10.9	1.31	0.258
3	0.5	48	18.99	0.391
4	0.41	10.9	1.31	0.258
5	0.41	5.93	0.69	0.258

Table II

Maximum and minimum of the constraints		
Variable	Min	Max
h_{hip}	0.68m	0.72m
q_3	-3°	3°
h_m	0	0.1m
T_i	-270N.m	270N.m

As Fig. 3 shows, the swing foot has almost parabolic trajectory and the hip height changes are limited creating smooth walking. It also illustrates that the biped robot has almost 0.3m/s constant transitional speed. Moreover, and more importantly, this figure shows that adaptive foot stepping is provided in order to guarantee proper balance to the biped. In other words, during the second step, where the external disturbance $T_d = 400\text{N.m}$ is applied to joint one, the NMPC forces the biped to land the swing foot sooner to maintain the stability. The two consecutive steps are equal to 0.43m and 0.32m, respectively. Fig. 4 shows the same conditions, but without the adaptive foot positioning, just like previously reported papers using NMPC for bipeds. As this figure shows the NMPC without adaptive foot positioning cannot maintain the stability of robot when there are external forces. Fig. 5 shows that control torques are limited by the NMPC to the desired values. An important condition is that the CoM ground projection remains in the stable region in both the SSP and the DSP. Fig. 6 reveals that the NMPC has provided the static stability to the biped.

One important point is the computation time, which is equal to 0.2 sec in average. Considering the sampling time equal to 0.02 sec., it might be concluded that the proposed method may not be applied in real time. However, by using lower level language programming (like C++) instead of the MATLAB environment, the computation time can be decreased substantially. Moreover, by using some other methods to solve the optimization problem, more computation time can be saved [16].

V. Conclusion

This paper proposed an NMPC method, where the need for the gait generation phase was omitted by defining as appropriate const function. Moreover, by defining an adaptive term to the cost function of the SSP, adaptive foot positioning was achieved for maintaining good balance and providing stability to the robot. Simulation results showed that this property is vital when the robot encounters external disturbances, like pushing or pulling the robot with large forces. The proposed method has the ability to walk in more complicated situations like climbing stairs. This will be addressed in future works.

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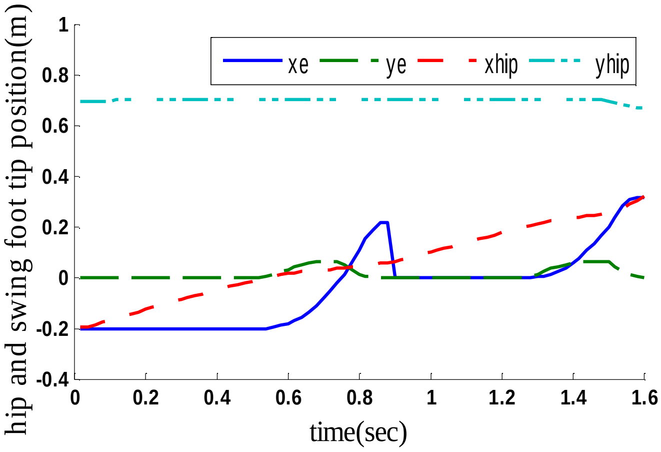

Fig. 3 Vertical and horizontal tip position of hip and swing foot in the adaptive foot length mode

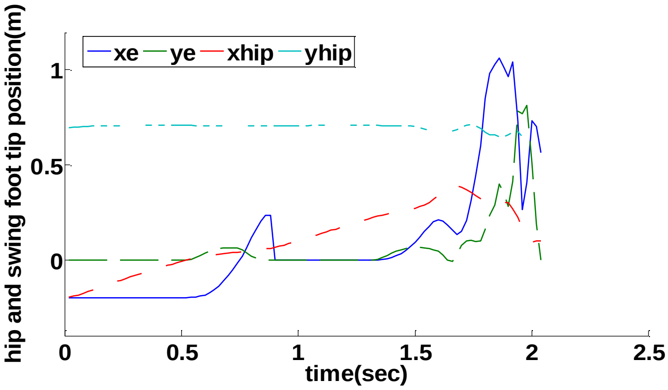

Fig. 4 Vertical and horizontal tip position of hip and swing foot in the fixed foot length mode

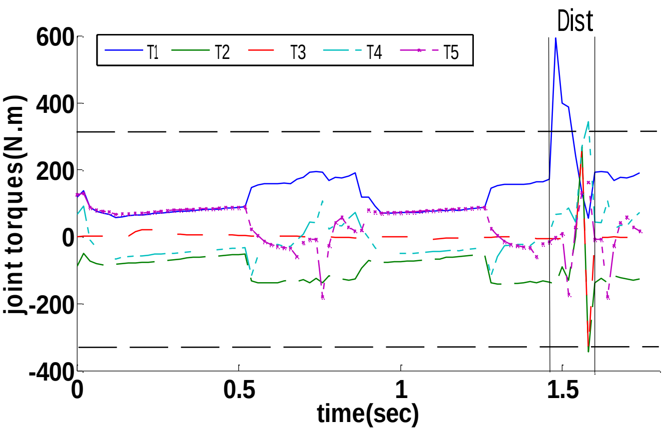

Fig. 5 Joint torques

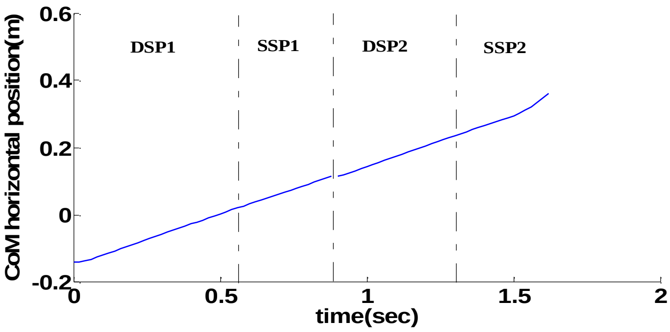

Fig. 6 CoM horizontal position

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