Robust adaptive control of uncertain nonlinear systems using neural networks

S. MOHAMMAD-HOSEINI[†], M. FARROKHI[†,††]and A. J. KOSHKOUEI [*]

[†]Department of Electrical Engineering, Iran University of Science and Technology, Tehran 16846, IRAN

[††] Centre of Excellence for Power System Operation and Automation, Iran University of Science and Technology, Tehran 16846, IRAN

[*] Control Theory and Applications Centre, Coventry University, Coventry, UK

E-mails: {sm_hoseini, farrokhi}@iust.ac.ir, a.koshkouei@coventry.ac.uk

Abstract

This paper presents a direct adaptive output feedback control design method for uncertain non-affine nonlinear systems, which does not rely on state estimation. The approach is applicable to systems with unknown but bounded dimensions and with known relative degree. A neural network is employed to approximate the unknown modeling error. A neural network is employed to approximate and adaptively make ineffective unknown plant nonlinearities. An adaptive law for the weights in the hidden layer and the output layer of the neural network are also established so that the entire closed-loop system is stable in the sense of Lyapunov. Moreover, the robustness of the system against the approximation error of neural network is achieved with the aid of an additional adaptive robustifying control term. In addition, the tracking error is guaranteed to be uniformly and asymptotically stable, rather than uniformly ultimately bounded, by using this additional control term. The proposed control algorithm is relatively straightforward and no restrictive conditions on the design parameters for achieving the systems stability are required. The effectiveness of the proposed scheme is shown through simulations of a non-affine nonlinear system with unmodelled dynamics, and is compared with a second-sliding mode controller.

Keywords: Discontinuous controllers, Output feedback; Nonlinear systems; Adaptive control; Neural networks

1. Introduction

Control system design for complex nonlinear systems has been widely studied in the last decade. Many remarkable results in this area have been reported, including feedback linearization techniques (Isidori 1989, Esfandiari and Khalil 1992), backstepping design (Krstic *et al.* 1995) for systems with unmatched uncertainties (Koshkouei *et al.* 2004). For uncertain systems, sliding mode control approaches have been developed by some researchers (Zinober 1990, 1994; Levant 2003, 2005). In these methods, the controller gains are computed using upper bound information on the system uncertainties, which is normally unavailable and there is no direct method to obtain them. Therefore, these methods may yield overestimation resulting from a conservative design. To overcome this problem, several adaptive schemes have been developed for affine nonlinear systems in order to deal with the problem of parametric uncertainties (Marino and Tomei 1995, Khalil 1996).

During the last decade, adaptive methods based on Neural Networks (NNs) have been developed to control uncertain nonlinear systems by removing the unknown nonlinear part of the system. Most of these approaches have been proposed for affine systems (Lewis *et al.* 1995, 1996) and some of them consider non-

‡ Corresponding author

affine systems, based upon the state feedback (Kim and Calise1997, Park and Park 2003) or output feedback (Ge *et al.* 1999, Ge and Zhang 2003, Hovakimyan *et al.* 2002). In particular, because of approximation errors inherent in NNs, when the number of neurons is limited or initialization of weights are not suitable, most of these methods can guarantee only uniformly ultimately bounded (UUB) stability. To remove this obstacle and to compensate the reconstruction errors, the method has been widely used in which an extra non-adaptive robustifying input term is considered based on information about bounding constants of system uncertainties which may be unavailable. This approach may results in a conservative design (Lewis *et al.*1996, Polycarpou and Mears 1998, Seshagiri and Khalil 2000, Park and Park 2003).
An adaptive method combined from the classical sliding mode (Koshkouei and Zinober, 1998) and neural-network techniques have been presented by Wai (2003) for the control of a rigid-link robot manipulator model. The method is based upon the equivalent control, which needs the observability of all system states and an estimate of the upper bound of uncertainties.

In this paper, an adaptive robustifying control term is proposed using the system output of the system which guarantees the robustness of system against approximation error of NN and assures the asymptotic stability of tracking error. Therefore, the overall proposed control law is based on output feedback control methods and estimation of states is not required. In addition, it is not necessary the plant dimension to be known a priori and for designing the controller, only the relative degree of the system is required. Since the control law comprises of stabilizer, adaptive and robustifying terms, the closed-loop system is robust against unmodelled dynamics and asymptotic stability of the system is assured. In addition, the method is applicable to a class of nonlinear systems with any relative degree. The method is based on strictly positive realness (SPR) condition of the closed-loop error dynamics, the Kalman-Yakobovich's lemma, as well as NN techniques.

The paper is organised as follows: Section 2 describes the class of nonlinear systems to be controlled and the problem of the tracking error. The controller design procedure and approximation properties of the NNs are addressed in Section 3. In Section 4, the stability of the closed-loop system is proved through analytical work. An example which illustrates the effectiveness of the proposed controller is presented in Section 5. In this section, the simulation results have been compared with a second-order sliding mode controller. Conclusions are given in Section 6.

2. Problem statement

Consider the nonlinear SISO system

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},u) \\ y = h(\mathbf{x}) \end{cases} \tag{1}$$

where $\mathbf{x} \in \Omega_x \subset R^n$ is the state vector on the compact set Ω_x as an operating region, $u \in \Omega_u \subset R$ is the input on the compact set Ω_u, and $y \in R$ is the output. The mapping $\mathbf{f}: R^{n+1} \to R^n$ is an unknown smooth vector field and $h: R^n \to R$ is a smooth and known real function. Assume that the relative degree of the system (1) is $r \le n$. Under this assumption, there is a diffeomorphism transformation

$$\Phi(\mathbf{x}) = [\mathbf{z}, \boldsymbol{\eta}] = \left[h(\mathbf{x}), L_f h, \ldots, L_f^{r-1} h, \varphi_1(\mathbf{x}), \ldots, \varphi_{n-r}(\mathbf{x}) \right]$$

which transforms the system (1) in to the following normal form with a new coordinate $[\mathbf{z}, \boldsymbol{\eta}] = [z_1, ..., z_r, \eta_{r+1}, ..., \eta_n]$ (Isidori 1995)

$$\begin{cases} \dot{z}_i = z_{i+1} & 1 \le i \le r-1 \\ \dot{z}_r = b_1(\mathbf{z}, \boldsymbol{\eta}, u) \\ \dot{\boldsymbol{\eta}} = \mathbf{w}(\mathbf{z}, \boldsymbol{\eta}) \\ y = z_1 \end{cases} \tag{2}$$

Assumption 1: *The zero dynamics of system (2),* $\dot{\boldsymbol{\eta}} = \mathbf{w}(\mathbf{0}, \boldsymbol{\eta})$ *are exponentially stable. Moreover, the desired trajectory and its derivates are bounded such that the internal dynamics remain bounded.*

Assumption 2: *Assume that* $b_u = \partial b_1(\mathbf{z}, \boldsymbol{\eta}, u)/\partial u \ne 0$. *This condition implies that the smooth function* b_u *is strictly either positive or negative on the compact set*

$$U = \left\{ (\mathbf{z}, \boldsymbol{\eta}, u) \middle| (\mathbf{z}, \boldsymbol{\eta}) = \Phi(\mathbf{x}), \mathbf{x} \in \Omega_x ;\ u \in \Omega_u \right\}$$

From (2), the input-output relation of system is

$$y^{(r)} = b_1(\mathbf{z}, \boldsymbol{\eta}, u) = b(\mathbf{x}, u). \tag{3}$$

This paper addresses the design of an output feedback control law that utilises the available measurement $y(t)$, to obtain system output tracking of a bounded trajectory $y_d(t)$ that is assumed to be r-times differentiable. The difference between unknown dynamics function b and its estimate $\hat{b}$ (i.e. the modelling error) is mapped using a Neural Network (NN). This mapping has to be based on measured input and output data only. The universal approximation property of NNs and the observability of the system are utilised to construct this mapping online using measured input/output time histories. Also, a robustifying part is designed to guarantee the robustness of the system against the mapping error. These various features of the proposed control design scheme are presented in the next section.

3. Controller design

3.1. Error dynamic and the control structure

Assume that $\hat{b}(y,u)$ is an appropriate approximation of $b(\mathbf{x},u)$. Consider the pseudo control

$$v = \hat{b}(y,u) \tag{4}$$

Then the modelling error is

$$\Delta(\mathbf{x},u) = \hat{b}(y,u) - b(\mathbf{x},u) \tag{5}$$

Using (3), (4) and (5) the system dynamic can be expressed as

$$y^{(r)} = \hat{b}(y,u) - \Delta(\mathbf{x},u) = v - \Delta(\mathbf{x},u) \tag{6}$$

Equation (6) represents the dynamic relation of r integrators between the pseudo control v and the system output y, where the modelling error $\Delta(\mathbf{x},u)$ acts as a disturbance signal. It is required that the system output y tracks a known and bounded reference y_d. Select the pseudo control

$$v = y_d^{(r)} + u_L + u_{ad} - u_R \tag{7}$$

where $y_d^{(r)}$ is the r-derivative of the desired output y_d, u_L stabilises the closed-loop system, u_{ad} is the adaptive part and it cancels out the modelling error $\Delta(\mathbf{x},u)$ whilst the control part u_R is proposed to achieve robust asymptotic stability. The robust control u_R could be continuous or discontinuous. In particular, one may consider a sliding mode control since it is robust in the presence of uncertainties.

Now, substituting (7) into (6), the closed-loop error dynamic can be presented as

$$e^{(r)} = -u_L + \left(\Delta(\mathbf{x},u) - u_{ad}\right) + u_R \tag{8}$$

It is important to point out that the model approximation function $\hat{b}(\cdot,\cdot)$ should be defined so that it is invertible with respect to u, allowing the actual control input to be computed by

$$u = \hat{b}^{-1}(y,v)$$

As stated, u_{ad} is designed to cancel the unknown modelling error $\Delta(\mathbf{x},u) = \Delta(\mathbf{x},\hat{b}^{-1}(y,v))$ whereas Δ depends on $u_{ad}(t)$ through v. So, there exists a fixed point problem

$$u_{ad}(t) = \Delta\left(\mathbf{x}(t), \hat{b}^{-1}(y, y_d^{(r)} + u_L + u_{ad} - u_R)\right) \tag{9}$$

The following assumption provides conditions that guarantee the existence and the uniqueness of a solution for u_{ad}.

Assumption 3: *The map $u_{ad} \to \Delta$ is a contraction over the entire input domain. This means, the following inequality should be satisfied:*

$$\left|\frac{\partial \Delta}{\partial u_{ad}}\right| < 1 \tag{10}$$

Using (4), (5), (7) and (10) implies

$$\left|\frac{\partial \Delta}{\partial u_{ad}}\right| = \left|\frac{\partial(\hat{b}-b)}{\partial u}\frac{\partial u}{\partial v}\frac{\partial v}{\partial u_{ad}}\right| = \left|\frac{\partial(\hat{b}-b)}{\partial u}\frac{\partial u}{\partial \hat{b}}\right| < 1$$

which can be rewritten as

$$\left|1 - \frac{\partial b/\partial u}{\partial \hat{b}/\partial u}\right| < 1 \tag{11}$$

Condition (11) is equivalent to the following two conditions:

$$\begin{aligned} &\operatorname{sgn}(\partial b/\partial u) = \operatorname{sgn}(\partial \hat{b}/\partial u) \\ &\left|\partial \hat{b}/\partial u\right| > \left|\partial b/\partial u\right|/2 > 0 \end{aligned} \tag{12}$$

Hence, $\hat{b}(y,u)$ should be selected such that it satisfies the conditions (12). When $b(\mathbf{x},u)$ is unknown, a simple choice for approximation is $\hat{b}(y,u) = cu$, where c is a suitable constant.

3.2. Construction of SPR Error Dynamic

In this section, a strictly positive realness (SPR) property of closed-loop error dynamic is studied. Assume that u_L is a suitable filter with the following structure

$$u_L = \frac{N_L}{D_L} e \tag{13}$$

and filtered error signal $\tilde{e}$ is defined as

$$\tilde{e} = G_{ad}(s)e = \frac{N_{ad}}{D_{ad}} e \tag{14}$$

where minimum phase filter $G_{ad}(s)$ is selected so that $G_{ad}(0) \neq 0$. This signal is constructed to ensure a realizable error signal that is used to adapt the NN weights, and for designing u_R .

Using (8) and based on the compensator defined in (13) and (14), the closed-loop transfer function of the system depicted in Figure 1, can be written as

$$\tilde{e}(s) = G(s)\left(\left(\Delta(\mathbf{x},u) - u_{ad}\right) + u_R\right)(s) \tag{15}$$

where

$$G(s) = \frac{D_L N_{ad}}{D_{ad}\left(s^r D_L + N_L\right)} \tag{16}$$

in which r is the same as in (2). Analysing the denominator of (16), the Routh-Hurwitz stability criterion implies that a necessary condition for the closed-loop system stability is that the polynomial $s^r D_L + N_L$ is complete, i.e. all of the polynomial coefficients should be nonzero. Therefore, the degree of the compensator numerator N_L (and hence D_L) should be at least $r-1$. Thus, k is defined as

$$k = \deg(D_L) \geq \deg(N_L) \geq r-1 \tag{17}$$

In addition, to simplify the design procedure, D_{ad} and D_L are selected such that

$$\deg(D_{ad}) = \deg(D_L) \tag{18}$$

Hence, the relative degree of $G(s)$ is

$$\rho = k + r - \deg(N_{ad}) \tag{19}$$

where $\deg(N_{ad}) \leq k$. Therefore, $\rho \geq r$. As it is shown in Section 4, the NN adaptation rules is realizable (i.e. dependent on available data only), the transfer function $G(s)$ must be strictly positive real (SPR). However, the relative degree of $G(s)$ is at least r. When the relative degree of $G(s)$ is 1, it can be made SPR by a proper construction of $N_{ad}(s)$. If $\rho > 1$, then $G(s)$ cannot be SPR (Narendra and Annaswamy 1989). To achieve SPR when $\rho > 1$, a stable low pass filter $T(s)$ is introduced such that

$$\deg(N_{ad}) + \deg(T) = k + r - 1 \tag{20}$$

Hence, the new filtered error dynamic is

$$\tilde{e}(s) = \bar{G}(s)T^{-1}(s)\left(\left(\Delta(\mathbf{x},u) - u_{\mathrm{ad}}\right) + u_R\right)(s) \tag{21}$$

where $\bar{G}(s)$ can be represented as

$$\bar{G}(s) = G(s)T(s) = \frac{b_1 s^{p-1} + b_2 s^{p-2} + \ldots + b_p}{s^p + a_1 s^{p-1} + \ldots + a_p}, \quad p = 2k + r \tag{22}$$

Since $\bar{G}(s)$ is a stable transfer function, its zeros (the roots of N_{ad} and $T(s)$) can be appropriately placed to make it SPR. Moreover, it is important to note that $T(s)$ should be designed such that the step response of $T^{-1}(s)$ has no overshoot. This is a mild constrain that is used in stability proof.

Hence, the state space model of closed-loop error dynamic (21) can be represented as

$$\dot{\xi} = \mathbf{A}_{cl}\xi + \mathbf{b}_{cl}\left[T^{-1}(s)\left(\left(\Delta(\mathbf{x},u) - u_{ad}\right) + u_R\right)\right]$$
$$\tilde{e} = \mathbf{c}_{cl}^T\xi \tag{23}$$

According to the Kalman-Yakobovich lemma, the strict positive realness of $\bar{G}(s)$ assures the existence of a matrix $\mathbf{P} = \mathbf{P}^T > 0$ which satisfies

$$\mathbf{A}_{cl}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{cl} = -\mathbf{Q} \tag{24}$$

and

$$\mathbf{P}\mathbf{b}_{cl} = \mathbf{c}_{cl} \tag{25}$$

where $\mathbf{Q} = \mathbf{Q}^T > 0$ is an arbitrary weighting matrix..

3.3. Neural Network-Based Approximation

In the following lemma, it is shown that the modelling error $\Delta(\mathbf{x},u)$ can be approximated by a neural network, based on input-output data only. Moreover, it is proved that if any non-affine system satisfies conditions (12), then it is unnecessary to use $u_{ad}(t)$ as an input signal to NN. So, it is possible to employ static NN rather than recurrent NN to approximate $u_{ad}(t)$.

Lemma 1*: If conditions (12) are satisfied, then, modelling error $\Delta(\mathbf{x},u)$ can be approximated by a static single hidden layer Multilayer Perceptron (MLP) as $\mathbf{w}^T\boldsymbol{\sigma}\left(\mathbf{V}^T\boldsymbol{\zeta}\right)$, where $\mathbf{w} \in R^m$ is a vector containing synaptic weights of the output layer, $\mathbf{V} \in R^{N\times m}$ is a matrix containing the weights for the hidden layer, $\boldsymbol{\sigma}(\cdot) \in R^m$ is a vector function containing the nonlinear functions in the neurons of the hidden layer, and $\boldsymbol{\zeta} \in R^N$ is the input vector, which is equal to $\boldsymbol{\zeta} = [1 \quad \bar{\mathbf{y}} \quad \bar{\mathbf{u}}_\alpha \quad \bar{\mathbf{u}}_{ad}]^T$, where*

$$\bar{\mathbf{y}} = [y(t) \quad \cdots \quad y(t - T_d(n_1 - 1))]$$
$$\bar{\mathbf{u}}_\alpha = [u_\alpha(t) \quad \cdots \quad u_\alpha(t - T_d(n_1 - r - 1))] \tag{26}$$
$$\bar{\mathbf{u}}_{ad} = [u_{ad}(t - T_d) \quad \cdots \quad u_{ad}(t - T_d(n_1 - r - 1))]$$

in which $u_\alpha = v - u_{ad} = y_d^{(r)} + u_L - u_R$.

Proof: Under the observability condition of system (1), it is shown that the continuous-time dynamic $\Delta(\mathbf{x},u)$ can be approximately reconstructed using delayed inputs and outputs (Lavertsky *et al.* 2003)

$$\Delta(\mathbf{x},u) = \mathrm{F}(\bar{y},\bar{v}) + \varepsilon_1 \tag{27}$$

where

$\bar{v} = [v(t) \quad v(t - T_d) \quad \cdots \quad v(t - T_d(n_1 - r - 1))]$, $n_1 \geq n$, and $|\varepsilon_1| \leq \varepsilon_{1M}$, in which ε_{1M} is proportional to sampling time interval T_d. Hence, ε_1 can be ignored by selecting T_d sufficiently small.

In addition, in Section 3 it was shown that the conditions (12) guarantee the existence and uniqueness of solution u_{ad} from the following equation:

$$M(\mathbf{x},u,u_{ad}) = \Delta(\mathbf{x},u) - u_{ad}(t) = 0 \tag{28}$$

Differentiating M with respect to u_{ad} and using (4) and (5) yields

$$\begin{aligned}\frac{\partial}{\partial u_{ad}}M(\mathbf{x},u,u_{ad}) &= \frac{\partial}{\partial u_{ad}}\left(\Delta(\mathbf{x},u) - u_{ad}\right)\\ &= \frac{\partial}{\partial u_{ad}}\left(v - b(\mathbf{x},u)\right) - 1\\ &= \frac{\partial}{\partial v}\left(v - b(\mathbf{x},u)\right)\frac{\partial v}{\partial u_{ad}} - 1\\ &= \left(1 - \frac{\partial}{\partial v}b(\mathbf{x},u)\right)(+1) - 1\\ &= -\left(\frac{\partial}{\partial u}b(\mathbf{x},u)\right)\left(\frac{\partial u}{\partial v}\right)\end{aligned} \tag{29}$$

which is nonzero by Assumption 2 and using (4). Thus, from (29), according to the implicit function theorem and using (27)

$$\Delta(\mathbf{x},u) - u_{ad}(t) = \mathrm{F}\left(\bar{y}, \bar{u}_\alpha + \bar{u}_{ad} + u_{ad}(t)\right) - u_{ad}(t) = 0 \tag{30}$$

which implies that there exists a unique solution for u_{ad} as

$$u_{\text{ad}}(t)=\Gamma(\bar{y},\bar{u}_{\alpha},\bar{u}_{ad}) \tag{31}$$

Using (31) in (30) yields

$$\Delta(\mathbf{x},u)=\Gamma(\zeta) \tag{32}$$

On the other hand, any sufficiently smooth function can be approximated on a compact set with an arbitrarily bounded error by a suitable large MLP (Hornik *et al.* 1989). Therefore, on the compact set Ω_{ζ}, a set of ideal weights $\mathbf{w}^*$ and $\mathbf{V}^*$ exist such that

$$\Delta(\mathbf{x},u)=\mathbf{w}^{*T}\boldsymbol{\sigma}(\mathbf{V}^{*T}\zeta)+\varepsilon_2 \tag{33}$$

where $|\varepsilon_2|\le\varepsilon_{2M}$ and ε_{2M} depends on the network architecture. The ideal constant weights $\mathbf{w}^*$ and $\mathbf{V}^*$ are defined as

$$\left(\mathbf{w}^*,\mathbf{V}^*\right):=\arg\min_{(\mathbf{w},\mathbf{V})\in\Omega_w}\left\{\sup_{\zeta\in\Omega_\zeta}\left|\mathbf{w}^T\boldsymbol{\sigma}\left(\mathbf{V}^T\zeta\right)-\Gamma(\zeta)\right|\right\} \tag{34}$$

in which $\Omega_{\text{w}}=\left\{(\mathbf{w},\mathbf{V})\middle|\|\mathbf{w}\|_{\text{F}}\le M_{\text{w}},\|\mathbf{V}\|_{\text{F}}\le M_{\text{V}}\right\}$ and M_{w} and M_{V} are positive numbers, and $\|\cdot\|_{\text{F}}$ denotes the Frobenius norm. □

However, in practice, the weights of neural network, which constructs u_{ad} to cancel out Δ, may be different from ideal ones, so an approximation error occurs.

Lemma 2: *The approximation error, which arises from the difference between (33) and output of NN satisfies the following equality:*

$$\Delta(\mathbf{x},u)-u_{ad}=\tilde{\mathbf{w}}\left(\boldsymbol{\sigma}-\dot{\boldsymbol{\sigma}}\mathbf{V}^T\zeta\right)+\mathbf{w}^T\dot{\boldsymbol{\sigma}}\tilde{\mathbf{V}}^T\zeta+\delta(t) \tag{35}$$

where

$$\begin{gathered}|\delta(t)|\le c_0+c_1\|\tilde{\mathbf{w}}\|\|\zeta\|+c_2\|\tilde{\mathbf{V}}\|_{\text{F}}\|\zeta\|\\ \tilde{\mathbf{w}}=\mathbf{w}^*-\mathbf{w}\\ \tilde{\mathbf{V}}=\mathbf{V}^*-\mathbf{V},\end{gathered} \tag{36}$$

$\dot{\boldsymbol{\sigma}}\in R^{m\times m}$ is the derivative of $\boldsymbol{\sigma}$ with respect to the input signals of all neurons in the hidden layer of NN, and c_i $(i=0,1,2)$ are positive constants.

Proof: Note that the disturbance term $\delta(t)$ is bounded with input of neural network, rather than filtered tracking error. Using the Taylor series expansion of $\boldsymbol{\sigma}(\mathbf{V}^{*T}\zeta)$ yields

$$\boldsymbol{\sigma}(\mathbf{V}^{*T}\zeta)=\boldsymbol{\sigma}(\mathbf{V}^T\zeta+\tilde{\mathbf{V}}^T\zeta)=\boldsymbol{\sigma}(\mathbf{V}^T\zeta)+\dot{\boldsymbol{\sigma}}(\mathbf{V}^T\zeta)\tilde{\mathbf{V}}^T\zeta+\mathbf{O}(\cdot) \tag{37}$$

where $\mathbf{HOT}\in R^m$ denotes the vector of high order terms. Note that, $\boldsymbol{\sigma}(\cdot)$ and its derivate are bounded, then $|\mathbf{O}(\cdot)|$ is also bounded and can be represented as

$$|\mathbf{O}(\cdot)|=\left|\boldsymbol{\sigma}(\mathbf{V}^{*T}\zeta)-\boldsymbol{\sigma}(\mathbf{V}^T\zeta)-\dot{\boldsymbol{\sigma}}(\mathbf{V}^T\zeta)\tilde{\mathbf{V}}^T\zeta\right|\le k_0+k_1\|\tilde{\mathbf{V}}\|_F|\zeta| \tag{38}$$

where k_0 and k_1 are positive constants. Using (36) and (37), the approximation error can be calculated as

$$\begin{aligned}\Delta(\mathbf{x},u)-u_{ad}&=\mathbf{w}^{*T}\boldsymbol{\sigma}(\mathbf{V}^{*T}\zeta)+\varepsilon_2-\mathbf{w}^T\boldsymbol{\sigma}(\mathbf{V}^T\zeta)\\&=\left(\mathbf{w}^T+\tilde{\mathbf{w}}^T\right)\left(\boldsymbol{\sigma}(\mathbf{V}^T\zeta)+\dot{\boldsymbol{\sigma}}(\mathbf{V}^T\zeta)\left(\mathbf{V}^{*T}-\mathbf{V}^T\right)\zeta+\mathbf{O}(\cdot)\right)-\mathbf{w}^T\boldsymbol{\sigma}(\mathbf{V}^T\zeta)+\varepsilon_2\\&=\tilde{\mathbf{w}}^T\left(\boldsymbol{\sigma}-\dot{\boldsymbol{\sigma}}\mathbf{V}^T\zeta\right)+\mathbf{w}^T\dot{\boldsymbol{\sigma}}\tilde{\mathbf{V}}^T\zeta+\tilde{\mathbf{w}}^T\dot{\boldsymbol{\sigma}}\mathbf{V}^{*T}\zeta+\mathbf{w}^{*T}\mathbf{O}(\cdot)+\varepsilon_2\end{aligned}$$

Define

$$\delta=\tilde{\mathbf{w}}^T\dot{\boldsymbol{\sigma}}\mathbf{V}^{*T}\zeta+\mathbf{w}^{*T}\mathbf{O}(\cdot)+\varepsilon_2$$

Using (34) and (38) it is obtained

$$|\delta|\le k_2M_{\text{V}}\|\tilde{\mathbf{w}}\||\zeta|+M_{\text{w}}\left(k_0+k_1\|\tilde{\mathbf{V}}\|_F|\zeta|\right)+\varepsilon_{2M}=\left(\varepsilon_{2M}+k_0M_{\text{w}}\right)+k_2M_{\text{V}}\|\tilde{\mathbf{w}}\||\zeta|+k_1M_{\text{w}}\|\tilde{\mathbf{V}}\|_F|\zeta|$$

Then

$$|\delta|\le c_0+c_1\|\tilde{\mathbf{w}}\||\zeta|+c_2\|\tilde{\mathbf{V}}\|_F|\zeta|$$ □

4. Stability analysis

In this section the asymptotic stability of the error system will be proved. Based on the results of subsection 3.3, the system (23) is first converted into a new form. Then, a lemma is presented which is needed for the proof of the system stability theorem.

Substituting (35) into (23), the closed-loop error dynamic can be represented as

$$\dot{\boldsymbol{\xi}} = \mathbf{A}_{cl}\boldsymbol{\xi} + \mathbf{b}_{cl}\left(T^{-1}(s)\tilde{\mathbf{w}}^T\left(\boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}\mathbf{V}^T\boldsymbol{\zeta}\right) + T^{-1}(s)\mathbf{w}^T\dot{\boldsymbol{\sigma}}\tilde{\mathbf{V}}^T\boldsymbol{\zeta} + \delta_f(t) + u_{Rf}\right)$$
$$\tilde{e} = \mathbf{c}_{cl}^T\boldsymbol{\xi}$$

where

$$\delta_f(t) = T^{-1}(s)\,\delta(t)$$
$$u_{Rf}(t) = T^{-1}(s)\,u_R(t)$$

Now, define

$$\boldsymbol{\psi} = \boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}\mathbf{V}^T\boldsymbol{\zeta}$$
$$\boldsymbol{\Psi} = \boldsymbol{\zeta}\,\mathbf{w}^T\dot{\boldsymbol{\sigma}}$$

and consider the discontinues control signal

$$u_R = -g\,\varphi\,\mathrm{sgn}(\tilde{e}) \tag{39}$$

where φ is an adaptive gain and g is a function of NN weights and input vector $\boldsymbol{\xi}$. Using the equality $\mathbf{w}^T\dot{\boldsymbol{\sigma}}\tilde{\mathbf{V}}^T\boldsymbol{\zeta} = \mathrm{tr}(\tilde{\mathbf{V}}^T\boldsymbol{\zeta}\mathbf{w}^T\dot{\boldsymbol{\sigma}})$, the closed-loop error dynamic is now as

$$\dot{\boldsymbol{\xi}} = \mathbf{A}_{cl}\boldsymbol{\xi} + \mathbf{b}_{cl}\left(T^{-1}(s)\tilde{\mathbf{w}}^T\boldsymbol{\psi} + \mathrm{tr}\left(T^{-1}(s)\tilde{\mathbf{V}}^T\boldsymbol{\Psi}\right) + \delta_f(t) - T^{-1}(s)g\varphi\,\mathrm{sgn}(\tilde{e})\right)$$
$$\tilde{e} = \mathbf{c}_{cl}^T\boldsymbol{\xi} \tag{40}$$

Here, $\tilde{\mathbf{V}}, \tilde{\mathbf{w}}$ and $\varphi\,\mathrm{sgn}(\tilde{e})$ are time varying signals. Hence, the transfer function operator in (40) is not commutable. Now, consider the following error terms

$$\delta_w = T^{-1}(s)\tilde{\mathbf{w}}^T\boldsymbol{\psi} - \tilde{\mathbf{w}}^T T^{-1}(s)\boldsymbol{\psi}$$
$$\delta_V = \mathrm{tr}\left(T^{-1}(s)\tilde{\mathbf{V}}^T\boldsymbol{\Psi}\right) - \mathrm{tr}\left(\tilde{\mathbf{V}}^T T^{-1}(s)\boldsymbol{\Psi}\right) \tag{41}$$
$$\delta_\varphi = T^{-1}(s)g\,\varphi\,\mathrm{sgn}(\tilde{e}) - \varphi\,\mathrm{sgn}(\tilde{e})T^{-1}(s)g$$

for which the following bounds can be defined

$$|\delta_{\mathrm{w}}| \le c_3\|\tilde{\mathbf{w}}\|,\quad |\delta_{\mathrm{V}}| \le c_4\|\tilde{\mathbf{V}}\|_{\mathrm{F}},\quad |\delta_\varphi| \le c_5|\varphi| \tag{42}$$

With the positive numbers $c_3,\ c_4$ and c_5. Substituting (41) in (40) yields

$$\dot{\boldsymbol{\xi}} = \mathbf{A}_{cl}\boldsymbol{\xi} + \mathbf{b}_{cl}\left(\tilde{\mathbf{w}}\boldsymbol{\psi}_f + \mathrm{tr}\left(\tilde{\mathbf{V}}^T\boldsymbol{\Psi}_f\right) - \varphi\,\mathrm{sgn}(\tilde{e})T^{-1}g + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} + \delta_f - \delta_\varphi\right)$$
$$\tilde{e} = \mathbf{c}_{cl}^T\boldsymbol{\xi} \tag{43}$$

where

$$\boldsymbol{\psi}_f = T^{-1}(s)\,\boldsymbol{\psi}$$
$$\boldsymbol{\Psi}_f = T^{-1}(s)\,\boldsymbol{\Psi} \tag{44}$$

In order to show that the system is asymptotically stable, using the proposed control method, the following lemma is needed.

Lemma 3: *The following inequality holds:*

$$\left|\delta_f + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} - \delta_\varphi\right| \le \varphi^* T^{-1}(s)g \tag{45}$$

where, $g = \left(2 + \|\mathbf{V}\|_{\mathrm{F}} + \|\mathbf{w}\|\right)\left(1 + \|\boldsymbol{\zeta}\|\right) + |\varphi|$ *and* φ^* *is a constant.*

Proof: Assume that $\left|T^{-1}(s)\right| \le 1$. Using (36) and (42) and considering

$$\left|\delta_f + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} - \delta_\varphi\right| \le \left|T^{-1}\delta\right| + |\delta_{\mathrm{w}}| + |\delta_{\mathrm{V}}| + |\delta_\varphi|$$
$$\le |\delta| + |\delta_{\mathrm{w}}| + |\delta_{\mathrm{V}}| + |\delta_\varphi|$$

one can obtain

$$\begin{aligned}
\left|\delta_f + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} - \delta_\varphi\right| &\le c_0 + c_1\|\mathbf{w}^* - \mathbf{w}\||\boldsymbol{\zeta}| + c_2\|\mathbf{V}^* - \mathbf{V}\|_F|\boldsymbol{\zeta}| + c_3\|\mathbf{w}^* - \mathbf{w}\| + c_4\|\mathbf{V}^* - \mathbf{V}\|_{\mathrm{F}} + c_5|\varphi| \\
&\le c_0 + c_1\|\mathbf{w}^*\||\boldsymbol{\zeta}| + c_1\|\mathbf{w}\||\boldsymbol{\zeta}| + c_2\|\mathbf{V}^*\|_{\mathrm{F}}\|\boldsymbol{\zeta}\| + c_2\|\mathbf{V}\|_{\mathrm{F}}\|\boldsymbol{\zeta}\| + c_3\|\mathbf{w}^*\| + c_3\|\mathbf{w}\| + c_4\|\mathbf{V}^*\|_{\mathrm{F}} + c_4\|\mathbf{V}\|_{\mathrm{F}} + c_5|\varphi| \\
&\le c_0 + c_1M_{\mathrm{w}}|\boldsymbol{\zeta}| + c_1\|\mathbf{w}\||\boldsymbol{\zeta}| + c_2M_{\mathrm{V}}\|\boldsymbol{\zeta}\| + c_2\|\mathbf{V}\|_{\mathrm{F}}\|\boldsymbol{\zeta}\| + c_3M_{\mathrm{w}} + c_3\|\mathbf{w}\| + c_4M_{\mathrm{V}} + c_4\|\mathbf{V}\|_{\mathrm{F}} + c_5|\varphi| \\
&\le \varphi_1\left(\left(2 + \|\mathbf{V}\|_{\mathrm{F}} + \|\mathbf{w}\|\right)\left(1 + \|\boldsymbol{\zeta}\|\right) + |\varphi|\right)
\end{aligned}$$

where

$$\varphi_1 = \max\left\{c_0, c_1M_{\mathrm{w}}, c_1, c_2M_{\mathrm{V}}, c_2, c_3M_{\mathrm{w}} + c_4M_{\mathrm{V}}, c_3, c_4, c_5\right\}.$$

Assume that g is a positive signal. Then the low pass filter $T^{-1}(s)$ is designed such that its step response has no overshoot and $T^{-1}(s)g$ remains positive. By suitable initialization of filter states, $0<\lambda<1$ is found such that $g \le \frac{T^{-1}(s)g}{\lambda}$. Consequently

$$\left|\delta_f + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} - \delta_\varphi\right| \le \frac{\varphi_1}{\lambda} T^{-1}(s)g \tag{46}$$

Therefore, φ^* may be selected as

$$\varphi^* = \frac{\varphi_1}{\lambda}$$

□

Theorem: *Consider the discontinuous control (39) and select the adaptation laws for NN weights, and the gain of the robustifying term* φ *as*

$$\begin{aligned}\dot{\mathbf{w}} &= \gamma_{\mathrm{w}} \tilde{e}\, \boldsymbol{\psi}_f \\ \dot{\mathbf{V}} &= \gamma_{\mathrm{V}} \tilde{e}\, \boldsymbol{\Psi}_f \\ \dot{\varphi} &= \gamma_\varphi \left|\tilde{e}\right| \left(T^{-1}g\right)\end{aligned} \tag{47}$$

Then, the closed-loop tracking error is asymptotically stable and the weights of NN remain bounded.

Proof: Define the Lyapunov function

$$L = \frac{1}{2}\boldsymbol{\xi}^T \mathbf{P}\boldsymbol{\xi} + \frac{1}{2\gamma_{\mathrm{w}}}\|\tilde{\mathbf{w}}\|^2 + \frac{1}{2\gamma_{\mathrm{V}}}\|\tilde{\mathbf{V}}\|_{\mathrm{F}}^2 + \frac{1}{2\gamma_\varphi}|\tilde{\varphi}|^2 \tag{48}$$

where $\mathbf{P}$ is the unique positive-definite symmetric solution of (24) for $\mathbf{Q}=q\,\mathbf{I}$ ($q>0$), and $\tilde{\varphi}=\varphi^*-\varphi$. Moreover, assume that $\mathbf{w}^*$ and $\mathbf{V}^*$ are ideal constant weights defined in (34). Then, from (36) $\dot{\mathbf{w}}=-\dot{\tilde{\mathbf{w}}}$, $\dot{\mathbf{V}}=-\dot{\tilde{\mathbf{V}}}$. Using (43), the time-derivative of L is

$$\dot{L} \le -\frac{1}{2}q\|\boldsymbol{\xi}\|^2 + \boldsymbol{\xi}^T\mathbf{P}\mathbf{b}_{cl}\left[\tilde{\mathbf{w}}^T\boldsymbol{\psi}_f + \mathrm{tr}\left(\tilde{\mathbf{V}}^T\boldsymbol{\Psi}_f\right) - \varphi\,\mathrm{sgn}(\tilde{e})T^{-1}g + \delta_{\mathrm{w}} + \delta_{\mathrm{V}} + \delta_f - \delta_\varphi\right] - \frac{1}{\gamma_{\mathrm{w}}}\tilde{\mathbf{w}}^T\dot{\mathbf{w}} - \frac{1}{\gamma_{\mathrm{V}}}\mathrm{tr}\left(\tilde{\mathbf{V}}^T\dot{\mathbf{V}}\right) - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi} \tag{49}$$

From (23) and (25)

$$\tilde{e} = \boldsymbol{\xi}^T\mathbf{P}\mathbf{b}_{cl} \tag{50}$$

Substituting (50) into (49) and using Lemma 3, yields

$$\begin{aligned}\dot{L} \le &-\frac{1}{2}q\|\boldsymbol{\xi}\|^2 + \tilde{\mathbf{w}}^T\left(\tilde{e}\,\boldsymbol{\psi}_f - \frac{1}{\gamma_{\mathrm{w}}}\dot{\mathbf{w}}\right) + \mathrm{tr}\left(\tilde{\mathbf{V}}^T\left(\tilde{e}\,\boldsymbol{\Psi}_f - \frac{1}{\gamma_{\mathrm{V}}}\dot{\mathbf{V}}\right)\right) \\ &-\tilde{e}\varphi\,\mathrm{sgn}(\tilde{e})T^{-1}(s)g + |\tilde{e}|\varphi^* T^{-1}(s)g - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi}\end{aligned}$$

Using the adaptation laws in (47) gives

$$\begin{aligned}\dot{L} &\le -\frac{1}{2}q\|\boldsymbol{\xi}\|^2 + |\tilde{e}|\varphi^* T^{-1}g - |\tilde{e}|\varphi T^{-1}g - \frac{1}{\gamma_\varphi}\tilde{\varphi}\dot{\varphi} \\ &= -\frac{1}{2}q\|\boldsymbol{\xi}\|^2 + |\tilde{e}|\left(\varphi^*-\varphi\right)T^{-1}g - \frac{1}{\gamma_\varphi}\dot{\varphi}\tilde{\varphi} \\ &= -\frac{1}{2}q\|\boldsymbol{\xi}\|^2 + \left(|\tilde{e}|T^{-1}g - \frac{1}{\gamma_\varphi}\dot{\varphi}\right)\tilde{\varphi} \\ &= -\frac{1}{2}q\|\boldsymbol{\xi}\|^2\end{aligned} \tag{51}$$

Since L is a positive function and $\dot{L}\le 0$, so $\|\boldsymbol{\xi}\|, \|\tilde{\mathbf{V}}\|, \|\tilde{\mathbf{w}}\|$ and $|\tilde{\varphi}|$ are bounded. In addition, from (34), $\mathbf{V}^*$ and $\mathbf{w}^*$ are bounded, therefore, according to (36), $\mathbf{V}$ and $\mathbf{w}$ remain bounded. Moreover, by integrating (51)

$$\int_0^\infty \|\boldsymbol{\xi}(t)\|^2 dt \le \frac{2}{q}\left(L(t)\big|_{t=0} - L(t)\big|_{t=\infty}\right) \tag{52}$$

Since, the right-hand side of (52) is bounded, then, according to the Barbalet's lemma

$$\lim_{t\to\infty}\|\boldsymbol{\xi}\|^2 = 0 \tag{53}$$

Since $\tilde{e} = \mathbf{c}_{cl}^T\boldsymbol{\xi}$, then

$$\lim_{t\to\infty}\tilde{e}(t) = 0 \tag{54}$$

According to the final value theorem and using (14), it yields

$$\lim_{s\to 0} s\,\tilde{e}(s) = \lim_{s\to 0} s\,G_{\text{ad}}(s)e(s) = 0$$

Since $G_{\text{ad}}(0) \neq 0$, one can conclude that

$$\lim_{s\to 0} s\,e(s) = 0$$

so

$$\lim_{t\to\infty} e(t) = 0 \tag{55}$$

which completes the proof.

Remark 1: The proposed control law in (7) consists of two parts: $u_c = y_d^{(r)} + u_L + u_{ad}$ and a robustifying term u_R. When the system dynamics is exactly known i.e. $\Delta(x,u) = \hat{b} - b(x,u)$, then one may select $u_{ad} = \Delta(x,u)$. Then, (15) implies $\tilde{e} = 0$. In this case, from (14) $N_{ad}(s)\,e = 0$. Since $N_{ad}(s)$ is a Hurwitz polynomial, the asymptotic stability of the system can be achieved. On the other hand, in the presence of parameters variation or unmodeled dynamics and/or external disturbances, the system dynamics includes these uncertainties.. In this case one can not find an explicit form for u_{ad}. Hence, a neural network is used to approximate u_{ad}. However, due to the approximation error inherent in neural networks, it is impossible to guarantee $\tilde{e} = 0$. To overcome this problem, a discontinuous part $u_R = -\varphi\, g\, \text{sgn}(\tilde{e})$ is considered with u_c which ensures the stability of the error dynamics $\tilde{e}$. The amplitude of this part is proportional to the approximation error. Now, according to given theorem, applying $u_c + u_R$ to the system yields $\tilde{e} \to 0$ as $t \to \infty$. Hence, the error system trajectories tend to an equilibrium point. To this end, the gain φ is selected sufficiently large. □

Remark2: When a discontinuous control is applied to a system, the phenomenon, called chattering, appears. Many methods have been proposed in the literature to reduce chattering including continuous approximation of the discontinuous control. A continuous approximation of $\text{sgn}(\tilde{e})$ in (39) is the saturation function

$$\text{sat}(\tilde{e}) = \begin{cases} \text{sgn}(\tilde{e}) & \text{if } |\tilde{e}| \geq \varepsilon \\ \dfrac{\tilde{e}}{\varepsilon} & \text{otherwise.} \end{cases} \tag{56}$$

Alternatively, one may consider the smoothing function $\tanh\left(\tilde{e}/\varepsilon\right)$ or $\dfrac{\tilde{e}}{|\tilde{e}| + \delta}$ where $\varepsilon > 0$ and $0 < \delta < 1$ as an approximation of $\text{sgn}(\tilde{e})$.

Figure 2 shows the block diagram of the system with the proposed control method in which TDL stands for the tapped delay line.

Note that, if there is a finite time t_s such that $\tilde{e} = 0$ for all $t \geq t_s$, then the system trajectories move towards the sliding surface $\tilde{e} = 0$ and tends along this surface to an equilibrium point (Koshkouei and Zinober 1998). The control (39) guarantees the robustness of the method in the presence of disturbances or unmodelled dynamics provided that the gain φ is selected sufficiently large.

5. Example

The performance of the proposed controller is illustrated by considering the following non-affine nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_2 + x_1^2 x_2 + (2x_1 x_2 + u)^3 \\ \dot{x}_3 = x_1 - 0.8x_3 \\ y = x_1 + x_3 \end{cases}$$

The relative degree of the system with output y is r = 2. In fact, the zero dynamic of the system is $\dot{x}_3 = -0.8x_3$, which is asymptotically stable. Therefore, in practice it is assumed that the system is modelled as a second order nonlinear plant, whose realization consists of states $x1$ and $x2$ (state $x3$ is omitted) and the output is modelled as $y = x_1$. Hence, the system without unmodelled dynamic can be represented as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_2 + x_1^2 x_2 + (2x_1 x_2 + u)^3 \\ y = x_1 \end{cases}$$

Note that Assumption 2 is satisfied; i.e.

$$\frac{\partial b(x,u)}{\partial u} = 3(2x_1 x_2 + u)^2 > 0 \quad \forall \mathbf{x} \in \Omega_{\mathbf{x}}, u \in \Omega_u$$

An appropriate available approximation of $b(\mathbf{x},u)$ is selected as $v = \hat{b}(y,u) = cu$, where c is a constant whose value depends on y_d and $\dot{y}_d$, and should be selected such that condition (12) is satisfied. In this example $c = 2$ is selected.

The second order compensator

$$\frac{N_L}{D_L} = \frac{18s^2 + 16s + 12}{s(s+7)}$$

is selected to stabilize the linear second order system $\ddot{e} = -u_L$.

Now, based on the assumptions on N_{ad} and D_{ad} in Section 3, the following filter is used to construct the error signal $\tilde{e}$

$$\frac{N_{\text{ad}}}{D_{\text{ad}}} = 50\frac{s^2 + 6s + 6}{(s+10)(s+20)}.$$

It is desired that the above filter has high bandwidths. Finally, $T(s) = 0.5s + 1$ is selected based on SPR property of $\bar{G}$.

The NN is of MLP type and has 20 neurons in the hidden layer with tangent hyperbolic activation functions. The weights are initialised randomly with small numbers. The input to the NN for $n_1 = 4 \geq n$ is $\xi = [1, y(t), y(t - T_d), y(t - 2T_d), y(t - 3T_d), u_\alpha(t), u_\alpha(t - T_d), u_{ad}(t - T_d)]^T$ with $T_d = 2$ msec . Also, the learning constants are chosen as $\gamma_{\mathbf{W}} = \gamma_{\mathbf{V}} = 0.9$ and $\gamma_\varphi = 0.1$. The reference y_d is a step signal, which is passed through a second order filter with transfer function $2/(s^2 + 1.6s + 2)$. And finally, to avoid chattering, $\tanh(\tilde{e}/0.9)$ is used as a continuous approximation of $\operatorname{sgn}(\tilde{e})$.

Simulation results are depicted in Figures 3-10. First, the controlled system performance is evaluated without the unmodelled mode dynamics. Figure 3, shows the system response when only the linear controller is applied. It is clear that the system response has high oscillatory behaviour and almost unstable because of the nonlinearities in the system.

Figure 4, compares the system response with two different control laws. First, simulations have been performed with the linear u_L and adaptive control term u_{ad} but without the robustifying term u_{R} . Then, both the adaptive and the robustifying terms have been used together with u_L . As Figure 4 shows, by using the adaptive term, the oscillations are almost eliminated but there is a tracking error because u_{ad} is not completely able to cancel out Δ (Figure 5).

By adding the robustifying part, the desired tracking is asymptotically achieved. Figure 6 shows the action of the control signals. At the stating time of the simulation, a large approximation error has occurred because of unsuitable weight initialization. In order to compensate this error, the robustifying control term becomes large and chattering phenomena emerges out. But, after this transition, the approximation error is reduced and the chattering is removed from the control signal. The norms of weights are depicted in Figure 7, which shows that the weights remain bounded.

Next, the effect of the unmodelled dynamics is examined and the simulation result is shown in Figure 8, which demonstrates that the proposed robust adaptive control law can compensate the effect of unmodelled dynamics appropriately.

Finally the proposed robust adaptive controller is compared with remarkable new output feedback second-order sliding mode control method, presented by Levant (2005). The sliding mode controller has been designed as in Levant (2005) with $\alpha = 1$ which is the gain of the sliding mode controller, and differentiator parameter L=40. Figures 9 and 10 show, the both controllers yield good tracking and are robust in the presence of uncertainties and unmodelled dynamics. But, the gain of the second-order sliding mode control is non-adaptive and is designed based on upper bound of uncertainties so this conservative design causes that the control signal contains chattering while the robust adaptive approach has smoother control signal.

6. Conclusions

In this paper, a direct adaptive output feedback control method has been developed for uncertain non-affine nonlinear systems that do not rely on state estimation. Moreover, it has been shown that the use of an additional robustifying part of the control guarantees the uniform asymptotic stability of the tracking error system. Without this control part, only the uniform ultimately boundedness of the tracking error system is demonstrated. The proposed control algorithm is relatively simple and requires no restrictive conditions on the design constants for the stability. The efficiency of the proposed scheme has been shown using the simulation of a nonlinear system with unmodelled dynamics. The simulation results showed the effectiveness of the proposed control method as compared to linear, linear-adaptive and second-order sliding mode controllers. Although in the proposed control method, there are some parameters and functions, which should be appropriately defined. Nevertheless the proposed method is not very sensitive to these constants and functions.

References

Esfandiari, F. and Khalil, H., 1992 Output feedback stabilization of fully linearisable system. *Int. J. Control*, **56**, 1007-1037.

Ge, S. S. Hang, C. and Zhang, T., 1998, Nonlinear adaptive control using neural network and its application to CSTR systems. *J. Process Control*, **9**, 313–323.

Ge, S. S. Hang, C. and Zhang, T., 1999, Adaptive neural network control of nonlinear system by state and output feedback, *IEEE, Trans. System Man. Cyber,* **29**, 818-828.

Ge, S. S. and Zhang, T., 2003, Neural network control of non-affine nonlinear with zero dynamics by state and output feedback*, IEEE, Trans. on Neural Networks*, **14,** 900–918

Hornik, K., Stinchombe, M. and White, H., 1989, Multilayer feed-forward networks are universal approximatros, *Neural Networks*, **2**, 359-366.

Isidori, A., 1995, Nonlinear Control Systems. 3rd Ed. London, Springer-Verlag.

Koshkouei, A. J., Zinober A. S. I. and Burnham, K.. 2004, Adaptive sliding mode backstepping control of nonlinear systems with unmatched uncertainty. *Asian Journal of Control*, **6**, 447-457.

Koshkouei, A. J. and Zinober, A. S. I., 1998, Sliding mode controller-observer design for SISO linear systems. *Int. J. Systems Science,* **29**, 1363-1373.

Krstic, M.., Kanellakopoulos, I. and Kokotovic, P., 1995, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, New York.

Levant, A., 2003, Higher-order sliding modes, differentiation and output feedback control, *Int. J. Control*, **76**, 924-941.

Levant, A., 2005, Homogeneity approach to high-order sliding mode design, *Automatica*, **41**, 823-830.

Marino, R. and Tomei, P., 1995, *Nonlinear Adaptive Design: Geometric, Adaptive and Robust*, Prentice-Hall, London.

Khalil, H., 1996, Adaptive output feedback control of nonlinear system represented by input-output models, *IEEE Trans. Automat. Control*, **41**, 177-188.

Lewis, F., Yesildirek, A. and Liu, K., 1996, Multilayer neural-net robot controller with guaranteed tracking performance, *IEEE Trans. Neural Nets.*, **7**, 388-399.

Lewis, F. Yesildirek A. and Liu, K., 1995, Feedback linearization using neural networks, *Automatica*, **31**, 1659-1664.

Hovakimyan, N., Nardi, F. and Calise, A. J., 2002, A novel error observer based adaptive output feedback approach for control of uncertain systems, *IEEE Trans. Automat. Control,* **47**, 1310 – 1314.

Kim B. S. and Calise, A. J., 1997, Nonlinear flight control using neural networks, *J. of Guidance, Control and Dynamics*, **20**, 26-33.

Park, J. H. and Park, G. T., 2003, Robust adaptive fuzzy controller for non-affine nonlinear systems with dynamic rule activation, *Int. J. Robust Nonlinear Control*, **13**, 117-139.

Polycarpou, M. U. and Mears, M. J., 1998, Stable adaptive tracking of uncertain systems using nonlinearly parameterized on-line approximatros, *Int. J. Control*, **70**, 363–384.

Seshagiri, S. and Khalil, H., Output feedback control of nonlinear systems using RBF neural networks, *IEEE Trans. on Neural Nets.,* **11**, 69-79, 2000.

Narendra, K. S. and Annaswamy, A. M., 1989, *Stable Adaptive System*, Prentice-Hall, London.

Lavertsky, E., Calise, A. J and Hovakimyan, N., 2003, Upper bounds for approximation of continuous-time dynamics using delayed outputs and feedforward neural networks, IEEE *Trans. on Automat. Control*, **48**, 1606-1610.

Zinober, A., Deterministic Control of Uncertain Systems, 1990, (Stevenage, UK: Peter Peregrinus).

Zinboer, A., 1994, Variable Structure and Lyapunov Control (Berlin: Springer-Verlag).
R. J. Wai, “Tracking control based on neural network strategy for robot manipulator”, Neurocomputing, 51, pp. 425-445, 2003.

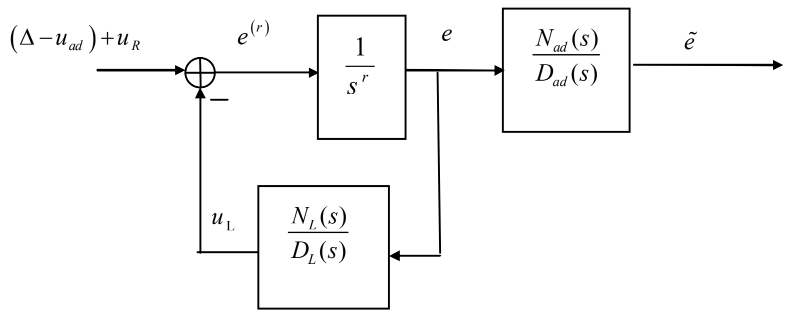

Figure 1: Block diagram of error dynamic

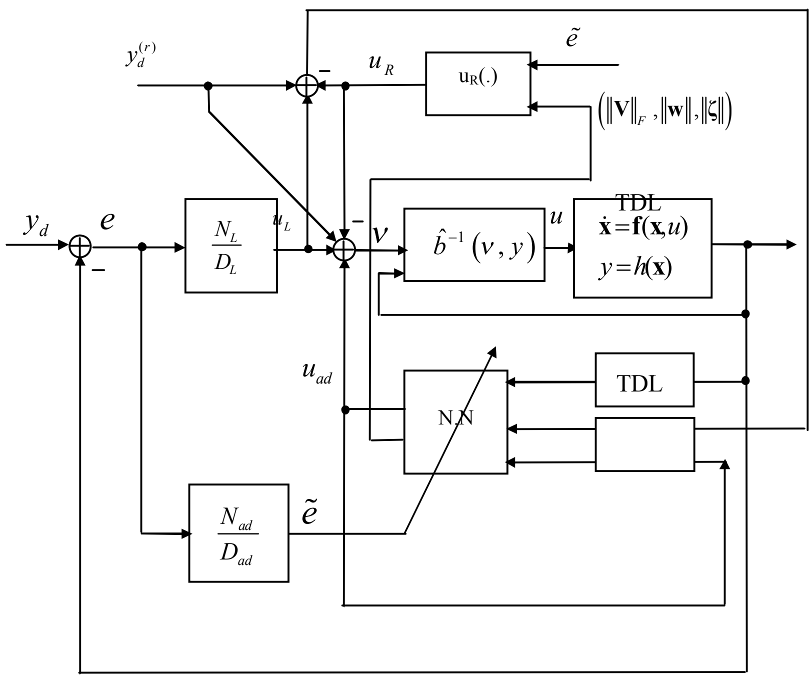

Figure 2: Block diagram of the proposed control method.

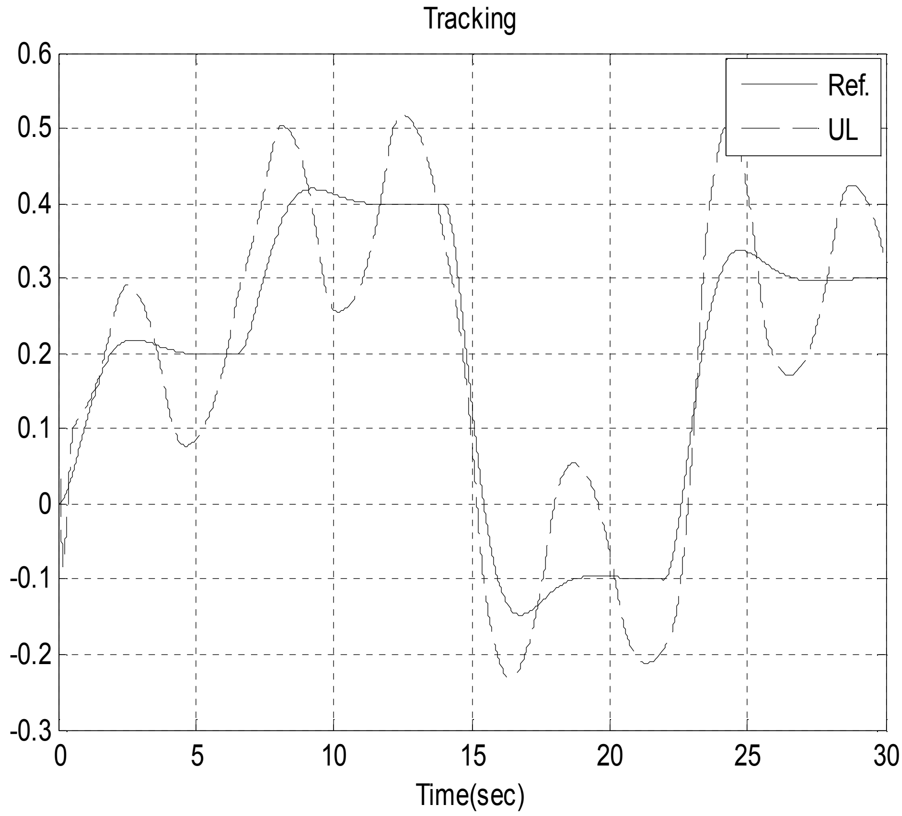

Figure 3: System without unmodelled dynamics: response with linear compensator.

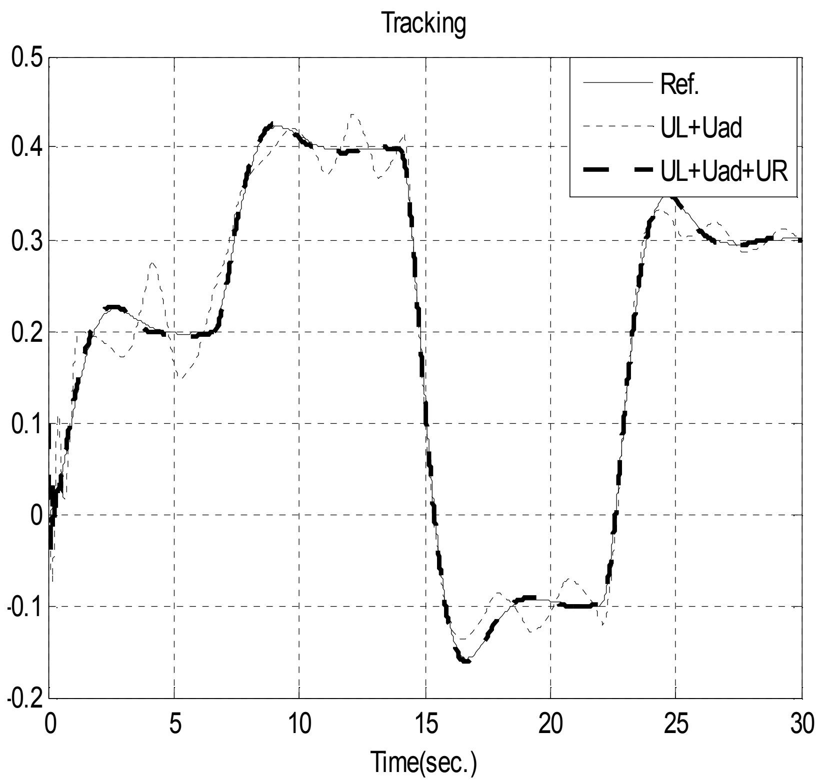

Figure 4: System tracking with adaptive and robust adaptive controls, for system without unmodelled dynamic.

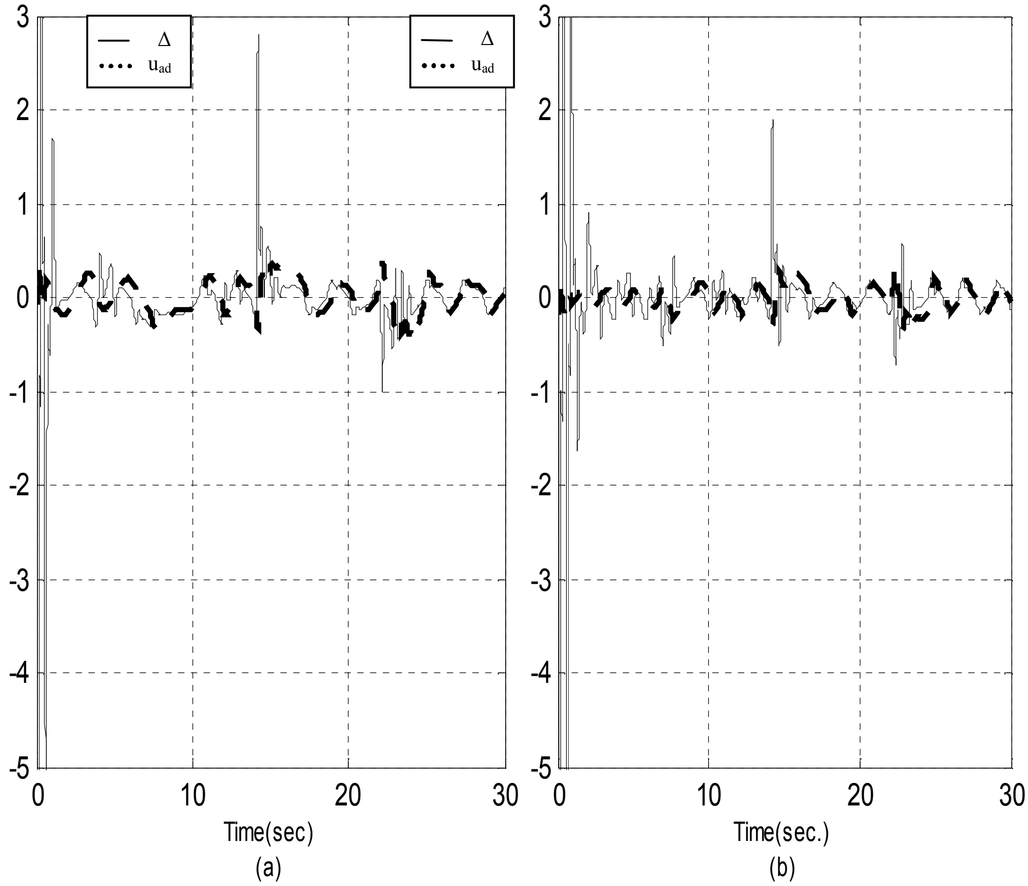

Figure 5: NN output (u_{ad}) and modelling error Δ. (a) system without unmodelled dynamic; (b) system with unmodelled dynamic.

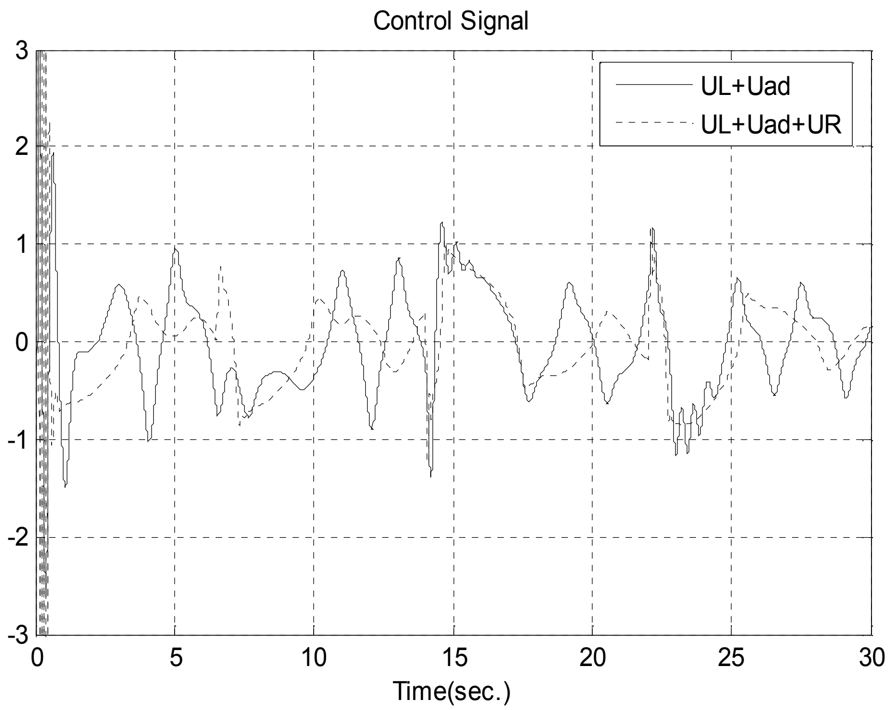

Figure 6: Control signals for system without unmodelled dynamics.

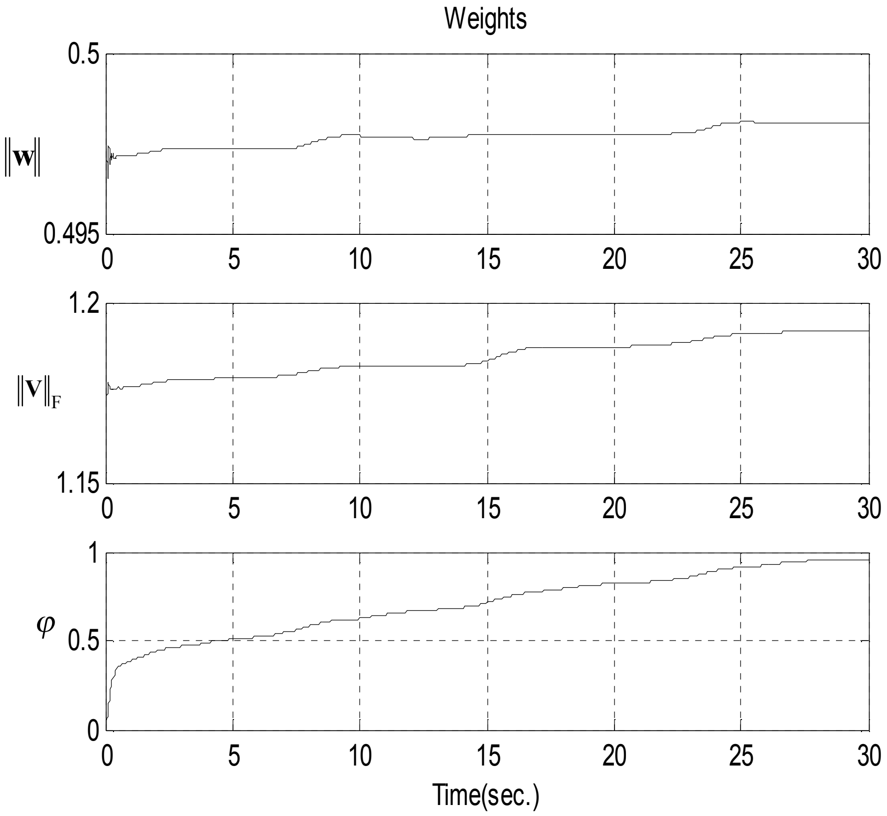

Figure 7: Norm of weights for system without unmodelled dynamics.

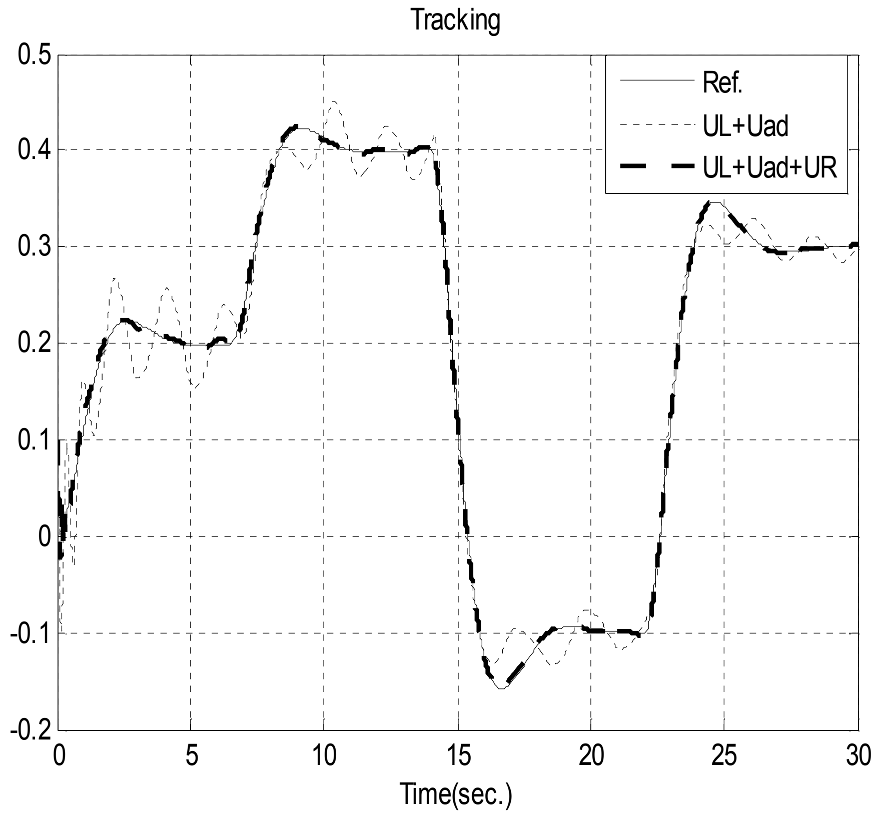

Figure 8: System tracking with adaptive and robust adaptive controls, for system with unmodelled dynamic.

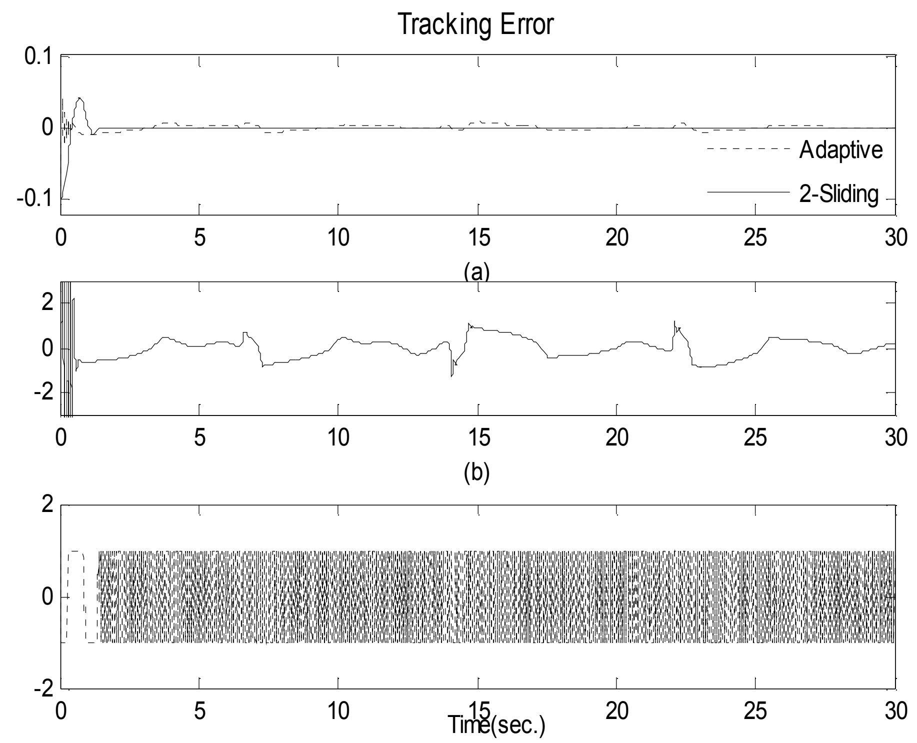

Figure 9: System without unmodelled dynamics, comparison of robust adaptive and 2-sliding mode controllers. (a) tracking error; (b) robust adaptive control signal; (c) 2-sliding mode control signal.

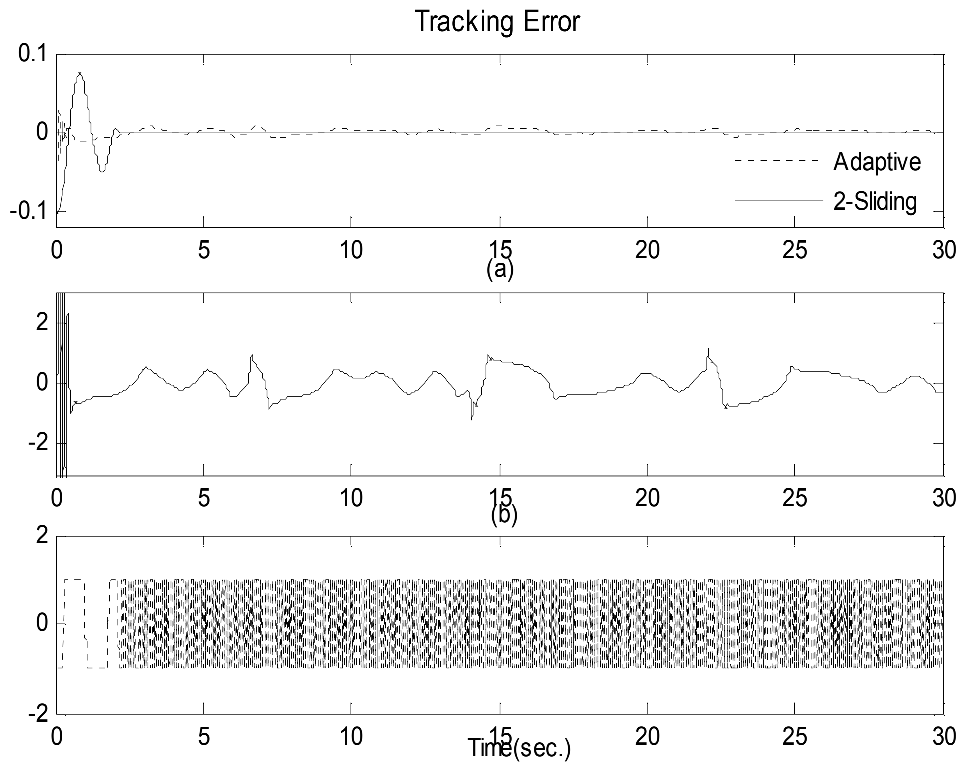

Figure 10: System with unmodelled dynamics, comparison of robust adaptive and second-order sliding controllers. (a) tracking error; (b) robust adaptive control signal; (c) 2-sliding mode control action.