Viscous Flow in Ducts

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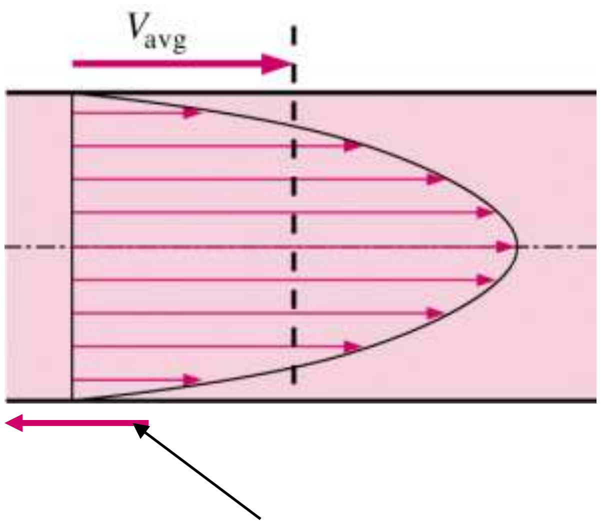
Objectives

1. Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow

2. Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements

Introduction

- Average velocity in a pipe
  - Recall - because of the no-slip condition, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and friction along the pipe walls



Friction force of wall on fluid

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c$$

For a circular tube

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R u(r) r\, dr$$

- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of Mass

$$\dot{m} = \rho V_{avg} A = constant$$

same same same

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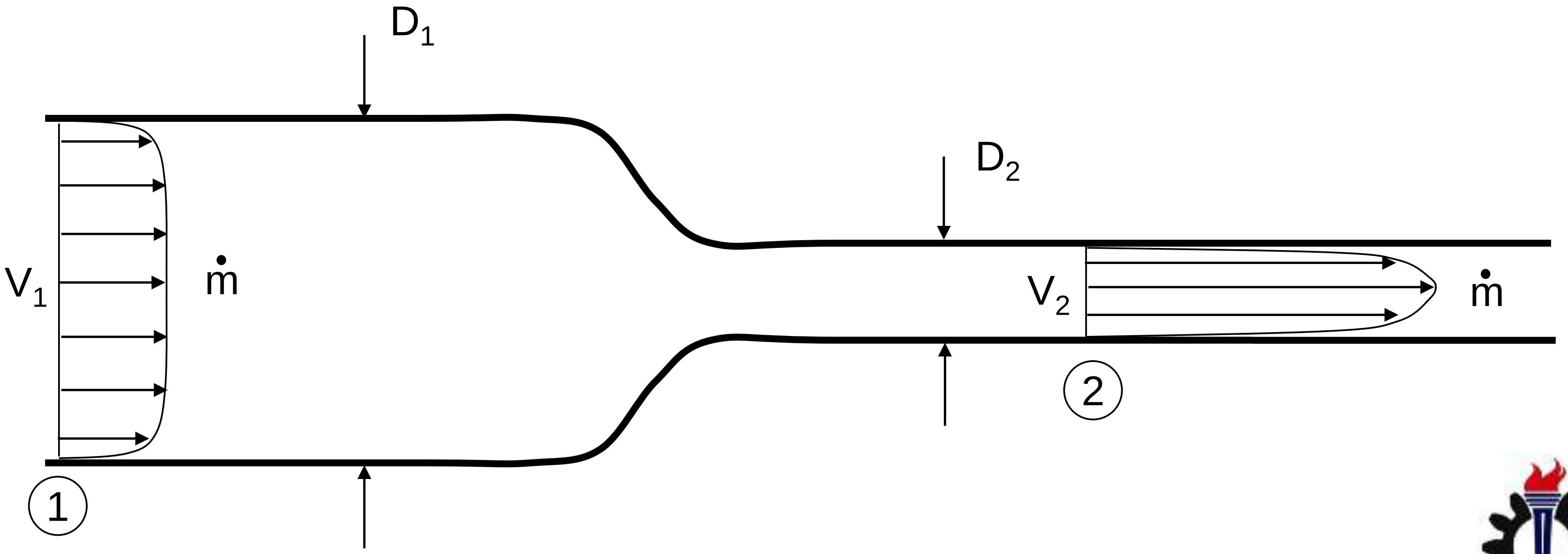
Introduction

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



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Laminar and Turbulent Flows

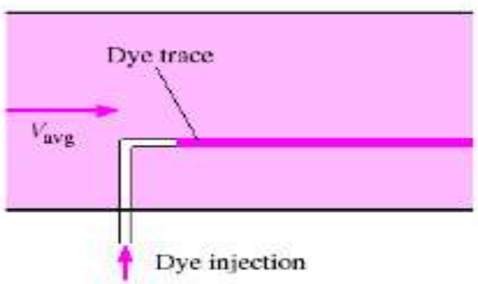
Laminar Flow

Can be steady or unsteady.

(Steady means that the flow field at any instant in time is the same as at any other instant in time.)

Can be one-, two-, or three-dimensional.

Has regular, *predictable* behavior



Analytical solutions are possible

Occurs at *low* Reynolds numbers.

Turbulent Flow

Is always *unsteady*.

Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.

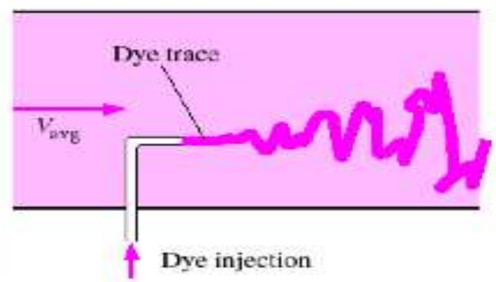
Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.

Is always *three-dimensional*.

Why? Again because of the random swirling eddies, which are in all directions.

Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.

Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.

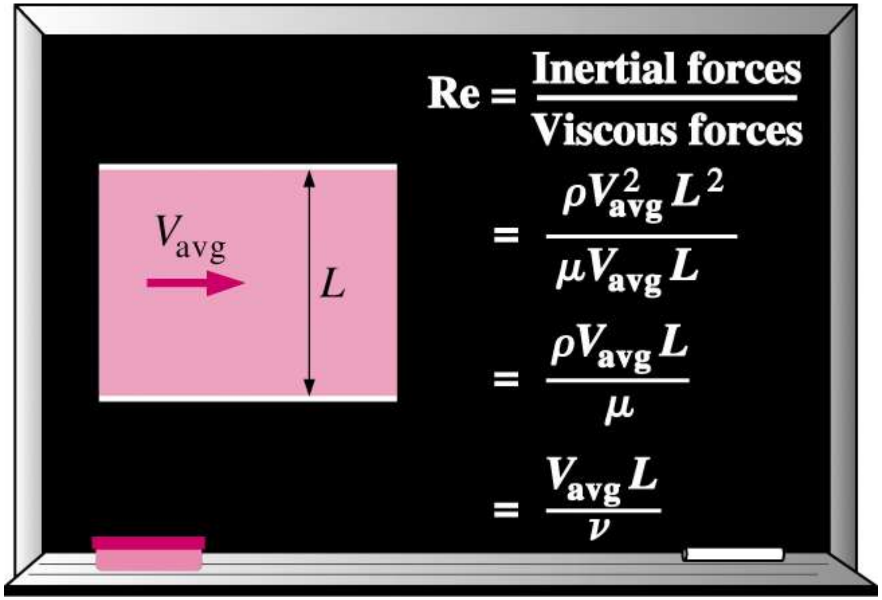


No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)

Occurs at *high* Reynolds numbers.

Laminar and Turbulent Flows

Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe

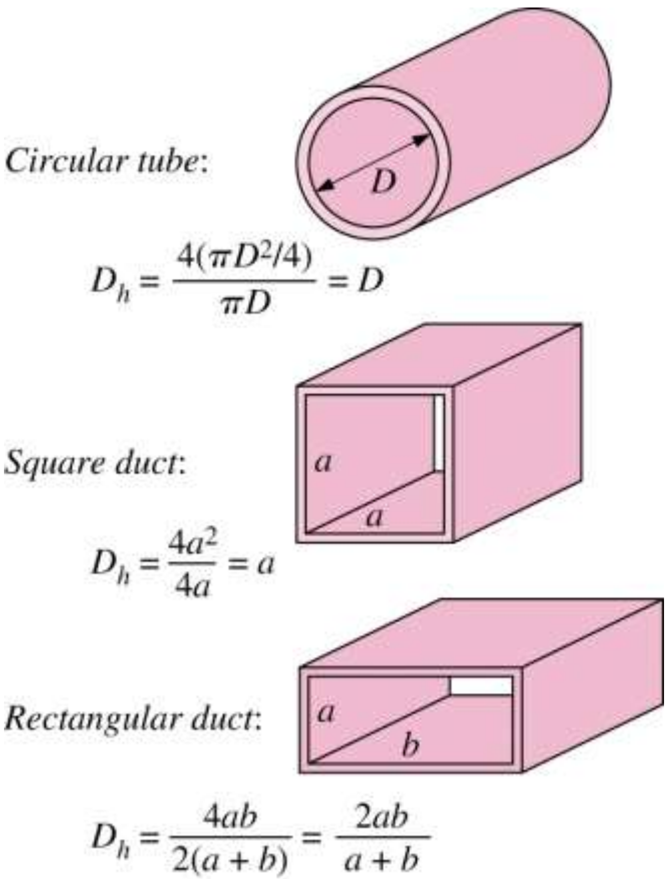
  $Re < 2300 \Rightarrow$ laminar

  $2300 \le Re \le 4000 \Rightarrow$ transitional

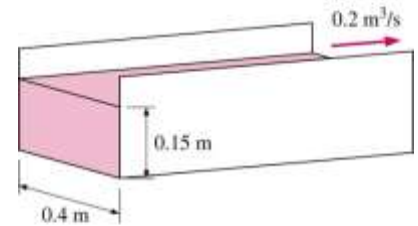
  $Re > 4000 \Rightarrow$ turbulent

- Note that these values are approximate.
- For a given application, Re_{cr} depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

Laminar and Turbulent Flows



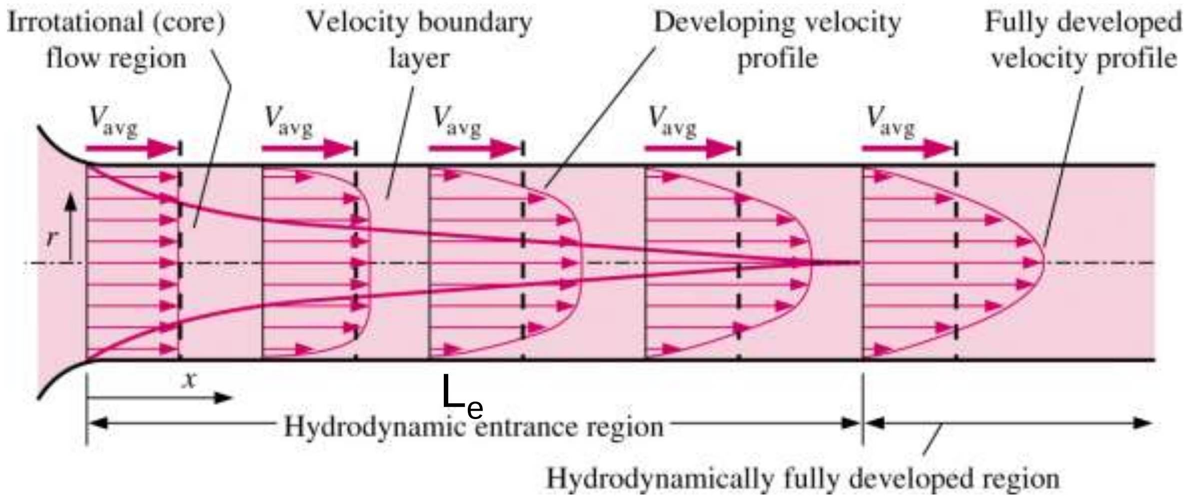
- For <u>non-round</u> pipes, define the hydraulic diameter
  $D_h = 4A_c/P$
  A_c = cross-section area
  P = wetted perimeter



- Example: open channel
  $A_c = 0.15 * 0.4 = 0.06\text{m}^2$
  $P = 0.15 + 0.15 + 0.4 = 0.7\text{m}$
  Don't count free surface, since it does not contribute to friction along pipe walls!
  $D_h = 4A_c/P = 4*0.06/0.7 = 0.34\text{m}$
  <u>What does it mean?</u> This channel flow is equivalent to a round pipe of diameter 0.34m (approximately).

The Entrance Region

- Consider a round pipe of diameter D. The flow can be laminar or turbulent. In either case, the profile develops downstream over several diameters called the *entry length* L_e. L_e/D is a function of Re.



$$\textit{Hydrodynamically fully developed:} \qquad \frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

The Entrance Region

- Dimensional analysis shows that the Reynolds number is the only parameter affecting entry length

$$L_e = f(d, V, \rho, \mu) \qquad V = \frac{Q}{A}$$

$$\frac{L_e}{d} = g\left(\frac{\rho V d}{\mu}\right) = g(\text{Re})$$

$$\frac{L_e}{d} \approx 0.06\ \text{Re} \qquad \text{laminar}$$

In turbulent flow the boundary layers grow faster, and L e is relatively shorter

$$\frac{L_e}{d} \approx 4.4\ \text{Re}_d^{1/6} \qquad \text{turbulent}$$

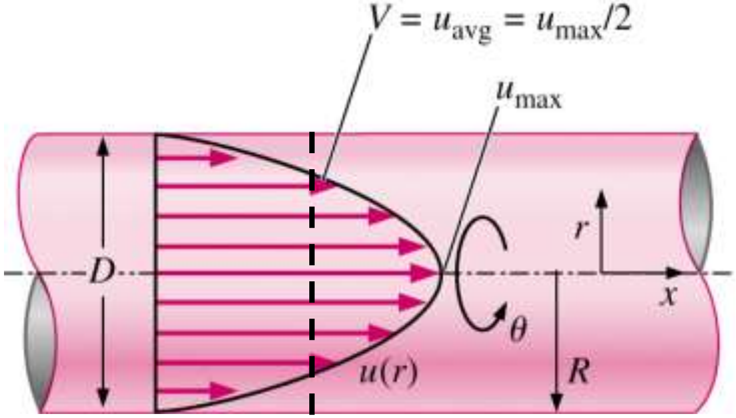
Fully Developed Pipe Flow

- Comparison of laminar and turbulent flow

  There are some major differences between laminar and turbulent fully developed pipe flows

  <u>Laminar</u>

  - Can solve exactly
  - Flow is steady
  - Velocity profile is parabolic
  - Pipe roughness not important



It turns out that $V_{avg} = 1/2U_{max}$ and $u(r) = 2V_{avg}(1 - r^2/R^2)$

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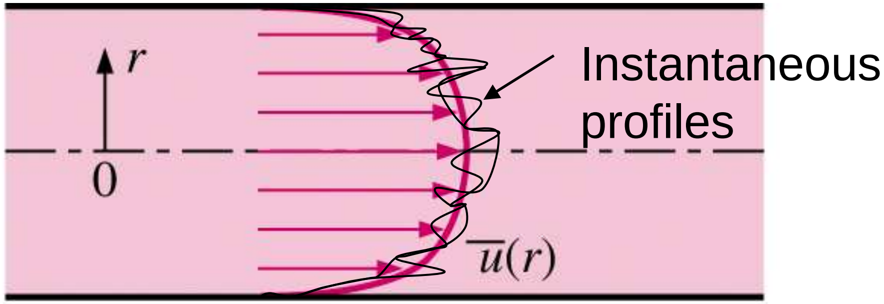
Fully Developed Pipe Flow

Turbulent

- *Canno*t solve exactly (too complex)
- Flow is unsteady (3D swirling eddies), but it is steady in the mean
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- Pipe roughness is very important



- V_{avg} 85% of U_{max} (depends on Re a bit)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape. See text
  - Logarithmic law (Eq. 8-46)
  - Power law (Eq. 8-49)

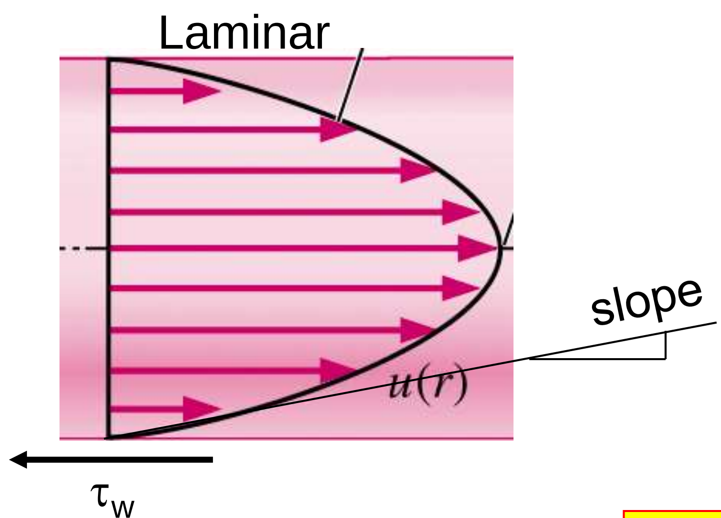
Fully Developed Pipe Flow
Wall-shear stress

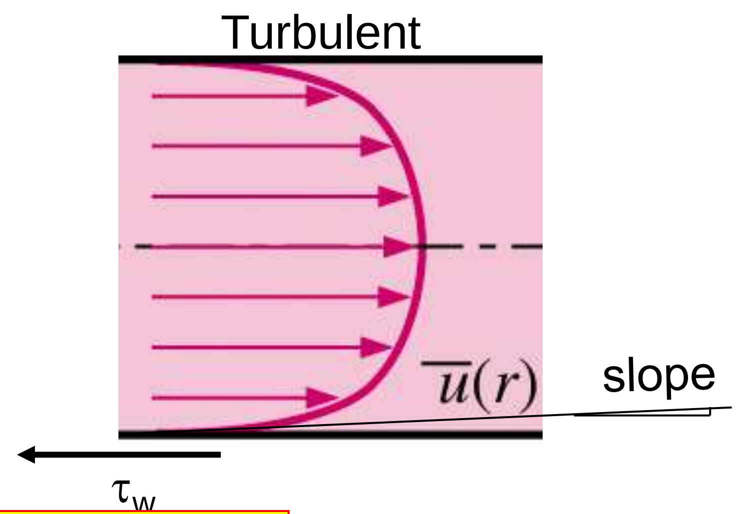
- Recall, for simple shear flows *u=u(y)*, we had

$$\tau = \mu du/dy$$

- In fully developed pipe flow, it turns out that

$$\tau = \mu du/dr$$



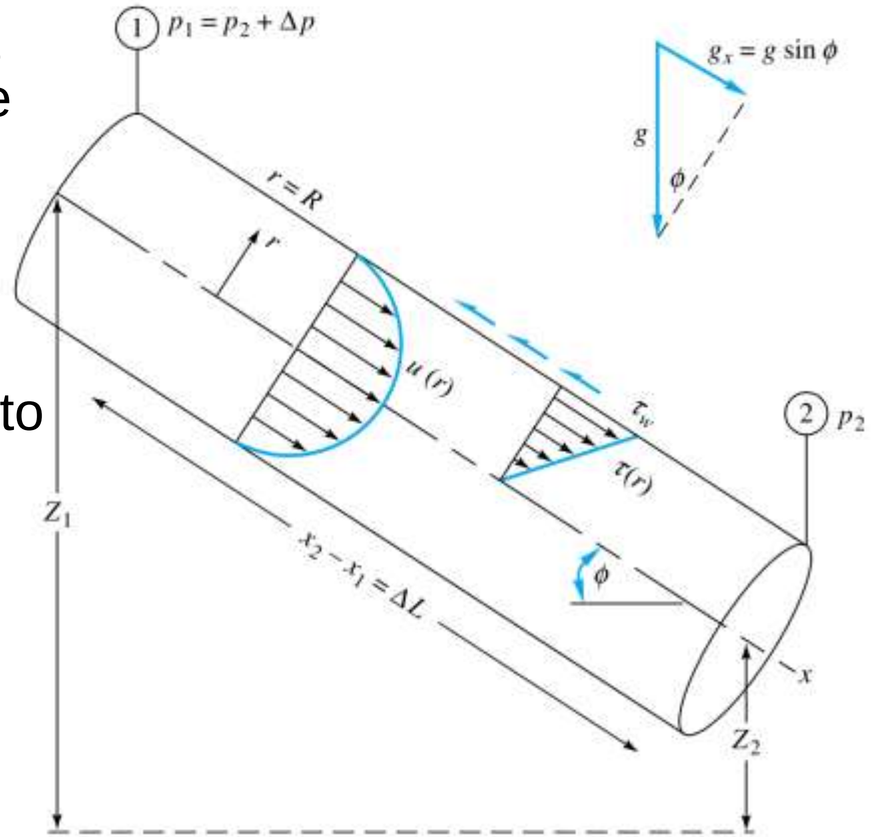


τ_w = shear stress at the wall, acting on the fluid

$$\tau_{w,turb} > \tau_{w,lam}$$

Fully Developed Pipe Flow
Pressure drop

- There is a direct connection between the pressure drop in a pipe and the shear stress at the wall
- Consider fully developed, and incompressible flow in a pipe
- Let's apply conservation of mass, momentum, and energy to this CV (good review problem!)



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Fully Developed Pipe Flow
Pressure drop

- Conservation of Mass

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$Q_1 = Q_2 = \text{const}$$

$$V_1 = \frac{Q_1}{A_1} = V_2 = \frac{Q_2}{A_2} \rightarrow \boxed{V_1 = V_2}$$

- Conservation of x-momentum

$$\Delta p \ \pi R^2 + \rho g(\pi R^2)\ \Delta L \sin\phi - \tau_w(2\pi R)\ \Delta L = \dot{m}(V_2 - V_1) = 0$$

$$\Delta z = \Delta L \sin\phi \Rightarrow \boxed{\Delta z + \frac{\Delta p}{\rho g} = \frac{2\tau_w}{\rho g}\frac{\Delta L}{R}}$$

Fully Developed Pipe Flow
Pressure drop

- <u>Conservation of Energy</u>

  there are no shaft-work or heat-transfer effects

$$\frac{p_1}{\rho} + \frac{1}{2}\alpha_1 V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}\alpha_2 V_2^2 + gz_2 + gh_f$$

since $V_1 = V_2$ and $\alpha_1 = \alpha_2$ (shape not changing) now reduces to a simple expression for the friction-head loss h_f

$$h_f = \left(z_1 + \frac{p_1}{\rho g}\right) - \left(z_2 + \frac{p_2}{\rho g}\right) = \Delta\left(z + \frac{p}{\rho g}\right) = \Delta z + \frac{\Delta p}{\rho g}$$

Fully Developed Pipe Flow
Friction Factor

- From momentum CV analysis

$$\Delta z + \frac{\Delta p}{\rho g} = \frac{2\tau_w}{\rho g}\frac{\Delta L}{R}$$

- From energy CV analysis

$$\Delta z + \frac{\Delta p}{\rho g} = h_f$$

- Equating the two gives

$$h_f = \frac{4\tau_w}{\rho g}\frac{\Delta L}{D}$$

- To predict head loss, we need to be able to calculate τ_w. How?
  - Laminar flow: solve exactly
  - Turbulent flow: rely on empirical data (experiments)
  - In either case, we can benefit from dimensional analysis!

Fully Developed Pipe Flow Friction Factor

- τ_w = func(ρ, V, μ, D, ε)

ε = average roughness of the inside wall of the pipe

- Π-analysis gives

$$\Pi_1 = f \qquad f = \frac{8\tau_w}{\rho V^2}$$

$$\Pi_2 = Re \qquad Re = \frac{\rho V D}{\mu}$$

$$\Pi_3 = \frac{\epsilon}{D} \qquad \epsilon/D = \text{roughness factor}$$

$$\Pi_1 = func(\Pi_2, \Pi_3) \qquad f = func(Re, \epsilon/D)$$

Fully Developed Pipe Flow
Friction Factor

- Now go back to equation for h_L and substitute f for τ_w

$$h_f = \frac{4\tau_w}{\rho g}\frac{L}{D}$$

$$f = \frac{8\tau_w}{\rho V^2} \rightarrow \tau_w = f\rho V^2/8$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

- Our problem is now reduced to solving for Darcy friction factor f
  - Recall $f = func(Re, \epsilon/D)$
  - Therefore
    - Laminar flow: f = 64/Re (exact)
    - Turbulent flow: Use charts or empirical equations (Moody Chart, a famous plot of f vs. Re and ε/D)

But for laminar flow, roughness does not affect the flow unless it is huge

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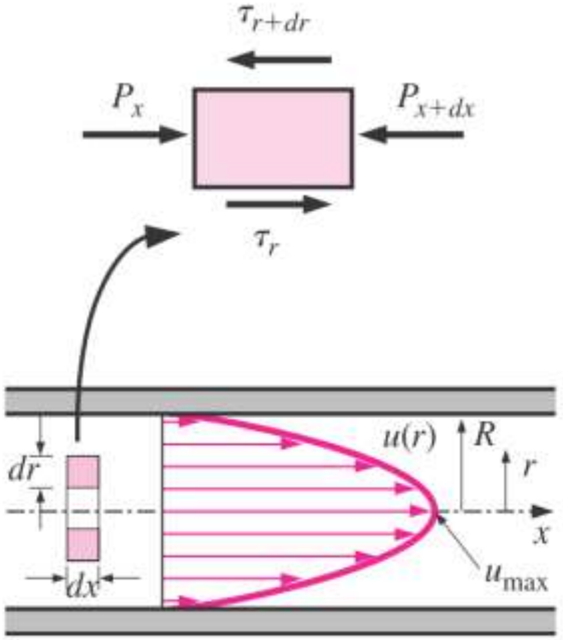
Fully Developed Pipe Flow
Laminar Flow

- We would like to find the velocity profile in laminar flow



$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$dr,\ dx \rightarrow 0$ gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \& \qquad \tau = -\mu(du/dr)$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

- *dP/dx = const.* $\quad \partial u/\partial r = 0$ at $r = 0$ and *u=0* at *r=R*

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \quad \Longrightarrow \quad u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Fully Developed Pipe Flow
Laminar Flow

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right) r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right) \Rightarrow u(r) = 2V_{avg}\left(1-\frac{r^2}{R^2}\right) \qquad u_{max} = 2V_{avg}$$

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \qquad \Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2} \quad \text{(I)}$$

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} \quad \text{(II)} \qquad \text{and} \qquad f = \frac{8\tau_w}{\rho V_{avg}^2}$$

(I) & (II) $\Rightarrow$

Circular pipe, laminar: $\quad f = \dfrac{64\mu}{\rho D V_{avg}} = \dfrac{64}{\text{Re}}$

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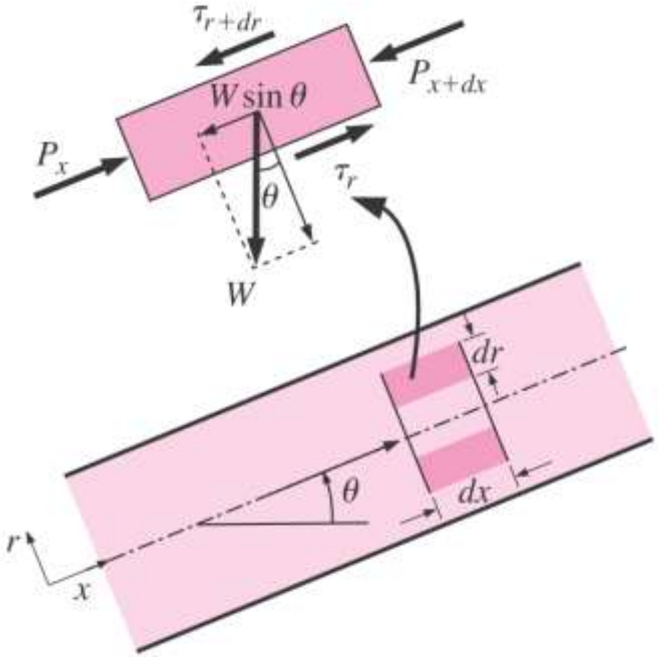
Fully Developed Pipe Flow
Laminar Flow – Inclined Pipe

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} - \rho g(2\pi r\,dr\,dx)\sin\theta = 0$$

$$\Rightarrow \quad \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

$$\Rightarrow \quad \boxed{u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g\sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)}$$



$$V_{avg} = \frac{(\Delta P - \rho g L\sin\theta)D^2}{32\mu L} \quad \text{and} \quad Q = \frac{(\Delta P - \rho g L\sin\theta)\pi D^4}{128\mu L}$$

Uphill flow: $\theta > 0$ and $\sin\theta > 0$

Downhill flow: $\theta < 0$ and $\sin\theta < 0$

The results already obtained for horizontal pipes can also be used for inclined pipes provided that ΔP is replaced by: $\Delta P - \rho g L \sin\theta$

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Example 1

Oil at 20°C (ρ=888 kg/m³ and μ= 0.800 kg/m·s) is flowing steadily through a D= 5 cm diameter L=40 m long pipe. P_{in} = 745 kPa, P_{out} =97 kPa. Determine the Q assuming the pipe is (a) horizontal, (b) inclined 15° upward, (c) inclined 15° downward. Also verify that the flow through the pipe is laminar.

Solution:

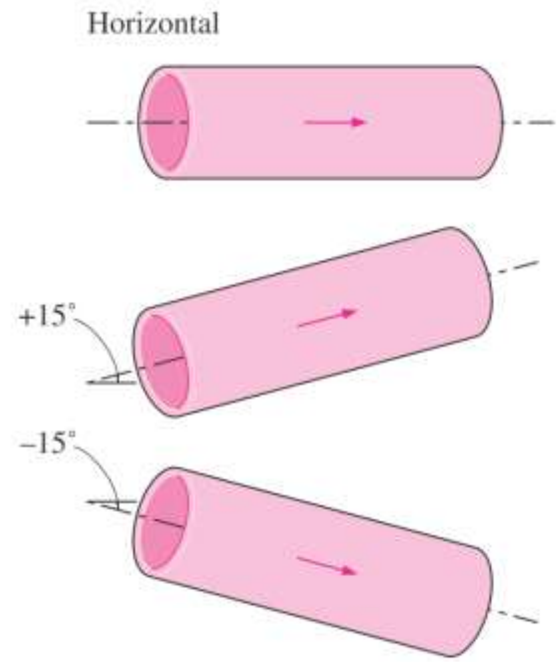
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f$$

$$\left.\begin{aligned}V_1&=V_2\\ L\sin\theta&=z_2-z_1\end{aligned}\right\}$$

$$\rightarrow\frac{P_1-P_2}{\rho g}-L\sin\theta=f\,\frac{L}{D}\frac{V^2}{2g}=\frac{64\mu}{\rho VD}\frac{L}{D}\frac{V^2}{2g}=\frac{64\mu}{\rho D^2}\frac{L}{2g}V$$

$$V=\frac{Q}{A}=\frac{4Q}{\pi D^2}\rightarrow\Delta P-\rho gL\sin\theta=\frac{128\mu LQ}{\pi D^4}$$

$$\rightarrow\boxed{Q=\frac{\pi D^4\left(\Delta P-\rho gL\sin\theta\right)}{128\mu L}}$$



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Example 1

a) Horizontal

$$\theta = 0 \rightarrow \sin\theta = 0$$

$$\rightarrow Q = \frac{\pi D^4 \Delta P}{128\mu L} = \frac{\pi (0.05)^4 (745-97)\times 10^3}{128(0.8)40} = 0.00311\,\text{m}^3/\text{s}$$

b) Uphill θ=15º

$$\theta = 15^\circ$$

$$\rightarrow Q = \frac{\pi D^4 (\Delta P - \rho g L \sin\theta)}{128\mu L}$$

$$= \frac{\pi (0.05)^4 (745-97-888\times 9.81\times 40 \sin 15^\circ)\times 10^3}{128(0.8)40} = 0.00267\,\text{m}^3/\text{s}$$

c) Downhill θ=-15º

$$\theta = 15^\circ$$

$$\rightarrow Q = \frac{\pi (0.05)^4 (745-97+888\times 9.81\times 40 \sin 15^\circ)\times 10^3}{128(0.8)40} = 0.00354\,\text{m}^3/\text{s}$$

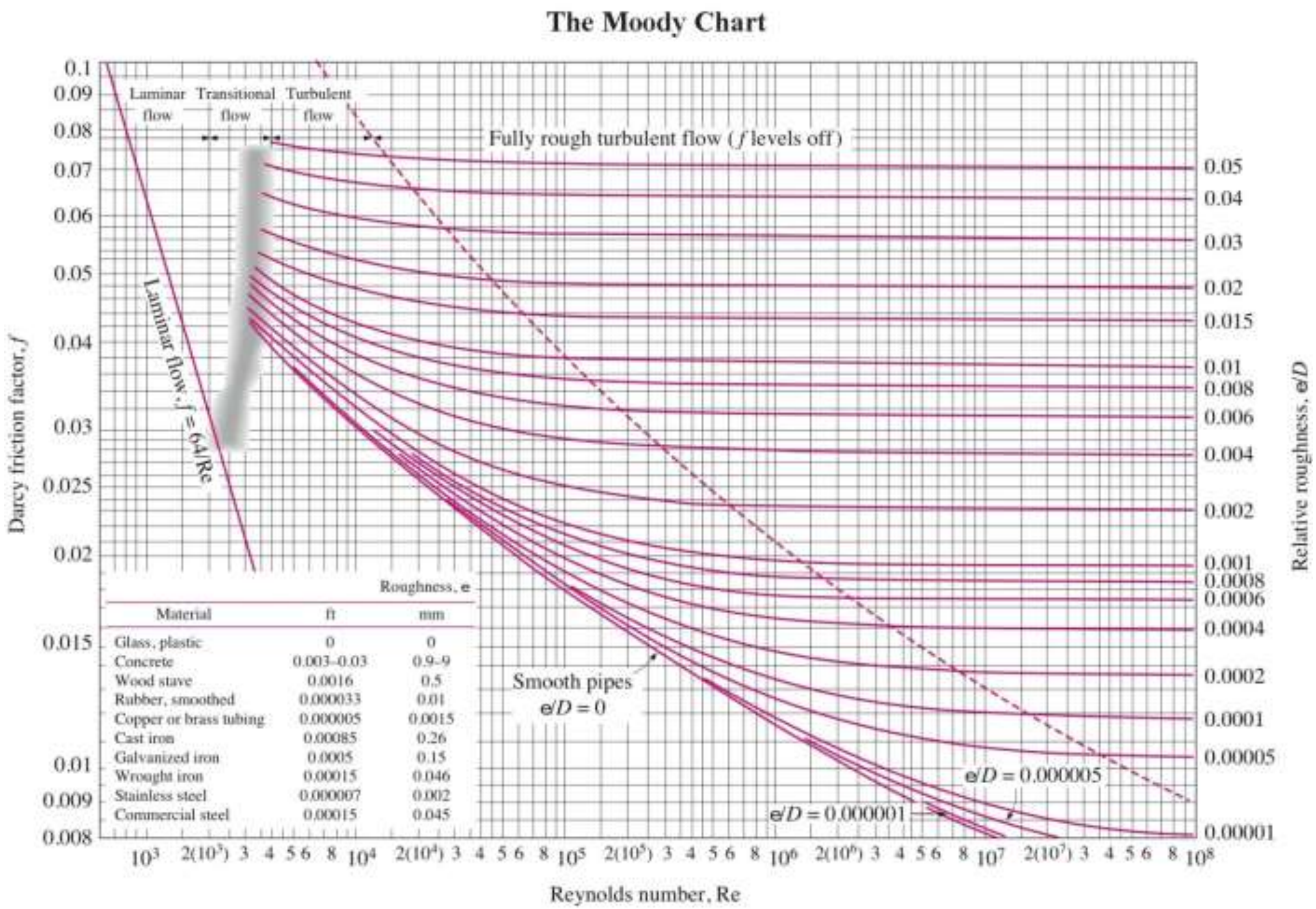
$$Q_{\max} = 0.00354\,\text{m}^3/\text{s}$$

$$\rightarrow \text{V}_{avg} = \frac{4Q}{\pi D^2} = 1.8\,\text{m/s}$$

$$\text{Re}_D = \frac{\rho V_{avg} D}{\mu} = 100 < 2300$$

The flow is always Laminar

The Moody Diagram



Fully Developed Pipe Flow
Friction Factor

- Moody chart was developed for circular pipes, but can be used for non-circular pipes using hydraulic diameter
- Colebrook equation is a curve-fit of the data which is convenient for computations

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Implicit equation for f which can be solved using the root-finding algorithm in EES

- Both Moody chart and Colebrook equation are accurate to ±15% due to roughness size, experimental error, curve fitting of data, etc.

Example 2

Oil, with $\rho = 900$ kg/m^3 and $\nu = 0.00001$ m^2/s, flows at 0.2 m^3/s through 500 m of 200-mm-diameter cast-iron pipe. Determine (*a*) the head loss and (*b*) the pressure drop if the pipe slopes down at 10° in the flow direction.

Solution

$$V = \frac{Q}{\pi R^2} = \frac{0.2 \text{ m}^3/\text{s}}{\pi (0.1 \text{ m})^2} = 6.4 \text{ m/s} \longrightarrow \text{Re}_d = \frac{Vd}{\nu} = \frac{(6.4 \text{ m/s})(0.2 \text{ m})}{0.00001 \text{ m}^2/\text{s}} = 128{,}000$$

For cast iron ε=0.26 mm

$$\frac{\epsilon}{d} = \frac{0.26 \text{ mm}}{200 \text{ mm}} = 0.0013 \xrightarrow{\text{from the Moody diagram}} f \approx 0.0225$$

$$h_f = f \frac{L}{d} \frac{V^2}{2g} = (0.0225) \frac{500 \text{ m}}{0.2 \text{ m}} \frac{(6.4 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \boxed{117 \text{ m}}$$

$$h_f = \frac{\Delta p}{\rho g} + z_1 - z_2 = \frac{\Delta p}{\rho g} + L \sin 10°$$

$$\Delta p = \rho g [h_f - (500 \text{ m}) \sin 10°] = \rho g (117 \text{ m} - 87 \text{ m})$$

$$= (900 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(30 \text{ m}) = 265{,}000 \text{ kg/(m} \cdot \text{s}^2) = \boxed{265{,}000 \text{ Pa}}$$

Types of Fluid Flow Problems

- In design and analysis of piping systems, 3 problem types are encountered
  1. Determine **Δp (or h_f)** given L, D, V (or flow rate)

     Can be solved directly using Moody chart and Colebrook equation
  2. Determine **V (or Q)**, given L, D, Δp
  3. Determine **D**, given L, Δp, V (or flow rate)
- Types 2 and 3 are common engineering design problems, i.e., selection of pipe diameters to minimize construction and pumping costs
- However, iterative approach required since both *V* and *D* are in the Reynolds number.

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Example 3

Oil, with $\rho = 950\ \text{kg/m}^3$ and $\nu = 2\ \text{E-5 m}^2\text{/s}$, flows through a 30-cm-diameter pipe 100 m long with a head loss of 8 m. The roughness ratio is $\epsilon/d = 0.0002$. Find the average velocity and flow rate.

Known:

$$\rho = 950\,\text{kg/m}^3,\ \nu = 2\times 10^{-5}\,\text{m}^2/\text{s},\ d = 0.3\,\text{m}$$

$$L = 100\,\text{m},\ h_f = 8\,\text{m},\ \varepsilon / d = 0.0002 \qquad Q = ?,\ V = ?$$

Iterative Solution:

$$f = h_f \frac{d}{L}\frac{2g}{V^2} = (8\ \text{m})\left(\frac{0.3\ \text{m}}{100\ \text{m}}\right)\left[\frac{2(9.81\ \text{m/s}^2)}{V^2}\right] \quad \text{or} \quad fV^2 \approx 0.471$$

we only need to guess *f*, compute *V*, then get Re_d, compute a better *f* from the Moody chart, and repeat.

Example 3

Guess $f \approx 0.014$, then $V = \sqrt{0.471/0.014} = 5.80$ m/s and $\mathrm{Re}_d = Vd/\nu \approx 87{,}000$. At $\mathrm{Re}_d = 87{,}000$ and $\epsilon/d = 0.0002$, compute $f_{\text{new}} \approx 0.0195$

New $f \approx 0.0195$, $V = \sqrt{0.481/0.0195} = 4.91$ m/s and $\mathrm{Re}_d = Vd/\nu = 73{,}700$. At $\mathrm{Re}_d = 73{,}700$ and $\epsilon/d = 0.0002$, compute $f_{\text{new}} \approx 0.0201$

Better $f \approx 0.0201$, $V = \sqrt{0.471/0.0201} = 4.84$ m/s and $\mathrm{Re}_d \approx 72{,}600$. At $\mathrm{Re}_d = 72{,}600$ and $\epsilon/d = 0.0002$, compute $f_{\text{new}} \approx 0.0201$

We have converged to three significant figures. Thus our iterative solution is

$$V = 4.84 \text{ m/s}$$

$$Q = V\left(\frac{\pi}{4}\right)d^2 = (4.84)\left(\frac{\pi}{4}\right)(0.3)^2 \approx 0.342 \text{ m}^3/\text{s}$$

Example 4

Work example 3 backward, assuming Q=0.342 m^3/s and ε=0.06mm are known but that d is unknown. Recall L=100m, ρ=950 kg/m^3, ν=2E-5 m^2/s and h_f=8m.

Known:

$$\rho = 950\,\text{kg/m}^3,\ \nu = 2\times10^{-5}\,\text{m}^2/\text{s},\ Q = 0.342\,\text{m}^3/\text{s}$$

$$L = 100\,\text{m},\ h_f = 8\,\text{m},\ \varepsilon = 0.06\ \text{mm} \qquad\qquad d = ?$$

Iterative Solution:

$$f = \frac{\pi^2}{8}\frac{(9.81\ \text{m/s}^2)(8\ \text{m})d^5}{(100\ \text{m})(0.342\ \text{m}^3/\text{s})^2} = 8.28d^5 \qquad \text{or} \qquad d \approx 0.655 f^{1/5}$$

Also write the ***Re*** and ***ε/d*** in terms of ***d***:

$$\text{Re}_d = \frac{4(0.342\ \text{m}^3/\text{s})}{\pi(2\ \text{E-5 m}^2/\text{s})d} = \frac{21{,}800}{d}$$

$$\frac{\epsilon}{d} = \frac{6\ \text{E-5 m}}{d}$$

Example 4

We guess *f*, compute *d* then compute Re_d *and* ε/d and compute a better *f* from the Moody chart:

1st try:

$$f \approx 0.03 \qquad d \approx 0.655(0.03)^{1/5} \approx 0.325 \text{ m}$$

$$\text{Re}_d \approx \frac{21{,}800}{0.325} \approx 67{,}000 \qquad \frac{\epsilon}{d} \approx 1.85 \text{ E-4}$$

2nd try:

$$f_{\text{new}} \approx 0.0203 \qquad \text{then} \qquad d_{\text{new}} \approx 0.301 \text{ m}$$

$$\text{Re}_{d,\text{new}} \approx 72{,}500 \qquad \frac{\epsilon}{d} \approx 2.0 \text{ E-4}$$

Last try:

$$f_{\text{better}} \approx 0.0201 \qquad \text{and} \qquad d = 0.300 \text{ m}$$

Minor Losses

- For any pipe system, in addition to the Moody-type friction loss computed for the length of pipe, there are additional so-called minor losses due to
  1. Pipe entrance or exit
  2. Sudden expansion or contraction
  3. Bends, elbows, tees, and other fittings
  4. Valves, open or partially closed
  5. Gradual expansions or contractions

$$h_m = K\frac{V^2}{2g}$$

- ***h_m*** is minor losses.
- ***K*** is the loss coefficient which:

  is different for each component.

  is assumed to be independent of Re.

  typically provided by manufacturer or generic table.

Minor Losses

- Total head loss in a system is comprised of major losses h_f (in the pipe sections) and the minor losses h_m (in the components)

$$\Delta h_{tot} = h_f + \sum h_m$$

$$\Delta h_{tot} = \underbrace{\sum_i f_i \frac{L_i}{d_i} \frac{V_i^2}{2g}}_{\text{i th pipe section}} + \underbrace{\sum_j K_j \frac{V_j^2}{2g}}_{\text{j th componet}}$$

- If the piping system has constant diameter

$$\Delta h_{tot} = h_f + \sum h_m = \frac{V^2}{2g}\left(\frac{fL}{d} + \sum K\right)$$

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Minor Losses

Typical commercial valve geometries:

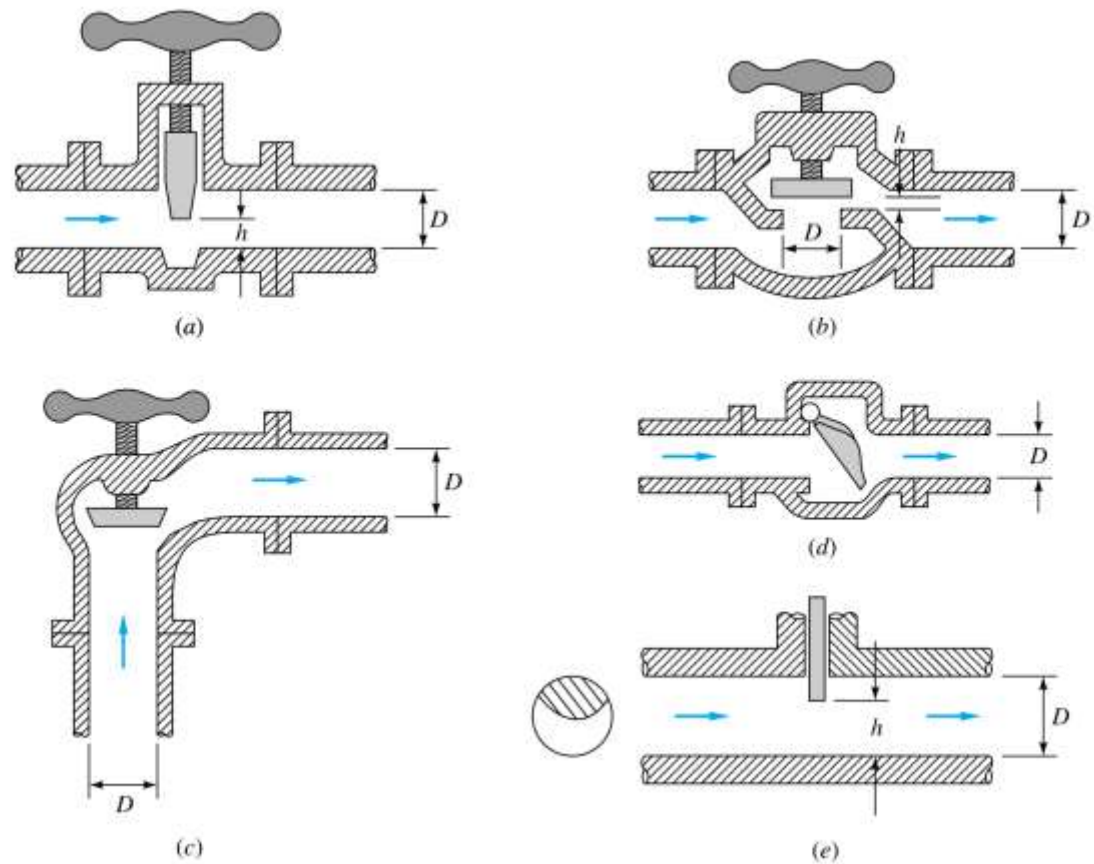
(a) gate valve

(b) globe valve

(c) angle valve

(d) swing-check valve

(e) disk-type gate valve



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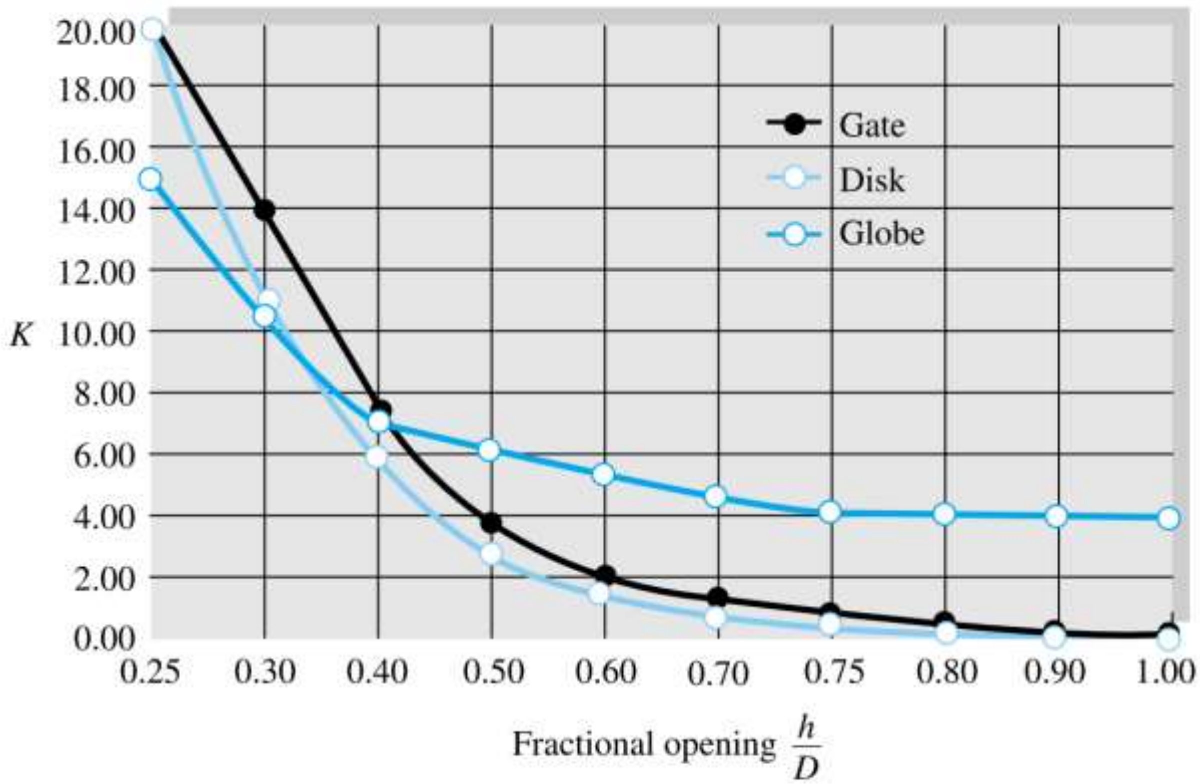
Minor Losses

Resistance coefficients ***K*** for open valves, elbows, and tees:

	Nominal diameter, in								
	Screwed				Flanged				
	$\frac{1}{2}$	1	2	4	1	2	4	8	20
Valves (fully open):									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows:									
45° regular	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° regular	2.0	1.5	0.95	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees:									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

This table represents losses averaged among various manufacturers, so there is an uncertainty as high as ±50%. loss factors are highly dependent upon actual design and manufacturing factors.

Average-loss coefficients for partially open valves:



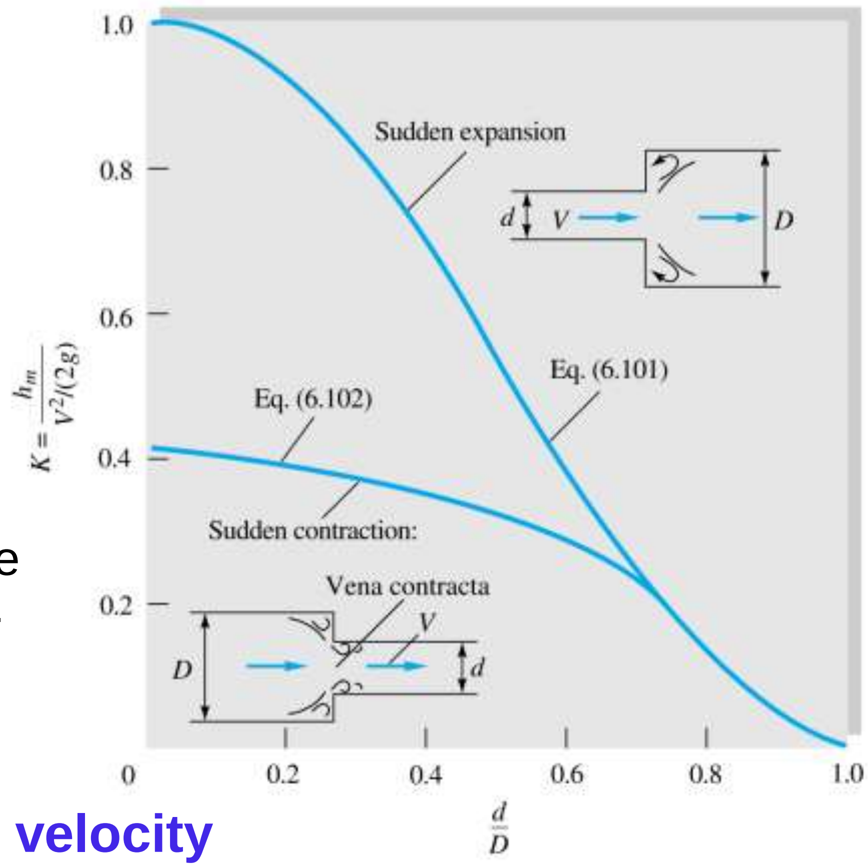
Minor Losses

Sudden Expansion (SE) and Sudden Contraction (SC):

$$K_{SE} = \left(1 - \frac{d^2}{D^2}\right)^2 = \frac{h_m}{V^2/(2g)}$$

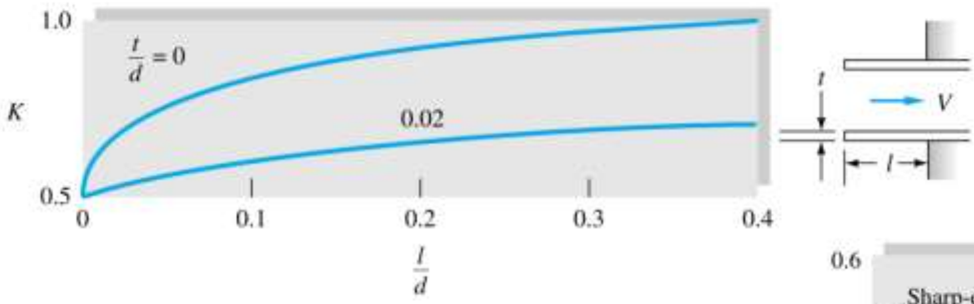
$$K_{SC} \approx 0.42\left(1 - \frac{d^2}{D^2}\right)$$

up to the value d/D = 0.76, above which it merges into the sudden-expansion prediction

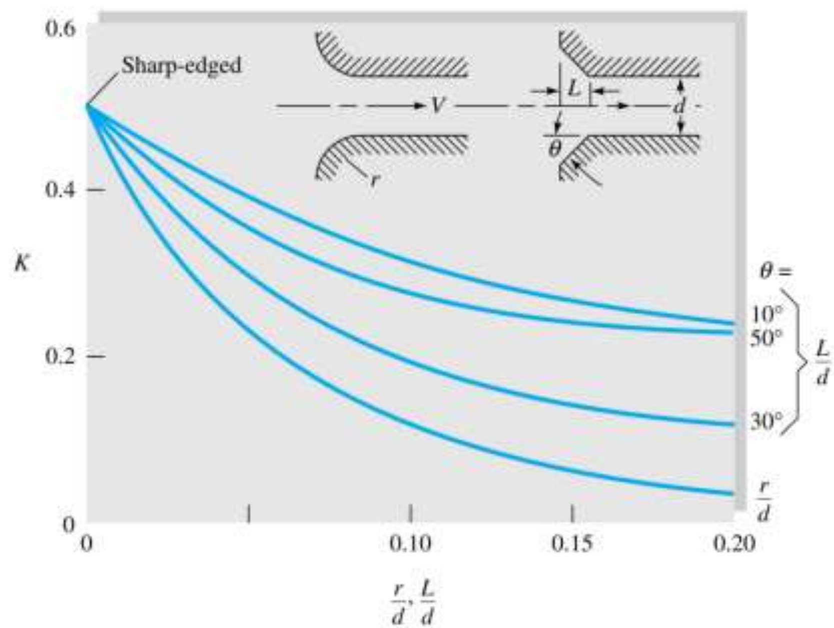


Note that *K* is based on the velocity in the small pipe.

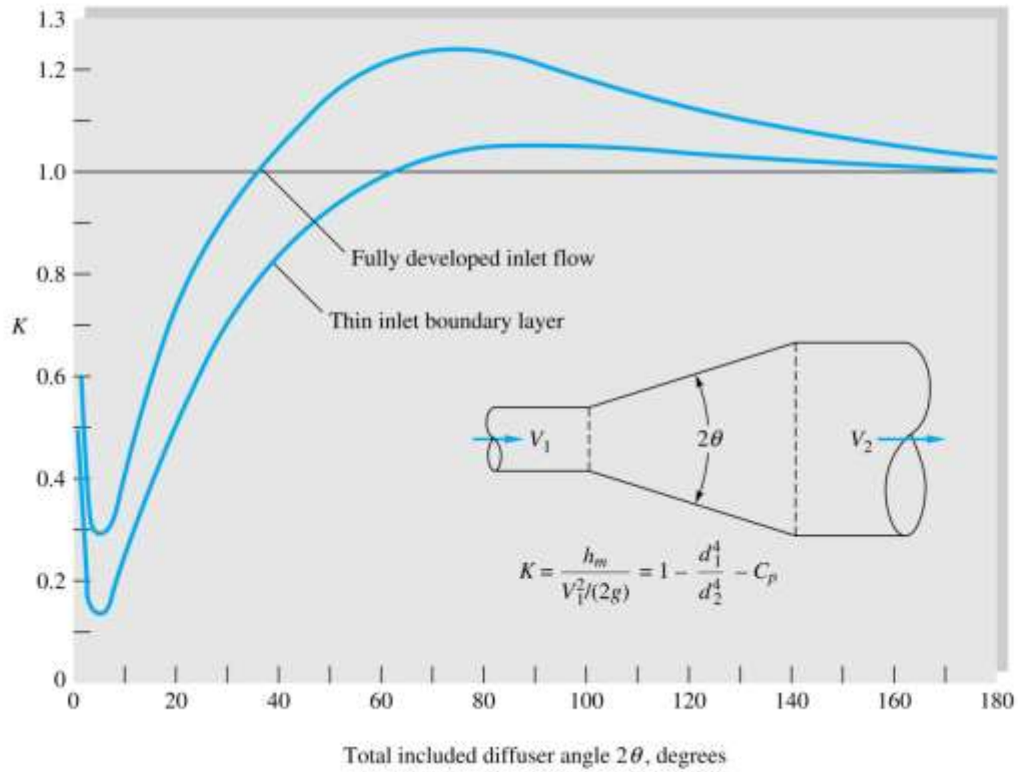
Entrance and Exit loss coefficients:



Exit losses are K≈1.0 for all shapes of exit to large reservoir



Minor Losses



Note that *K* is based on the velocity in the small pipe.

Gradual Expansion

Gradual Contraction

Contraction cone angle 2θ, deg	30	45	60
K for gradual contraction	0.02	0.04	0.07

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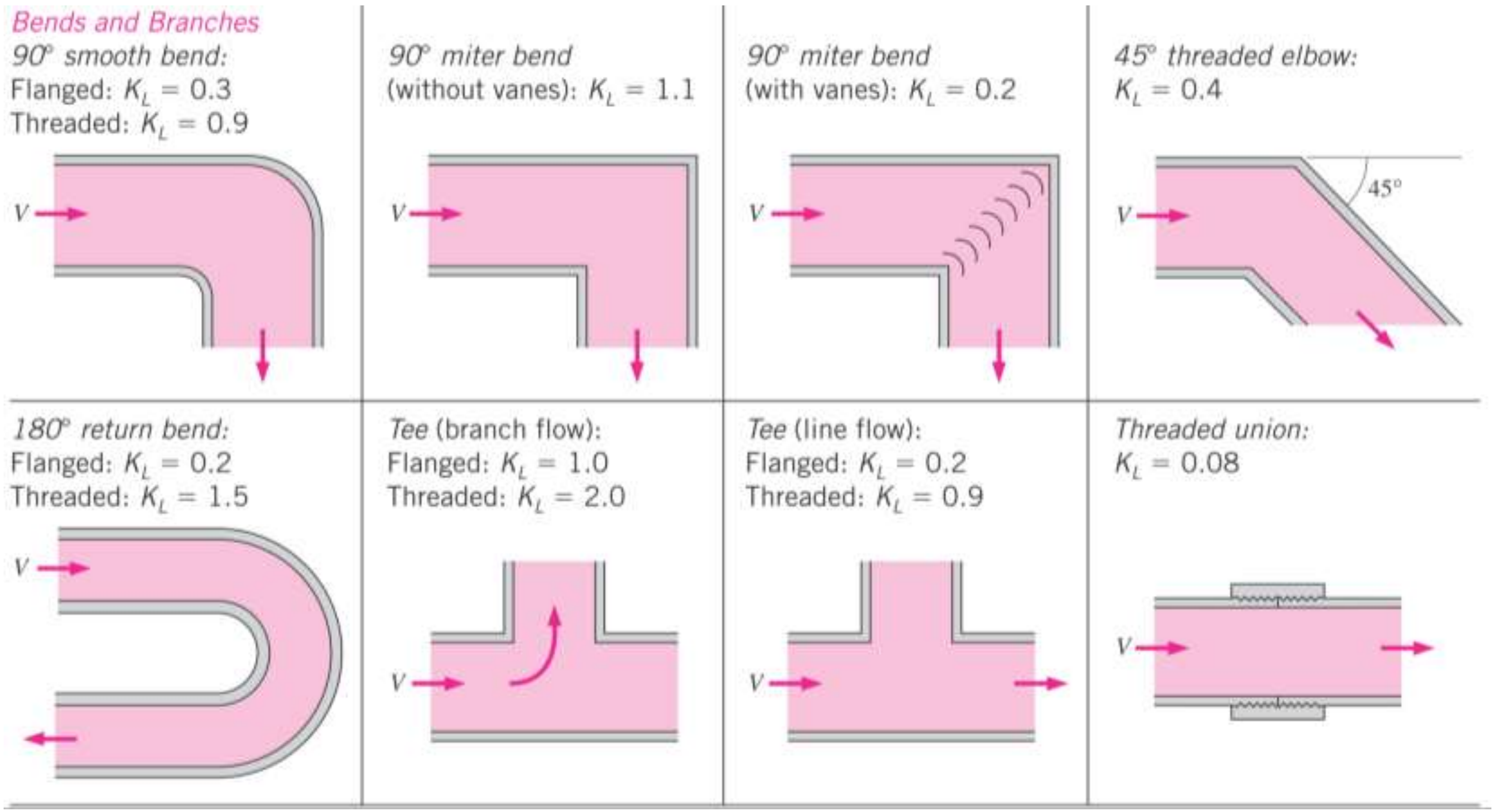
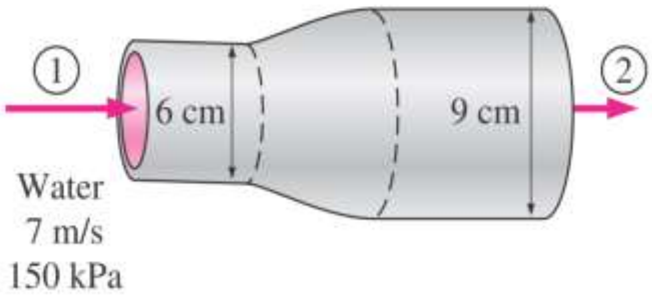
Minor Losses



Example 5

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe. The walls of the expansion section are angled 30° from the horizontal. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.



Assumptions:

1- Steady & incompressible flow.

2- Fully developed and turbulent flow with $\alpha \approx 1.06$.

Properties:

$\rho_{water} = 1000\ kg/m^3$

The loss coefficient for gradual expansion of θ=60° total included angle is
$K_L = 0.07$

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Example 5

Solution:

$$\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \rightarrow V_2 = \frac{A_1}{A_2} V_1 = \frac{D_1^2}{D_2^2} V_1 \rightarrow V_2 = \frac{(0.06)^2}{(0.09)^2}(7) = 3.11\,\text{m/s}$$

$$h_m = K\frac{V_1^2}{2g} = (0.07)\frac{7}{2\times 9.81} = 0.175\,\text{m}$$

$$\frac{P_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_{turbine}} - \cancel{h_{pump}} + \Delta h_{tot}$$

$$\rightarrow \frac{P_1}{\gamma} + \alpha_1\frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \alpha_2\frac{V_2^2}{2g} + h_m$$

$$\rightarrow P_2 = P_1 + \rho\left\{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2} - g h_m\right\} = 150 + 1000\left\{\frac{1.06\left(7^2 - 3.11^2\right)}{2} - 9.81(0.175)\right\}$$

$$\boxed{P_2 = 169\,\text{kPa}}$$

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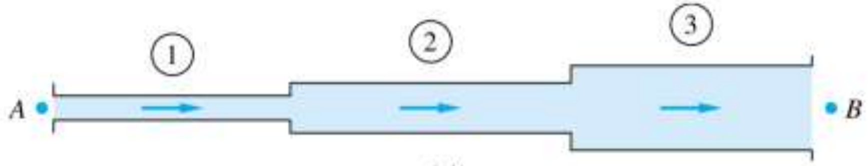
Multiple Pipe Systems

Pipes in Series

- Volume flow rate is constant
- Head loss is the summation of parts

$$Q_1 = Q_2 = Q_3 = \text{const}$$

$$V_1 d_1^2 = V_2 d_2^2 = V_3 d_3^2$$



(a)

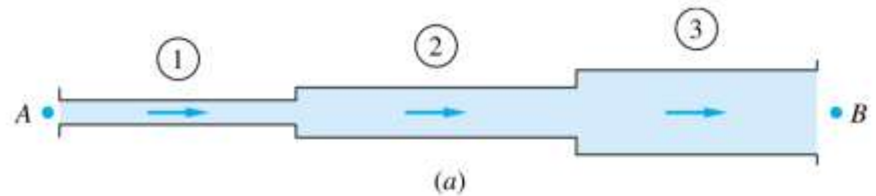
$$\Delta h_{A\to B} = \Delta h_1 + \Delta h_2 + \Delta h_3$$

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}\left(\frac{f_1 L_1}{d_1} + \sum K_1\right) + \frac{V_2^2}{2g}\left(\frac{f_2 L_2}{d_2} + \sum K_2\right) + \frac{V_3^2}{2g}\left(\frac{f_3 L_3}{d_3} + \sum K_3\right)$$

Since V_2 and V_3 are proportional to V_1 based on the mass conservation equation:

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}(\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3)$$

Pipes in Series



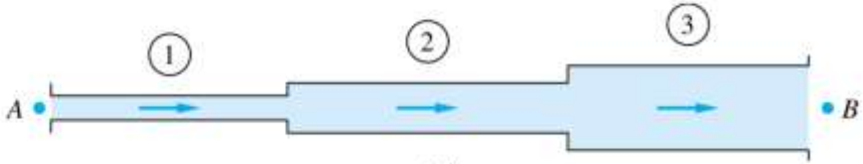
$$\Delta h_{A\to B} = \frac{V_1^2}{2g}(\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3)$$

- ✓ *If Q is given*, the right-hand side and hence the total head loss can be evaluated.
- ✓ If the head loss is given, a little iteration is needed, since f_1, f_2, and f_3 all depend upon V_1 through the Reynolds number. Begin by calculating f_1, f_2, and f_3, assuming fully rough flow, and the solution for V_1 will converge with one or two iterations.

Example 6

Given is a three-pipe series system. The total pressure drop is P_A-P_B =150000Pa, and the elevation drop is z_A-z_B=5m. The pipe data are as the table. The fluid is water, ρ =1000 kg/m^3 and ν =1.02×10 $^{-6}$ m^2/s. Calculate the ***Q*** in m^3/h through the system.

Pipe	*L*, m	*d*, cm	ε, mm	ε/*d*
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005



(a)

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Example 6

Solution:

The total head loss across the system is: $\Delta h_{A\to B} = \frac{p_A - p_B}{\rho g} + z_A - z_B = \frac{150{,}000}{1000(9.81)} + 5\text{ m} = 20.3\text{ m}$

From the continuity relation:

$$V_2 = \frac{d_1^2}{d_2^2} V_1 = \frac{16}{9} V_1 \qquad V_3 = \frac{d_1^2}{d_3^2} V_1 = 4V_1$$

and:

$$\text{Re}_2 = \frac{V_2 d_2}{V_1 d_1}\text{Re}_1 = \frac{4}{3}\text{Re}_1 \qquad \text{Re}_3 = 2\text{ Re}_1$$

Neglecting minor losses and substituting into the head loss Eq., we obtain:

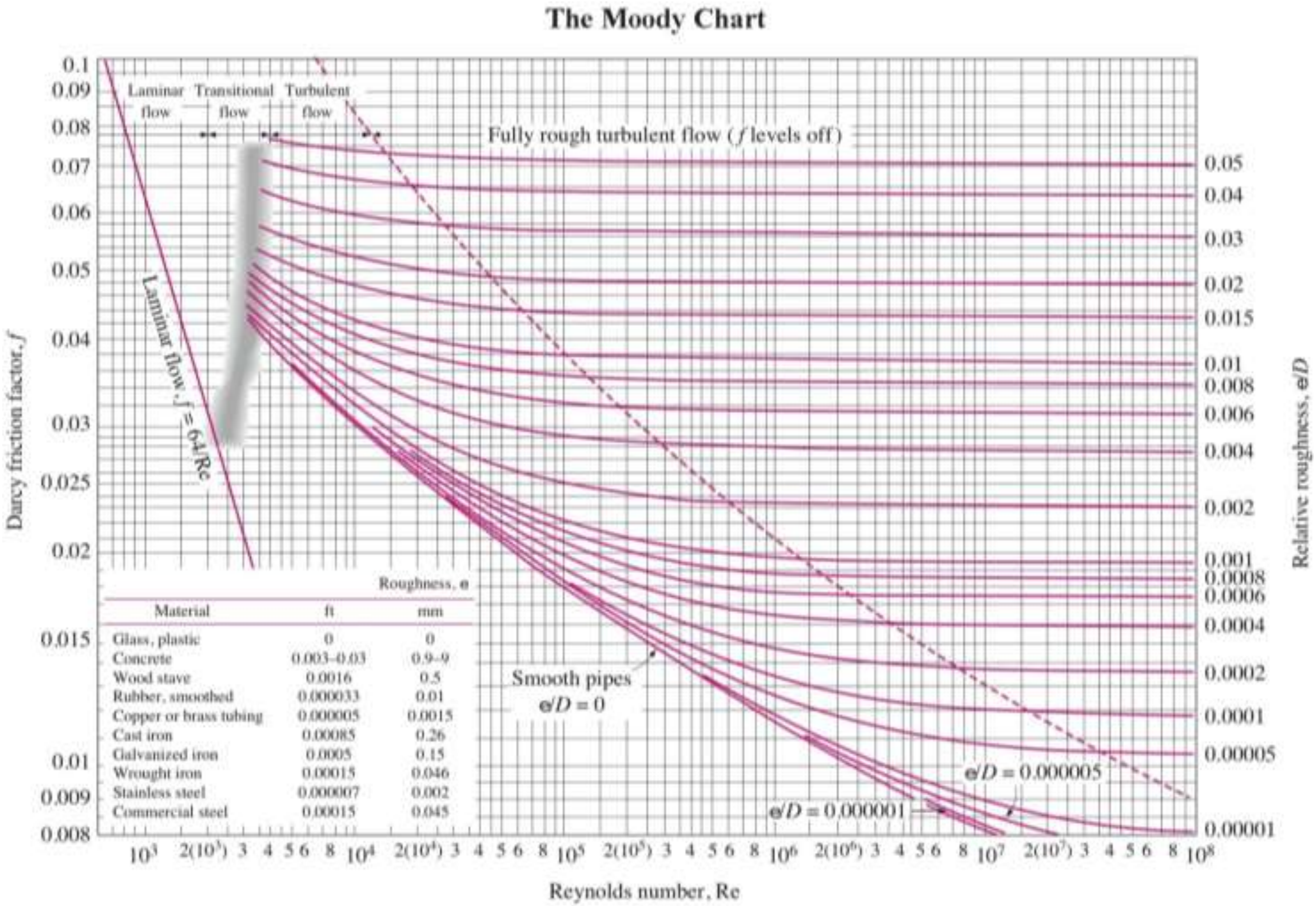
$$\Delta h_{A\to B} = \frac{V_1^2}{2g}(\alpha_0 + \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3) \longrightarrow \Delta h_{A\to B} = \frac{V_1^2}{2g}\left[1250 f_1 + 2500\left(\frac{16}{9}\right)^2 f_2 + 2000(4)^2 f_3\right]$$

$$20.3\text{ m} = \frac{V_1^2}{2g}(1250 f_1 + 7900 f_2 + 32{,}000 f_3) \qquad \textbf{(1)}$$

Begin by estimating f_1, f_2, & f_3 from the Moody-chart fully rough regime

$$f_1 = 0.0262 \qquad f_2 = 0.0234 \qquad f_3 = 0.0304$$

The Moody Diagram



Example 6

Solution (cont.):

$$20.3 \text{ m} = \frac{V_1^2}{2g}(1250f_1 + 7900f_2 + 32{,}000f_3) \quad \textbf{(1)}$$

Substitute in Eq. (1) to find: $V_1^2 \approx 2g(20.3)/(33 + 185 + 973)$

Thus the first estimate is $V_1 = 0.58\ m/s$ from which:

$$\text{Re}_1 \approx 45{,}400 \qquad \text{Re}_2 = 60{,}500 \qquad \text{Re}_3 = 90{,}800$$

Hence, using the Re number and roughness ratio and from the Moody chart,

$$f_1 = 0.0288 \qquad f_2 = 0.0260 \qquad f_3 = 0.0314$$

Substitution into Eq. (1) gives the better estimate

$$V_1 = 0.565 \text{ m/s} \qquad Q = \tfrac{1}{4}\pi d_1^2 V_1 = 2.84 \times 10^{-3} \text{ m}^3/\text{s}$$

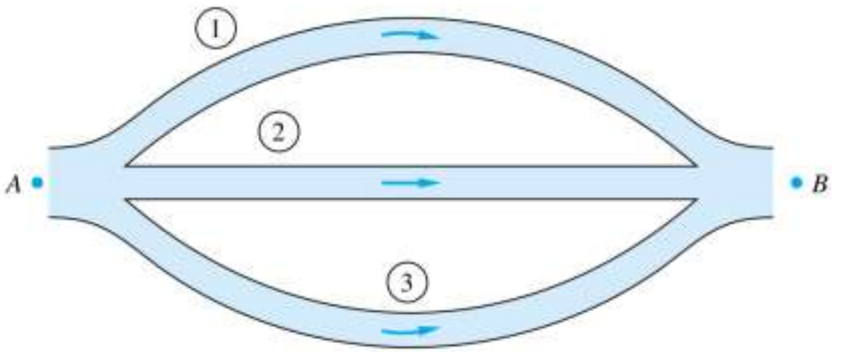
$$Q_1 = 10.2 \text{ m}^3/\text{h}$$

A second iteration gives ***$Q=10.22\ m^3/h$***, a negligible change

Multiple Pipe Systems

Pipes in Paralel

- Volume flow rate is the sum of the components
- Pressure loss across all branches is the same



$$\Delta h_{A\rightarrow B} = \Delta h_1 = \Delta h_2 = \Delta h_3$$

$$Q = Q_1 + Q_2 + Q_3$$

✓ If the Δh is known, it is straightforward to solve for Q_i in each pipe and sum them.

✓ The problem of determining Q_i when h_f is known, requires iteration. Each pipe is related to h_f by the Moody relation $h_f = f\,(L/d)\left(V^2/2g\right) = f\,Q^2/C$, where $C = \left(\pi^2 g d^5/8L\right)$ Thus head loss is related to total flow rate by:

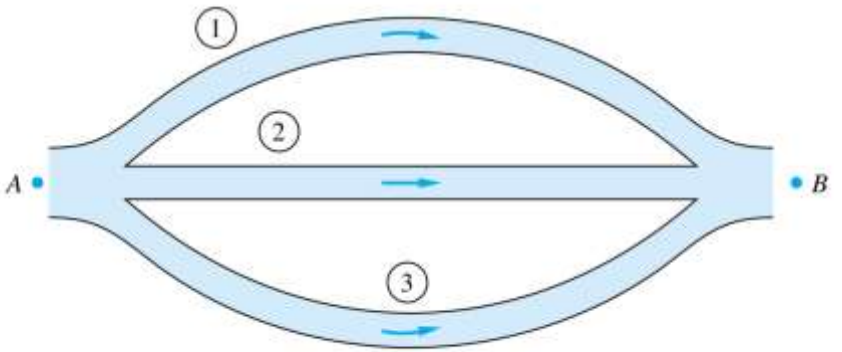
$$h_f = \frac{Q^2}{\left(\sum\sqrt{C_i/f_i}\right)^2} \qquad \text{where } C_i = \frac{\pi^2 g d_i^5}{8L_i}$$

Multiple Pipe Systems

Pipes in Paralel

$$h_f = \frac{Q^2}{\left(\sum \sqrt{C_i/f_i}\right)^2} \qquad \text{where } C_i = \frac{\pi^2 g d_i^5}{8L_i}$$

$$Q_i \approx \sqrt{\frac{C_i h_f}{f_i}}$$

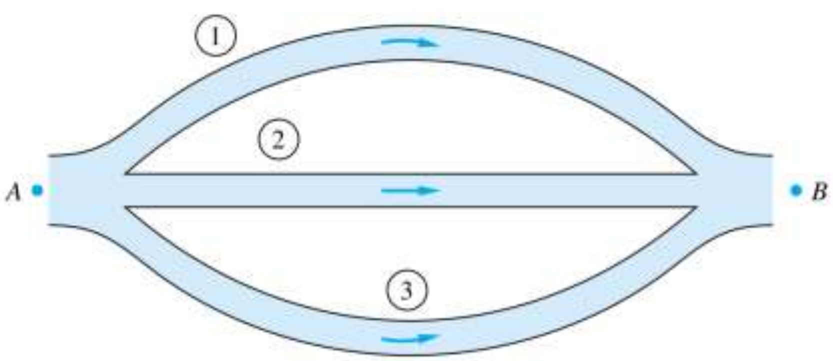


Since the f_i vary with Reynolds number and roughness ratio, one begins the above Eq. by guessing values of f_i (fully rough values are recommended) and calculating a first estimate of h_f. Then each pipe yields a flow-rate estimate Q_i and hence a new Reynolds number and a better estimate of f_i. Then repeat this Eq. to convergence.

Example 7

Assume that the same three pipes in Example 6 are now in parallel with the same total head loss of 20.3 m. Compute the total flow rate Q, neglecting minor losses.

Pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005



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Example 7

Solution:

$$\Delta h_{A\to B} = \Delta h_1 = \Delta h_2 = \Delta h_3$$

From the above equation we have:

$$20.3\text{ m} = \frac{V_1^2}{2g}1250f_1 = \frac{V_2^2}{2g}2500f_2 = \frac{V_3^2}{2g}2000f_3$$

Guess fully rough flow in pipe 1: f_1 =0.0262, V_1 =3.49 m/s; hence Re_1=273,000.

From the Moody chart read f_1 =0.0267; recompute V_1 =3.46 m/s, Q_1 =62.5 m^3/h.

Next guess for pipe 2: f_2 =0.0234, V_2 =2.61m/s; then Re_2=153,000, and hence f_2=0.0246, V_2 =2.55 m/s, Q_2 =25.9 m^3/h.

Finally guess for pipe 3: f_3 =0.0304, V_3=2.56 m/s; then Re_3=100,000, and hence f_3=0.0313, V_3 =2.52 m/s, Q_3 =11.4 m^3/h.

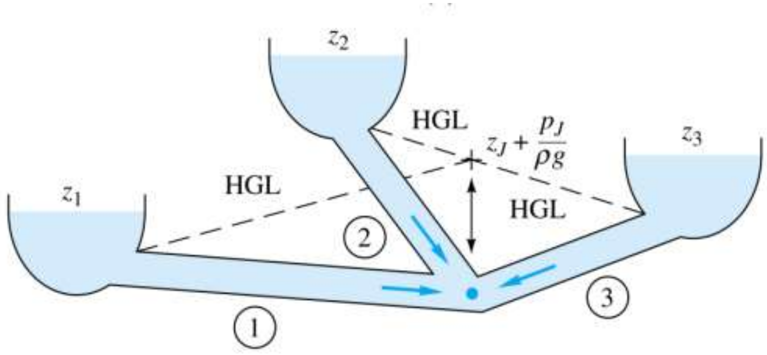
$$Q = Q_1 + Q_2 + Q_3 = 62.5 + 25.9 + 11.4 = \boxed{99.8\text{ m}^3/\text{h}}$$

Multiple Pipe Systems

Pipes in Junction

If all flows are considered positive toward the junction, then

$$Q_1 + Q_2 + Q_3 = 0$$



The pressure must change through each pipe so as to give the same static pressure p_J at the junction.

Let the HGL at the junction have the elevation

$$h_J = z_J + \frac{p_J}{\rho g}$$

Assuming $p_1=p_2=p_3=0$ (gage)

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

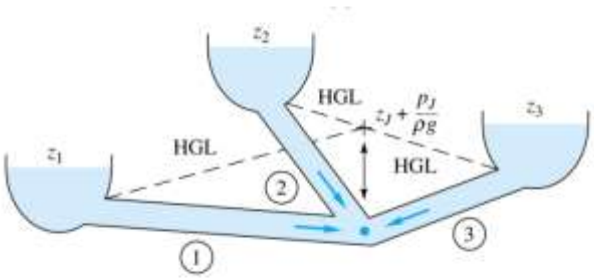
Pipes in Junction

We guess the position h_J and solve the following Eq. for V_1 ,V_2 & V_3 and hence Q_1 ,Q_2 & Q_3 :

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J \qquad \Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J \qquad \Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

iterating until the flow rates balance at the junction according to Eq.

$$Q_1 + Q_2 + Q_3 = 0$$



If we guess h_J too high, the sum flow sum will be negative and the remedy is to reduce h_J ,and vice versa.

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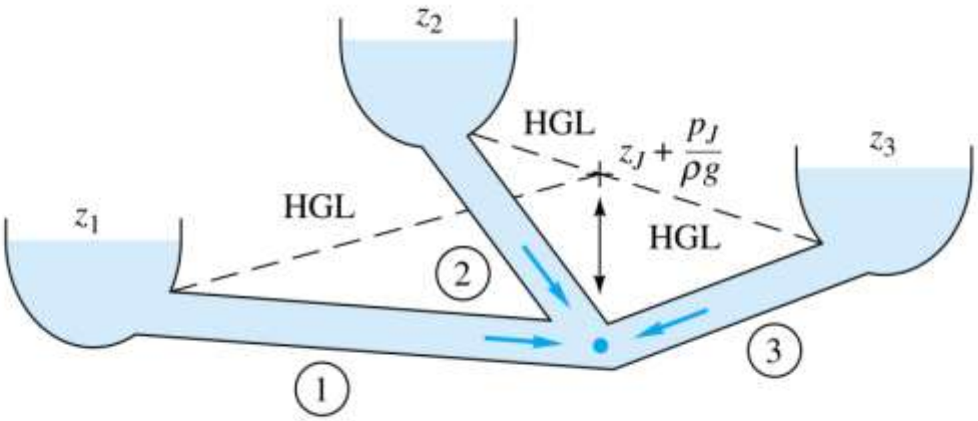
Example 8

Take the same three pipes as in Example 7, and assume that they connect three reservoirs at these surface elevations. Find the resulting flow rates in each pipe, neglecting minor losses.

$$z_1 = 20 \text{ m} \qquad z_2 = 100 \text{ m} \qquad z_3 = 40 \text{ m}$$



Example 8

Solution:

As a first guess, take h_J equal to the middle reservoir height, $z_3 = h_J = 40$ m.

This saves one calculation ($Q_3 = 0$)

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h	L_i/d_i
1	40	−20	0.0267	−3.43	−62.1	1250
2	40	60	0.0241	4.42	45.0	2500
3	40	0		0	0	2000
					$\sum Q = -17.1$	

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Example 8

Solution (cont.):

Since the sum of the flow rates toward the junction is negative, we guessed h_J too high. Reduce h_J to 30 m and repeat:

$$\Delta h_1 = \frac{V_1^2}{2g}\frac{f_1 L_1}{d_1} = z_1 - h_J$$

$$\Delta h_2 = \frac{V_2^2}{2g}\frac{f_2 L_2}{d_2} = z_2 - h_J$$

$$\Delta h_3 = \frac{V_3^2}{2g}\frac{f_3 L_3}{d_3} = z_3 - h_J$$

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m³/h
1	30	−10	0.0269	−2.42	−43.7
2	30	70	0.0241	4.78	48.6
3	30	10	0.0317	1.76	8.0
					$\sum Q = 12.9$

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Example 8

Solution (cont.):

This is positive $\sum Q=12.9$, and so we can linearly interpolate to get an accurate guess: $h_J \approx 34.3$ m.

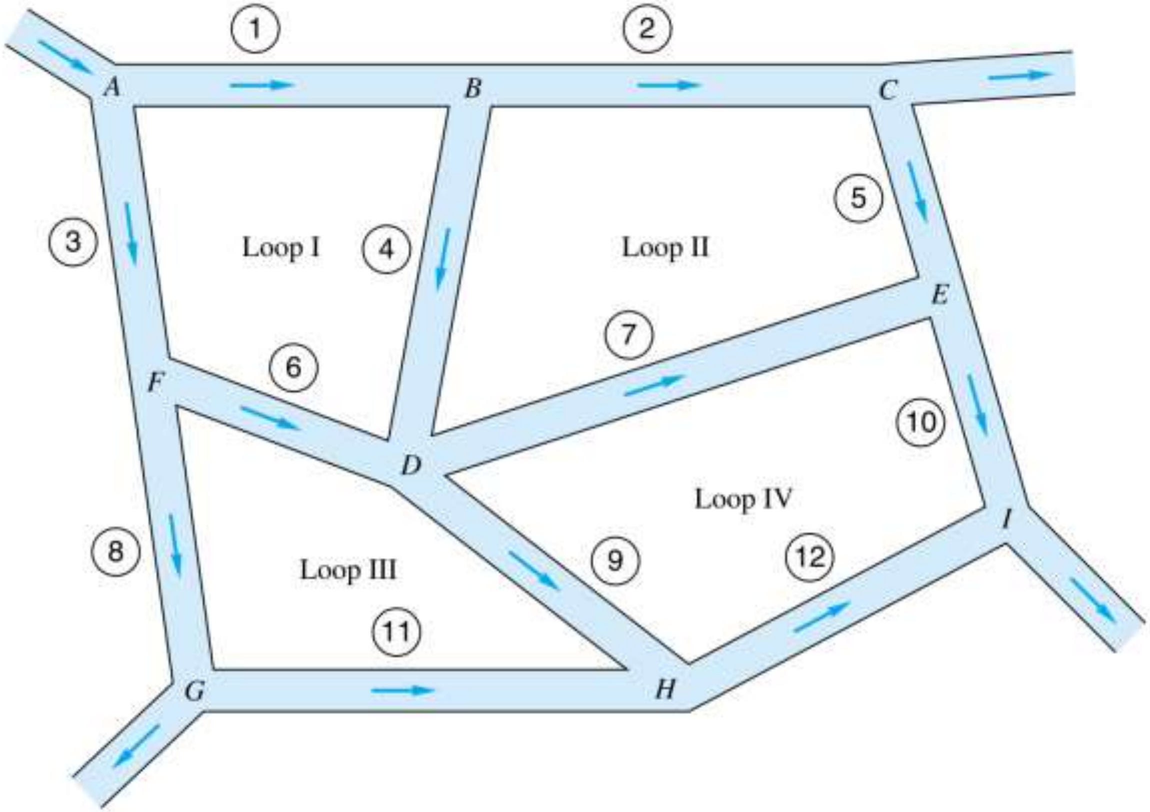
Make one final list:

Reservoir	h_J, m	$z_i - h_J$, m	f_i	V_i, m/s	Q_i, m^3/h
1	34.3	−14.3	0.0268	−2.90	−52.4
2	34.3	65.7	0.0241	4.63	47.1
3	34.3	5.7	0.0321	1.32	6.0
					$\sum Q = 0.7$

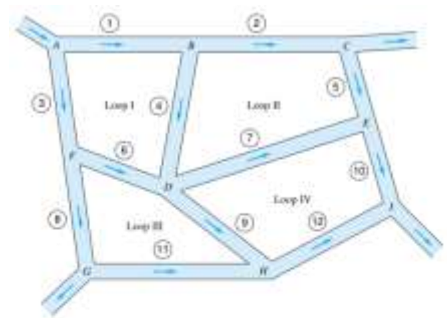
we calculate that the flow rate is 52.4 m^3/h toward reservoir 3, balanced by 47.1 m^3/h away from reservoir 1 and 6.0 m^3/h away from reservoir 3.

Multiple Pipe Systems

- Piping network



Piping network



This network is quite complex algebraically but follows the same basic rules:

1. The net flow into any junction must be zero.
2. The net head loss around any closed loop must be zero. In other words, the HGL at each junction must have one and only one elevation.
3. All head losses must satisfy the Moody and minor-loss friction correlations.

By supplying these rules to each junction and independent loop in the network, one obtains a set of simultaneous equations for the flow rates in each pipe leg and the HGL (or pressure) at each junction.