Numerical and Experimental Analysis of Nonlinear Parabolic Springs Employed in Suspension System of freight cars

D.Younesian*, M. S. Fallahzadeh

1-Associate Professor 2- MSc Student Department of Railway Engineering, Iran University of Science and Technology.

* Younesian@iust.ac.ir

Abstract

Nonlinear vibration of parabolic springs employed in suspension system of a freight car has been studied in this paper. First, dynamical behavior of the springs is investigated by using finite element method and the obtained results are then used in vibration analysis of a railway freight car. For this purpose, dynamics of a parabolic spring subjected to a cyclic excitation has been studied in the frequency range of 2 to 15 Hz. By utilizing an experimental setup, equivalent static and dynamic stiffness and damping of the spring have been obtained and compared with theoretical results. Different classes of rail irregularities are taken into account to excite the vehicle. Bond Graph method is employed to extract the equations of motion of the system and validity of the obtained equations is investigated. Finally, a parametric study is carried out and the influence of vehicle velocity and rail irregularity on vertical acceleration of the freight car has been examined.

Keywords: *Parabolic spring, railway vehicle, nonlinear vibration, Bond Graph*

Introduction

Leaf springs are one of the significant mechanical components of heavy vehicles. These components are widely used in the primary suspension system of the railway vehicles. Due to the extent of the applications, many efforts have been made to improve performance of the leaf springs. Although coil and air springs have been remarkably used in different suspension systems in the recent years, improving dynamical performance of the leaf springs can result in further application of them for railway vehicles. Different investigations have been performed so far to study the dynamics of the leaf springs. Generally, these scientific projects can be categorized based on the analysis procedures, static or dynamic experiments and modal or fatigue predictions.Hazrati and Azadi [1] examined the effect of thickness and number of layers on the performance of leaf springs.Tavakoli et al. [2] analyzeda leafspring bush by using a finite element model. Composite leaf springs have been recently employed in different industries due to the higher stiffness and natural frequency and lower weight and cost. Load carrying capacity, stiffness and weight of a composite leaf spring have been compared with the characteristics of a steel leaf spring by using finite element method [3, 4].Tambaca [5] derived the equations of a leaf spring utilizing a linear theory. He showed that the stiffness of spring is a function of geometric and material properties of the system. Li et al. [6] used a contact finite element algorithm and classic beam theory to study the performance of leaf springs. Moon and Song [7] modeled a leaf spring by using Adams software and compared the results with those obtained from experiments. Finite element method is one of the procedures frequently employed in vibration analysis of the springs.Krishan and Aggarwall [8] have modeled a multi leaf spring system in ANSYS.The weak joint bending of the stack of slim non-uniform curved beams has been considered in [9]. Friction between the spring layers has been neglected in this study and an analytical procedure is utilized to examine the problem. Large deformation of a beam has been analyzed by Darvizeh et al. [10] using finite element method. Influence of contact between different layers of a leaf spring on the energy dissipated in the system has been investigated in [11]. Moon and Song [12] studied the hysteretic characteristics of leaf springs in commercial vehicles. They used an experimental method to check the validity of their simulations in ADAMS software.Many research works have been conducted by scientific researchers to investigate the static behavior of leaf springs. However, some authors have studied the dynamic behavior of the springs subjected to dynamic loading [13, 14].

In the present study, dynamic characteristics of a parabolic leaf spring of a freight wagon are investigated by using ABAQUS software and a theory presented in [15]. Response of the railway vehicle is then obtained and a parametric study is carried out to examine the effect of different parameters of the model on the response of the vehicle.

Leaf Springs

Generally, leaf springsare categorized as mono and multi spring systems.They also can be classified base on their layers thickness. Leaf spring systems composed of layers with variable thickness are known as parabolic springs. Leaf spring systems have some advantages and disadvantages. Higher structural damping with respect to the other types of springs is one of the remarkable characteristics of these systems.Lower longitudinal elastic property and poorelasto-plastic properties of the leaf springs are also some of the disadvantages.Parabolic leaf spring of the freight wagon employed in this study is illustrated in Figures 1 and 2.

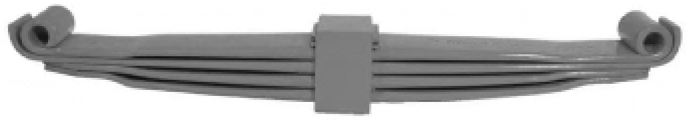


Fig1. A parabolic leaf spring system

Fig2. Parabolic suspension system employed in a freight wagon

Finite Element Analysis of Parabolic Springs

ABAQUS software is utilized to develop a finite element model for the system. Preloading of 6.5 tons is considered at first and the model is then subjected to a harmonic displacement according to the following equation:

$$x = Asin(2\pi ft) \qquad (1)$$

inwhich A and f are amplitude and frequency, respectively. In order to solve the model, dynamic explicit analysis is used [16].Finite element model has been developed in ABAQUS to analyze the system and displacement and stress are obtained in a static analysis. The model and obtained results are illustrated in Figure 3.

Validity of the Model

In order to dynamically analyze the parabolic spring system, validity of the model has to be checked at first. For this purpose, two different experiments are considered and the results obtained from the finite element model have been compared with those obtained by the experiments. In the first experiment, deflection of a leaf spring system subjected to a static force is measured and friction coefficient between the layers is obtained in an impact test as well as total damping of the mechanical system. The experimental setup for static test and the measured stiffness of the system have been shown in Figures 4 and 5, respectively.

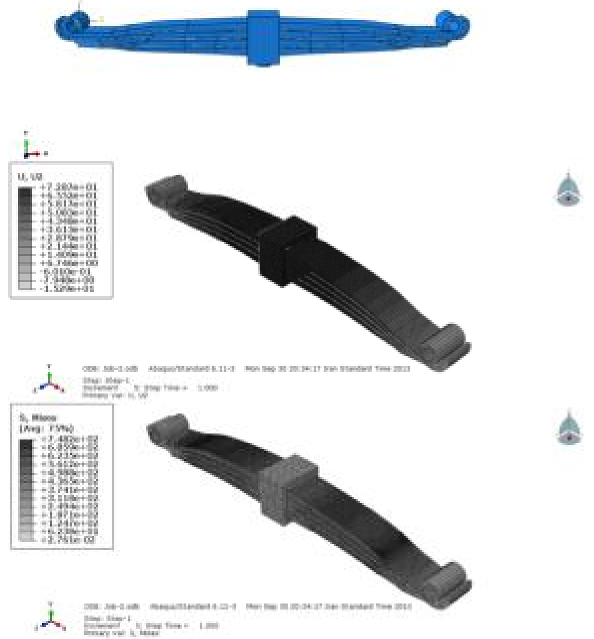


Fig3.. Finite element model and the obtained results

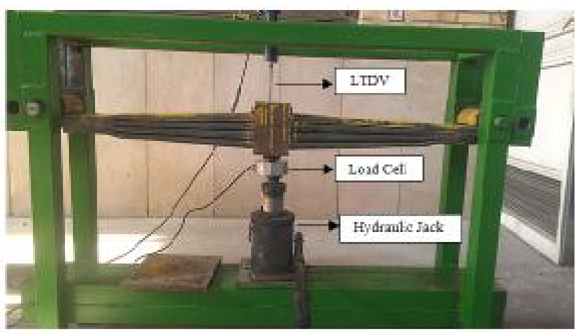


Fig4. Experimental setup used for static test

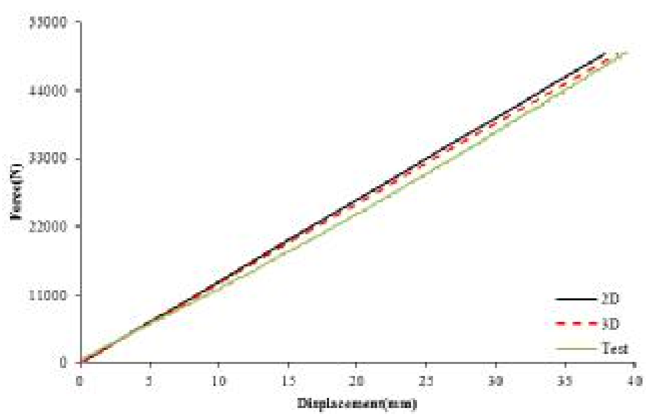


Fig5. Stiffness of the spring obtained from finite element models and experiment

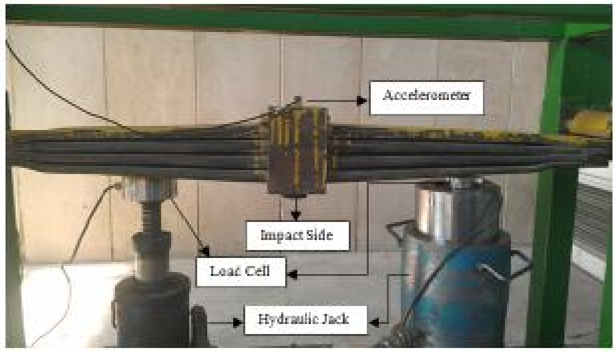
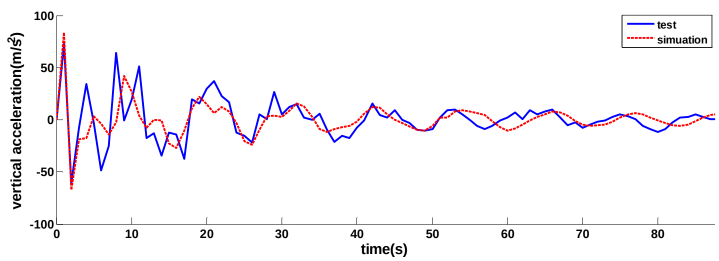
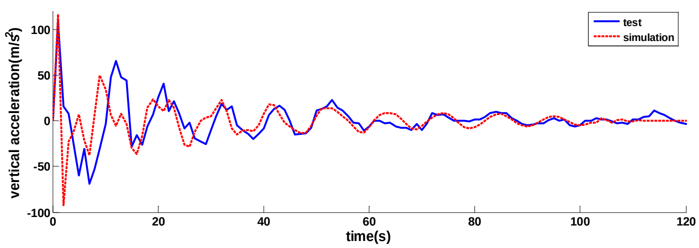


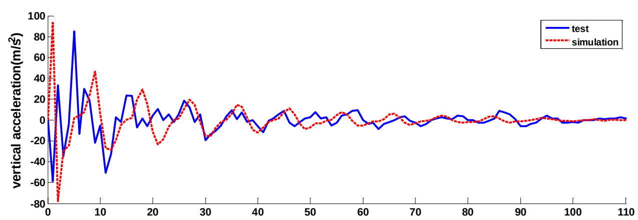
Fig6. Experimental setup used for impact test



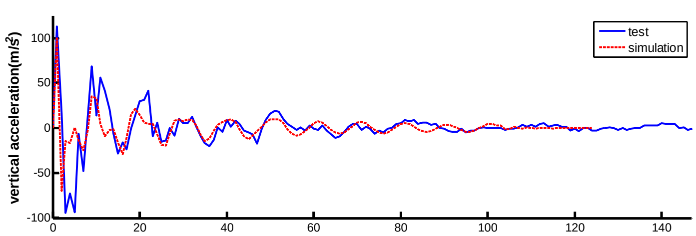
Preload: 2 tons



Preload: 3 tons



Preload: 4 tons



Preload: 5 tons

Fig7. Vertical acceleration obtained for different preloads

As seen in Figure 5, finite element model can accurately predict the stiffness of the parabolic spring. An impact test is considered for the second experiment and acceleration is measured for the spring excited by a hammer.In Figure 6, the experimental setup for the impact test is composed of a hydraulic jack, load cells, accelerometers, a hammer and a data logger. Preload values are 2, 3, 4 and 5 tons in four different experiments. Friction coefficient between the layers is experimentally obtained using a method presented in [17]. The obtained results for acceleration have been illustrated in Figure 7.As seen in this figure, there is a good agreement between the experimental and numerical results. The friction coefficient determined in different experiments (Preloads) is 0.3. By comparing the results obtained from finite element analysis, static test and dynamic test, it can be concluded that the finite element model of the parabolic spring accurately predicts thestatic and dynamic behaviors of the system.

Dynamic behavior of the parabolic spring

Influence of amplitude and frequency on the dynamic behavior of the parabolic spring has been investigated in this section. According to Berg's theory [15] and figure 8, dynamic stiffness and damping of the spring can be obtained using characteristics of hysteretic loops, when the system is subjected to a cyclic loading. Therefore, one can utilize the following equations:

$$S = \frac{F_0}{x_0}[KN/m] \tag{2}$$

$$D = \frac{E}{F_0 x_0} \tag{3}$$

In the latter equations S and D are dynamic stiffness and damping of the leaf spring, respectively. Hysteretic loops for cyclic loadings of different amplitudes and frequencies are illustrated in figure 9.

Effect of frequency on the stiffness and damping is shown in figures 10 and 11. Variation of the force with respect to the displacement is also obtained for different amplitudes and illustrated in figure 12.

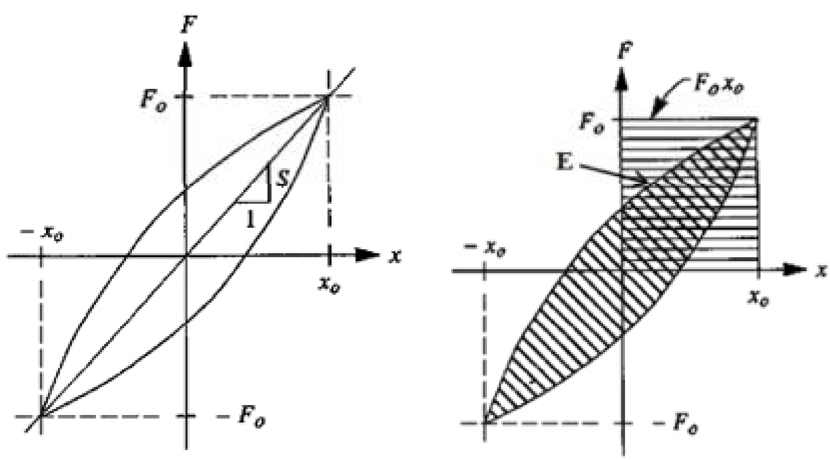


Fig8.. Dynamic stiffness and damping [15]

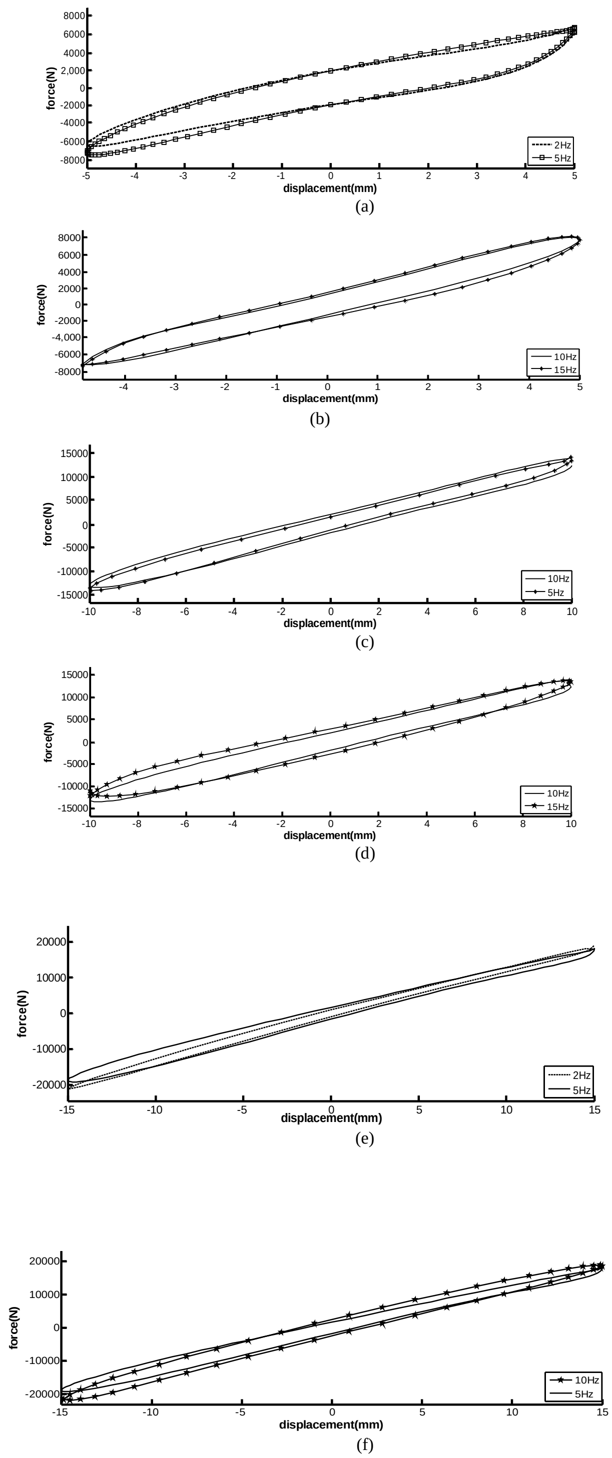


Fig9.. Variations of the force with respect to the amplitude; (a), (b) 5 mm; (c), (d) 10 mm; (e),(f) 15 mm

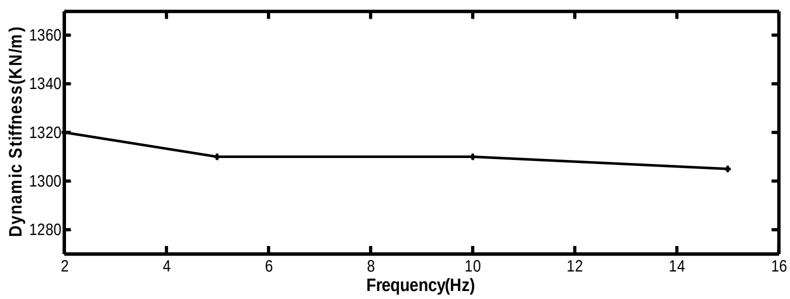


Fig10. Effect of frequency on the damping; Amplitude: 10 mm

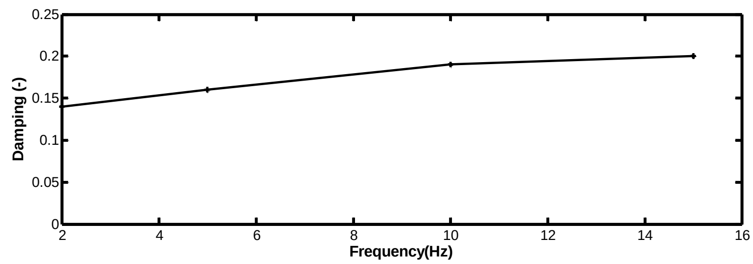
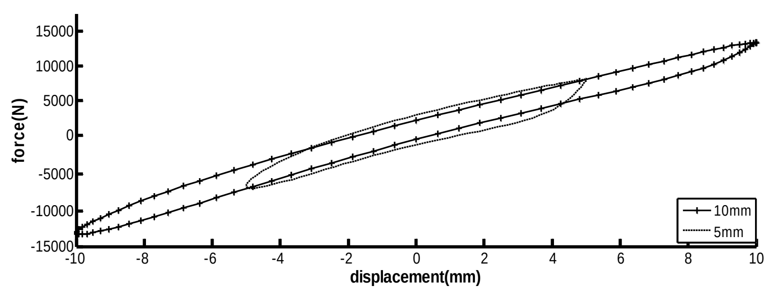
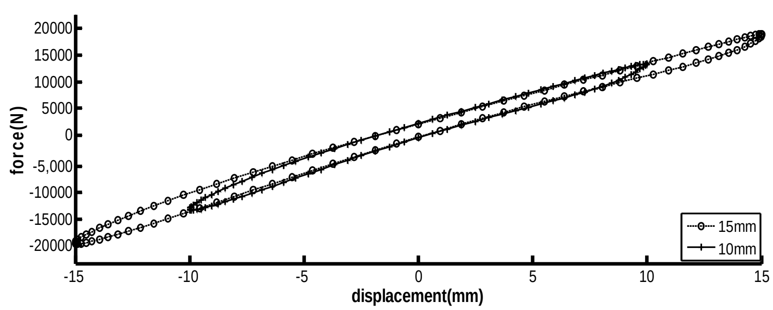


Fig11. Effect of frequency on the damping; Amplitude: 10 mm

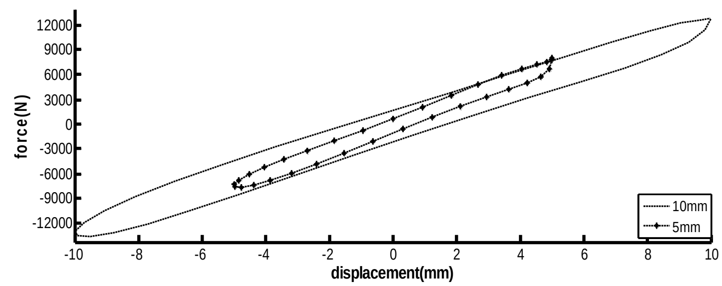


f = 5 Hz

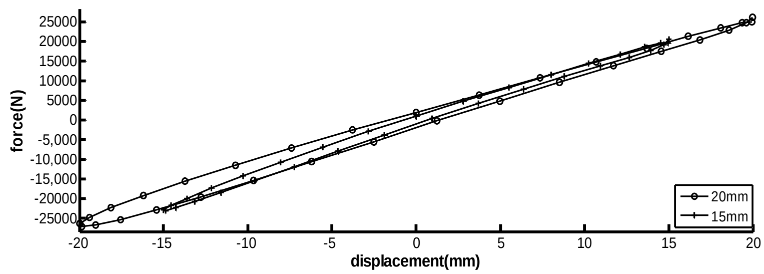


f = 5 Hz

Fig12. Variation of the force with respect to the displacement(Continuous)



f = 15 Hz



f = 15 Hz

Fig12. Variation of the force with respect to the displacement

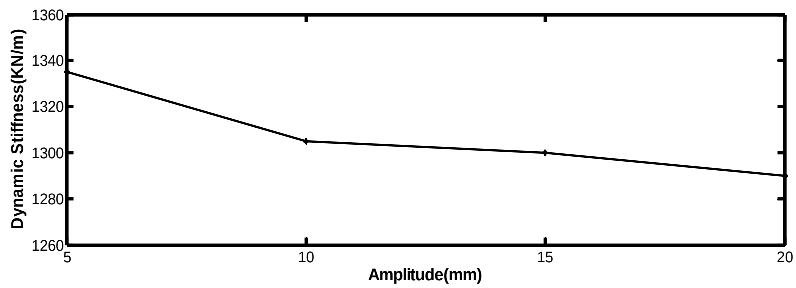


Fig13. Variation of dynamic stiffness with respect to displacement; f = 10 Hz

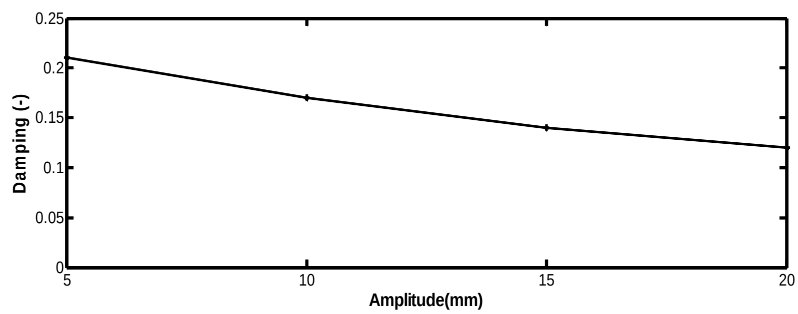
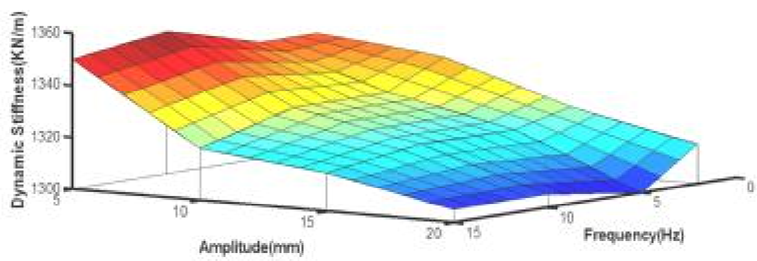


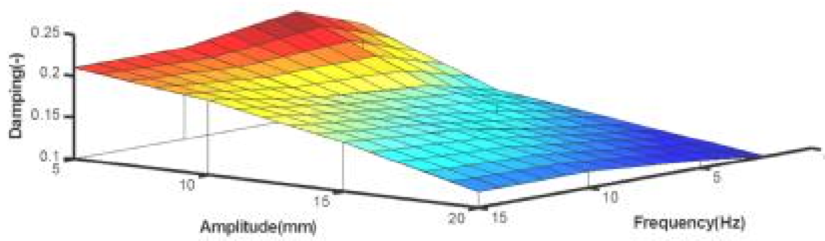
Fig14. Variation of damping with respect to displacement; f = 10 Hz

In a similar way, corresponding force is plotted against displacement for frequency of 10 Hz and results are illustrated in figures 13 and 14.

In order to simultaneously observe the relation between the spring properties, amplitude and frequency, three dimensional graphs are taken into account in this study. these graphs are illustrated in figure 15.



(a)



(b)

Fig15. Variation of (a) dynamic stiffness and (b) damping with respect to amplitude and frequency

Table 1. Definition of $\boldsymbol{A_v}$ for different classes of rail irregularity

Railway Class	A_v
1	15.52×10^{-8}
2	8.84×10^{-8}
3	4.91×10^{-8}
4	2.75×10^{-8}
5	1.55×10^{-8}
6	0.88×10^{-8}

Rail irregularities

As mentioned before, vertical vibrations of a freight wagon are studied in this paper. Rail irregularity is one of the significant sources of the vibrations in railway vehicles. Different classes of the rail irregularities are taken into account in this section. Power spectral density of the irregularities is defined as follow:

$$G_{rr} = \frac{A_v \grave{u}_2{}^2(\grave{u}^2 + \grave{u}_1{}^2)}{\grave{u}^4(\grave{u}^2 + \grave{u}_2{}^2)} \tag{4}$$

where $\grave{u}_1$ and $\grave{u}_2$ are 0.131 and 0.0233, respectively. Moreover, for different classes of rail quality $A_v$has been defined in Table 1.

Also, rail irregularity can be generated by using the following function:

$$r^d(x) = \sum_{k=1}^{N} a_k \cos(\grave{u}_k x + \ddot{o}_k) \tag{5}$$

wherea_k is the amplitude of cosine function which can be obtained utilizing following equations:

$$a_k = 2\sqrt{G_{rr}(\grave{u})\Delta\grave{u}} \tag{6}$$

$$\grave{u}_k = \grave{u}_1 + (K - .5)\Delta\grave{u} \tag{7}$$

$$\Delta\grave{u} = (\grave{u}_u - \grave{u}_1)/N \tag{8}$$

in which$\grave{u}_u$ and $\grave{u}_1$ are upper and lower limits. For example, some rail irregularities generated by the latter equations have been illustrated in figure 16. A comparison between amplitudes of the different classes is also carried out and corresponding results are illustrated in figure 17.

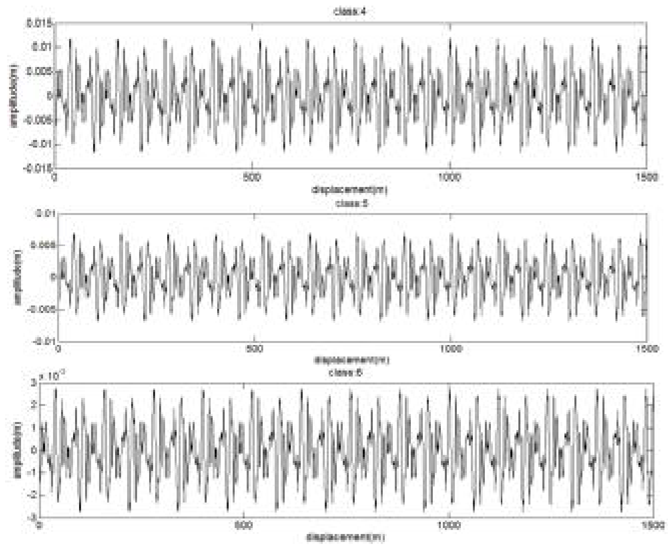


Fig16. Generated rail irregularities for different classes

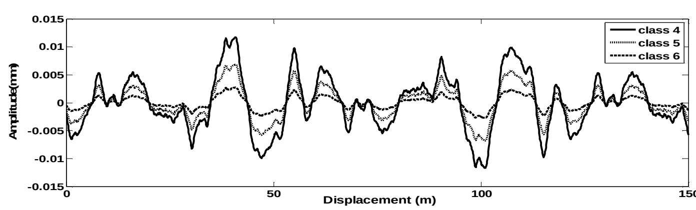


Fig17. A comparison between the amplitudes of different classes of rail irregularity

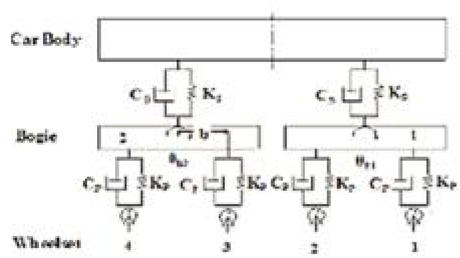


Fig18. A schematic of the model

Railway vehicle model

The railway vehiclecomposed of two bogies and a freight wagon is modeled as six degrees of freedom system. A schematic of the model is shown in figure 18.The freight wagon is connected to the bogies through center pivot mechanism. According to reference [18], a high-stiffness torsional spring is considered at the center of connection illustrated in figure 19. Different parameters of the railway vehicle have been presented in Table 2.

Bond graph method is employed to find the space-state form of the equations of motion. All the parameters of the model are constant, except for c1 and k1. In fact the model is nonlinear due to the nonlinear relation between these two parameters, amplitude and frequency. Values of c1 and k1 have been determined in different amplitudes and frequencies by using the three dimensional graphs presented in figure 15. However, damping parameter (ξ) in figure 15 denotes damping ratio and damping coefficient of the system can be consequently obtained utilizing the following equation:

$$\xi = \frac{C}{C_{cr}} = \frac{C}{2m\grave{u}_n} = \frac{C}{2\sqrt{K.m}} \qquad (9)$$

Values of c1 and k1 depend on the amplitude and frequency of the displacement of the parabolic spring. Consequently, values of the amplitude and frequency must be simultaneously obtained from the displacement. For this purpose, short-time Fourier transform has been used. It should be mentioned that the acceleration of center of the freight wagon can be averagely obtainedfrom the acceleration of the center of the bogies. Acceleration of bogies can be determined utilizing space-state equations of motion.

Validity of the equations

Validity of the equations of motion extracted by Bond graph method has been checked in this section.For this purpose, an irregularity is considered in figure 20 and it is assumed that a train of speed 10 m/s is passing the irregularity. Vertical acceleration response of the wagon is shown in figure 21. Dynamic behavior of the vehicle must be similar to the results obtained in [18].

It can be observed from figure 21 that the wagon is vertically stable. It is also seen that the results obtained from the Bond graph method are accurate.Only bond graph model is illustrated in figure 22 and the equations are omitted in this paper.Bonds are classified to those represent bi-directional exchange of energy and those represent the uni-directional flow of information (from an element to other elements). Results determined for a wagon of speed 10 and 20 m/s are illustrated in figure 23. In order to perform a comparison between the results, two different cases have been taken into account. In the first case, parameters of the primary suspension system are assumed to be constant. In the second case, these parameters are functions of amplitude and frequency. In order to find the constant values of the parameters, stiffness and damping are found from averaging. Response of the model has been shown in figure 23 for two linear and nonlinear cases. As seen in this figure, there is no significant difference between the responses of the vehicle for linear and nonlinear cases. Two reasons can be mentioned regarding the similarity of the responses. Firstly, as seen in figure 24, the acceleration response of the wagon is similar to the input acceleration of the model (figure 24) and the effect of rail irregularity is not large enough to show the difference between the linear and nonlinear model. Secondly, the differences between the characteristics of the suspension system obtained for different amplitude and frequency are not noticeable. In order to investigate the effect of classes of the rail irregularity on the shape of the acceleration response of the vehicle model, time history of the responses is obtained for class 4 of the rail irregularity and illustrated in figure 25.

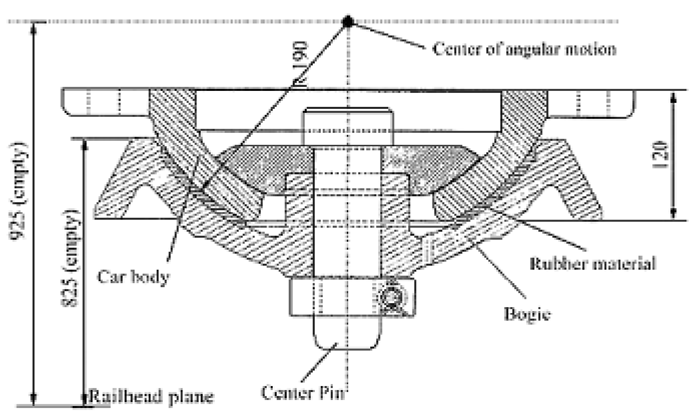


Fig19. Connection between the freight wagon and bogies

Table 2. Characteristics of the railway vehicle

characteristic	Align	Unit	Value
Overall mass of body and load	M_w	Kg	**50000**
Mass of Bogie	m_b	Kg	4750
Half of distance between the center pivot	l_w	m	4.5
Half of length of Bogie	l_b	m	1.7
Body Moment	J_w	Kg.m^2	986840
Bogie Moment	J_b	Kg.m^2	1590
Torsional stiffness	k_s	Nm/rad	10000000
Torsional damping	c_s	N.m.s/rad	2000

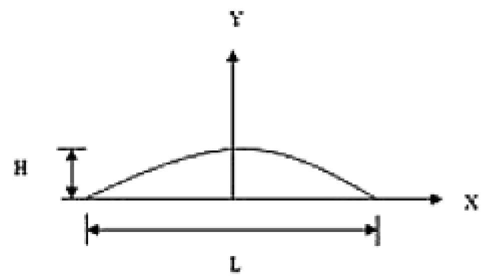


Fig20. Assumed irregularity model; H= 20 mm, L= 1 m

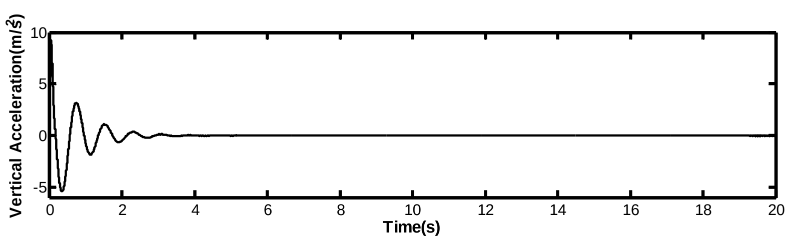


Fig21. Time history of the vertical acceleration of the wagon

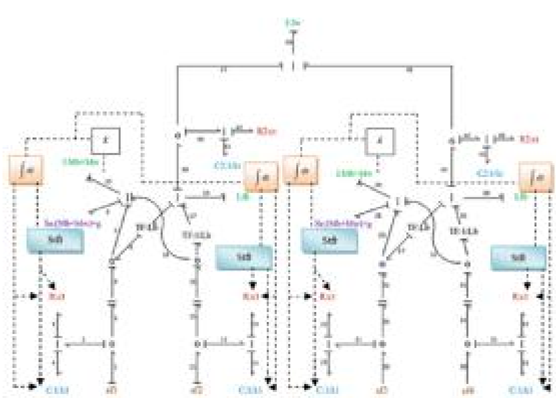


Fig22. Bond graph model for the freight wagon

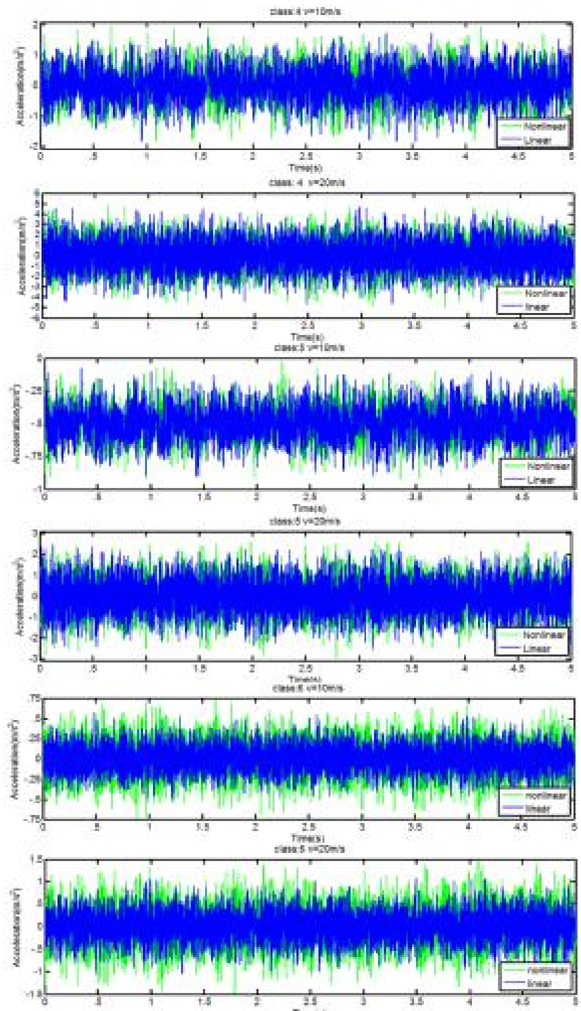


Fig23. Linear and nonlinear vertical acceleration of the wagon for different velocities and irregularities

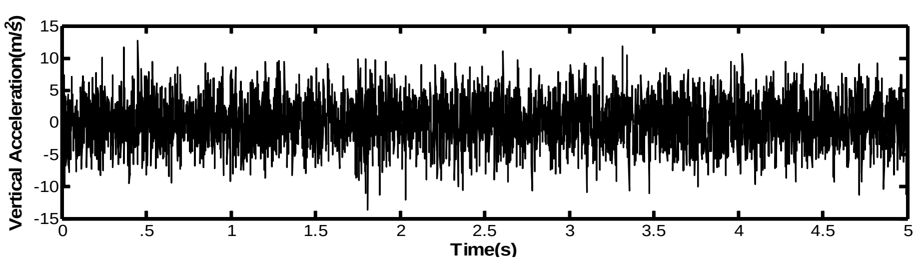
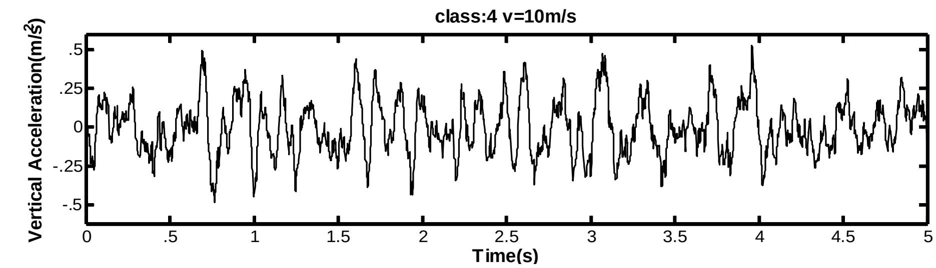
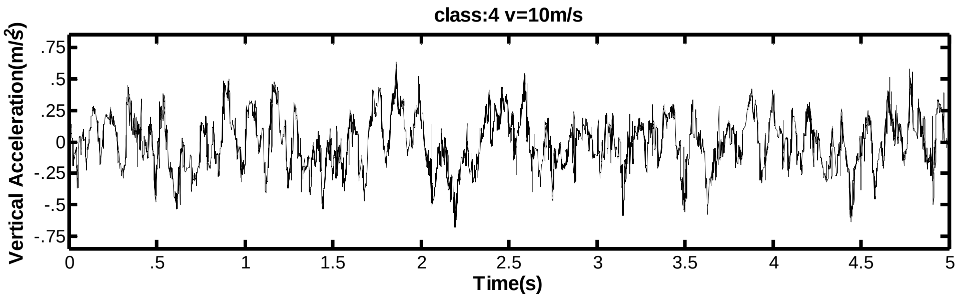


Fig24. Input acceleration signal; V = 10 m/s



Linear model



Nonlinear model

Fig25. Input acceleration signal; V = 10 m/s

Conclusions

Dynamic characteristics of a parabolic leaf spring of a freight wagon were investigated by using finite element method. Vertical acceleration of the railway vehicle was obtained and a parametric study was carried out to examine the effect of different parameters of the model on the response of the vehicle. Based on the obtained results,the main conclusions of this study are listed below:

Frequency has a negligible effect on the dynamic stiffness of the parabolic spring. However, increasing the value of frequency results in an increase for damping of the spring.

Effect of the amplitude on the characteristics of the parabolic spring is more significant than the frequency.

For different frequency and amplitudes, damping values are higher in the cases that the dynamic stiffness of the spring is noticeable.

Variations of the stiffness and damping depend on geometric properties of the parabolic spring.

Shape of the vertical acceleration response of the wagon is similar to shape of the input acceleration of the model generated by rail irregularities.

Due to the nature of the parabolic springs, there is no significant difference between the linear and nonlinear suspension systems.

Vertical acceleration is higher in the case of nonlinear model. This can be explained by this fact that the dynamic stiffness is larger than the corresponding average value utilized for linear model.

There is no significant increase in vertical acceleration of the wagon, for the vehicle velocities larger than 30 m/s.

References

[1]. I.HazratiAshtiani, Sh.Azadi, A new method to model the performance of leaf springs in multi-body software, Second international conference on the recent advances in railway engineering, ICRARE2009.

[2]. S.Tavakkoli,F.Aslani,D.S.Rohweder,"analytical prediction of leaf spring bushing loads using msc/nastran and mdi/adams", Ford Motor Company Dearborn, Michigan.

[3]. B.V.Lakshmi,I.Satyanarayana,"static and dynamic analysis on composite leaf spring in heavy vehicle",Advanced Engineering Research and Studies,Vol2,pp.80-84,2012.

[4]. M.Venkatesan, D.H. Devaraj," design and analysis of composite leaf spring in light vehicle", Modern Engineering Research,Vol2,pp.213-218,2012.

[5]. J.Tambaca,"Derivation of a model of leaf spring",Proceedings of the Conference on Applied Mathematics and Scientific Computing,pp.305-315,2005.

[6]. Q.Li,W.Li,"A contact finite element algorithm for the multileaf spring of vehicle suspension systems",Automobile Engineering,Vol218,pp.305-314,2004.

[7]. W.K.Moon,C.K.Song,"Hysteretic characteristics of leaf springs in commericial vehicle",Transaction of the Korean Society Of Automotive Engineers,Vol16,pp.99-105,2008.

[8]. K.Krishan, M.L.Aggarwal,"A finite element approach for analysis of a multi leaf spring using cae tools",International Science Congress Association,Vol1,pp.92-96,2012.

[9]. M.A.Osipenko,Y.I.Nyashin,R.N.Rudakov,"A contact problem in the theory of leaf spring bending", International Journal of Solids and Structures,Vol40,pp.3129–3136.2003.

[10]. M.Darvizeh, A.Darvizeh, Reza Ansari, Ali Alijani, 1-D and 2-D analysis of large displacements of a bar using continuum mechanics theorem, Journal of Modarres Mechanical Engineering, 11 (4): 33-40, 2011.

[11]. KeyvanAsadi, Hamid Ahmadian, Hassan Jalai, Analysis of energy dissipation of a minor slip in joints, Journal of Modarres Mechanical Engineering, 11(4) 53-63, 2011.

[12]. W.K.Moon,C.K.Song,"Hysteretic characteristics of leaf springs incommericial vehicle",Transaction Of The Korean Society Of Automotive Engineers,Vol16,pp.99-105,2008.

[13]. W.K.Moon,C.K.Soon," Hysteretic Characteristics of Leaf Springs in Commercial Vehicles",Transactions of the Korean Society of Automotive Engineers,Vol16,pp.99-105,2008.

[14]. P.S.Fancher,R.D.Ervin,C.C.MacAdam,C.B.Winkler(1982)"Measurement And Representation Of The Mechanical Properties Of Truck Leaf Springs,National Highway Traffic Safety Administration".

[15]. M.Berg,"A Non-Linear Rubber Spring Model For Rail Vehicle Dynamics Analysis",Vehicle System Dynamics,Vol30,pp.197-212,1998.

[16]. M.A.Rahman,M.T.Siddiqui,M.A.Kowser,"Design And Non-LinearAnalysis Of A Parabolic Leaf Spring",Mechanical Engineering,Vol37,pp.47-51,2007.

[17]. I.D.Moon,H.S.Yoon,C.Y.Oh,"A Flexible multi-body dynamic model for analyzing the hysteretic characteristics and the dynamic stress of ataper leaf spring",Mechanical Science and Technology,Vol20,pp.1638-1645,2006.

[18]. N.Banerjee,A.K.Saha,R.Karmakar,R.Bhattacharyya,"Bond graph modeling of a railway truck on curved track",Vol 17,pp.22-34,2009.