



## An optimal consensus to guarantee the stability and crash avoidance of large-scale traffic flow in presence of time delay

Hossein Chehardoli<sup>1\*</sup>, Ali Ghasemi<sup>2</sup> and Mohammad Daneshian<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering, Ayatollah Boroujerdi University, Boroujerd, Iran.

<sup>2</sup>Department of Mechanical Engineering, Faculty of Engineering, North Tehran Branch, Islamic Azad University.

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### ABSTRACT

A new safe optimal consensus procedure is presented to guarantee the asymptotic and string stability as well as crash avoidance of large-scale non-identical traffic flow. Since time delay is an inherent characteristic of physical actuators and sensors, measurement delay and lags are involved in the upper-level control structure. A third-order linear model is employed to define the 1-D motion of each automated vehicle (AV) and the constant time headway plan is employed to regulate the inter-AV distance. It is assumed that the network structure is decentralized look ahead (DLA) and each AV has access to relative position and velocity regarding with the front AV. A linear control law is introduced for each AV and by performing the stability analysis in frequency domain, the necessary conditions guaranteeing string stability and crash avoidance for large-scale traffic flow are derived. Afterwards, to calculate the optimal control parameters guaranteeing the best performance, an objective function combining all mentioned conditions as well as maximum overshoot, settling time and stability margin is introduced. The genetic algorithm (GA) technique is employed to optimize the presented objective function and obtain the optimal control parameters. Various numerical results are proposed to demonstrate the efficiency of this method.

## 1. Introduction

From the past few years, the traffic jam has been recognized as a crucial environmental, social and economic problem by many governments. The traffic jam has numerous undesirable occurrences such as increasing air pollution, traveling time, fuel consumption and decreasing safety, highway capacity, etc. [1, 2]. The intelligent transportation systems (ITS) are useful and applicable solutions for the problems arisen by traffic jam [3-5]. The idea of multiple connected automated vehicles (MCAV) is a useful tool to achieve the idea of ITS [6-8]. The main purpose of connecting the automated vehicles (AVs) is organizing motion of the traffic flow with an identical velocity and as small as possible inter-AV distances [9-11].

In general, three plans are used to regulate the inter-AV distance in convoys of AVs. 1) Constant distance plan (CDP): the inter-AV distance is controlled to be

distance plan (CDP): the inter-AV distance is controlled to be always constant [6, 10, 12]. 2) Constant time headway plan (CTHP): the inter-AV distance is a function of leader AV velocity [3, 13] and 3) mixed distance plan (MDP): which is a combination of CDP and CTHP [14, 15]. The network structure of MCAV can be centralized [16] or decentralized [17]. If the leader AV be in communication with all following AVs, network is centralized and otherwise is decentralized [18].

Two major stability analyses are investigated in MCAV. A convoy of AVs is asymptotic stable if the distance error between consequent AVs tends to zero asymptotically [19, 20]. Moreover, a convoy of AVs is string stable if the amplitude of distance error will not increase along the convoy by applying an external disturbance on leader AV [21, 22]. The assurance of string stability completely depends on network communication structure. Decentralized look ahead (DLA) networks with CDP cannot achieve the string stability [20].

\*Corresponding Author

Email Address: [h.chehardoli@abru.ac.ir](mailto:h.chehardoli@abru.ac.ir)  
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The huge amount of relevant literature on MCAV can be classified to different categories. 1) Communication structure: Decentralized look ahead [23], centralized look ahead [10], decentralized bi-directional [24], centralized bi-directional [25], decentralized multi ahead AVs following [18], centralized multi ahead AVs following [26] and non-uniform structure [12, 27]. 2) Linear control scheme including: scalability [19], crash avoidance [28], time delay [12, 29], model predictive control [23], partial differential equation approximation [30]. 3) Nonlinear control: adaptive [3] and robust [31] control. 4) Identical [32] and non-identical [13] MCAV. 5) String stability analysis [33, 34] and 6) optimal performance [35].

Beside the previous works on MCAV, a comprehensive method guaranteeing four purposes asymptotic stability, string stability, crash avoidance and optimal control parameter scheduling has not been offered. In these works, at most two purposes are satisfied simultaneously for example asymptotic and string stability [7, 10], asymptotic stability and crash avoidance [36] and asymptotic stability and optimal control [37]. Motivated by previous researches, in this paper a comprehensive optimal consensus procedure is presented to satisfy asymptotic stability, string stability and crash avoidance and optimizing the control parameters of large-scale non-identical traffic flows. Since time delay is an intrinsic feature of physical actuators and sensors, measurement delay and lags are considered in consensus procedure and analyzing the stability. A linear control procedure employing relative position and velocity regarding with the front AV is introduced for each following AV. The necessary constrains on control parameters guaranteeing asymptotic stability, string stability and crash avoidance are obtained by deriving the closed-loop dynamics of each following AV. To calculate the optimal values of control parameters, a new objective function including important features stability index, maximum overshoot, string stability, crash avoidance conditions and settling time is introduced. Afterwards, the genetic algorithm (GA) technique is employed to optimize the presented objective function. It will be shown that under this

optimal control law, the asymptotic stability, string stability and crash avoidance of large-scale non-identical traffic flow under the measurement delay and lags are guaranteed. In summary, the most important novelties of the current study are enumerated as follows. 1) Presenting a comprehensive consensus procedure assuring simultaneously the crash avoidance, asymptotic stability and string stability of large-scale non-identical traffic flow in presence of measurement delay and actuator lag and 2) presenting an optimal consensus procedure to optimize the control parameters and consequently, the control effort.

The remain of the current study is structured as follows. In part 2, the upper-level dynamics of AVs is introduced. In part 3, the asymptotic and string stability are discussed. In part 4, crash avoidance analysis is performed. In part 5, the optimal control parameters assuring asymptotic and string stability and crash avoidance are calculated by using genetic algorithm. In part 6, numerical results are presented to illustrate the efficiency of the presented algorithm. Lastly, this study is concluded in part 7.

## 2. Upper-level dynamics of AVs

A non-identical traffic flow can be modeled as cooperative non-identical convoys according to Fig. 1. Each convoy consists of a leader AV and some following AVs. In a convoy of AVs,  $x_i, v_i$  and  $a_i$  are position, velocity and acceleration of  $i$ -th AV, respectively. Moreover,  $x_0, v_0$  and  $a_0$  denote the position, velocity and acceleration of leader AV, respectively.

The upper-level dynamics of the  $i$ -th AV is presented as follows [6, 7, 12, 20]:

$$\tau_i \dot{a}_i + a_i = u_i \quad (1)$$

where  $u_i$  and  $\tau_i$  are the upper-level controller and engine time constant of the  $i$ -th AV, respectively. The control architecture of an AV consists of two levels [20].

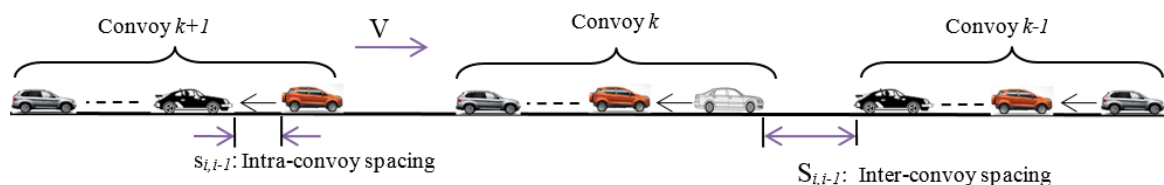


Figure 1: Large-scale non-identical traffic flow.

A lower-level controller compensates the nonlinear 1-D dynamics and an upper-level controller determines the desired acceleration of AV. In this paper, only the upper-level control is designed and we assume that the lower-level control has been already designed.

### 3. Analyzing of asymptotic and string stability

#### 3.1. Analyzing of asymptotic stability

The desired distance between two consecutive AVs is considered as  $S_{i-1,i} = h_i v_i + S_{\min} + l_{i-1}$  where  $h_i, S_{\min}$  and  $l_{i-1}$  are constant time headway of the  $i$ -th AV, minimum safe distance and length of the front AV. The distance error between consecutive AVs is defined as:

$$\delta_i = x_{i-1} - x_i - l_{i-1} - h_i v_i - S_{\min} \quad (2)$$

According to DLA network structure, the following control law is defined for the  $i$ -th AV

$$u_i(t) = k_1 [x_{i-1}(t-d) - x_i(t-d) - l_{i-1} - h_i v_i - S_{\min}] + k_2 [v_{i-1}(t-d) - v_i(t-d) - h_i a_i] \quad (3)$$

where  $k_1$  and  $k_2$  are control parameters and  $d$  is the communication time delay. Lag is an inherent feature of mechanical actuators. By considering the engine's lag ( $\Pi_i$ ), the control law (3) will be in the following form:

$$u_i(t - \Pi_i) = k_1 [x_{i-1}(t - \bar{\Pi}_i) - x_i(t - \bar{\Pi}_i) - l_{i-1} - h_i v_i(t - \Pi_i) - S_{\min}] + k_2 [v_{i-1}(t - \bar{\Pi}_i) - v_i(t - \bar{\Pi}_i) - h_i a_i(t - \Pi_i)] \quad (4)$$

where  $\bar{\Pi}_i = \Pi_i + d$ . By defining the desired position of the  $i$ -th AV as  $x_i^d = x_0 - \sum_{j=1}^i (S_{j-1,j} + l_{j-1})$ , the distance error of the  $i$ -th AV will be as follows:

$$e_i = x_i - x_i^d \Rightarrow \dot{e}_i = \dot{x}_i - \dot{x}_i^d \Rightarrow \ddot{e}_i = \ddot{x}_i \quad (5)$$

By employing (1), (4) and (5) and using  $x_{i-1}^d - x_i^d = h v_0 - l_{i-1}$ , the closed-loop dynamics of the  $i$ -th AV is derived as (6).

$$\tau_i \ddot{e}_i + \dot{e}_i = k_1 [e_{i-1}(t - \bar{\Pi}_i) - e_i(t - \bar{\Pi}_i) - h_i \dot{e}_i(t - \Pi_i)] + k_2 [\dot{e}_{i-1}(t - \bar{\Pi}_i) - \dot{e}_i(t - \bar{\Pi}_i) - h_i \ddot{e}_i(t - \Pi_i)] \quad (6)$$

Taking the Laplace transform of (6) with zero initial conditions yields

$$[\tau_i s^3 + s^2 + (k_1 + k_2 s) e^{-\bar{\Pi}_i s} + (k_1 + k_2 s) h_i s e^{-\Pi_i s}] E_i = (k_1 + k_2 s) e^{-\bar{\Pi}_i s} E_{i-1} \quad (7)$$

Therefore, we will have

$$\frac{E_i}{E_{i-1}} = \frac{(k_1 + k_2 s) e^{-\bar{\Pi}_i s}}{\tau_i s^3 + s^2 + (k_1 + k_2 s) e^{-\bar{\Pi}_i s} + (k_1 + k_2 s) h_i s e^{-\bar{\Pi}_i s}} = \frac{P_i(s)}{R_i(s)} = Q_i(s) \quad (8)$$

We can conclude that a large-scale traffic flow with DLA communication structure is asymptotic stable if and only if the transfer function  $Q_i(s)$  be asymptotically stable or  $R_i(s)$  be Hurwitz.

#### 3.2. Analyzing of string stability

The transfer function  $Q_i(s)$  refers to the distance error propagation of two subsequent AVs  $i-1$  and  $i$ . It can be proved that under the subsequent condition, the string stability of a traffic flow is guaranteed [3, 4, 6, 12, 20].

$$|Q_i(j\omega)| = \left| \frac{E_i(j\omega)}{E_{i-1}(j\omega)} \right| < 1, \quad \forall \omega > 0 \quad (9)$$

**Theorem 1.** A large-scale non-identical traffic flow with control law (3) is string stable under the following condition.

$$k_1 \geq \frac{2}{h_i^2} \quad (10)$$

**Proof.** According to (9), if  $|R_i(j\omega)|^2 - |P_i(j\omega)|^2 > 0$ , the string stability is guaranteed. We can write

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$$\begin{aligned}
 |P_i(j\omega)|^2 &= k_1^2 + k_2^2 \omega^2 \\
 R_i(j\omega) &= (-\omega^2 + k_1 \cos \bar{\Pi}_i \omega + k_2 \omega \sin \bar{\Pi}_i \omega \\
 &+ h_i k_1 \omega \sin \Pi_i \omega - h_i k_2 \omega^2 \cos \Pi_i \omega) + \\
 &j(-\tau_i \omega^3 - k_1 \sin \bar{\Pi}_i \omega + k_2 \omega \cos \bar{\Pi}_i \omega \\
 &+ k_1 h_i \omega \cos \Pi_i \omega + h_i k_2 \omega^2 \sin \Pi_i \omega)
 \end{aligned} \quad (11)$$

By doing some algebraic calculations, we will have

$$\begin{aligned}
 |R_i(j\omega)|^2 - |P_i(j\omega)|^2 &= [\text{Im}^2(R_i(j\omega)) + \\
 \text{Re}^2(R_i(j\omega))] - (k_1^2 + k_2^2 \omega^2) &= \\
 \tau_i \omega^6 + (1 + h_i^2 k_2^2) \omega^4 + (k_2^2 + h_i^2 k_1^2 - k_2^2) \omega^2 & \\
 + 2\tau_i k_1 \omega^3 \sin \bar{\Pi}_i \omega - 2\tau_i k_2 \omega^4 \cos \bar{\Pi}_i \omega - & \\
 2\tau_i k_1 h_i \omega^4 \cos \Pi_i \omega - 2\tau_i k_2 h_i \omega^5 \sin \Pi_i \omega & \\
 - 2k_1 \omega^2 \cos \bar{\Pi}_i \omega - 2k_2 \omega^3 \sin \bar{\Pi}_i \omega - & \\
 2k_1 h_i \omega^3 \sin \Pi_i \omega + 2k_2 h_i \omega^4 \cos \Pi_i \omega &
 \end{aligned} \quad (12)$$

By employing the subsequent math expressions

$$\begin{aligned}
 \forall \psi > 0: \quad \sin \psi \leq \psi &\Rightarrow -\sin \psi \geq -\psi; \\
 \cos \psi \leq 1 &\Rightarrow -\cos \psi \geq -1
 \end{aligned} \quad (13)$$

Eq. (12) is shortened as (14):

$$\begin{aligned}
 |R_i(j\omega)|^2 - |P_i(j\omega)|^2 &= \\
 (\tau_i^2 - 2\tau_i h_i k_2 \Pi_i) \omega^6 + (1 + h_i^2 k_2^2 - 2h_i k_2 - & \\
 2k_1 h_i \Pi_i - 2k_2 \Pi_i - 2\tau_i k_2 - 2\tau_i k_1 \bar{\Pi}_i - & \\
 2\tau_i k_1 h_i) \omega^4 + k_1 (k_1 h^2 - 2) \omega^2 > 0 &
 \end{aligned} \quad (14)$$

In [38], it is proved that the most energy of distance errors is in the low frequency area. Therefore, this area is very prominent for string stability. Accordingly, if the coefficient of  $\omega^2$  be positive or equivalently (10) holds, we conclude that the string stability is guaranteed and the proof is complete.

### 4. Crash avoidance analysis

The asymptotic and the string stability conditions could not guarantee the crash avoidance of a MCAV during accelerating/decelerating motions of leader AV. The following theorem introduces the sufficient conditions guaranteeing crash avoidance.

**Theorem 2.** Consider the polynomial  $A_n(x) = b_n x^n + b_{n-1} x^{n-1} + \dots + b_0$ ,  $n \geq 2$  with positive coefficients  $b_n, b_{n-1}, \dots, b_0$ . Under the

following condition,  $A_n$  has only distinct real roots [39].

$$b_i^2 - 4b_{i-1} b_{i+1} > 0; \quad i = 1, 2, \dots, n-1 \quad (15)$$

Consider the following transfer function between the separation distance with the front AV and the velocity of subsequent vehicle AV.

$$\Xi_i(s) = \frac{\varepsilon_i(s)}{v_{i-1}(s)} \quad (16)$$

where  $\varepsilon_i(s)$  is the Laplace transform of  $\varepsilon_i(t) = x_{i-1} - x_i - l_{i-1} - S_{\min}$ . If the impulse response of  $\Xi_i(s)$  preserves its sign, the crash between consecutive AVs is avoided. The following theorem deals with the sufficient conditions guaranteeing crash avoidance of large-scale non-identical traffic flow.

**Theorem 3.** Under the following conditions, the crash avoidance of large-scale non-identical traffic flow is guaranteed.

$$\begin{aligned}
 (1 + k_2 h_i)^2 - 4\tau_i (k_2 + k_1 h_i) &> 0 \\
 (k_2 + k_1 h_i)^2 - 4k_1 (1 + k_2 h_i) &> 0
 \end{aligned} \quad (17)$$

**Proof.** By employing (1) and (3), the closed-loop dynamics of  $i$ -th AV without delay will be as (18):

$$\tau_i \dot{a}_i + a_i = k_1 (\dot{\varepsilon}_i(t) - h_i v_i) + k_2 (\dot{\varepsilon}_i(t) - h_i a_i) \quad (18)$$

By taking Laplace transform from (18), we will have

$$\begin{aligned}
 \varepsilon_i(s) &= \frac{s(1 + \tau_i) + h_i (k_1 + k_2 s)}{k_1 + k_2 s} v_i(s) \\
 \Rightarrow \frac{\varepsilon_i(s)}{v_i(s)} &= \frac{s(1 + \tau_i) + h_i (k_1 + k_2 s)}{k_1 + k_2 s}
 \end{aligned} \quad (19)$$

Eq. (18) can be revised as (20):

$$\begin{aligned}
 \tau_i \ddot{v}_i + (1 + k_2 h_i) \dot{v}_i + (k_2 + k_1 h_i) v_i + k_1 v_i & \\
 = k_1 v_{i-1} + k_2 \dot{v}_{i-1} &
 \end{aligned} \quad (20)$$

Taking Laplace transform of (20) yields

$$\frac{v_i(s)}{v_{i-1}(s)} = \frac{k_1 + k_2 s}{\tau_i s^3 + (1 + k_2 h_i) s^2 + (k_2 + k_1 h_i) s + k_1} \quad (21)$$

By employing (19) and (21), we obtain that

$$\Xi_i(s) = \frac{\varepsilon_i(s)}{v_{i-1}(s)} = \frac{\varepsilon_i(s)}{v_i(s)} \frac{v_i(s)}{v_{i-1}(s)} = \frac{s(1 + \tau_i s) + h_i(k_1 + k_2 s)}{s^2(1 + \tau_i s) + (1 + h_i s)(k_1 + k_2 s)} \quad (22)$$

Since all poles of  $\Xi_i(s)$  are real, we can write

$$\Xi_i(s) = \frac{n_1}{s + q_1} + \frac{n_2}{s + q_2} + \frac{n_3}{s + q_3} \quad (23)$$

where  $n_1 = \frac{k_1 - q_1 k_2}{\tau_i q_1 (q_2 - q_1)(q_3 - q_1)}$ ,

$n_2 = \frac{k_1 - q_2 k_2}{\tau_i q_2 (q_1 - q_2)(q_3 - q_2)}$ , and

$n_3 = \frac{k_1 - q_3 k_2}{\tau_i q_3 (q_1 - q_3)(q_2 - q_3)}$ .

We assume that  $q_1 > q_2 > q_3$ . Moreover, the impulse response of  $\Xi_i(s)$  is assumed in the following form

$$\xi(t) = n_1 e^{-q_1 t} + n_2 e^{-q_2 t} + n_3 e^{-q_3 t} \quad (24)$$

Since  $q_3$  is the minimum pole of the  $\Xi_i(s)$ ,  $n_3$  has the main weight in  $\xi(t)$ . By doing some mathematical manipulations, it is concluded that  $n_1 + n_2 + n_3 > 0$  and  $n_1 n_2 n_3 < 0$ . Since  $n_1 n_2 n_3 < 0$ , two possibilities can be considered for coefficients  $n_i$ . 1) All  $n_1, n_2$  and  $n_3$  have negative sign which is in conflict with  $n_1 + n_2 + n_3 > 0$ . 2) Two of  $n_1, n_2$  and  $n_3$  have positive signs. Based on the assumption  $q_1 > q_2 > q_3$ , we will have  $k_1 - q_3 k_2 > k_1 - q_2 k_2 > k_1 - q_1 k_2$  and the coefficients  $n_1, n_2, n_3$  have positive, negative and positive signs, respectively. So that,  $k_1 - q_3 k_2$  must be positive else, two of coefficients  $n_i$  are negative that is in conflict with  $n_1 n_2 n_3 < 0$ . Hence,  $n_3$  is always positive. In other words, if the control parameters are selected so as to conditions (17) are satisfied, the crash avoidance will be guaranteed.

### 5. Optimal control parameter scheduling using genetic algorithm technique

The control parameters  $k_1$  and  $k_2$  should satisfy the asymptotic stability, string stability and crash avoidance simultaneously. But if we can find optimal values for these parameters, the control effort will be optimized. To this aim, the genetic algorithm (GA) is employed to calculate optimal

values for  $k_1$  and  $k_2$ . The objective function is defined as a linear function of important features: stability index of the  $Q_i(s)$ ,  $Q_i(s)$ , conditions of string stability (10), conditions of crash avoidance (17), settling time and maximum overshoot of step response of  $Q_i(s)$ . Therefore, we define the following objective function

$$OF = w_1 Mp + w_2 St + w_3 Si + w_4 |f| + w_5 |g_1| + w_6 |g_2| \quad (25)$$

where  $Mp$  is the maximum overshoot,  $St$  is the settling time,  $Si$  is the stability index defined as  $Si = \frac{1}{Mrp(Q(s))}$  where  $Mrp(Q(s))$  is the maximum real part of poles of  $Q(s)$ ,  $f = k_1 h_i^2 - 2 - \alpha_f$ ,  $g_1 = (1 + k_2 h_i)^2 - 4 \tau_i (k_2 + k_1 h_i) - \alpha_{g_1}$  and  $g_2 = (k_2 + k_1 h_i)^2 - 4 k_1 (1 + k_2 h_i) - \alpha_{g_2}$  where  $\alpha_f, \alpha_{g_1}$  and  $\alpha_{g_2}$  are positive values should be designed. Also,  $w_i, i = 1, 2, \dots, 6$  are the positive real weights.

In order to minimize the objective function (25), the GA is employed with the following characteristics. Variables should be optimized are  $k_1$  and  $k_2$ , the maximum iteration to reach the optimal solution is 100 iterations, number of initial populations is considered as 50, the crossover percentage is 0.7 and the percentage of mutation is considered as 0.2.

### 6. Numerical studies

In this part, a traffic flow consisting of identical and non-identical convoys is considered. The measurement delay, minimum distance and constant time headway are considered as  $d = 0.01s, S_{min} = 5m$ , and  $h = 2s$ , respectively. For the identical convoy, the following values are considered:  $\Pi = 0.13s, \tau = 0.1s$  and  $l = 4m$ . Moreover, the constant weights  $w_i, i = 1, 2, \dots, 6$  are chosen as  $w_1 = 1.2, w_2 = 7.4, w_3 = 1.1, w_4 = 0.2, w_5 = 0.5, w_6 = 0.5$  in both identical and non-identical convoys. Since settling time and maximum overshoot are more important than other features, their coefficients are selected greater than other coefficients. To study the performance of asymptotic and string stability in presence of external disturbance, we assume that the leader AV motion is according to acceleration profile

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$$a_0(t) = \begin{cases} -2, & 40 \leq t \leq 50 \\ 1, & 120 \leq t \leq 130 \\ 0, & \text{otherwise} \end{cases}$$

with initial steady velocity  $40\text{m/s}$ .

### 6.1. Convoy of identical AVs

By using the GA technique, the optimal control parameters which minimize the objective function (25) are derived as  $k_1 = 1.42$  and  $k_2 = 0.43$ . Fig. 2 displays the behavior of objective function. According to this figure, the optimal value of objective function by passing 100 repetitions is 12.18. Moreover, the optimal values of maximum overshoot, settling time and stability margin are calculated as  $Mp = 0$ ,  $Ts = 1.302\text{s}$  and  $Mrp(Q(s)) = 1.745$ , respectively.

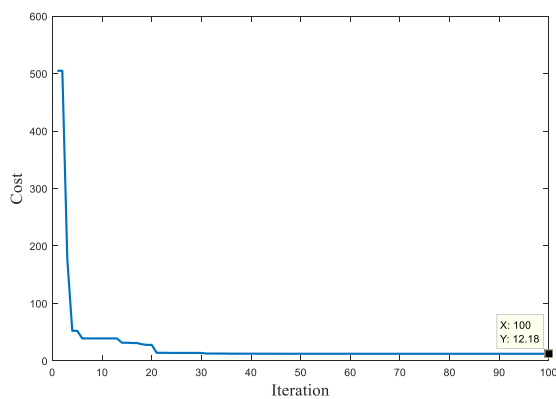


Figure 2: Variation of objective function.

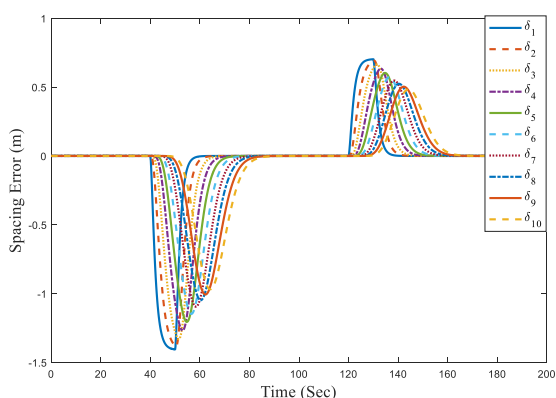


Figure 3: Distance error of identical convoy.

Fig. 3 shows the distance error between neighbor vehicles of convoy. According to this figure, since the distance error vanishes asymptotically, the convoy is internal (asymptotic) stable. On the other hand, the maximum of distance errors of following AVs decreases along the convoy during accelerating and decelerating motions. So that, the convoy of identical AVs is string stable. The velocity of AVs are illustrated in Fig. 4. According to this figure, since the convoy is internal stable, all AVs track the leader AV velocity. To study the crash avoidance performance of identical convoy, a hard and sudden braking maneuver is assumed for the leader AV. Fig. 5 depicts the velocity of convoy during a drastic braking. As this figure indicates, AV's velocities behave monotonically during braking maneuver. Fig. 6 shows the inter-AV distance of identical convoy during a drastic braking. According to this figure, the inter-AV distance after emergency stop is always positive therefore, the safety is assured and crash avoidance is achieved.

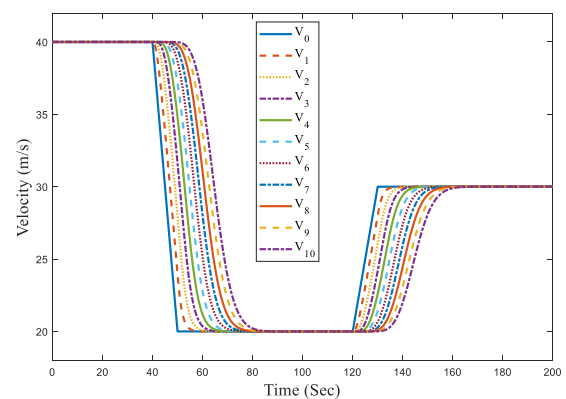


Figure 4: Velocity of identical convoy.

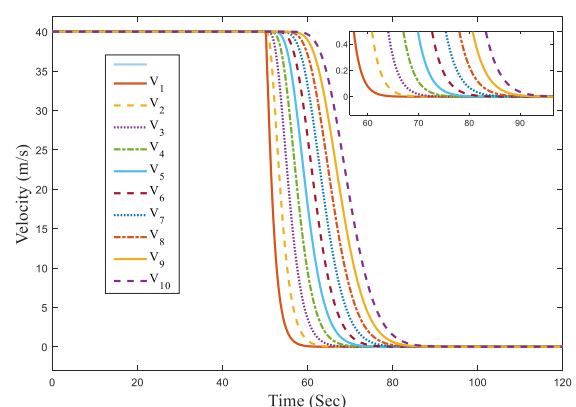
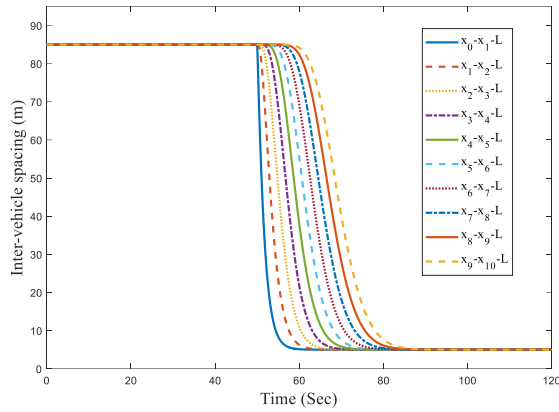


Figure 5: Velocity of identical convoy during emergency braking.



**Figure 6:** Inter-AV distance of identical convoy during a drastic braking.

Table 1 shows a comparison of important features for optimal (case 1) and non-optimal (case 2) control parameters. According to these results, the case 2 presents a weak performance. The maximum of overshoot in case 1 is zero, while in case 2 is %46.2 which dramatically introduces an undesirable response. The settling time of case 1 is smaller than case 2. Since the stability index of case 1 is larger than case 2, it shows a better time response. Finally,

**Table 1:** Performance of close-loop dynamics with optimal and non-optimal control parameters

Important features	$k_1$	$k_2$	$Mp$	$St(s)$	$Si$	$\max \ \delta_1(t)\ , (m)$
Optimal control parameters	1.42	0.43	0	2.62	0.66	1.38
Non-optimal control parameters	2.18	1.17	46.2	3.64	0.95	2.21

**Table 2:** Parameters of non-identical AV convoy

AV number	$\Pi_i(s)$	$\tau_i(s)$	$l(m)$	$k_1$	$k_2$
1	0.1	0.1	4	1.45	0.47
2	0.12	0.08	4.1	1.48	0.39
3	0.15	0.13	3.9	1.4	0.43
4	0.08	0.15	4.2	1.38	0.4
5	0.1	0.18	4	1.37	0.39
6	0.05	0.07	3.7	1.33	0.35
7	0.12	0.2	4.1	1.38	0.39
8	0.13	0.1	3.5	1.42	0.43
9	0.09	0.14	4.4	1.44	0.41
10	0.14	0.18	4.3	1.36	0.35

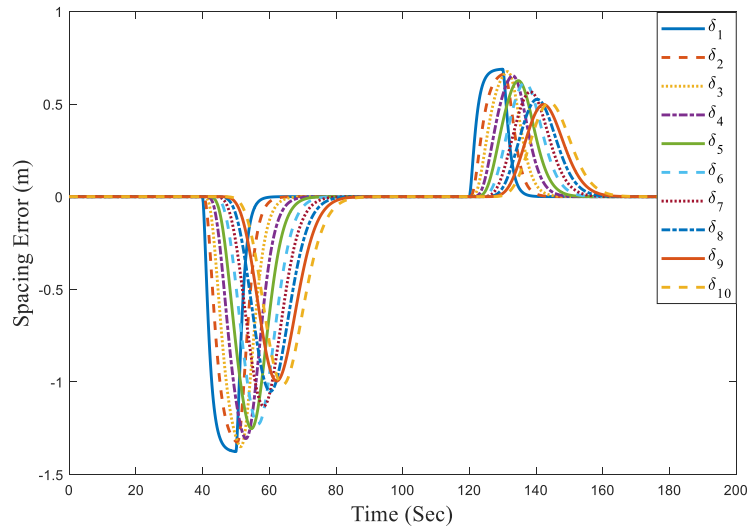
the maximum value of spacing error in case 1 is larger than case 2. Therefore, the length of platoon in case 2 is larger than case 1 which means that the traffic capacity is smaller in case 2.

### 6.2. Non-identical convoy of AVs

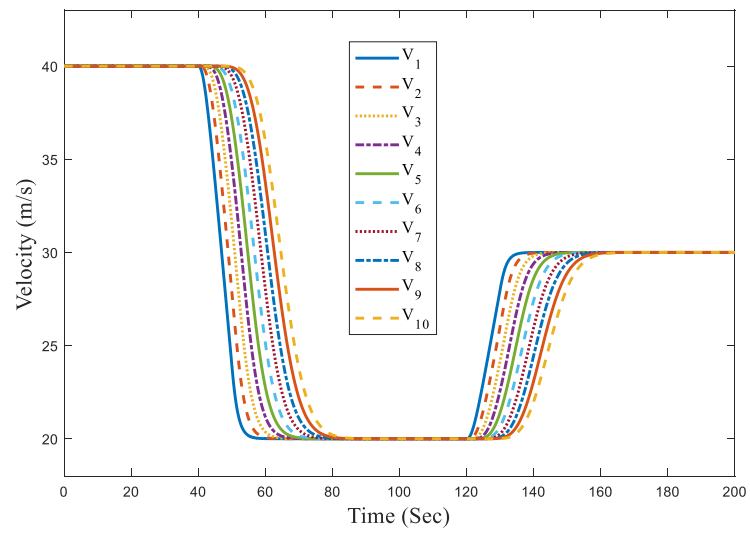
The system parameters and optimal control parameters calculated by genetic algorithm are presented in Table 2. These values are employed in simulation results.

The distance error and convoy velocity are depicted in Figs. 7 and 8, respectively. According to these figures, since the distance error vanishes asymptotically, the vehicle convoy is internal stable. Moreover, the amplitude of distance error has a decrement trend implying that the convoy is string stable. Figs. 9 and 10 depict the behavior of non-identical convoy during emergency braking. As these figures show, the distance error is always positive and consequently, the non-identical convoy is safe and crash avoidance is assured.

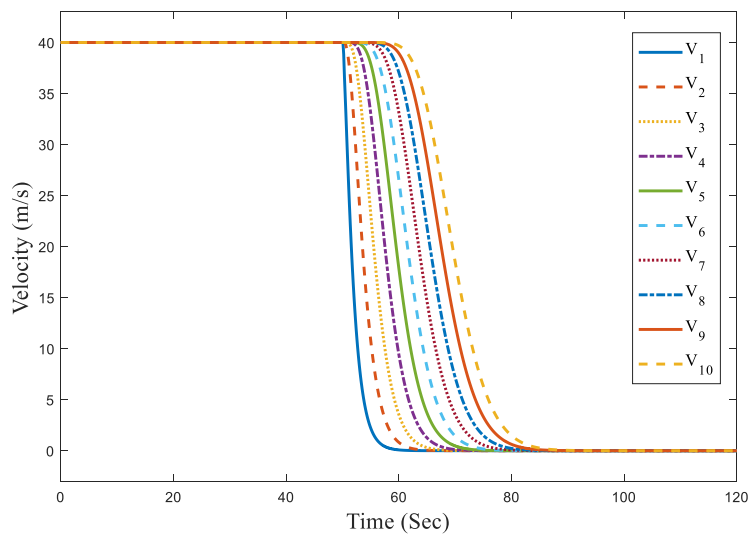
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**Figure 7:** Distance error of non-identical convoy.

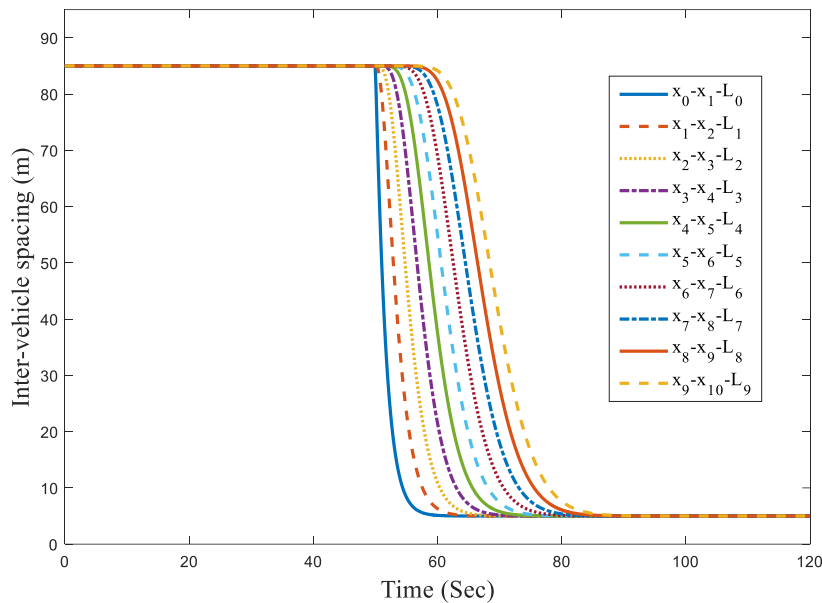


**Figure 8:** Velocity of non-identical convoy.



**Figure 9:** Velocity of non-identical convoy during emergency braking.





**Figure 10:** Inter-AV distance of non-identical convoy in emergency braking.

## 7. Conclusion

In this paper, a decentralized safe optimal consensus procedure was presented to achieve an optimal performance of large-scale non-identical traffic flow. Measurement delay and lag were investigated in control structure and stability analysis. Constant time headway plan was used to regulate the inter-AV distance and a linear consensus procedure by using the relative position and velocity regarding with the front AV was presented for each following AV. Necessary conditions on control parameters assuring asymptotic stability, string stability and crash avoidance were derived. To achieve an optimal control performance, the genetic algorithm technique was used to calculate the optimal values of control gains. We proved that the proposed method is a comprehensive method guaranteeing asymptotic stability, crash avoidance, string stability and optimal traffic flow behavior, simultaneously. Various numerical studies were presented to demonstrate the efficiency of the offered method.

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