Research Paper

Techniques to carry weight loads and resist against bending in conical shells, cases in Kashan

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Abstract

Little has been written about the structural behavior of conical shells, while conical shells' construction techniques due to the high rise and small span has always required special attention throughout the history of Islamic architecture. This paper aims to examine how conical shells bear weight loads and resist against bending through a qualitative analysis. After proving that 'rise/span' ratio strongly affects weight loads and bending in external shells, the main question of the paper is "what construction techniques architects adopted to carry weight loads, and how these techniques differed in domes with various 'rise/span' ratios". To find out about paper's main question, a qualitative approach for structural analysis has been adopted. First, architectural maps and dimensions of six cases were documented, and then, according to the documentation, some ratios and parameters have been defined to find their probable correlation with 'rise/span' ratio. Results show that to restrain weight loads in external shell, design of stiffeners and thickness of the conical shell play a crucial role to keep structure balanced. According to the results, 'rise/span' ratio has a positive correlation with 'thickness/span', 'height of stiffeners/span', 'stiffeners' sectional area/span', 'gradual decrease of thickness' and "number of stiffeners/ number of flat planes". In contrast, 'Rise/span' ratio does not have any meaningful relationship with 'stiffeners' end point' and 'struts' end point', and has a negative correlation with "the number of flat plains.

1. INTRODUCTION

A great deal is known about the construction of domes, but much is still uncertain about the interplay between structural and architectural concepts in the erection of conical shells. Conical shells are mainly exposed to weight loads and bending, especially due to the high rise and small span. The structural analysis is the process by which the authors determine how conical shells restrain weight loads and resist against bending they are subject to. The aim of this paper is to identify and analyze all crucial parameters in stability notion of external shells through adopting a qualitative approach.

1.1. Hints on Conical Dome's History and Structure

Conical shells were considered as common appearances of the Seljuk architecture [1] with pinnacle in a tapered or cane form without any curved line [2]. Shells can be round, flanged or polygonal [3]; they are also called conical faceted or conical circular shells. During the fourteenth and fifteenth century, the smooth conical roof form, made of a number of flat planes meeting at the apex, is replaced by a pyramid form [4].

The idea of the double dome for conical shells was introduced when architects decided to show both the external appearance of the dome and the aesthetics of the interior of the domed space, resulting in high external shells with shallower interior domes showing ornaments [4]. Moreover, conical or polygonal shells protect the inner dome from climatic changes [5], rain and snow, and add significantly to the crowning glory of the tomb. Regarding the studies about typological and morphological features of double shell domes in general and conical domes in particular, studies by [6-10] are worth noting. Morphologically, discontinuous double-shell domes consist of the external shell (the most importance component and the most visible part of dome), high drum, internal shell, and radial stiffeners within the wooden struts [8]. Yet, the structural aspects of conical domes in general and conical external shells in particular, have not been fully explored.

It is an undeniable fact that the structural role of domes in Islamic architecture has been noticeable over

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centuries [11]. Domes as one of the earliest types of buildings are one of the few structural forms which have had an evolutionary process and a continuous history to the present time [12]. When considering conical domes a considerable number of aesthetic, historical, symbolic, construction and technical issues should be considered [13]. Among those aspects, construction and technical matters demonstrate how the building is structurally stable [14]. Different strategies have been adopted for

vaults and domes to restrain loads and remain structurally competent.

For instance, in order to construct semicircular domes, the most important construction matter is increasing the amount of mortar as dome circularly goes higher, guaranteeing the stability of these domes. Indeed, the mortar binds the blocks and transmits the tension forces, so these domes span large areas with no intermediate supports (Fig. 1 & 2).

Fig. 1 Left, constructing semicircular domes by increasing the amount of mortar as dome circularly goes higher, Interior View, Meibod Ice House, Yazd, Photo: by Authors, 2007

Fig. 2 Right, semicircular dome of Hai Rajab Ali Mosque in Ashkzar, Yazd, ninth century AD, Photo: by authors, 2005

To construct vaults which were the most important element for transmitting the shell's loads to columns and foundation, the thickness of vaults was gradually reduced at 22.5, 45 and 67.5 angles for decreasing extra weights (Fig. 3).

Another strategy has been adopted to bear the loads in *Ourchin* or Pineapple domes, which is through the change in the form of external shell and plan (Fig 4). Actually, "the arrangement of the above floor on the beneath floor is in a way that the outer facet of the beneath floor is along the inner facet of the above floor" (Fig. 4) [2]. Due to step-like form of the external shell and geometrical relationships of the floors, the whole structure remains balanced. Accordingly, stiffeners or any other external support is not required to transmit the loads to the transmitting load system.
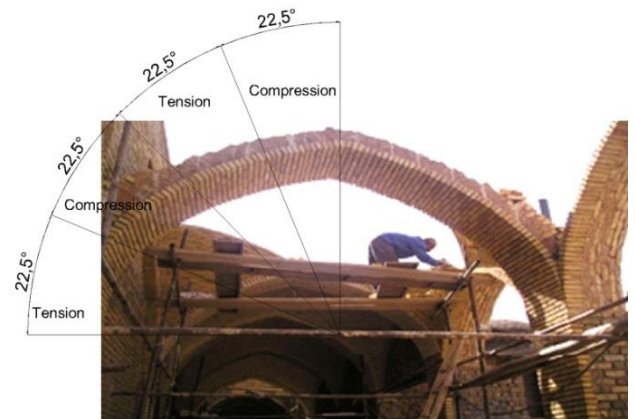

Fig. 3 Reducing the thickness of a vault gradually in Kashan's bazaar Photo: by authors, 2012
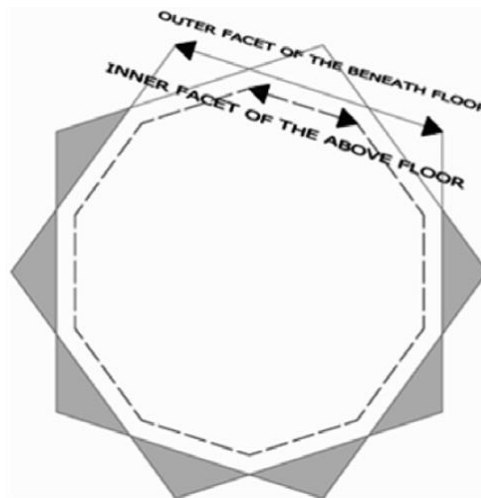

Fig. 4 Left, transmitting loads in Ourchin domes through the change in the form of external shell and in the form of plan [2]

Regarding construction techniques of conical shells, the predominating load usually consists only of the weight of the external shell as they are roofing, not supporting, elements [14]. To bear weight loads in external shells, internal stiffeners were introduced in the 14th century when both structural and architectural sciences progressed [10]. They were used to join the external shell to the lower components [8], to prevent the collapse of the external shell [9], and to meet the structural requirements and balance of the whole system, especially against earthquakes [10]. Their arrangement and sizes are strongly influenced by both vernacular architecture agreements and the scale of span [10].

2.1. Definitions and Terms (Rise/Span Ratio)

Generally, domes are all characterized by a thrust whose intensity and angle may disturb the stability of the whole. The dome's thrust is composed of its weight and the horizontal thrust of the basic arch section. In conical shells, the horizontal thrust is minimized due to the higher rises compared to other types of domes. According to Fig. 5, 'rise' should be considered the distance from the apex of the shell to spring, and it varies as the height and span of the dome vary. Authors argue that 'rise' should not be mistaken for "height" of the conical shells for which two reasons have been discussed. First, as can be seen in Fig. 5, while the "height" of these two conical profiles is exactly the same, dimension of the "rises" are different. Second, it is an undeniable fact that stiffeners are designed

to keep the "rise" of external shell balanced and not the "height" of it, so where the stiffeners are constructed and placed, can help us verify where the "rise" really is. Indeed, stiffeners lean back against the "rise" of the dome to control bending and carry weight loads. In a word, "rise" of the conical shells is totally different from "height" of them, and these terms should be carefully applied in scientific papers.

Fig. 5 shows line of thrust (Inclined component) in conical shells with two different 'rise/span' ratios. As can be inferred from Fig 5, "rise" cannot solely indicate the difficulty of restraining and transferring weight loads since by increasing spans, the structure becomes more balanced. Indeed, although 'rise' in the right profile is higher than the one in the left profile, larger span in the right profile makes it easier to spread loads (Fig 5). Generally, as the "rise" gets higher and the "span" gets smaller (as 'rise/span' ratio becomes larger), it becomes more difficult to bear weight loads and resist against bending. Since we try to find out the way weight forces are restrained, transferred and balanced in conical shells, the 'rise/span' ratio can be applied to show the difficulty in restraining weight loads and bending in external shell. It is expected that when a number of conical shells have different 'rise/span' ratios, some other designing parameters and details are different in these shells as well. The aim of this paper is to first find all those parameters that vary as 'rise/span' ratio or "difficulty in restraining weight loads and bending" vary and to find their probable correlation with this ratio.
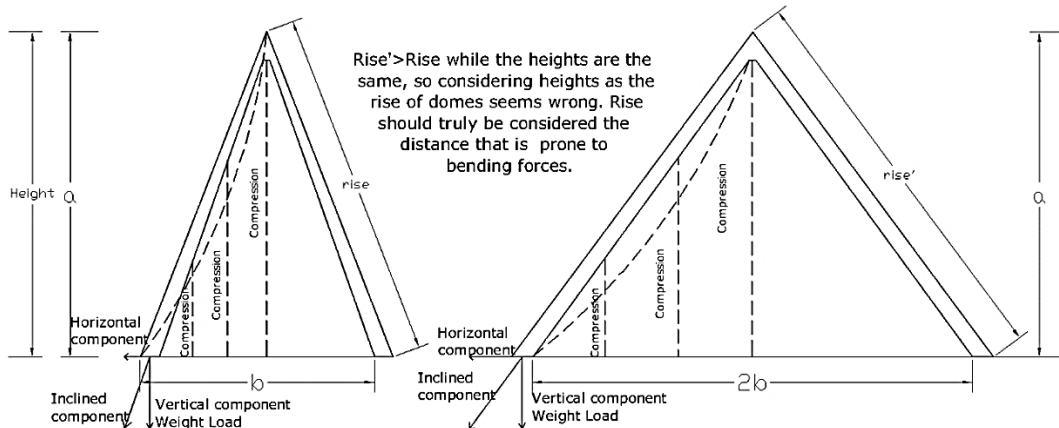

Fig. 5 Two conical profiles with different proportions, showing "rise", "height", "span", and "thrusts" (drawn by authors, 2015)

2. METHODOLOGY

A qualitative approach has been adopted to analyze the shells structurally which has been categorized into two main stages. In the first place, case studies have been documented and their architectural maps and photos have been presented. Moreover, dimensions of shells' components have been presented in a Table.

In the next stage and after extracting designing dimensions, some ratios and parameters have been introduced and calculated to be compared with "rise/span" ratio.

2.1. Studying Case Studies

Taking into account the possibility of access to the space between external shell and internal dome, a sample of six dominant faceted conical domes was selected in Kashan, Iran, so the number of cases was limited. Architectural maps of shells including plans, sections (Figs. 6-11) have been documented by a team of students under the supervision of authors. Figs. 6-11 also illustrate the photos of external shells and stiffeners which are taken by authors. Table 1 presents dimensions of conical shells and their components based on the rise of external shell, from the highest rise to the lowest one.
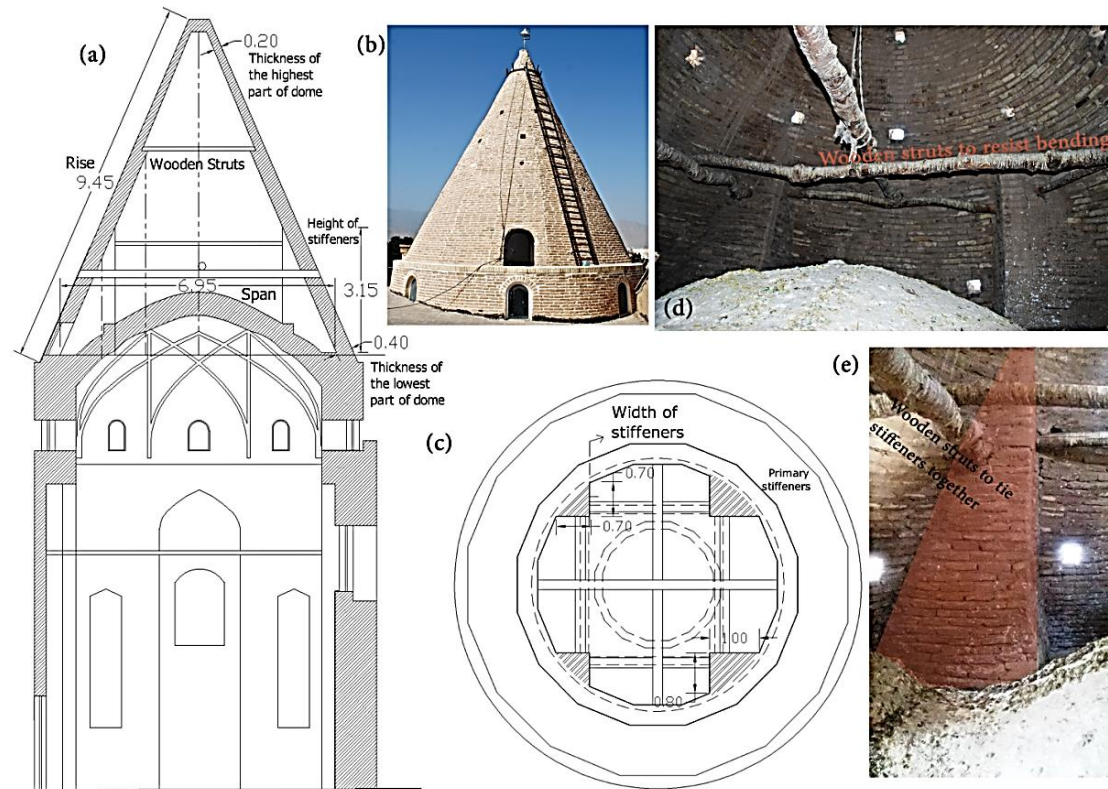
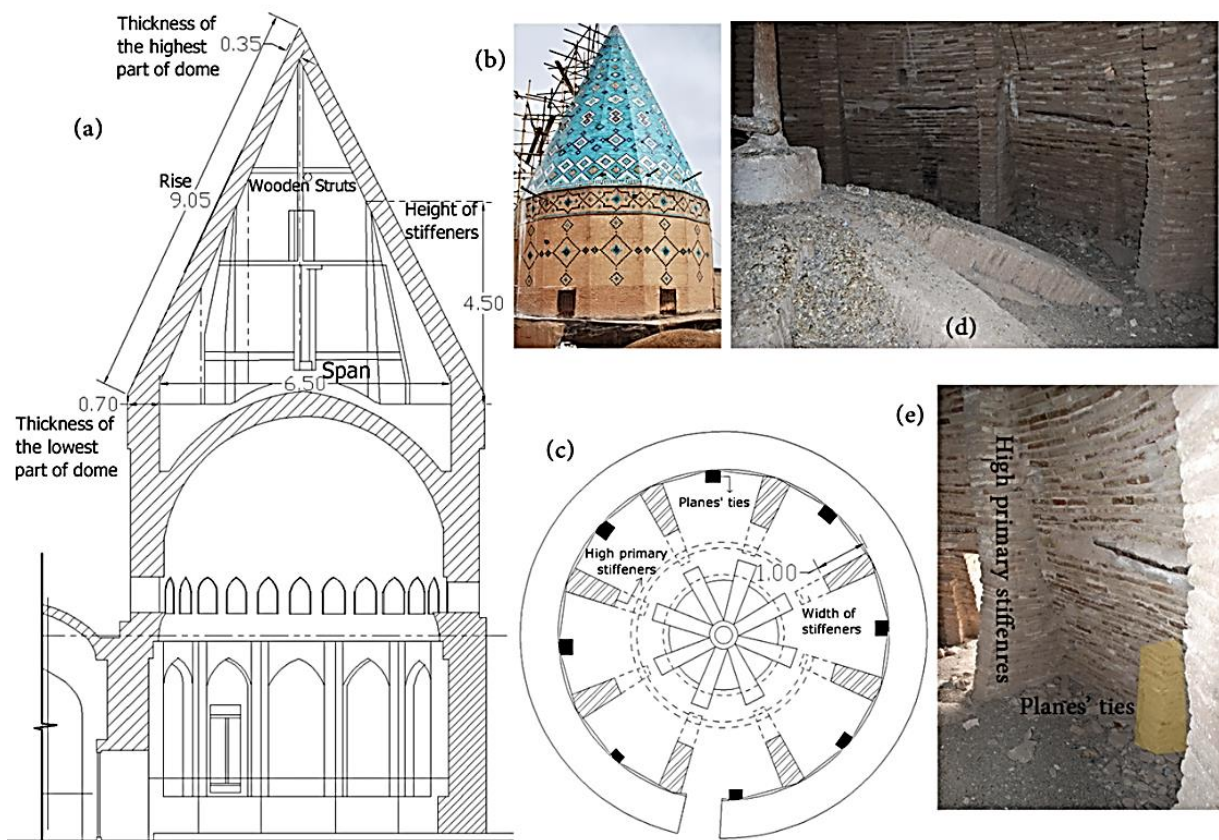**Fig. 6** Cheheltan, Safavi (1501-1723 AD). a) section of the shell b) exterior view c) plan, d & e) stiffeners

Fig. 7 Mirneshane, Safavi (1501-1723 AD). a) section of the shell b) exterior view c) plan, d & e) stiffeners
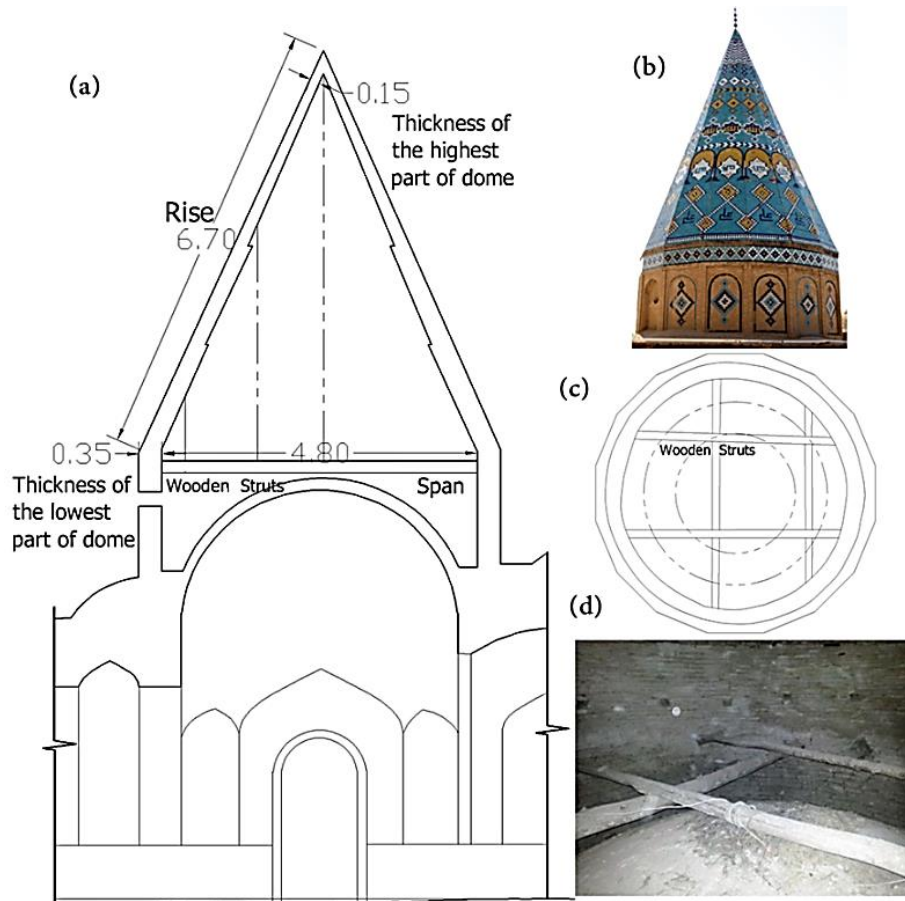
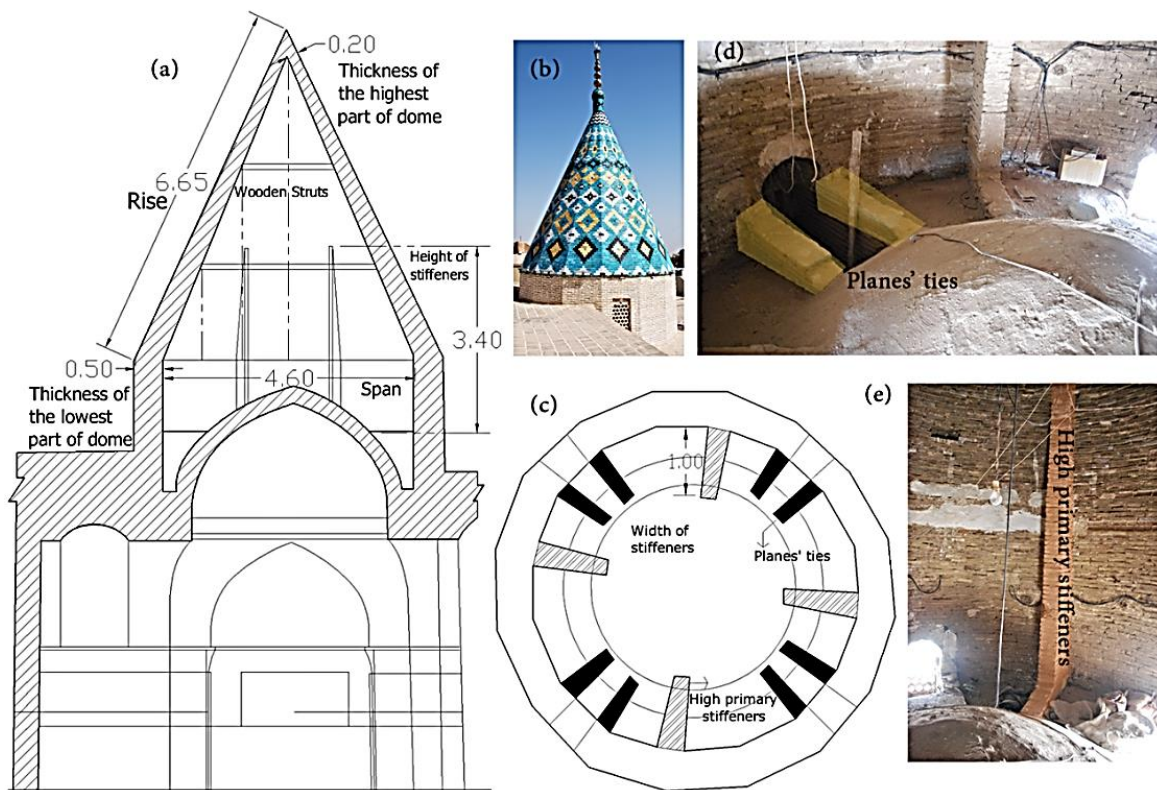**Fig. 8** Shahzade Ebrahim, Qajar (1795-1927 AD). a) section of the shell b) exterior view c) plan, d) stiffeners

Fig. 9 Soltan Amir Ahmad, Safavi (1501-1723 AD). a) section of the shell b) exterior view c) plan, d & e) stiffeners
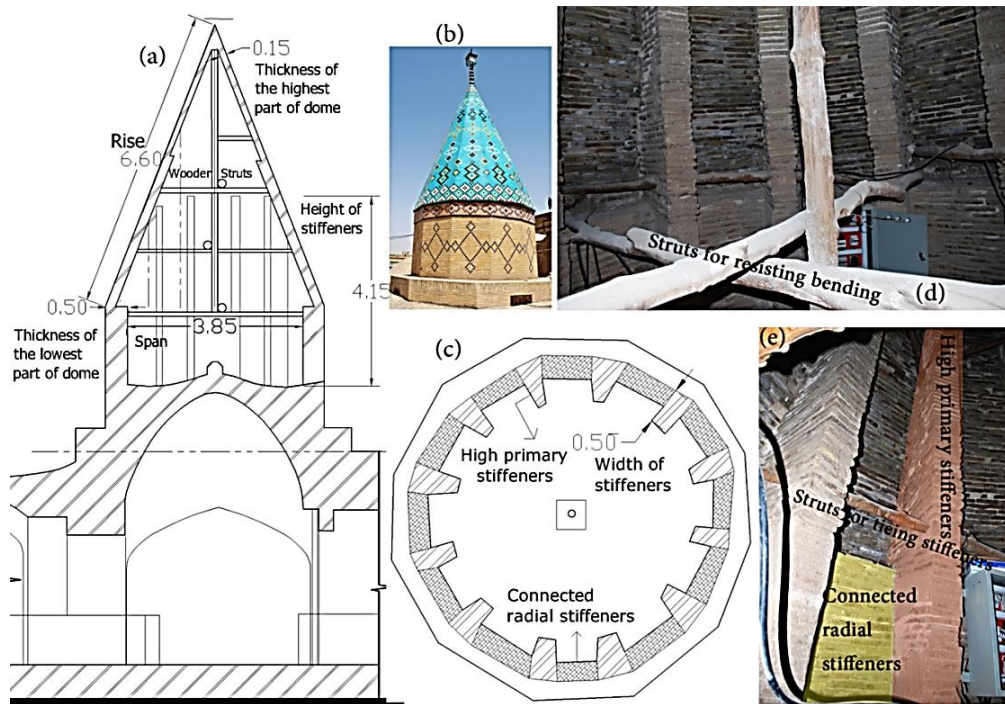
Fig. 10 Panjeh shah, Safavi (1501-1723 AD). a) section of the shell b) exterior view c) plan, d & e) stiffeners
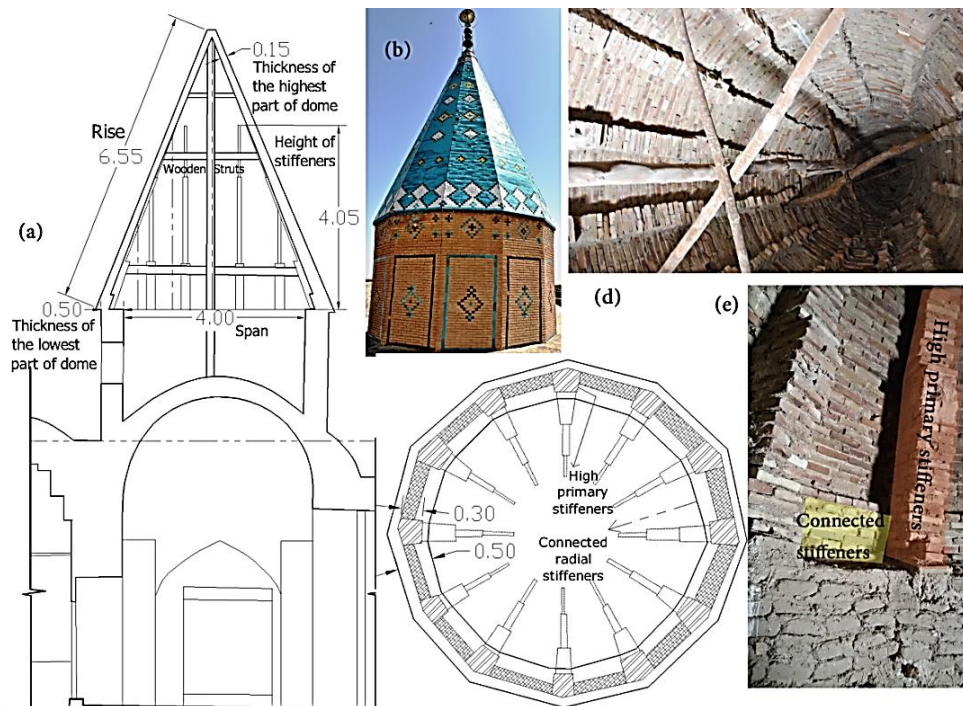

Fig. 11 Ata Bakhsh, Safavi (1501-1723 AD). a) section of the shell b) exterior view c) plan, d & e) stiffeners

Table 1 Dimensions of shells and their components

External Shells	Rise (m)	Number of planes	Span	Thickness (m) (the lowest point)	Thickness (m) (the highest point)	Number of Stiffeners	Height of stiffeners (m)	width of stiffeners (m)	Cross sectional area of stiffeners (m²)
1. Cehltan	9.45	16	6.95	0.4	0.2	4	3.15	1 & 0.7	1.3
2. Mirneshane	9.05	16	6.5	0.7	0.35	8	4.5	1	3.44
3. Shahzade ebrahim	6.7	16	4.8	0.35	0.15	No stiffener	No stiffener	No stiffener	No stiffener
4.Soltan Amir Ahmad	6.65	16	4.6	0.5	0.2	4	3.4	1.1	2.65
5. Panje Shah	6.60	12	3.85	0.5	0.15	12	4.15	0.5	4.2
6. Atabakhsh	6.55	12	4	0.5	0.15	12	4.05	0.3	3.2

2.2. Introducing Designing Parameters and Ratios

Examining case studies, dimensions of shells and their components helps us to introduce ratios and designing details which may have a correlation with 'rise/span' ratio. These ratios and designing details include 'dome's thickness/span' (the thickness of the lowest part of the shell where load transition happens), 'stiffeners' height/span', 'stiffeners' cross sectional area/span', 'decrease in shell's thickness', 'number of stiffeners/ number of flat planes', "stiffeners' end point" and 'number of flat plains'.

Table 2 lists these parameters, ratios and details based on 'rise/span' ratio, from the highest 'rise/span' ratio to the lowest one to make comparison possible.

Table 2 List of ratios, designing parameters and details

External Shells	Rise/ Span (A)	Thickness /span (B)	decrease of thickness (C)	Stiffeners' sectional area /span (D)	Number of stiffeneners/ number of flat planes (E)	Height of stiffeners/ span (F)	Stiffeners' endpoint (G)	struts' endpoint (H)	Number of planes (I)
Panjeshah	1.71	0.13	0.35	1.09	1	1.07	Middle	Upper	12
Atabakhsh	1.63	0.12	0.35	0.8	1	1.01	Upper	Upper	12
Soltan Amir Ahmad	1.44	0.1	0.3	0.57	0.25	0.74	Middle	Upper	16
Mirneshane	1.4	0.1	0.35	0.52	0.5	0.69	Middle	Upper	16
Shahzade Ebrahim	1.39	0.07	0.2	-	-	-	-	Lower	16
Cheheltan	1.35	0.05	0.2	0.18	0.25	0.45	Middle	Upper	16

3. RESULTS

By examining Table 2, Figs. 12 and 14, it is clear that 'rise/span' ratio and "shell's thickness/span" are positively correlated (+0.87), A & B, suggesting that thickness plays a crucial role in bearing weight loads. 'Panjeh Shah' and 'Ata Baksh' shells with the highest 'rise/span' ratios, achieve the highest ratios of "shell's thickness/span" while 'Shahzade Ebrahim' and 'Cheheltan' shells with the lowest 'rise/span' ratios have the lowest ratios of 'shell's thickness/span'. 'Soltan Amir Ahmad' and 'Mirneshaneh' with average ratios stand in the middle of the Table 2.

Moreover, as "rise/span" ratio increases, the thickness of the shell decreases more from the lowest point to highest point. Although these two ratios, A & C, are positively correlated (+0.61), Figs. 12 and 14, their relationship is not a strong one, and the diagram does not show a significant trend, Table 2. As can be seen in Fig 12, 'Cheheltan' and 'Shahzade Ebrahim' shells have the lowest decrease in the thickness and 'Panje Shah' and 'Ata Bakhsh' shells have the highest decrease in the thickness, but 'Mirneshane' does not follow the trend, resulting in not a strong correlation.
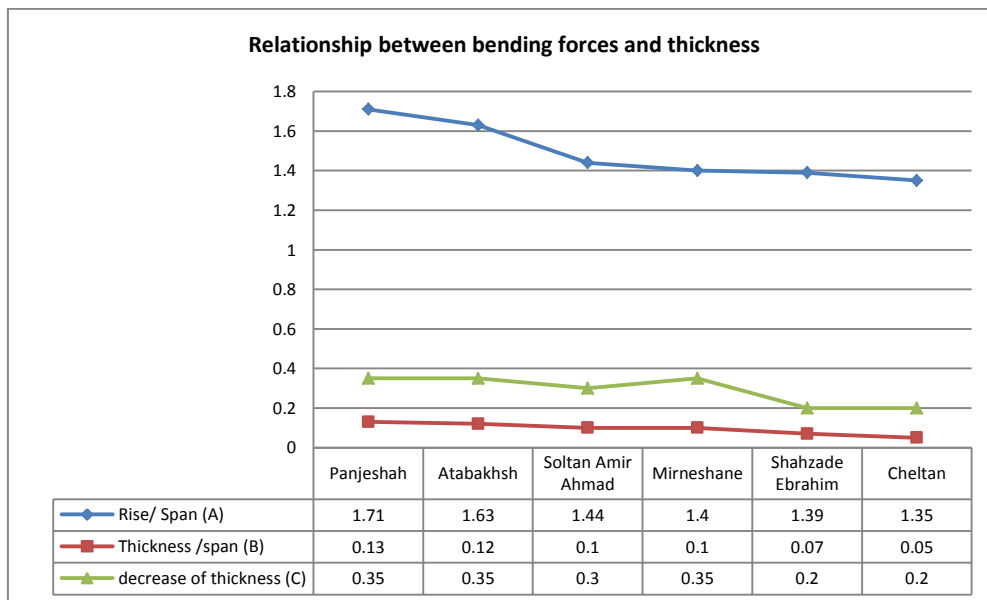

Fig. 12 Comparing 'Rise/span' ratio with 'Thickness/span' ratio and 'decrease of thickness'

Regarding stiffeners, 'Shahzade Ebrahim' has been removed from analysis since it does not have any stiffener. The case of 'Shahzade Ebrahim' has been constructed without stiffeners and through decrease in the thickness of external shell due to its relatively lower "rise/span" ratio.

The cases without stiffeners have been presented in a separate diagram, Fig. 13 and the correlation of all ratios and parameters with "rise/span" ratio has been presented in Fig 14. As can be verified from Fig. 13 and 14, there is a strong relationship between 'rise/span' ratio and

'Stiffener's sectional area/span' ratio (+0.91, A & D). 'Panjeh Shah' and 'Ata Bakhsh' shells with highest 'rise/span' ratio have 'connected radial stiffeners' (Figs. 10 & 11), concluding that these stiffeners are so effective in bearing weight loads especially at the lowest part of the external shell where weight loads are at the maximum. 'Connected radial stiffeners' also fasten high primary stiffeners together to avoid their collapse (Figs 10 & 11). Moreover, the number of stiffeners and the number of flat planes are equal in these two conical shells (Figs 10 & 11, Table 2) (one stiffener for each plane) to better restrain weight loads and resist against bending. Secondary low stiffeners in 'Mirneshaneh' and 'Soltan Amir Ahmad' shells have been used to tie the flat planes of external shells (Figs. 7 & 9). By increasing the sectional area of stiffeners, planes' ties spread loads more easily. By defining 'stiffeners' cross sectional area/span' ratio, authors have been able to take into account the type, number and dimensions of stiffeners.

As can be seen in Fig. 13 and Fig. 14, 'rise/span' and 'stiffener's height/span' ratios are positively and strongly correlated (A & F, +0.93). Needless to say, higher stiffeners control rise of external shell more effectively.

As can be seen in Table 2, in all cases except 'Ata Bakhsh' shell, stiffeners' end points are at the middle part of the external shell. To specify stiffeners' end points, external shells have been divided into three equal parts (Figs. 6-11). All parts of shells are exposed to weight loads, especially the lowest parts where the weight of materials increases. That is why stiffeners decrease in both width and thickness as they gradually reach the higher parts of the shell. According to the fact that bending forces

are higher in the middle part of the shell, stiffeners that end in the middle part of the shell can restrain both bending and weight forces. Accordingly, there is no need to have higher stiffeners to cover upper parts of shells where weight loads are insignificant; just 'Ata Bakhsh' shell with a high 'rise/span' ratio has stiffeners that end in the upper part of the dome. As a result, it can be concluded that there is no relationship between 'rise/span' ratio and stiffeners' end point (A & G).

To avoid stiffeners' collapse, wooden struts have been placed in upper parts of the conical shells and tie them together, except for 'Shahzade Ebrahim', Table 2. These wooden struts also help a more gradual spread of loads. Not having stiffeners in 'Shahzade Ebrahim', wooden struts are just placed at the lowest part of the shell, Fig 8 & Table 2. As in all cases except for 'Shahzade Ebrahim', the wooden struts are placed in the upper parts of conical shells, no correlation can be found between "rise/span" ratio and "struts' end point" (A & H).

As can be inferred from Table 2, 'rise/span' ratio is negatively correlated (-0.91) to the number of flat planes (A & I), Table 2 and Fig 14. Its reason can be suggested by examining case studies; In 'Panjeh Shah' and 'Ata Bakhsh' shells with lower number of flat planes (12), one stiffener has been constructed for each plane, while in shells with higher number of flat planes (16) it is less possible to construct one stiffener for each plane. As a result, in shells with higher 'rise/span' ratios, lower number of flat planes makes it possible to have one stiffener for each plane. For the same reason, 'rise/span' ratio is positively (+0.86) correlated with "number of stiffeners/ number of flat planes" (A & E), Table 2 and Fig 14.
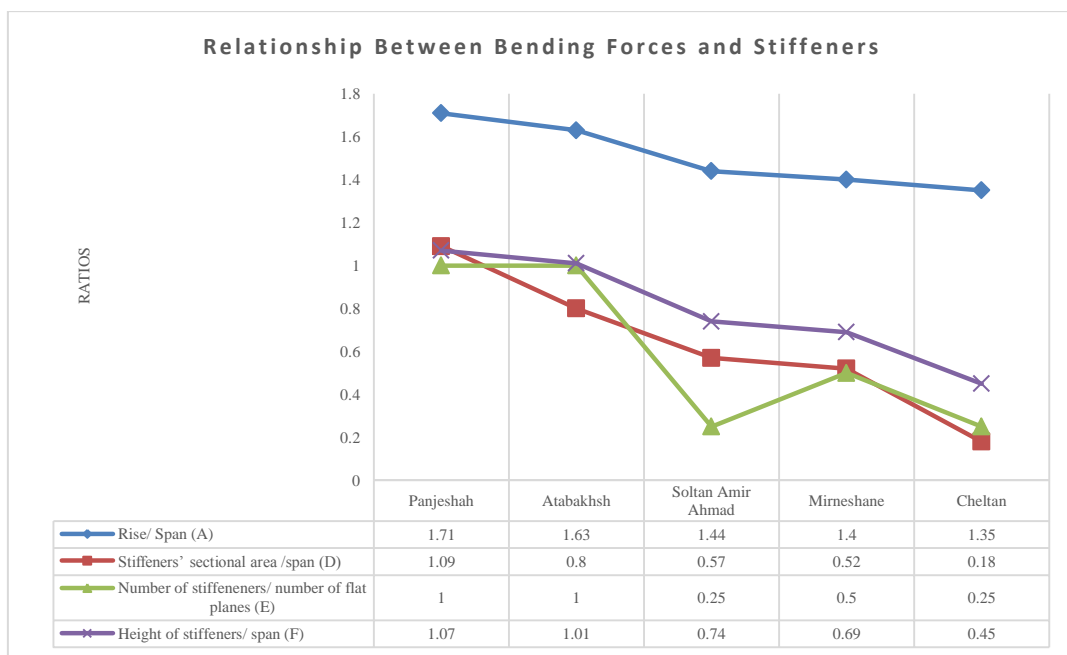

	Panjeshah	Atabakhsh	Soltan Amir Ahmad	Mirneshane	Cheltan
Rise/ Span (A)	1.71	1.63	1.44	1.4	1.35
Stiffeners' sectional area /span (D)	1.09	0.8	0.57	0.52	0.18
Number of stiffeneners/ number of flat planes (E)	1	1	0.25	0.5	0.25
Height of stiffeners/ span (F)	1.07	1.01	0.74	0.69	0.45

Fig. 13 The relationship between "rise/span" ratio and stiffeners
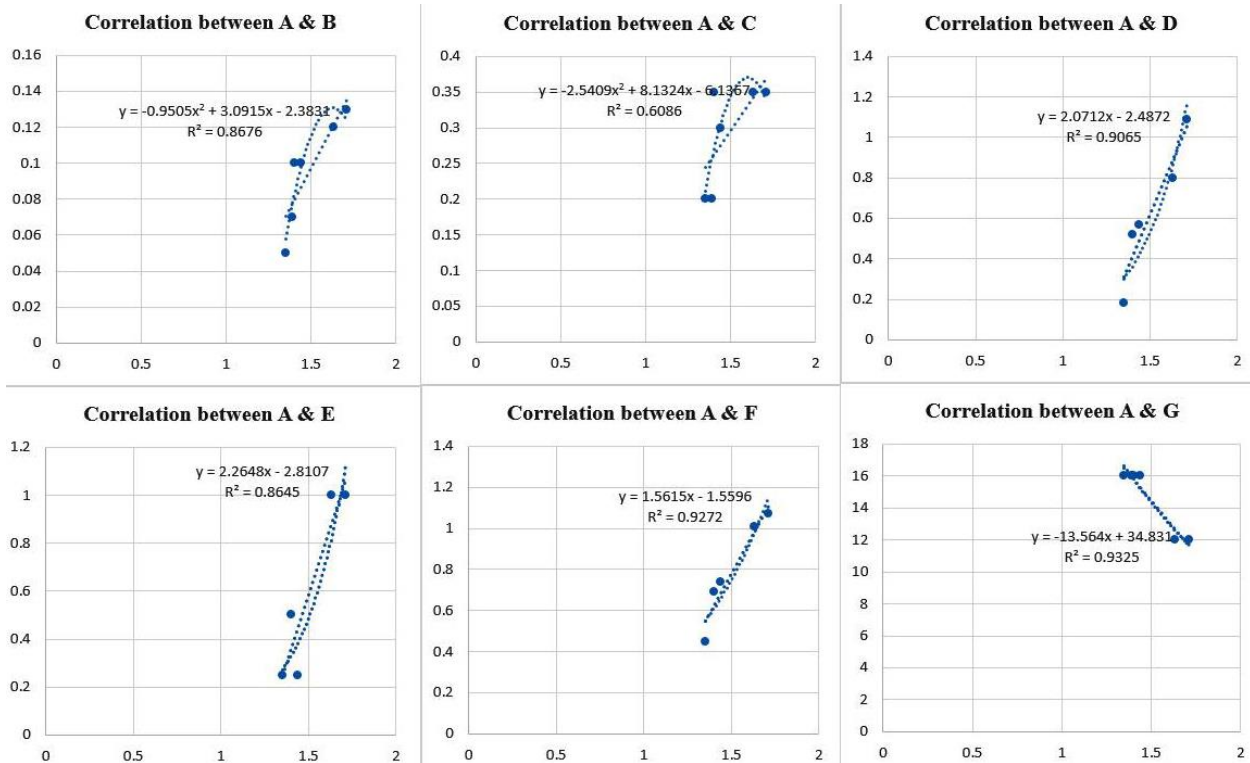
Fig. 14 The correlation between "rise/span" ratio=A with other ratios

4. CONCLUSION

In this paper, authors applied structural analysis to extant conical shells to understand how they carry the loads imposed upon them. As rise of external shells increase and their span decreases, resisting against bending forces due to weight loads becomes more challenging, so the significance of "rise/span" ratio shows how difficult it is to carry weight loads and resist against bending.

Results show that by applying two general strategies, architects have been able to spread loads in the external shell. The first strategy is through properly designing the thickness of the external shell, and the second strategy is through properly designing the stiffeners. Height, dimensions, type of stiffeners, their end points and the way they have been tied together are all important factors in bearing and spreading loads, so stiffeners should be carefully analyzed and studied. These strategies can be applied to extant conical shells which suffer from weakness in construction.

CONFLICT OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

REFERENCES

[1] Saoud R. Muslim Architecture under Seljuk Patronage (1038-1327). Foundation for Science, Technology, and Civilization (FSTC). http://www. muslimheritage. com/features/default. cfm, 2003.

[2] Saeidian A, Gholi M, Zamani E, Ansari M, Bemanian MR. Identifying the formation of ourchin dome, a symbol of tomb architectural structure in southern part of Iran, 2012.

[3] Gacek T, Pstrusińska J. Proceedings of the ninth conference of the european society for central asian studies, Cambridge Scholars Publishing, 2009.

[4] Petersen A. Dictionary of Islamic architecture, Psychology Press, 1996.

[5] Smith EB. The dome: A study in the history of ideas, Princeton University Press Princeton, NJ, 1971.

[6] Ashkan M, Ahmad Y. Persian domes: history, morphology, and typologies, International Journal of Architectural Research, 2009, Vol. 3, No. 3, pp. 98-115.

[7] Ashkan M, Ahmad Y. The significant of Iranian domes: Analysis of Morphology and Typologies, 2009.

[8] Ashkan M, Ahmad Y. Discontinuous double-shell domes through Islamic eras in the middle east and central Asia: History, morphology, typologies, geometry, and construction, Nexus Network Journal, 2010, Vol. 12, No. 2, pp. 287-319.

[9] Ashkan M, Ahmad Y. Significance of conical and polyhedral domes in persia and surrounding areas: morphology, typologies and geometric characteristics, Nexus Network Journal, 2012, Vol. 14, pp. 275-290.

[10] Ashkan M, Ahmad Y, Arbi E. Pointed dome architecture in the middle east and central asia: evolution, definitions of morphology, and typologies, International Journal of Architectural Heritage, 2012, Vol. 6, No. 1, pp. 46-61.

[11] Rahman MM. Islamic architecture and arch, International Journal of Built Environment and Sustainability, 2015, Vol. 2, No. 1.

[12] Farshad M. On the shape of momentless tensionless masonry domes, Building and Environment, 1977, Vol. 12, No. 2, pp. 81-85.

[13] Grabar O. The Islamic dome, some considerations, The Journal of the Society of Architectural Historians, 1963, pp. 191-198.

[14] Gye D. Arches and domes in Iranian Islamic buildings: An Engineer's Perspective, Iran, 1988, pp. 129-144.

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