

# Dilation and particle breakage effects on the shear strength of calcareous sands based on energy aspects

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*Abstract: In this paper shear behavior of two calcareous sands having different physical properties are investigated using drained and undrained triaxial tests. The investigated sands are obtained from two different zones located in Persian Gulf, Kish Island and Tonbak region. Analysis based on energy aspects show that friction angle in these soils, having crushable particles, is formed of three components: substantial internal friction angle, dilation and particle breakage angle. Dilation component is available in the two investigated sand. Particle breakage component is a function of grains hardness, structure and geometry shape. Particles breakage decreases the volume of sample during drained tests and creates positive pore water pressure during undrained tests. Two investigated sands show different amount of dilation and particle breakage under similar conditions. Simultaneous dilation and particles crushing and different amount of them result in different shear behavior of the two studied sands. Energy aspects are used to determine the effect of particle crushing on the shear strength. There is a suitable compatibility between relative breakage of grains and consumed energy ratio for particle breakage.*

*Key Words: calcareous soil, triaxial test, dilation, particle breakage, energy*

## Introduction

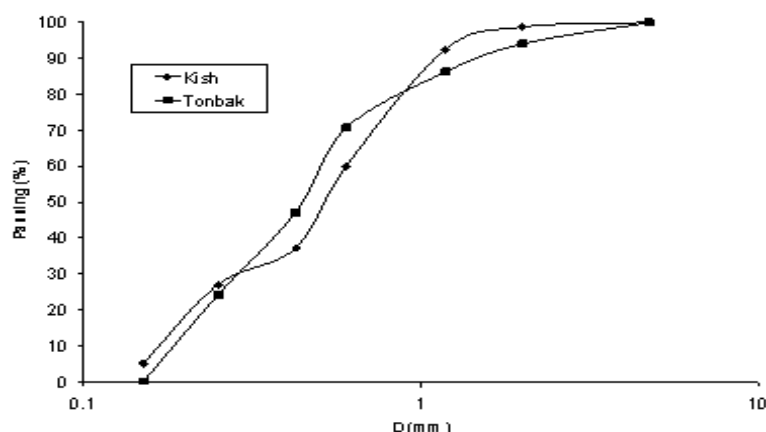
Low hardness, porous grains and different origin of calcareous soils results to different shear behavior compared with siliceous soils especially against driving piles. Usually laboratory tests show high shear strength for calcareous sands. However, these sands have shown some problems during constructions and pile drivings. Different origins and different deposition conditions creates calcareous soils having different characteristics. Information about these soils such as density, stress history, dimension, shape and strength of grains are very limited.

Investigations of Jewell and Andrews (1988), Coop and Lee (1993), Jewell and Khorshid (1988), Kaggwa (1998), Coop (1990), Airey and Fahey (1991), Hyodo et al (1998), Al-Shafei (1999), Salehzadeh (2000), Ismail et al (2002), Luzzani and Coop (2002), Coop et al (2004) and Sharma et al (2006) are some examples of conducted studies on calcareous soils.

Perhaps, Chandler presented the most complete idea about calcareous sands having crushable

grains in 1985 that improved by Baharom and Stalleberass in 1998. They told that critical state in sands having crushable grains in triaxial tests is in fact equilibrium between contractions due to particles breakage and dilation resulted from rearrangement of grains. The question is, if contraction due to particle breakage and dilation resulted from particles rearrangement is not equal and balanced, how is the critical state? When the particles breakage is ceased? Luzzani and Coop (2002) and Coop et al (2004) conducted some ring shear tests up to very large shear strains and presented that particles breakage is continued by continuing the shear strains to very large strains. They verified that critical state in triaxial tests is in fact a balance between volume reduction due to particles breakage and dilation resulted from particles rearrangement. This is a transient state since stable grading is not reached.

In this paper monotonic shear behavior of two calcareous sands having different physical properties are investigated using consolidated-drained and consolidated- undrained triaxial tests. The studied calcareous sands are obtained from two different regions of Persian Gulf and



**Fig.1** Particle size distribution of the studied sands.

south of Iran.

At first, the results of consolidated-drained triaxial tests on two sands are analyzed using Row theory and its improved version by Chen et al (2000). Test results show that two calcareous sands having crushable grains have some differences and some similar features. In these sands, shear strength and then internal friction angle has formed from three components: substantial internal friction, dilation and particle breakage during shearing. Dilation and breakage friction angle is a function of initial relative density, initial confining pressure, initial grading, texture and hardness of sand particles. Based on test results, dilation and particles breakage are different for two investigated calcareous sands. Like CD tests, the CU tests show some differences and similar features like pore water pressure development and then shear strength of samples for the two studied sands.

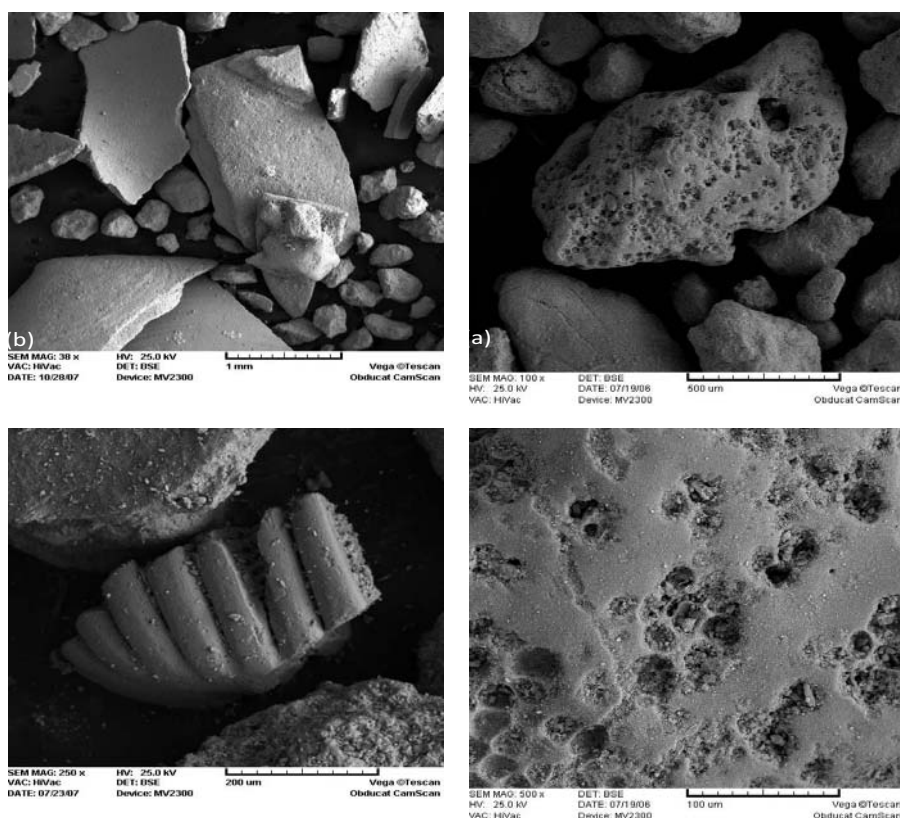
## Materials

As mentioned before, two calcareous sands are studied in this paper. Kish Island sand, obtained from east of Kish Island, located in Persian Gulf in the south of Iran and Tonbak sand obtained from depth of 2 to 9m of sea bed where the depth of water equals 10m, located in Persian Gulf and south of Iran. Particle size distribution of the two sands is shown in figure 1. According to this figure, Kish Island sand is somehow finer than Tonbak sand. Electronic microscopic pictures

(SEM) of the two sands are shown in figure 2. According to this figure, the two sands have different textures. Kish Island sand grains are biological and have porous structure with bulky and subangular shape. However Tonbak sand grains are porous, angular, weaker and some of them are platy and with thin walls. It seems that Tonbak sand grains are more fragile than Kish Island grains. Then it's may be the volume reduction due to particles breakage in Tonbak sand during loading is more than Kish Island sand. Physical properties of the two sands are summarized in table 1. It's expected that two sand, have different shear behavior regarding to their different grains texture and physical properties.

## Sample Preparation and Tests

For sample preparation, at first some determined sand (oven dried) is poured into a membrane inside the triaxial apparatus mold of sample. Then, the determined sand is placed inside the mold by uniform tamping to the mold. Then a sample with specific relative density is created. The prepared samples have 38mm diameter and 76mm height. After sample preparation, a low negative pressure as much as 10kPa is applied to the sample and the mold is removed. After putting the sample inside of the triaxial cell, a small confining cell pressure as much as 20kPa is applied to the sample. For saturation of the sample, at first CO<sub>2</sub> gas is passed through the sample for about 10 to 15minutes, then the de-



**Fig.2** Microscopic electronic pictures of (a) Kish Island sand and (b) Tonbak sand.

**Table 1** Physical Properties of Kish Island and Tonbak sand

sand	G <sub>s</sub>	$\gamma_d(\min)$ ( $kN/m^3$ )	$\gamma_d(\max)$ ( $kN/m^3$ )	$e_{\max}$	$e_{\min}$	$e_{\max} - e_{\min}$	CaCO <sub>3</sub> (%)
Kish Island	2.68	15.6	17.8	0.72	0.51	0.21	93
Tonbak	2.69	13.4	15.6	0.99	0.71	0.28	95.5

aired water is passed through the sample. In all processes a confining effective pressure of about 10kPa is kipped. Using a back-pressure and keeping a confining effective pressure of about 10kPa, when the air bubbles are solved inside the pore water and the B value is 0.95, the sample is saturated. According to the described method, all tests are prepared and tested using a computer controlled triaxial apparatus. The performed CD and CU tests are listed in table 2.

### Drained Tests

According to table 2, fourteen CD tests were performed on the Kish Island, seven having relative density of %20 and 7 having relative

density of %80. Initial confining pressure was selected from 50kPa to 600kPa. Six CD tests were performed on Tonbak sand having relative density of %20, with confining pressure from 50kPa to 600kPa.

Diagram of  $\sigma'_1/\sigma'_3$  and volumetric strains against axial strain are shown in figures 3 and 4 for Kish Island sand in loose and dense state respectively. As shown in these figures, Kish Island sand shows a dilative behavior. In fact after a contraction portion, all samples are dilated. Dilation is more in dense state relative to loose state. However, it seems that the Kish Island sand has a substantial interlocking that even in loose state shows a dilative behavior. As expected, increasing the initial confining pressure reduces

**Table 2** The list of performed CD and CU tests

Sand	Dr (%)	Test	$\sigma_3$ (kPa)	Code
Kish Island	20	CD	50	KL50CD
Kish Island	20	CD	100	KL100CD
Kish Island	20	CD	200	KL200CD
Kish Island	20	CD	300	KL300CD
Kish Island	20	CD	400	KL400CD
Kish Island	20	CD	500	KL500CD
Kish Island	20	CD	600	KL600CD
Kish Island	80	CD	50	KD50CD
Kish Island	80	CD	100	KD100CD
Kish Island	80	CD	200	KD200CD
Kish Island	80	CD	300	KL300CD
Kish Island	80	CD	400	KL400CD
Kish Island	80	CD	500	KL500CD
Kish Island	80	CD	600	KL600CD
Kish Island	20	CU	100	KL100CU
Kish Island	20	CU	300	KL300CU
Kish Island	20	CU	500	KL500CU
Kish Island	80	CU	100	KD100CU
Kish Island	80	CU	300	KD300CU
Kish Island	80	CU	500	KD500CU
Tonbak	20	CD	50	TL50CD
Tonbak	20	CD	100	TL100CD
Tonbak	20	CD	200	TL200CD
Tonbak	20	CD	300	TL300CD
Tonbak	20	CD	500	TL500CD
Tonbak	20	CD	600	TL600CD
Tonbak	20	CU	100	TL100CU
Tonbak	20	CU	300	TL300CU
Tonbak	20	CU	500	TL500CU

the dilation.

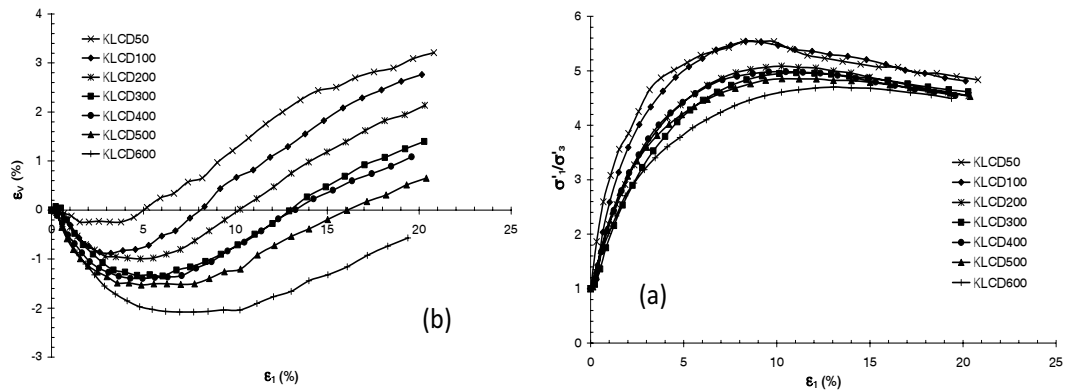
However, the sand shows the contraction behavior just in loose state and confining pressure of 600kPa.

Almost, all samples show the volume increasing to strains up to %20. It means that, it's not happen the critical state to strains up to %20. Perhaps in this soil, the critical state is achieved in strains greater than %20. Apparently, diagrams of  $\sigma'_1/\sigma'_3$  against axial strains show the constant shear strength just in confining pressure greater than 500kPa.

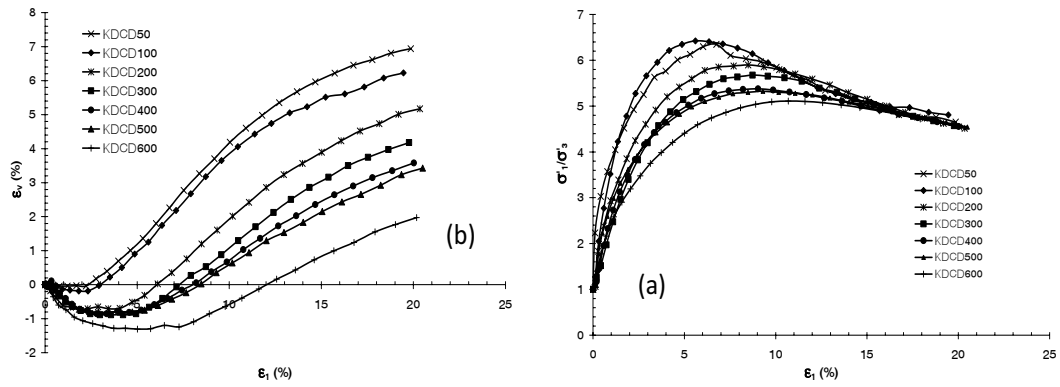
Diagrams of  $\sigma'_1/\sigma'_3$  and volume strains against axial strains are shown in figure 5 for Tonbak sand in loose state. As shown in this figure in low confining stresses, Tonbak sand has dilation behavior. Increasing of confining pressure

reduces the dilation rate and samples show the contraction behavior. Apparently, in this soil and high confining pressures, the critical state is clearer. Therefore, regarding to the structure of the two sands (figure 2); it can be said that 1) calcareous sands have substantially dilative behavior due to inherent interlocking and 2) sands having weak structure have low dilation potential. In other words, dilation and contraction resulted from particles breakage complexe the critical state concept in these soils.

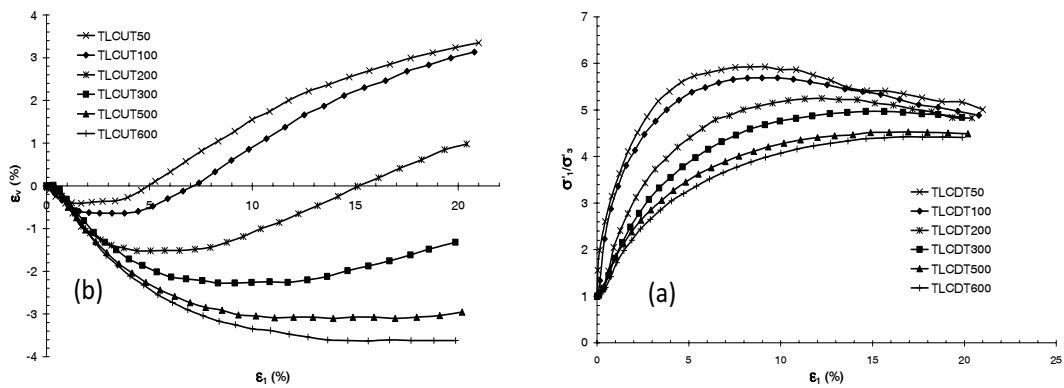
Comparing figures 3b and 5b, it is observed that in low confining pressure (50kPa) the Kish Island sand has low dilation in comparison with Tonbak sand. In high confining pressure (greater than 500kPa) the dilation of Tonbak sand is omitted completely. Regarding the structure of the two sand grains, probably this behavior shows that the particles breakage of Tonbak sand is more



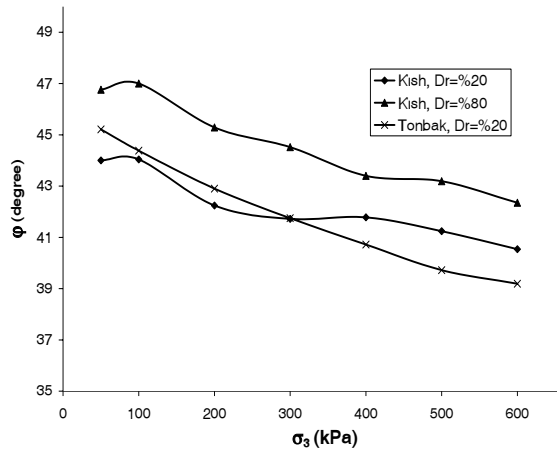
**Fig. 3** Triaxial test results on Kish Island sand in loose state (a)  $\sigma'_1/\sigma'_3$  (b) volumetric strain against axial strain



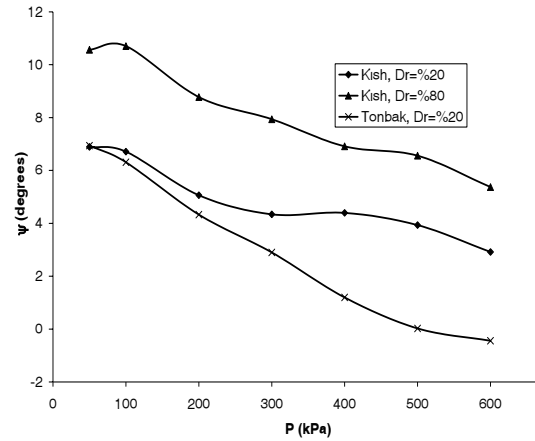
**Fig.4** Triaxial test results on Kish Island sand in dense state (a)  $\sigma'_1/\sigma'_3$  (b) volumetric strain against axial strain



**Fig.5** Triaxial test results on Tonbak sand in loose state (a)  $\sigma'_1/\sigma'_3$  (b) volumetric strain against axial strain



**Fig.6** Total internal friction angle of Kish Island and Tonbak sands against confining pressure



**Fig.7** Dilation angle of Kish Island and Tonbak sands against confining pressure

than Kish Island sand. It's noted that particles breakage is accompanied with volume reduction.

### Particle breakage and energy aspects

The Rowe stress- dilation theory in triaxial test is as:

$$\frac{\sigma'_1}{\sigma'_3} = \left(1 + \frac{d\varepsilon_v}{d\varepsilon_1}\right) \tan^2\left(45 + \frac{\varphi_\mu}{2}\right) \quad (1)$$

Where  $\sigma'_1$  is the maximum principal stress,  $\sigma'_3$  is the minimum principal stress,  $d\varepsilon_v$  is the volumetric strain increment (dilation positive),  $d\varepsilon_1$  is the axial strain increment (compression positive) and  $\varphi_\mu$  is the internal friction angle between soil grains. Equation 1 has been obtained based on the principle of minimum ratio of the rate of work done on the soil by  $\sigma'_1$  to that done by the soil against  $\sigma'_3$ .

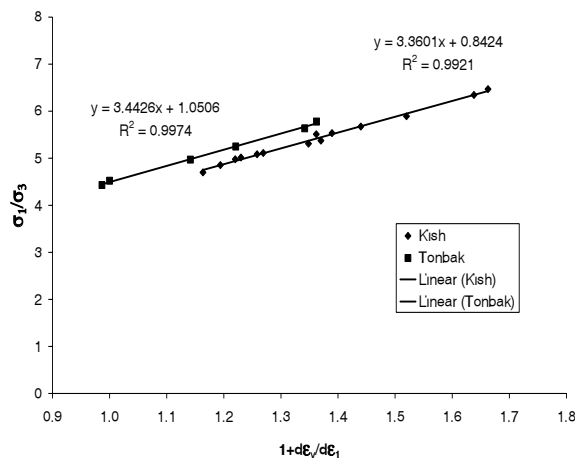
TZOU-SHIN UENG and TSE-JEN CHEN (2000) showed that if the soil grains crush during loading in triaxial test, the Rowe's equation is modified as:

$$\frac{\sigma'_1}{\sigma'_3} = \left(1 + \frac{d\varepsilon_v}{d\varepsilon_1}\right) \tan^2\left(45 + \frac{\varphi_\mu}{2}\right) + \frac{dE_B}{\sigma'_3 d\varepsilon_1} (1 + \sin \varphi_\mu) \quad (2)$$

Where  $dE_B$  is the energy consumed rate to grains crushing.

Rowe based on experimental tests showed that  $\varphi_\mu$  in equation 1 should be replaced by  $\varphi_f$ , that varies from  $\varphi_\mu$  in the maximum relative density to  $\varphi_{cv}$  in the loose interlocking and constant volume change. This differences between  $\varphi_f$  and  $\varphi_\mu$  is due to the energy consumed during rearrangement of soil grains. Therefore, in this paper  $\varphi_f$  is used instead of  $\varphi_\mu$  that  $\varphi_f$  is the internal friction angle without considering the dilation and particle breakage.

Considering the dilation and particles breakage in internal friction angle (equation 1 without  $d\varepsilon_v/d\varepsilon_1$ ), results of CD tests are shown in figure 6 as the total friction angle against initial confining pressure. According to this figure, increasing the confining pressure reduces the internal friction angle. Furthermore, as it's expected, increasing the initial relative density increases the internal friction angle (Kish Island sand). As shown in figure 6, it should be noted, although calcareous sands have shown some problems and weaknesses during constructions such as pile driving. It seems that they show a very suitable strength using usual laboratory triaxial tests. However, the two investigated sands have similar mineralogy, but they show different strengths. Tonbak sand having brittle structure shows a greater internal friction angle in confining pressure less than 300kPa and lower internal friction angle in confining pressures higher than 300kPa in comparison with Kish Island sand.

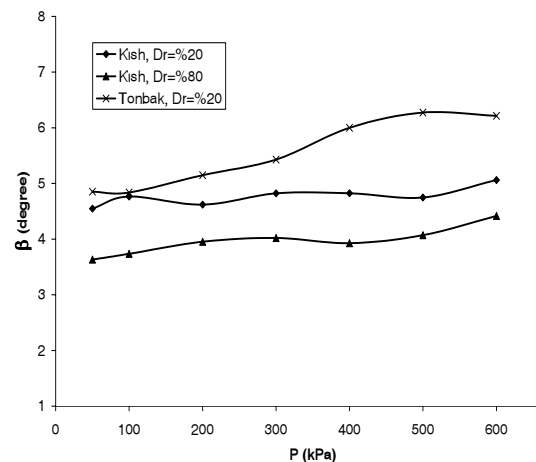


**Fig.8** Effect of particle breakage on the shear strength envelop of Kish Island and Tonbak sands

Dilation angle (differences between total friction angle and the internal friction angle from equation 1) of Kish Island sand in both loose and dense states and Tonbak sand is shown in figure 7 against initial confining pressure. It's observed that calcareous sands have substantially a considerable dilation angle even in loose state. Increasing the confining pressure reduces the dilation angle. Beside this, regarding the brittle structure of Tonbak sand grains, its dilation angle is lower than Kish Island sand especially in high confining pressure. Tonbak sand in confining pressure greater than 500kPa shows a negative dilation angle (contraction).

Consolidated-drained test results are shown in figure 8 as  $\sigma'_1/\sigma'_3$  against  $(1+d\varepsilon_v/d\varepsilon_1)$ . According to Rowe's theory, it is expected that the best fitting line to the test results pass through the origin. However, this doesn't occur. Comparing the equation 2 and figure 8, it's observed that particles breakage has apparently changed the shear strength envelop in the investigated sands. It is noted that the effect of particles breakage is different in two studied sands. According to figure 8 the shear strength envelopes of the two investigated sands are as:

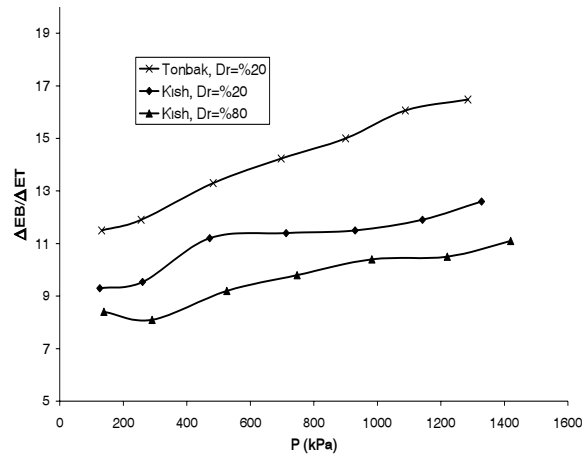
$$\begin{aligned} \frac{\sigma'_1}{\sigma'_3} &= 3.3601\left(1 + \frac{d\varepsilon_v}{d\varepsilon_1}\right) + 0.8424 \\ \frac{\sigma'_1}{\sigma'_3} &= 3.4426\left(1 + \frac{d\varepsilon_v}{d\varepsilon_1}\right) + 1.0506 \end{aligned} \quad (3)$$



**Fig.9** Particles breakage angle in the two studied sands

Where the second terms show the effect of particles breakage and are different for the two sands.

It can be said that the consumed energy increment (equation 2) to particle breakage is a function of both the amount of breakage and crushability potential or crushing strength of grains. Then, regarding to weaker structure, it's expected that Tonbak sand has greater grains crushing. Considering that particles breakage is accompanied by volume reduction, it reduces the dilation. The internal friction angle without dilation and particle breakage ( $\phi_f$  in equation 2) is achieved using the experimental equations obtained for the two sands (equation 3). Therefore, internal friction angles ( $\phi_f$ ) of the two studied sands are 32.6 and 33.4 for Kish Island and Tonbak sand respectively. It's clear that the difference between these angles and the amount obtained from equation 2 are the particles breakage portion. Figure 9 shows the particles breakage portion from total internal friction angle against confining pressure. According to this figure, the particles breakage portion in loose samples is greater than dense samples (Kish Island sand). It probably means the loose samples crushability potential is greater than dense samples. Furthermore, particles breakage in Tonbak sand is greater than Kish Island sand. Totally, unlike dilation, increasing the confining



**Fig.10** The consumed energy for particles breakage to total applied energy on samples ratio in peak point

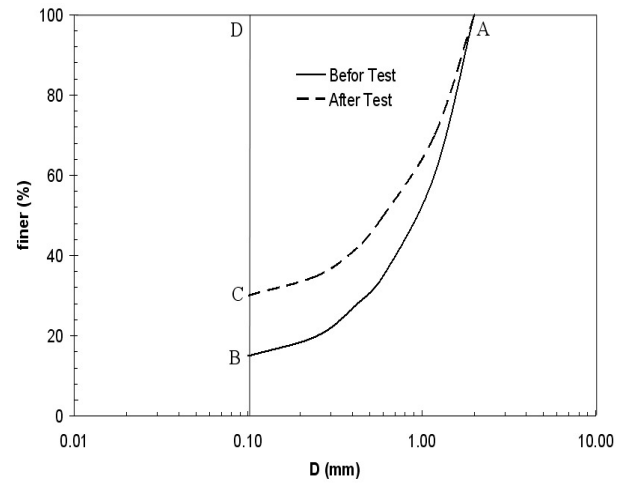
pressure increases the particles breakage angle. Regarding that the analysis is performed in peak point, it is possible to calculate the consumed energy to particles breakage by using the experimental shear failure envelop (figure 8). Using this figure and equation 2 we have:

$$\begin{aligned}\Delta EB &= \sum b \sigma'_3 \Delta \varepsilon_1 / (1 + \sin \varphi_f) \\ \Delta ET &= \Delta EB + \sum (\sigma'_3 \Delta \varepsilon_v + \sigma'_1 \Delta \varepsilon_1)\end{aligned}\quad (4)$$

That  $b$  parameter is obtained by comparing equation 2 with figure 8. The  $b$  parameter is 0.84 and 1.051 for Kish Island and Tonbak sand respectively. The consumed energy for particles breakage to total applied energy on the sample ratio ( $\Delta EB/\Delta ET$ ) in peak point is shown in figure 10 against mean confining pressure ( $P$ ). As shown in this figure, increasing the confining pressure and reducing the initial relative density increases the consumed energy to particles breakage (Kish Island sand). As expected, in Tonbak sand having weaker structure, the consumed energy to particles breakage is greater than Kish Island sand.

### Grain size distribution

Grain size distribution of samples before and after test is compared to evaluation of particles



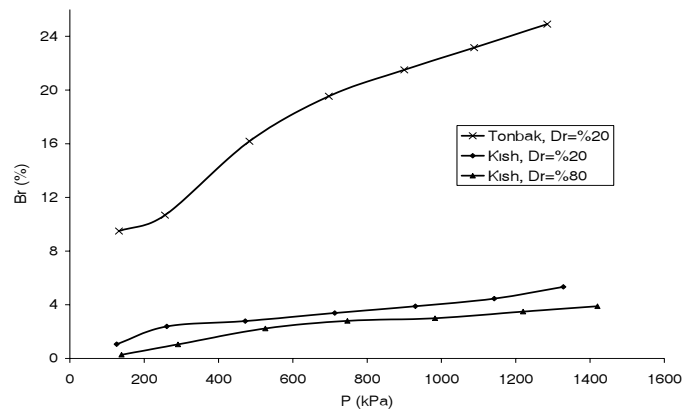
**Fig.11** definition of particle breakage ratio ( $Br = \frac{S_{ABC}}{S_{ABD}} \times 100$ )

breakage during consolidation and shearing. Considering particles breakage ratio definition ( $Br$ ) according to figure 11, test results for the two studied sands are shown in figure 12. Due to particles erosion effects in sieving, the obtained grading curves are accompanied by some errors despite similar method of grading the samples. As shown in figure 12, the particles breakage of Tonbak sand is greater than Kish Island sand. Also it is observed that increasing the confining pressure and reducing the initial relative density increases the particle breakage ratio. It is clear that the particles breakage of Tonbak sand is about 5 to 6 times of Kish Island sand in similar conditions. However, the consumed energy to particles breakage in Tonbak sand is less than 1.5 times of Kish Island sand. Therefore, it can be said that the consumed energy to particles breakage is a function of both the amount of particles breakage and particles breakage potential or particles breakage strength. Particle strength against breakage can be a function of particle hardness, shape and its internal porosity. Then Tonbak sand grains are experienced more crushing regarding to weaker and brittle structures.

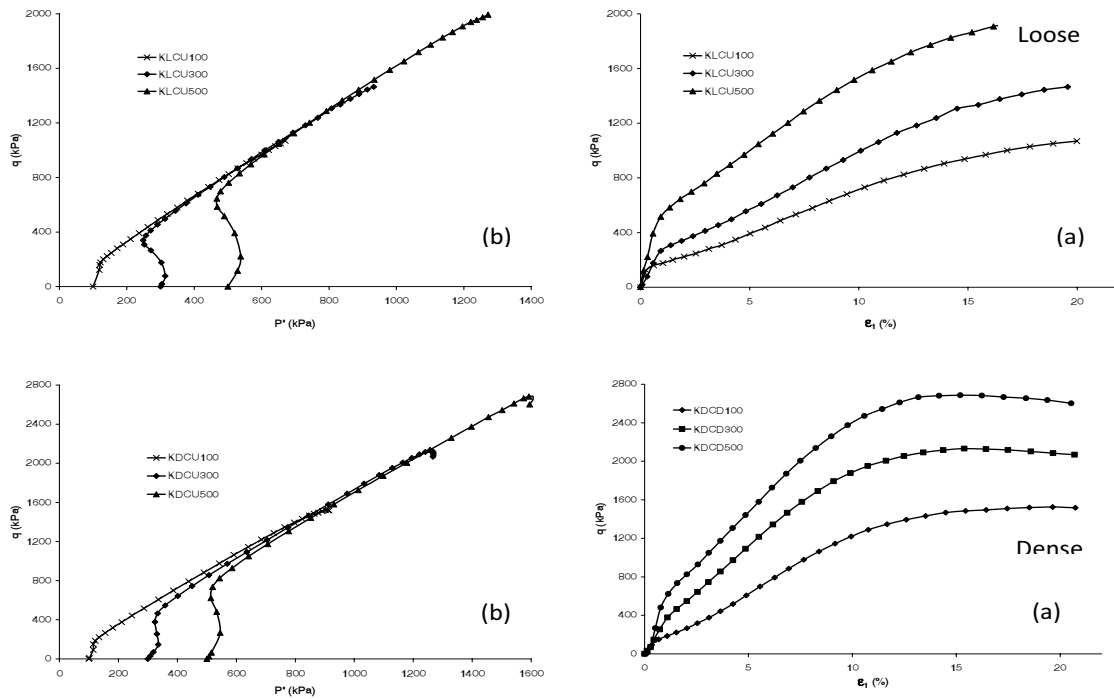
### Discussion

Coop (2004) performing ring shear tests on

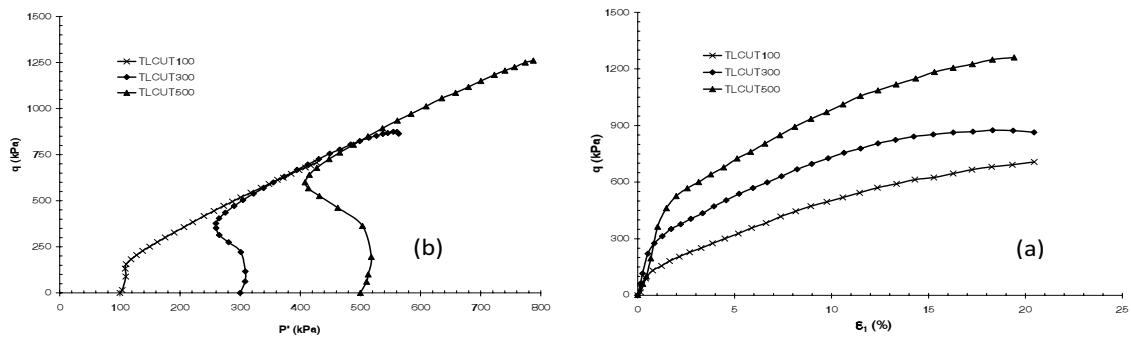




**Fig.12** Particles breakage ratio of the two sand grains during CD tests



**Fig.13** CU tests on Kish Island sand in loose and dense state: (a)  $q$  against axial strain, (b)  $q$  against  $P'$



**Fig.14** CU tests on Tonbak sand in loose state: (a)  $q$  against axial strain, (b)  $q$  against  $P'$

calcareous sands, presented that particles breakage is not only the function of normal stresses but it is also a function of experienced shear strains. Regarding to limit strains in triaxial tests, it may not be to exactly simulate the phenomenon such as pile driving in calcareous sands. However, it may be to judge about other structures behavior such as shallow foundation or retaining walls. Considering simultaneous interaction of dilation and particles breakage, the critical state in triaxial tests is complex. According to the conducted triaxial tests on two different calcareous sands, the apparent critical state is observed only in Tonbak sand and confining pressures greater than 500kPa. As stated before, this state is in fact a transient and a balance between dilation and volume reduction resulted from particles breakage. In the studied sands and confining pressures less than 500kPa, it is not observed the critical state to strains up to %20. Regarding that the rate of volume increasing reduces by increasing the axial strains, it may be, the critical state is achieved in axial strains greater than %20. The situation in which dilation and particles breakage is ceased and constant grading and relative density is achieved. Probably, the grain crushing is continued to very large strains after termination of dilation. If we accept that such an event is happen during pile driving and the volume reduction is not compensated by the lateral deflection of the around soil, then the friction resistance falls between the pile surface and soil (for example in loose and weakly cemented calcareous sands). It may be say that in these sands, the real critical state is a function of grains crushability, initial grading, initial relative density and confining pressure.

### Undrained Tests

As listed in table 2, some CU triaxial tests were performed on Kish Island and Tonbak calcareous sands. Results of these tests are shown in figure 13 as  $q$  against axial strain and  $P'$  in both loose and dense states. According to figure 13a, it's observed that Kish Island dense samples have apparently two yield points. One of them is in the

contraction or positive pore water pressure development zone and strains about %1. Another is in the dilation or negative pore water pressure development zone and strains about % (12-13). The mentioned behavior is weakly observed for loose samples. According to figure 13b, the negative pore water pressure is developed after a small contraction and positive pore water pressure development. The difference of loose and dense samples is the amount of initial contraction and secondary dilation. It means that the dense samples have shown greater negative pore water pressure in which results to higher shear strength. However, it seems that the effect of inherent dilation is more efficient than the initial relative density.

Test results on Tonbak sand are shown in figure 14 as  $q$  against axial strain and  $P'$  for loose state. Regarding figure 5, it is expected that this sand shows lower positive pore water pressure especially in high confining pressures (figure 14). Considering of more brittle and weak structure of Tonbak sand grains, it's experienced greater particles breakage. As mentioned before, particles breakage results in volume reduction and then development of positive pore water pressure. Increasing the confining pressure increases the particles breakage and volume reduction of samples. If the volume reduction resulted from particles breakage is greater than the inherent dilation, positive pore water pressure is developed. However, in Kish Island sand the volume reduction resulted from particles breakage is less than the inherent dilation even to confining pressure up to 500kPa. Therefore, considering continues negative pore water pressure development especially in Kish Island sand, the concept of critical state is complex in CU tests.

### Conclusion

Based on drained and undrained triaxial tests on two calcareous sands obtained from Persian Gulf having different structures, it is observed that:

- Calcareous sands have substantial interlocking that result in dilative behavior of them in both

loose and dense state. May be the effect of dilation greater than the effect of initial relative density. Increasing the confining pressure decreases the dilation. Regarding the calcareous soil structure, it may be, increasing the confining pressure in the range of engineering stresses, dilation completely cease or exist.

- According to Rowe theory and its improved version for soils having crushable grains, some part of applied energy to the sample during the test is consumed to breakage of sand particles. Consumed energy to breakage of grains is a function of both grains crushing strength and crushing amount. Decreasing the relative density and weakening the sand grains, increases the ratio of consumed energy to breakage of particles.

- Internal friction angle in calcareous soils having crushable grains is formed from three components: substantial friction of grains, dilation and grains breakage. Increasing the confining pressure decreases the dilation component and increases the breakage component. Inherent dilation and crushing of grains result in complex the critical state concept. In fact, the critical state in calcareous soils having the mentioned two potential in triaxial tests is a transient state and a balance between dilation and contraction due to particle breakage.

- Regarding that particles breakage is related to applied strains and strain limitation of triaxial apparatus, prediction the derived piles bearing capacity using usual triaxial test results is not exact.

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