Multi-reservoir operation by adaptive pheromone re-initiated ant colony optimization algorithm

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Abstract: *Through a collection of cooperative agents called ants, the near optimal solution to the multi-reservoir operation problem may be effectively achieved employing Ant Colony Optimization Algorithms (ACOAs). The problem is approached by considering a finite operating horizon, classifying the possible releases from the reservoir(s) into pre-determined intervals, and projecting the problem on a graph. By defining an optimality criterion, the combination of desirable releases from the reservoirs or operating policy is determined. To minimize the possibility of premature convergence to a local optimum, a combination of Pheromone Re-Initiation (PRI) and Partial Path Replacement (PPR) mechanisms are presented and their effects have been tested in a benchmark, nonlinear, and multimodal mathematical function. The finalized model is then applied to develop an optimum operating policy for a single reservoir and a benchmark four-reservoir operation problem. Integration of these mechanisms improves the final result, as well as initial and final rate of convergence. In the benchmark Ackley function minimization problem, after 410 iterations, PRI mechanism improved the final solution by 97 percent and the combination of PRI and PPR mechanisms reduced final result to global optimum. As expected in the single-reservoir problem, with a continuous search space, a nonlinear programming (NLP) approach performed better than ACOAs employing a discretized search space on the decision variable (reservoir release). As the complexity of the system increases, the definition of an appropriate heuristic function becomes more and more difficult; this may provide wrong initial sight or vision to the ants. By assigning a minimum weight to the exploitation term in a transition rule, the best result is obtained. In a benchmark 4-reservoir problem, a very low standard deviation is achieved for 10 different runs and it is considered as an indication of low diversity of the results. In 2 out of 10 runs, the global optimal solution is obtained, where in the other 8 runs; results are as close as 99.8 percent of the global solution. Results and execution time compare well with those of well developed genetic algorithms (GAs).*

Key words: *Ant colony; Optimization; Reservoir operation; Multi-reservoir*

Introduction

Ant algorithms as metaheuristic algorithms were first proposed by Dorigo (1992) and Dorigo et al. (1996) as a multi-agent approach to different combinatorial optimization problems like the traveling salesman and the quadratic assignment problem. Later Dorigo and Di Caro (1999) introduced a general ant colony optimization (ACO) algorithm, namely ant colony metaheuristic, which enabled the algorithm to be applied to other engineering problems provided that the problem can be properly formulated. Recently, Dorigo et al. (2000) reported the successful application of ACO algorithms to a number of benchmark combinatorial optimization problems. Montgomery and Randall (2002a, 2002b) introduced several alternative pheromone applications in different problems. The application of ACO algorithms to water resources problems, however, is of quite recent origin. Abbaspour et al. (2001)

employed ACO algorithms to estimate hydraulic parameters of unsaturated soils. Maier et al. (2003) used ACO algorithms to find a near global optimal solution to a water distribution system, indicating that ACO algorithms may form an attractive alternative to genetic algorithms for optimal design of water distribution systems. Zecchin et al. (2003) compared the performance of the original ant system (Dorigo et al. 1996) with that of a min-max ant system, a modified version of the ant system proposed by Stützle and Hoos (1997a, 1997b), for optimization of water distribution networks. Simpson et al. (2001) discussed the selection of parameters employed in ant algorithms for optimizing pipe network systems. Jalali et al. (2004a) employed ACO algorithms to solve the problem of optimal reservoir operation. Also, Jalali et al. (2004b) introduced an improved version of the ACO algorithm in single reservoir operation optimization. They included pheromone promotion, explorer ants, and a local search technique in a standard ACO algorithm.

Application of ant algorithms to continuous optimization problems requires the transformation of a continuous search space to a discrete one by discretization of the continuous decision variable. Thus, the allowable continuous range of decision variables is discretized into a discrete set of allowable values and a search is then conducted over the resulting discrete search space for the optimum solution (Abbaspour et al. 2001). The ant algorithm has been shown to outperform other general purpose heuristic search algorithms including GAs for small-scale problems (Dorigo and Gambardella 1997). Performance of the method, however, deteriorates for problems of growing dimensions (Dorigo et al. 1996). For problems with a small search space, the algorithm is more likely to locate the optimal

solution before stagnation occurs. For large-scale problems, however, the method may encounter pre-mature convergence to a sub-optimal solution.

Optimum operation of multi-reservoir systems has received much attention during the last three decades (Becker and Yeh 1974; Mariño and Loaiciga 1985; Oliveira and Loucks 1997). Most of the works in this area have been dominated by dynamic programming (DP), due to the serial features of the multi-reservoir operation problem. The DP method, which is theoretically capable of locating the global optimal solution, suffers from the so-called curse of dimensionality and therefore, faces some serious limitations in large-scale real world multi-reservoir problems. Labadie (2004) presented a state-of-the-art review of the optimal operation of multi-reservoir systems with mathematical and heuristic optimization algorithms. He discussed some applications of genetic algorithms, artificial neural networks, and fuzzy-based approach to the multi-reservoir optimization problem; however, ACO algorithms were not included in the review.

Several DP-based methods such as incremental DP (IDP), discrete differential DP (DDDP), and IDP with successive approximation (IDPSA), have been developed to overcome the curse of dimensionality syndrome. In a recent paper, Mousavi and Karamouz (2003) introduced a computational improvement scheme by diagnosing infeasible combinations. Another approach applied to multi-reservoir systems employed an aggregation/decomposition (A/D) technique in which some kind of heuristic is used to build a suitable approximation model of the original DP model (Ponnambalam and Adams 1996).

Many algorithms have been developed to

tackle combinatorial optimization (CO) problems. These algorithms may be classified as either complete or approximate algorithms. In approximate methods one sacrifices the guarantee of finding optimal solutions for the sake of getting good solutions in a significantly reduced amount of time.

In the last 20 years, a new kind of approximate algorithm has been developed which basically tries to combine basic heuristic methods in higher level framework aiming to efficiently and effectively explore a search space. These methods are nowadays commonly called metaheuristics (Blum and Roli 2003). This class of algorithms includes, but is not restricted to, Ant Colony Optimization (ACO), Evolutionary Computation (EC) including Genetic Algorithm (GA), Iterated Local Search (ILS), Simulated Annealing (SA), and Tabu Search (TS) (Blum and Roli 2003).

There have been several applications of GAs to multi-reservoir operation problems (Esat and Hall 1994; Fahmy et al. 1994; Oliveira and Loucks 1997). Esat and Hall (1994) clearly demonstrated the advantages of GAs over standard dynamic programming techniques in terms of computational requirements. Recently, Wardlaw and Sharif (1999) applied GAs to four-reservoir system operation, concluding that algorithm with real value coding performs significantly faster than the one employs binary coding. They extended the formulation to a more complex ten-reservoir problem. Being at its early stages of development, Marriage Bees Optimization (MBO) metaheuristic algorithm was applied to a single reservoir operation problem with promising results (Bozorg Haddad and Afshar 2004).

In this paper, an adaptive refinement scheme

is incorporated into the original ACO algorithm to address the optimum operation of multi-reservoir systems. The proposed procedure employs Pheromone Re-Initiation (PRI) and Partial Path Replacement (PPR) offers an alternative method which might hopefully provide a remedy for the dimensionality problem inherent in DP. The model is successfully applied to a semi-benchmark multi-reservoir operation problem and the results for a single and multi-reservoir system are presented and compared with those of DDDP and well developed GA.

Ant Colony Optimization (ACO) Algorithms: General Aspects

An interesting and very important behavior of ant colonies is their foraging behavior, and in particular, their ability to find the shortest route between their nest and a food source, realizing that they are almost blind. The path taken by individual ants from the nest to the food source is essentially random (Dorigo et al. 1996). However, when they are traveling, ants deposit a substance called pheromone, forming a pheromone trail as an indirect means of communication. As more ants choose a path to follow, the pheromone on the path builds up, making it more attractive for other ants to follow.

In the ACO algorithm, artificial ants are permitted to release pheromone while developing a solution or after a solution has been fully developed, or both. As stated, the amount of pheromone deposited is made proportional to the goodness of the solution an artificial ant develops. Rapid drift of all ants toward the same part of the search space is avoided by employing the stochastic component of the choice decision policy and the numerous mechanisms such as

pheromone evaporation, explorer ants, and local search.

For the successful application of ACO algorithms to combinatorial optimization problems, one must project the problem on a graph. Consider a graph G= (D,L,C), in which D={d_i} is the set of decision points at which some decisions are to be made, L= {l_{ij}} is the set of options j =1…NC, at each decision point i=1…NT, and C={c_{ij}} is the set of costs associated with option L={l_{ij}}. A feasible path on the graph is called a solution $(\varphi)^k$ and the path with minimum cost is called the optimum solution $(\varphi*)^k$.

The transition rule used in the original ant system is defined as follows (Dorigo et al. 1996):

$$P_{ij}(k,t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum\limits_{j=1}^{J}[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}} & if \quad j \in N_k(i) \\ 0 & otherwise \end{cases}$$

(1)

where $P_{ij}(k,t)$ is the probability that ant k selects option l_{ij} for decision point i at iteration t; $\tau_{ij}(t)$ is the concentration of pheromone on arc (i,j) at iteration t; $\eta_{ij} = 1/c_{ij}$ is the heuristic value representing the cost of choosing option j at decision point i; $N_k(i)$ is the feasible neighborhood of ant k when located at decision point i; and α and β are two parameters that control the relative importance of the pheromone trail and heuristic value. The heuristic value η_{ij} is analogous to providing the ants with sight and is sometimes called visibility. This value, in static problems, is calculated once at the start of the algorithm and is not changed during the computation.

Let q be a random variable uniformly distributed over [0,1], and $q_0 \in [0,1]$ be a tunable parameter. The next node j that ant k chooses to go is (Dorigo and Gambardella 1997):

$$j = \begin{cases} \arg \max\limits_{l \in N_k(i)} \left\{ [\tau_{il}(t)]^{\alpha}[\eta_{il}]^{\beta} \right\} & if \quad q \leq q_0 \\ J & otherwise \end{cases}$$

(2)

where J is a value of a random variable selected according to the probability distribution of $P_{ij}(k,t)$ (Eq. 1). Equations (1) and (2) provide a probabilistic decision policy to be used by the ants to direct their search towards the optimal regions of the search space. The level of stochasticity in the policy and the strength of the updates in the pheromone trail determine the balance between the exploration of new points in the state-space and the exploitation of accumulated knowledge (Dorigo and Gambardella 1997). To simulate pheromone evaporation, the pheromone evaporation coefficient (ρ) is defined which enables greater exploration of the search space and minimizes the chance of premature convergence to sub-optimal solutions upon completion of a tour by all ants in the colony. The global trail updating is done as follows:

$$\tau_{ij}(t+1) = (1-\rho).\tau_{ij}(t) + \rho.\Delta\tau_{ij}(t)$$

(3)

where $\tau_{ij}(t+1)$ is the amount of pheromone trail on option j of the ith decision point at iteration $t+1$; $0 \leq \rho \leq 1$ is the coefficient representing the pheromone evaporation and $\Delta\tau_{ij}$ is the change in pheromone concentration associated with arc (i,j) at iteration t. The amount of pheromone τ_{ij} associated with arc (i,j) is intended to represent the learned desirability of choosing option j when at decision point i.

Various methods have been suggested for calculating the pheromone changes. The method used here was originally suggested by Dorigo and Gambardella (1997) in which

International Journal of Civil Engineerng. Vol. 5, No. 4, December 2007

287

only the ant which produced the globally best (gb) solution from the beginning of the trail is allowed to contribute to pheromone change:

$$\Delta\tau_{ij}(t) = \begin{cases} 1/G^{k^*_{gb}} & if \quad (i,j) \in \text{tour done by ant } k^*_{gb} \\ 0 & \text{otherwise} \end{cases}$$

(4)

where $G^{k^*_{gb}}$ is value of the objective function for ant k^*_{gb}, which is the ant with the best performance within the past total iterations.

Pheromone Re-initiation and Partial Path Replacement Algorithm

Realizing the possibility of premature convergence to a local optimum, or stagnation point, in original ACO algorithms, considerable research has been conducted to minimize this possibility. Even though Jalali et al. (2004b) employed a combination of explorer ants, local search, and the pheromone promotion techniques to minimize the possibility of the premature convergence syndrome, their results for 10 runs with different seeds revealed relatively high standard deviation, which may be considered as an indication of diversity of the results. To further reduce the possibility of premature convergence, an adaptive refinement procedure on pheromone concentration named "pheromone re-initiation" is proposed. In this algorithm, when the possibility of stagnation is increased (i.e., for a pre-defined number of iterations no improvement is achieved) pheromone concentrations in all paths are re-initialized by setting them equal to the initial value of τ_0. After pheromone re-initiation, the search continues as normal. Pheromone re-initiation assigns the same desirability for all paths to be followed by ants. Therefore, a new search space is generated which may

help jumping out of the stagnation point, hence improving the final results.

To further improve the best result and reduce the overall diversity of the final results for different runs, a "partial path replacement" (PPR) mechanism is also proposed and included in the algorithm. The PPR employed in the present algorithm is based on random displacement of some components of pairs of solutions in each iteration. To reduce the computational time, in each iteration, a number of ants are chosen and parts of their solutions are randomly displaced with those of the global best from the beginning of the trail (Fig. 1). A simple flow diagram of the proposed ACO algorithm with PRI and PPR mechanisms is depicted in Fig. 2. To test the effect of the adaptive pheromone re-initiation procedure along with PPR, two examples are considered. The first example is a single reservoir and the second one is a benchmark four-reservoir problem which has been studied by different researchers since 1968.

Model Application

a) Ackley Function

To test the performance of the proposed mechanisms, the well known continuous, multimodal, nonlinear Ackley function was selected as the first case example. General form of the function has been defined as (Gen and Cheng, 1997):

$$f(\underline{x}) = 20 + e - 20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)$$

$$-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)$$

(5)

$$-20 \le x_i \le 30 \quad (i=1,2,...,n)$$
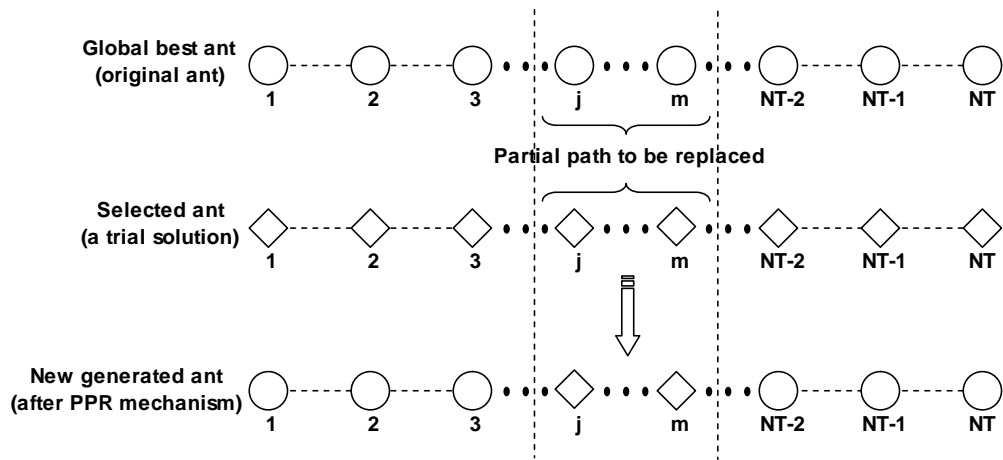
Due to existence of numerous local minimal

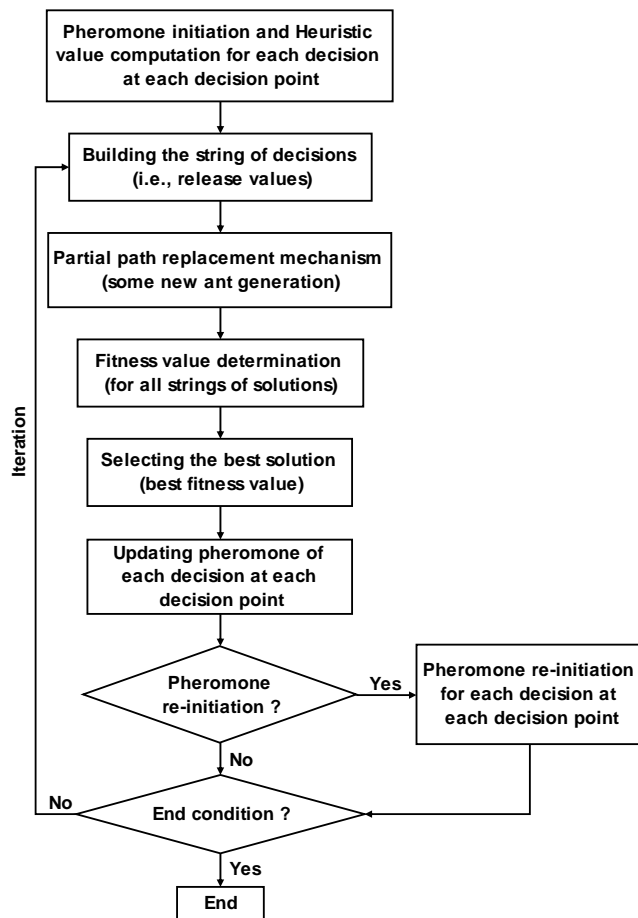**Fig. 1** Schematic of Partial Path Replacement (PPR) Mechanism

Fig. 2 Proposed ACO Algorithm with Partial Path Replacement and Pheromone Re-Initiation Mechanisms

Table 1 Statistical Measures of Ackley Function Resulting from 15 Different Runs of Standard and Proposed ACO.

Algorithm	Dimension	Mean	The Best	The Worst	S.D.*	C.V.**
Standard ACO	5	5.04	0.00	13.31	6.52	1.29
	10	15.63	10.78	18.55	2.08	0.13
	15	17.90	17.10	18.96	0.64	0.04
	20	18.02	16.30	18.53	0.66	0.04
	25	18.49	17.74	18.96	0.38	0.02
	30	18.65	18.10	19.13	0.37	0.02
ACO-PRI	5	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00
	15	0.00	0.00	0.00	0.00	0.00
	20	0.00	0.00	0.00	0.00	0.00
	25	6.27	0.00	12.00	5.44	0.87
	30	9.45	0.00	12.12	3.46	0.37
ACO-PRI-PPR	5	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00
	15	0.00	0.00	0.00	0.00	0.00
	20	0.00	0.00	0.00	0.00	0.00
	25	0.00	0.00	0.00	0.00	0.00
	30	0.00	0.00	0.00	0.00	0.00

* Standard deviation

** Coefficient of Variation

points, hill climbing based explicit optimization methods may trap in one of the local optimum points. On the other hand, search based methods of optimization, such as proposed approach, may explore wider search space leading to better solutions. It is quite clear that the global optimal of the Ackley function has an objective function of zero with all variables being equal to zero, too.

Ackley function with different dimensions, ranging from 5 to 30, was optimized using standard ACO with and without the proposed mechanisms. Feasible range of decision variable was uniformly discretized into 11 classes. Statistical measures for 15 different runs with 1000 iterations and 100 ants are presented in Table 1. As is clear, the proposed mechanisms have significantly improved the

quality of the final solution. Specifically, the fully developed algorithm has been converged to the global optimum for all 15 trial runs and all function ranging from 5 to 30 in dimensions. Graphical presentation of the effect of different proposed mechanisms in solution convergence is presented in Fig.3. Inclusion of the proposed mechanisms has significantly improved the rate of convergence as well as improving the quality of final solution.

b) Optimal Operating Policy for a Single Reservoir

To illustrate the performance of the proposed and tested algorithm, a real world single reservoir problem was selected. Dez reservoir is located in southern province of Khoozestan in Iran. Detail information and the data used in this study are available from
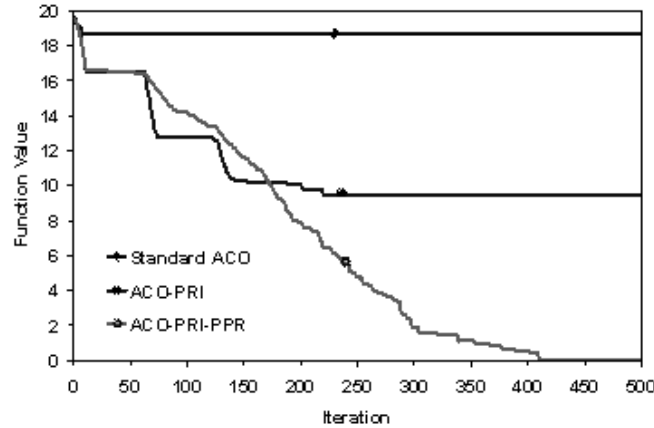
Fig. 3 Effect of Different Improving Mechanisms on the Rate of Convergence to Optimal Solution (Averaged Over 15 Runs for 30 Dimensional Ackley Function)

the authors and may be found elsewhere (Jalali 2005). To develop an optimal operating rule, the following general form was considered:

$$R(y,m)/R_{max} = a(m)+b(m).S(y,m)/S_{max}$$
$$+ c(m).I(y,m)/I_{max} \quad y=1...NY, m=1...12 \quad (6)$$

where $R(y,m)$, $S(y,m)$, and $I(y,m)$ define the release, storage volume, and inflow to the reservoir during year 'y' and month 'm', respectively. To normalize the inflow, release, and storage, values of maximum storage (S_{max}), maximum inflow (I_{max}), and maximum release (R_{max}) were used. Values of operating parameters, $a(m)$, $b(m)$, and $c(m)$ as decision variables may now be bounded between zero and one.

To apply ACO algorithms to a specific problem, several steps must be taken. Problem representation as a graph, assigning a heuristic information to each path, defining a fitness function, and selection of an ACO algorithm are the most important steps (Cordon et al. 2002).

The objective function is to minimize the total square deviation (TSD) from the periodic target demand and expressed as:

$$\underset{R(y,m)}{Min} \ TSD = \sum_{y=1}^{NY}\sum_{m=1}^{12}\left\{[R(y,m)-D(m)]/D_{max}\right\}^{2} \quad (7)$$

subject to:

$$S(y,m+1)=S(y,m)+Y(y,m)-R(y,m)$$
$$-LOSS(y,m) \qquad \forall \ y,m \qquad (8a)$$

$$R(y,m)/R_{max}= a(m)+b(m).S(y,m)/S_{max}$$
$$+ c(m).I(y,m)/Imax \qquad \forall \ y,m \qquad (8b)$$

$$S_{min} \leqq S(y,m) \leqq S_{max} \qquad \forall \ y,m \qquad (8c)$$

where $D(m)$ is the demand at month m, D_{max} is the maximum periodic demand, S_i is the reservoir storage volume at time period i, and $LOSS(y,m)$ is the loss (e.g., evaporation) at year y and month m.

The problem so formulated was solved defining 36 decision variables which were uniformly discretized into 11 classes with $\beta =0$ and $\alpha =1$.

The proposed complete algorithm, which now benefits from pheromone re-initiation, partial path replacement, and pheromone updating as defined by Jalali et al. (2004b), was applied to the above single reservoir problem with 60 periods (5 years).

International Journal of Civil Engineerng. Vol. 5, No. 4, December 2007

291

Table 2 Effect of Pheromone Evaporation Rate (ρ) on the Statistical Measures of the Solutions.

Parameter	ρ				
	0.10	0.25	0.50	0.75	0.90
Mean	1.082	1.104	1.078	1.066	1.091
The Best	1.033	1.013	1.011	1.014	1.027
The Worst	1.151	1.296	1.162	1.134	1.230
S.D.	0.046	0.085	0.051	0.046	0.067
C.V.	0.043	0.077	0.047	0.043	0.062

Table 3 Effect of Exploitation Probability (q_0) on the Statistical Measures of the Solutions.

Parameter	q_0				
	0.00	0.25	0.50	0.75	0.90
Mean	1.362	1.356	1.230	1.174	1.066
The Best	1.123	1.128	1.040	1.072	1.014
The Worst	1.858	1.795	1.513	1.419	1.134
S.D.	0.241	0.242	0.142	0.109	0.046
C.V.	0.177	0.179	0.115	0.093	0.043

Table 4 Statistical Measures of 10 Test Runs for Optimum Operating Policy Model.

Parameter	No. of Periods	
	60	480
Mean	1.036	56.394
The Best	0.998	53.441
The Worst	1.144	61.649
S.D.	0.046	2.535
C.V.	0.044	0.045

To select the best values for ρ and q_0, a sensitivity analysis with 100 ants and 1000 iterations were conducted. Results of the analysis are presented in Table 2 and 3. Reduced values of q_0 increase the effect of exploration, causing higher coefficient of variation and solution diversities for 10 test runs. To examine the performance of the algorithm to develop an optimal operating policy for an extended period of 480 months, values of 0.75 and 0.9 for ρ and q_0 were selected, respectively. To examine the effect of number (or percentage) of ants contributing in PPR mechanism, values of objective function and execution time for 0, 25, 50, 75, and 100 percent were tested. Results are presented in Fig. 4, in which

execution time increases more or less linearly with the number of ants selected in PPR mechanism. Even though the value of objective function improves as the number (or percentage) of ants in PPR mechanism, yet, the improvement is not significant when it exceeds from 50 percent. Therefore, the long term operating policy problem for 480 periods was solved using ρ =0.75, q_0=0.9 and 50 percent of population being permitted to contribute in PPR mechanism. Results of the study are presented in Table 4.

Rate of convergence to a near optimal solution for the best run and averaged over 10 test runs are presented in Figs. 5 and 6 for 60 and 480 periods, respectively. To test the
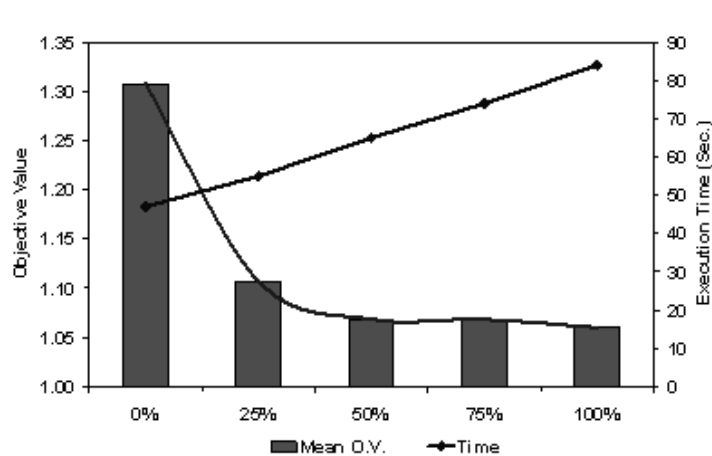
Fig. 4 Variation of Objective Value (O.V.) and Execution Time for Different Percent of Ants Contributing in PPR Mechanism (Averaged over 10 Different Runs).
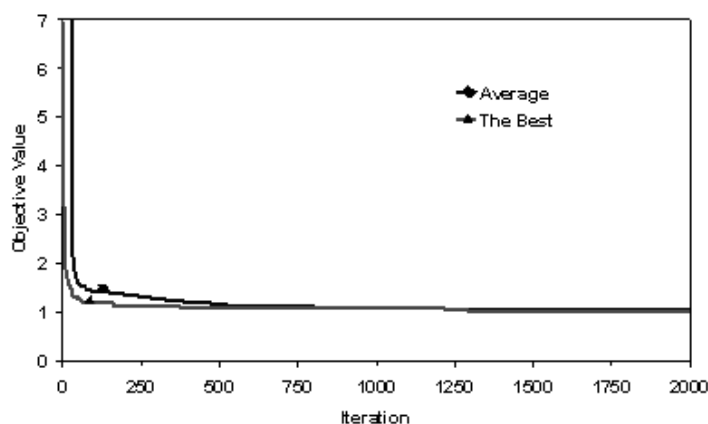

Fig. 5 Evolution of the Objective Function Value of the Optimum Operating Policy Problem Resulting from Proposed ACO Algorithm (60 Periods)
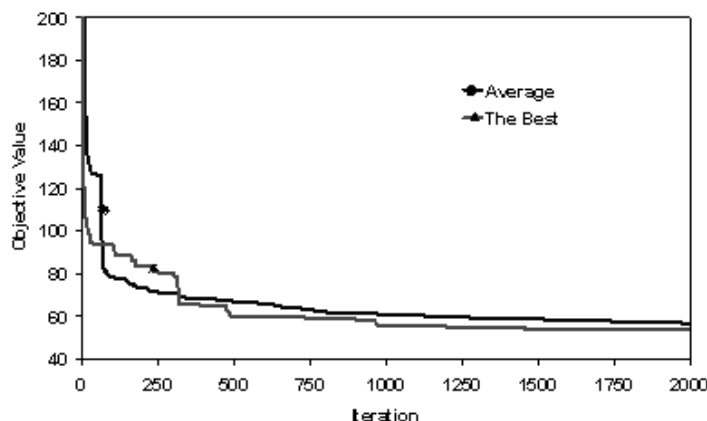

Fig. 6 Evolution of the Objective Function Value of the Optimum Operating Policy Problem Resulting from Proposed ACO Algorithm (480 Periods)

performance of two different policies developed with 60 and 480 periods, two 480 periods simulation model were run using operating parameters so developed. Values of 52.2 and 53.4 were resulted from simulation model for TSD with the operating parameters resulted from 60 and 480 period, respectively.
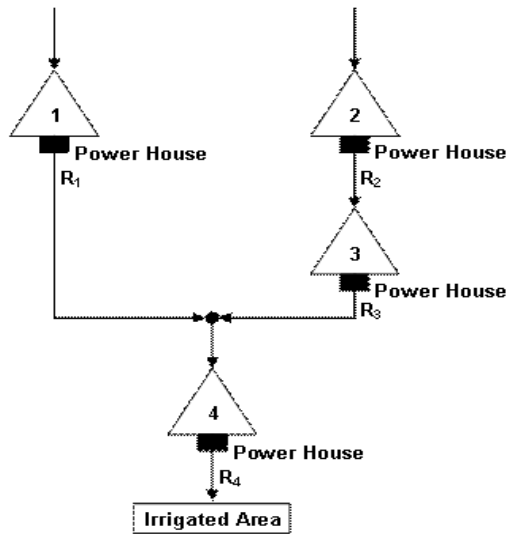
Fig. 7 Schematic of Four-Reservoir Problem

Four-Reservoir Problem

The four-reservoir problem was first formulated and solved by Larson (1968), and more recently by Esat and Hall (1994) and Wardlaw and Sharif (1999). This problem offered the opportunity to test the performance of ACOAs against a known global optimum and to perform sensitivity analysis. The system consists of four reservoirs, as shown in Fig. 7. The supplies from the system are used for hydropower generation and for irrigation. Hydropower generation is possible from each reservoir, and all discharges pass through the turbines. The outflow from reservoir four may be diverted for irrigation. Hydropower and irrigation benefits are quantified by linear functions of discharge. The objective is to maximize benefits from the system over 12 two-hour operating periods. There are inflows to the first and second reservoirs only, and these are 2 and 3 units, respectively, in all time periods. The initial storage in all reservoirs is 5 units. The fundamental constraints on reservoir storage are:

$$0.0 \le S_i^{(1)}, S_i^{(2)}, S_i^{(3)} \le 10 \qquad \forall\, i \qquad (9a)$$

$$0.0 \le S_i^{(4)} \le 15 \qquad \forall\, i \qquad (9b)$$

and on releases from the reservoirs through the turbines are:

$$0.0 \le R_i^{(1)} \le 3 \qquad \forall\, i \qquad (10a)$$

$$0.0 \le R_i^{(2)}, R_i^{(3)} \le 4 \quad \forall\, i \qquad (10b)$$

$$0.0 \le R_i^{(4)} \le 7 \qquad \forall\, i \qquad (10c)$$

The above constraints apply at all time steps. The continuity constraints for each reservoir over each operating period i are:

$$S_{i+1} = S_i + I_i + M.R_i \qquad \forall\, i \qquad (11)$$

where S_i is the vector of reservoir storages at time i in NR reservoirs; I_i is the vector of reservoir inflows in time period i to NR reservoirs; R_i is the vector of reservoir releases in time period i from NR reservoirs; and M is a $NR \times NR$ matrix of indices of reservoir connections:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix}$$

In addition to the above general constraints, there are target final storages for all reservoirs. There are five units for reservoirs one to three, and seven units for reservoir four. The objective function to be maximized can be written as:

$$\text{Max} \quad I = \sum_{n=1}^{4} \sum_{i=1}^{12} b_i^{(n)}.R_i^{(n)} + \sum_{i=1}^{12} b_i^{(5)}.R_i^{(4)} \quad (12)$$

The benefit function $b_i^{(n)}$ was tabulated by Larson (1968) and Heidari et al. (1971).

The original solution presented by Larson (1968) has been used for evaluation of the ACO approach discussed below. For the final target storages has been used a penalty function (Heidari et al. 1971). Taking the target ending storage in reservoir n to be $f^{(n)}$, the penalty function is expressed as

$$g^{(n)}[S_{13}^{(n)}, f^{(n)}] = -40[S_{13}^{(n)} - f^{(n)}]^2$$

for $\quad S_{13}^{(n)} \leq f^{(n)} \qquad n=1...NR \qquad (13)$
and

$$g^{(n)}[S_{13}^{(n)}, f^{(n)}] = 0$$

for $\quad S_{13}^{(n)} \geq f^{(n)} \qquad n=1...NR \qquad (14)$

The objective function is thus modified to

$$\begin{aligned}\text{Max} \quad I = &\sum_{n=1}^{4} \sum_{i=1}^{12} b_i^{(n)}.R_i^{(n)} + \sum_{i=1}^{12} b_i^{(5)}.R_i^{(4)} \\ &+ \sum_{n=1}^{4} g^{(n)}[S_{13}^{(n)}, f^{(n)}]\end{aligned} \quad (15)$$

The above semi-benchmark four-reservoir problem was solved by Wardlaw and Sharif (1999), employing a genetic algorithm. For the best combination of cross-over and mutation probabilities along with the best-fitted coding for the problem, the effect of different seeds was examined employing 750 generations and 100 population size. In 5 out of 10 runs, a global optimum of 401.3 units

was reported. Results of the other runs were as close as 99.8 percent of the global solution.

To solve a multi-reservoir problem with the proposed ACO algorithm, two minor modifications to the formulation used for the single reservoir model are needed. First, the period in each iteration must be extended to account for all reservoirs in the system. In other words, for a multireservoir system with NR reservoirs, and the total operation period of NT, the dimensions of the time-dependent variables (i.e., release and storage) must be defined with $NR \times NT$, resulting in a string with $NR \times NT$ cells or elements. The second modification must include the defined interrelation and/or interaction between the reservoirs in the system. In a typical multi-reservoir problem this interrelation may be well defined by the continuity equation between the reservoirs and the tributaries in the system (Eq. 11).

As the complexity of the system increases, definition of an appropriate heuristic function becomes more and more difficult. In fact, by defining an inappropriate heuristic function one may mislead the ants by providing them with wrong sight or vision. This was tested in the four-reservoir problem which is considered next. Realizing the objective function, one may define the heuristic function for the problem under consideration as follows:

$$\eta_{ij} = (BF)_i.r_j \quad i=1,...,NR \times NT, \ j=1,...,NC \quad (16)$$

in which BF_i is the benefit function for cell or element i in a trial solution string. As an example, referring to Heidari et al. (1971), BF_i for $i=1...12$ and BF_i for $i=13...24$ associate with $b_k^{(1)}$ and $b_k^{(2)}$ ($k=1,...,12$) for reservoir numbers 1 and 2. The heuristic
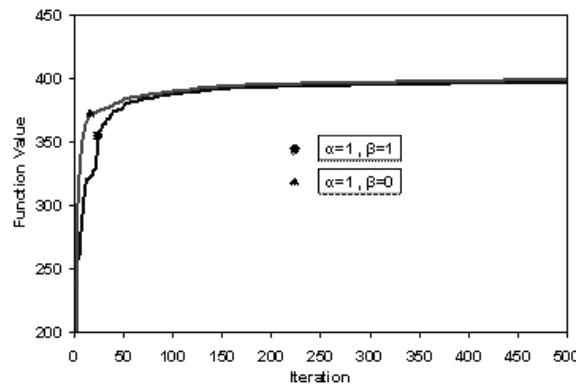
Fig. 8 Evolution of the Objective Function Value of the Four-Reservoir Problem Resulting from Proposed ACO Algorithm for Different Heuristic Value Amplification Options (Averaged Over 10 Different Runs)

values must be determined prior to the start of iterations for all paths and periods; hence it is not possible to include penalty terms in the heuristic value's determination. The average obtained for 10 different runs with the proposed ACO algorithm for $\alpha = 1$ and $\beta = 1$, employing the heuristic functions defined by Eq. (16), is presented in Table 2. Specifically, the best and the average for 10 runs with 200 ants and 1,000 iterations were found to be 398.1 and 396.9, respectively. It is worth mentioning that the global optimum for the problem has been reported as 401.3, which is 0.8 percent better than the best result obtained here.

Realizing the constraints on the maximum releases, interaction of the reservoirs' releases with the overall system performances, as well as the problems with the inclusion of the penalty terms in the heuristic function, it may be desirable to decrease the rate of contribution of the heuristic value on the transition rule, hence decreasing the weight of exploration in the final results. In fact, this idea was tested by assigning $\beta = 0$ in 10 different runs with the same 200 ants and 1,000 iterations. The average result is presented in Fig. 8. The best and the average results were determined to be 401.3 and 400.6, respectively (Table 2). In fact, in 2 out of 10 runs, the global optimum

of 401.3 was obtained. Average values of the objective function for 10 runs, along with the standard deviation of the results are presented in Table 2. The best and worst results are also given in the same table. Clearly, the average value of the objective function for 10 different runs (i.e., 400.6) is 99.8 percent of the global optimum (i.e., 401.3, presented by Larson 1968). The rate of the convergence of the results (average for 10 runs) is depicted in Fig. 8. It is interesting to note that with this algorithm an alternative global optimal solution to the solution of Larson (1968) has been obtained. To appreciate the differences, releases proposed by Larson (1968) and those of the present model are presented in Fig. 9. It can be noted that all scheduled releases are the same, except those of periods 10 and 11 from reservoir number 2. Regardless of these differences in release schedule, the same value of 401.3 for the objective function is obtained.

Concluding Remarks

Realizing the possibility of pre-mature convergence to a local optimum, or stagnation point, in original ACO algorithms, considerable research has been conducted to minimize this possibility. In this paper to
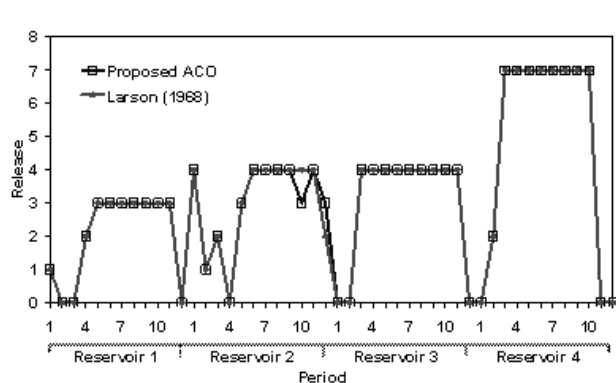
Fig. 9 Comparison between the best result of proposed ACO and result of Larson (1968) in four-reservoir problem (Data adapted from Larson 1968)

reduce the chance of premature convergence, an adaptive refinement procedure on pheromone concentration called pheromone re-initiation, along with a partial path replacement mechanism proved to be efficient. These mechanisms improve the performance of the standard ACO algorithm significantly. The improvement includes the final result, as well as initial and final rates of convergence. In the benchmark Ackley function minimization problem, after 410 iterations, PRI mechanism improved the final solution by 97 percent and the combination of PRI and PPR mechanisms reduced final result to global optimum. As the complexity of the system increases, a definition of an appropriate heuristic function becomes more and more difficult; this may lead the ants to a less desirable direction by providing them with wrong sight or vision. As an example, in the optimal operation of a multi-reservoir problem, a minimum weight assignment to the exploration item in the transition rule may end up with better results. Pheromone re-initiation assigns the same desirability for all paths to be followed by ants. Therefore, a new search space is generated which helps jumping out of the stagnation point. Integrating pheromone re-initiation and partial path replacement was shown to be quite effective in developing a near-optimal solution in the multi-reservoir operation problem. Application of the proposed

algorithm to a benchmark mathematical function as well as a benchmark four-reservoir problem provided very promising results. The algorithm also proved to be very efficient in developing optimal operating policy for long periods. In four-reservoir problem, even though all the solutions could have been converged to a possible large flat plateau of the response surface, yet convergence of 2 out of 10 solutions to the global optimum may be considered as an indication of convergence to more or less the best option. A very low standard deviation obtained for different runs could be considered as an indication of low diversity in the result. Results of the model compare well with global optimal and well-developed GA solutions.

Original ACO algorithms have been developed for discrete optimization, hence being quite suitable for discrete search spaces. Their application to continuous search space problems, such as reservoir operation may introduce discretization error. Improving the performance of ACOAs in a continuous search space is an ongoing research employing multi-colony approach.

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Notation

The following symbols are used in this paper:

BF_i = benefit function for cell or element i in a trial solution string.
C = set of costs associated with the options $\{c_{ij}\}$.
D = set of decision points $\{d_i\}$.
$D(m)$ = demand at month m.
D_{max} = maximum periodic demand.
G = Graph (D, L, C).
$'G^{k^*_{gb}}$ = value of the objective function for the ant with the best performance within the past total iterations.
I_i = inflow vector at time period i.
I_{max} = maximum inflow.
$I(y,m)$ = inflow to reservoir at year y and month m.
L = set of options $\{l_{ij}\}$.
$LOSS(y,m)$ = loss (e.g., evaporation) at year y and month m.
NC = number of release intervals(or classes)
NR = number of reservoirs
NT = number of time periods.
NY = number of years.
R_i = release vector at time period i.
R_{max} = maximum release.
$R(y,m)$ = release at year y and month m.
S_i = reservoir storage volume vector at time period i.
S_{max} = maximum storage allowed.
S_{min} = minimum storage allowed.
$S(y,m)$ = reservoir storage volume at year y and month m.
TSD = total square deviation from target demand.

α,β = parameters that control the relative importance of the pheromone trail versus a heuristic value.

η_{ij} = heuristic value representing the cost of choosing option j at decision point i.

ρ = pheromone evaporation coefficient.

τ_0 = initial value of pheromone.

$\tau_{ij}(t)$ = concentration of pheromone on arc (i,j) at iteration t.

$b_i^{(n)}$ = benefit coefficients of reservoir n at time period i

k^*_{gb} = ant with the best performance within the past total iterations.

$P_{ij}(k,t)$ = probability that ant k selects option l_{ij} for decision point i at iteration t.

q = random variable uniformly distributed over [0, 1].

q_0 = tunable parameter $\in$ [0, 1].