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Investigation of experimental and analytical shear strength of reinforced concrete deep beams

Abolfazl Arabzadeh[1,*], **Reza Aghayari**[2], **Ali Reza Rahai**[3]

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Abstract

An experimental-analytical investigation was conducted to study the behavior of high-strength RC deep beams; a total of sixteen reinforced concrete deep beams with compressive strength in range of 59 MPa $\leq f'_c \leq$ *65 MPa were tested under two-point top loading. The shear span-to-effective depth ratio a/d was 1.10; all the specimens were simply supported and reinforced by vertical, horizontal and orthogonal steel bars in various arrangements. The test specimens were composed of five series based on their arrangement of shear reinforcing. The general behavior of tested beams was investigated. Observations were made on mid-span and loading point deflections, cracks form, failure modes and shear strengths. The test results indicated that both vertical and horizontal web reinforcement are efficient in shear capacity of deep beams, also the orthogonal shear reinforcement was the most efficient when placed perpendicular to major axis of diagonal crack. Concentrating of shear reinforcement within middle region of shear span can improve the ultimate shear strength of deep beam. The test results were then compared with the predicted ultimate strengths using the ACI 318-08 provisions; ACI code tended to either unsafe or scattered results. The performed investigations deduced that the ACI code provisions need to be revised.*

Keywords: Deep beams, Strut-and-tie, Shear strength, Shear reinforcement.

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1. Introduction

The reinforced concrete deep beams have become an important structural elements having small span-to-depth ratio. The investigation of their behavior is a subject of considerable interest in RC structures researches.

In deep beams, according to shear span-to-depth ratio and web reinforcement the ultimate strength is generally controlled by shear rather than flexure, if the sufficient amount of longitudinal reinforcement is used. Several different failure modes have been identified from experimental studies, due to variability in failure, the determination of their shear strength and identification of failure mechanism are very complicated [1].

Strut-and-tie method is one of the most simple and applicable methods which can be used to simplify analysis and design of deep beams. In the research programs the influence of effective parameters on the behavior of deep beams was investigated. Siao [2] investigated the shear strength of short RC walls, corbels and deep beams. Most of reported studies are based on the analysis of experimental results using strut-and-tie model (STM). STM was introduced by Ritter [3] and then is developed by Morsch [3], Scaalich & Schafer [4], Marti [5], Mitchell-Collins [6], Rameriz [7]. The published papers of Tan et al [8], Kong et al [9], Oh [10] and Shin [11] are focused on effect of web reinforcing and their arrangement. In 2001 Arabzadeh proposed a developed truss model [12] to predict the shear capacity of RC deep beams. The Canadian standard CSA [13] provided an approach based on Modified Compression Field Theory (MCFT) and strut-and-tie model. Moreover, in the new versions of ACI code [14][15], the strut-and-tie model is added in general design provisions. The current approaches for analysis or design of deep beams consist of rational or semi-rational models which explained in some codes, is applicable for normal strength concrete (NSC) but their application should be developed for high-strength concrete (HSC).

A number of significant or impressive parameters on shear behavior of deep beams have been identified; including concrete compressive strength, span-to-depth ratio, amount and arrangement of shear reinforcement and amount of main

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[*] *Corresponding Author: arabzade@modares.ac.ir*

[1] *Assistant Professor of Structural Engineering Dept., Tarbiat Modarres University, Tehran, IRAN*

[2] *Assistant Professor of Civil Engineering Dept, Razi University, Kermanshah, IRAN*

[3] *Professor of the Faculty of Civil Engineering, Amir Kabir University of Technology, IRAN*

reinforcement. Several experimental studies were performed to investigate the efficiency of web reinforcement in shear behavior of deep beams. The objective of current study is to evaluate behavior and shear strength of RC deep beams and also to investigate the accuracy of ACI code formulas. Test variables are amount and arrangement of web reinforcement.

2. Experimental program

2.1. Specimen details

Test specimens consisted of sixteen simply supported concrete deep beams with different properties. They were classified in four series according to type of their web reinforcing:

Series A consist of six deep beams with variable vertical steel bars and uniform spacing (Fig.1).

Series B consist of three deep beams with variable vertical steel bars concentrated at center of shear span (Fig.1).

Series C consist of four deep beams reinforced by both variable horizontal and constant vertical web reinforcement (Fig.1).

Series D consist of three deep beams reinforced by diagonal steel bars which are placed perpendicular to diagonal cracks (Fig.1).

All specimens had a rectangular cross-section with 80×400 mm2. their overall and effective spans wee 1600 mm and 1200 mm, respectively. Fig.1 and Table.1 gives the additional details of specimens.

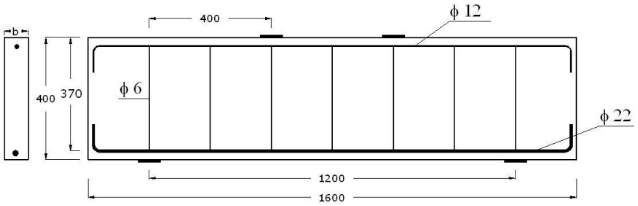


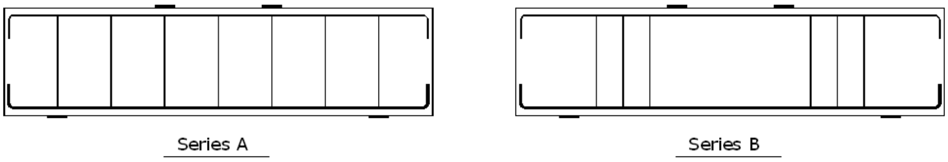
Fig. 1. a. Dimension of specimens

2.2. Material properties

The longitudinal steel reinforcements consist of 12D (12 mm diameter), 22D (22 mm dia.) and 25D (25 mm dia.) deformed steel bars, and also steel shear reinforcement include 6D (6 mm dia.) smooth round bars as indicated in Table.2. The concrete was prepared by Type II Portland cement and river fine aggregate. Maximum aggregate size was 12.5 mm (1/2 in) and the slump was approximately 90 mm. The concrete strength was defined based on the average value of three standard cylinders (300×150 mm).

2.3. Test setup and loading

The hardened specimens were white-washed to observe explicitly the cracks and failure throughout the tests. The beams were tested in setup as shown in Fig.2. All specimens were simply supported by using restrained and free roller and were loaded on the top face. The load was applied through a hydraulic jack to the center of a strong girder and divided to two-symmetric loads via its supports located on top face of specimens. The applied load and the reaction forces of specimens were distributed on top and bottom surfaces of beams through rectangular 130×80 mm^2 steel plates.


Series A

Series B

Fig. 1. b. The typical schema of specimens

Table 1. the tested specimens characteristics Reinforcement

			Reinforcement									
Series	ID	f_c'	Bottom		Top		Vertical		Horizontal		Inclined	
		MPa	No.	ρ'_s(%)	No.	ρ_s(%)	No.	ρ_v(%)	No.	ρ_h(%)	No.	ρ_i(%)
A	A-1	59	1-12D	0.40	1-22D	1.32	—	—	—	—	—	—
	A-2	60	1-12D	0.40	1-22D	1.32	6-6D	0.18	—	—	—	—
	A-3	61	1-12D	0.40	1-22D	1.32	10-6D	0.29	—	—	—	—
	A-4	60	1-12D	0.40	1-22D	1.32	16-6D	0.47	—	—	—	—
	A-5	65	1-12D	0.40	1-22D	1.32	21-6D	0.62	—	—	—	—
	A-6	60	1-12D	0.40	1-25D	1.66	28-6D	0.82	—	—	—	—
B	B-1	62.5	1-12D	0.40	1-25D	1.66	6-6D	0.18	—	—	—	—
	B-2	59	1-12D	0.40	1-22D	1.32	10-6D	0.29	—	—	—	—
	B-3	58	1-12D	0.40	1-22D	1.32	16-6D	0.47	—	—	—	—
C	C-1	58	1-12D	0.40	1-22D	1.32	10-6D	0.29	1-6D	0.10	—	—
	C-2	60	1-12D	0.40	1-22D	1.32	10-6D	0.29	2-6D	0.20	—	—
	C-3	60	1-12D	0.40	1-22D	1.32	10-6D	0.29	3-6D	0.30	—	—
	C-4	58	1-12D	0.40	1-22D	1.32	10-6D	0.29	4-6D	0.40	—	—
D	D-1	61	1-12D	0.40	1-22D	1.32	—	—	—	—	6-6D	0.42
	D-2	60	1-12D	0.40	1-22D	1.32	—	—	—	—	10-6D	0.70
	D-3	60	1-12D	0.40	1-22D	1.32	—	—	—	—	16-6D	1.00

Table 2. mechanical properties of used steel bars

Bar ID.	ε_y	ε_u	f_y (**MPa**)	f_u (**MPa**)	E (**MPa**)
25D	0.0027	0.271	577	577	214000
22D	0.0028	0.263	585	589	206000
12D	0.0021	0.200	433	491	208000
6D	0.0020	0.097	397	469	201000

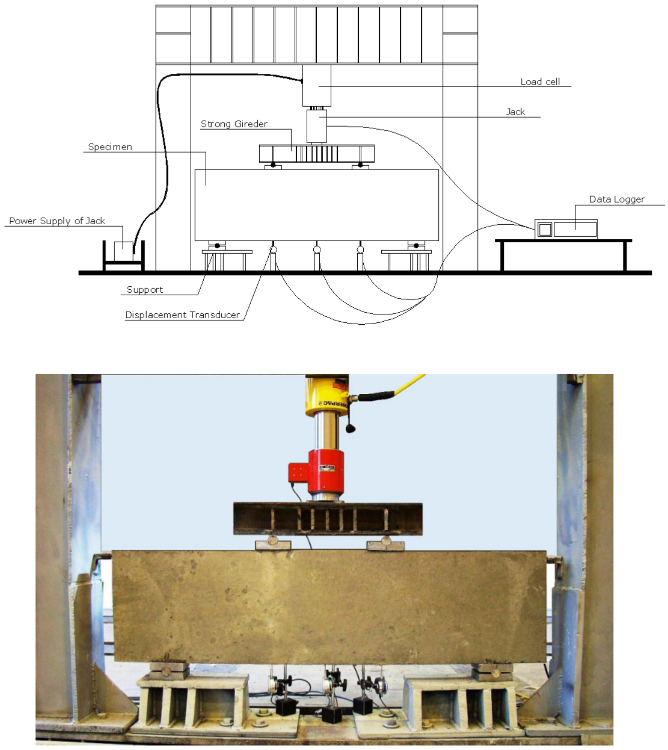


Fig. 2. Testing setup

3. Evaluation of test results

3.1. General behavior

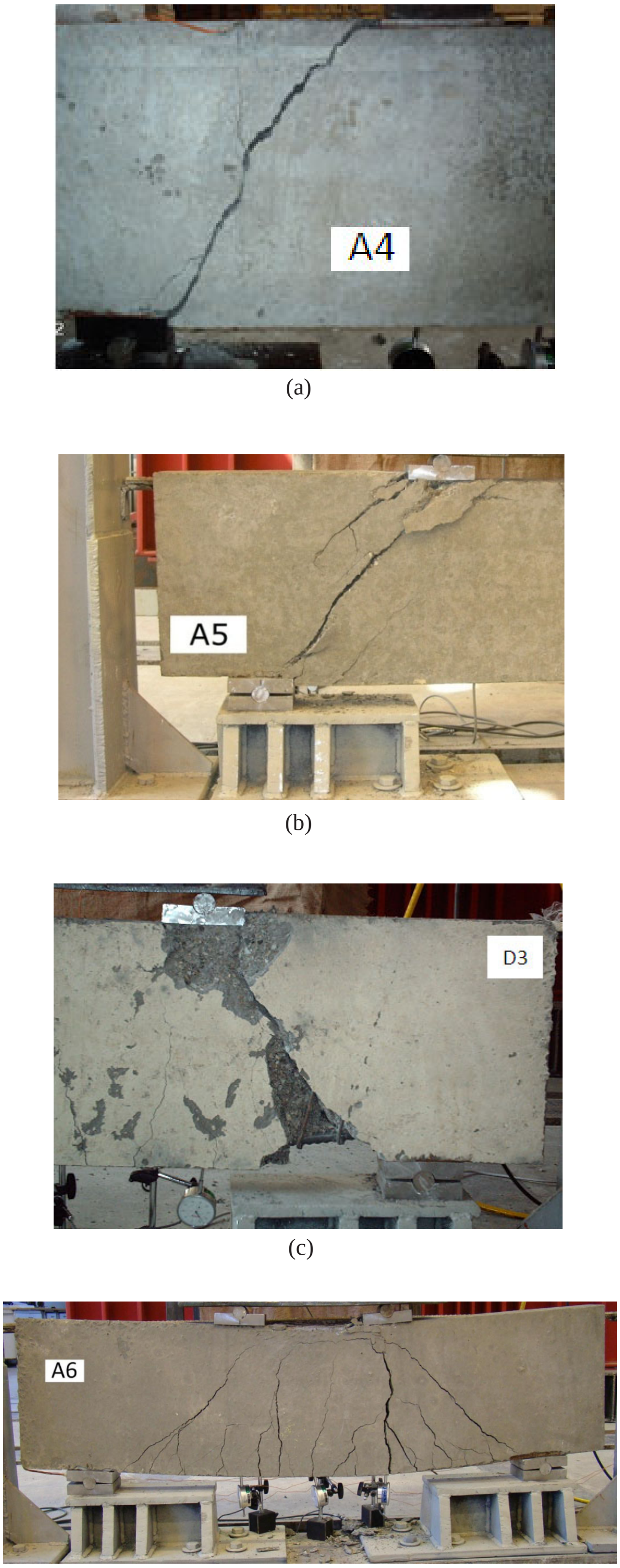
All the beam specimens showed a same response up to failure. In the early steps of loading, few vertical flexural cracks formed in the pure-bending region. As the load increased approximately to 30-50% ultimate load, generally the diagonal cracks appeared at the mid-height of beam within the clear shear span in the direction of the main strut and propagated rapidly toward the outside edge of the loaded point and the inside edge of the support. While the diagonal cracks were developing across length, their widths were propagating in the center of shear span. Failure for all specimens was brittle and their failing mechanism is identified as follows:

a) Diagonal splitting along the direction of main strut (Fig.3.a).

b) Strut crushing failure due to forming of several parallel diagonal cracks (Fig.3.b).

c) Shear-compression near the support or loading point, this type of failure only observed in A-1 specimen (without shear reinforcement) after forming of diagonal cracks at mid-height of the beam and propagating toward the supports or loading points (Fig.3.c).

d) Shear-flexure failure, only the specimen A-6 failed due to excessive opening of propagated flexural cracks and yielding of main longitudinal reinforcement (Fig.3.d).



(a)

(b)

(c)

(d)

Fig. 3. The various failure of specimens

3.2. Effect of shear reinforcement

Fig.4 shows the relationship between measured shear strength V_u and the ratio of shear reinforcement for all tested specimens of series A, B, C and D. As shown in Fig.4, by increasing the ratio of shear reinforcement from 0 to 0.84 percent, the ultimate shear strength tends to increase about 80 percent. On the other hand, in specimens of series D, the shear reinforcement was more efficient than those of other series with same amount of shear reinforcement; it means that, when the web orthogonal reinforcing is placed perpendicular to direction of main strut, its efficiency on the ultimate shear strength is more significant than when it is placed in vertical or horizontal direction. Moreover, because of high concrete confinement next to supports and loading points, the initial cracks form at the mid-height of beam within the shear span. Consequently, concentrating the shear reinforcement at central region of shear span is more efficient than uniform spacing of them, but with the same amount of shear reinforcement, the shear strength for specimens of series B is greater than of series A.

Fig.5 shows variations of shear strength versus ratio of horizontal shear reinforcement for specimens with constant ratio of vertical reinforcement ρ_v=0.29% . It shows that the effect of horizontal shear reinforcement is less than vertical shear reinforcement. Moreover, at a shear span-to-effective depth ratio closer to 1.0 the resistance of horizontal shear reinforcement occurs due to dowel action, but this has a very small effect compared to effect of main longitudinal reinforcement.

3.3. Load-deflection curves

Fig.6 compares the mid-span load-deflection curve for specimens of series A, B, C and D. All specimens present a nearly linear behavior up to failure, with a rapid decrease in the initial stiffness at the appearance of major diagonal cracks.

Applied load decreased suddenly once attained the peak point due to increasing the shear distortions. In addition beyond the formation of major diagonal cracking, the beams with bigger shear reinforcement ratio behave stiffer than those reinforced with less web reinforcement. This means, before forming of diagonal cracks, shear reinforcement has not a considerable efficiency on beam stiffness, but beyond the formation of major diagonal cracks appear the improving effect of shear reinforcement on beam stiffness is dominant and increases with increasing the amount of web reinforcing.

3.4. Strain in longitudinal reinforcement

Three strain gauges were attached to the bottom steel bar of specimens, to investigate the variation of strain in flexural reinforcement. Those gauges were placed at mid-span, under left point load and left support. Fig.7 shows the variation of measured strain across the half of beam span for A-2, A-4 and A-6 specimens. The strain variation in longitudinal reinforcement was similar for all tested specimens and the formation of a tie-action is observed with a nearly uniform strain distribution within shear span. According to Fig. 7, as amount of shear reinforcement increases the strain of main tensile reinforcement raises due to increasing the ultimate capacity of beam, also due to moment redistribution toward out of the pure-bending region in A-6 specimen, against the other specimens, bottom longitudinal bar was subjected to considerable tensile strain at supports.

3.5. Strain in web reinforcement

Through the test of all specimens, strain of web steel bars was recorded via attached strain-gauges. Measurement and observation show the significant effectiveness of web reinforcing on preventing the cracks opening. The strain response of shear reinforcement for all beams was the same, and can be summarized to:

a. Strains of web reinforcement at middle region of shear span were higher than those which were near the supports or loading point.

b. Strains of horizontal web bars were lower than of vertical reinforcement.

c. Strain of shear reinforcements increases significantly beyond the formation of diagonal cracks.

d. Strain of shear reinforcement in specimens with small ratio

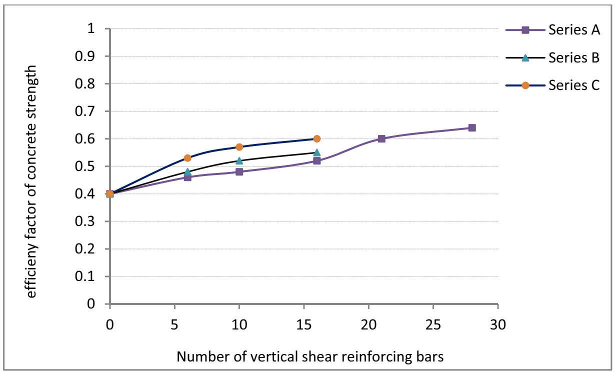


Fig. 4. Effect of shear reinforcement on efficiency factor of concrete strength

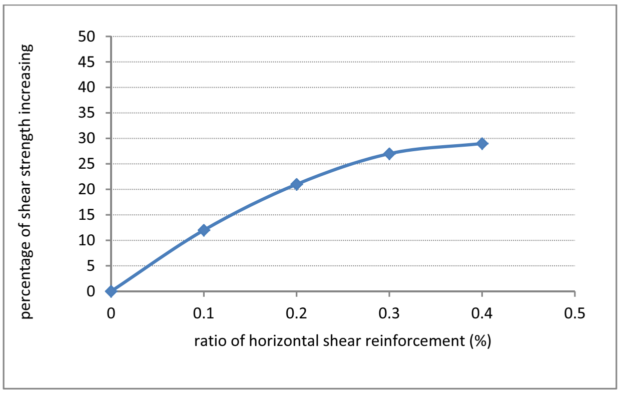
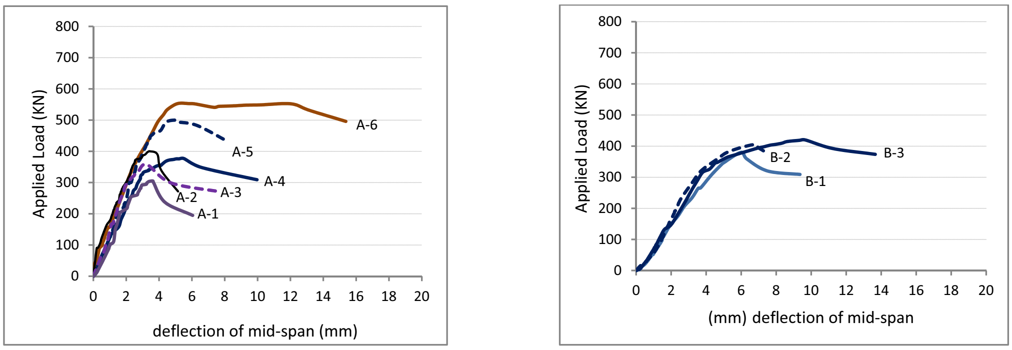
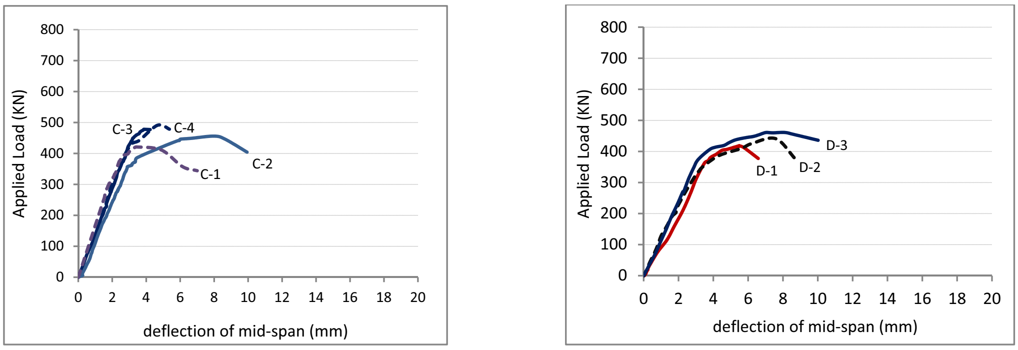


Fig. 5. Effect of horizontal shear reinforcement on shear strength



a. Series A *b. Series B*



c. Series C *d. Series D*

Fig. 6. Load-deflection curves of specimens for mid-span

of shear reinforcing were bigger than those which were reinforced heavily, as in A-5 and A-6 specimens, the mean tensile stress in web bars could not attain f_y (tensile yielding stress). Fig.8 shows a simply strain variation in shear reinforcement of A-4 and C-4 specimens.

4. Prediction the shear strength based on ACI 318-08 code

One of the principal purposes of current study is to evaluate the reliability the relations in appendix A of ACI 318-08 provided for strut-and-tie method. The nominal compressive strength of a concrete strut is taken as (ACI 318-08, Eq.(A-2)):

$$f_{ns}=f_{cu}A_c \qquad (1)$$

Where A_c is the smaller cross-sectional area of strut at both ends of that and f_{cu} is the effective compressive strength of concrete given by (ACI 318-08, Eq.(A-3))

$$f_{cu}=0.85\beta_s f_c' \qquad (2)$$

Where β_s=0.75 for a bottle-shape strut, if the minimum required reinforcing is satisfied, and β_s=0.60 for a bottle-shape strut, without the minimum required reinforcing. Also f_c' is the specified compressive strength of concrete.ACI 318-08 in appendix A provides two different efficiency factors, for bottle-shaped struts with sufficient reinforcement the higher coefficient is obtained. In section A.3.3 of ACI code, the sufficient steel reinforcing is required to resist against transverse tensile force occurred perpendicular to strut axis. Therefore ACI code assumes, the strut compression force is spreads out at a slope of 2:1 (two units along the axis of strut and one unit transverse to that axis)

$$\frac{F_D}{2} = C\cos\theta \qquad (3)$$

$$F=2C\sin\theta \qquad (4)$$

$$T = 2\frac{F_D}{2} = F/\tan\theta = F/m \qquad (5)$$

Where, C is the compressive force of diagonal concrete strut, F

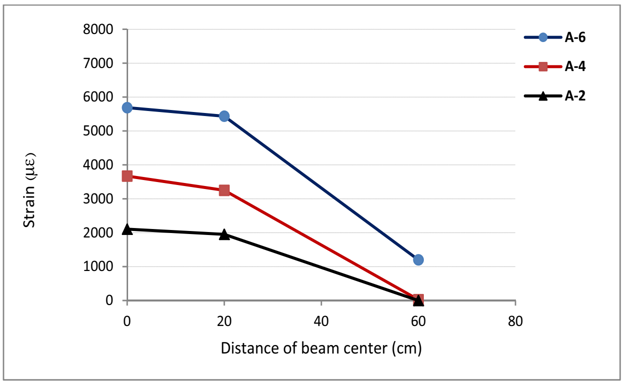


Fig. 7. Strain variation across length of beam

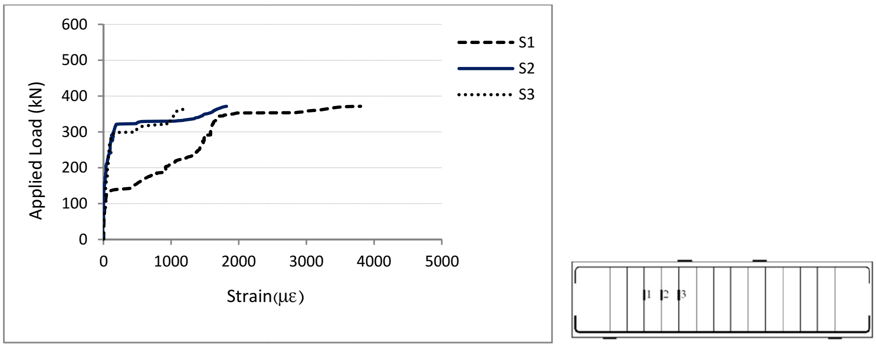


Fig. 8. a. Variation of strain in shear reinforcement of specimen A-4

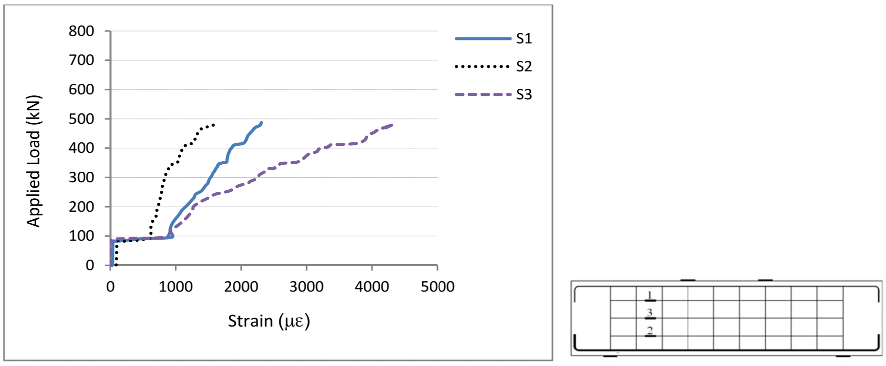


Fig. 8. b. Variation of strain in shear reinforcement of specimen C-4

is the applied compressive force, T is the total transverse tensile force and m is the slope of dispersion of compression force. Eq.5 presents the resulted tensile transverse force according to assumed slope for dispersion of compression. In Ref. [1] a new parameter was proposed namely ρ_D or "Equivalent perpendicular reinforcement ratio", this parameter is determined by Eq.6 as the components of the shear reinforcing perpendicular to the axis of splitting crack divided by the width of the beam and the length of diagonal concrete strut. Eq.7 presents the minimum requirement shear reinforcement by assuming the strut compression force spreads out at a slope of m.

$$\rho_D=\rho_h \sin^2\theta+\rho_v \cos^2\theta \tag{6}$$

$$\rho_{\mathrm{Dmin}}=\frac{T}{f_y b L_s} \tag{7}$$

Where ρ_h, ρ_v are the ratios of horizontal and vertical reinforcement respectively, θ is the angle between horizontal shear reinforcement and strut axis, ρ_D is the Equivalent perpendicular reinforcement ratio, b and L_s are the width of beam and length of concrete diagonal strut respectively, ρ_{Dmin} is the minimum requirement shear reinforcement and f_y is the yield strength of shear bars.

For members with a compressive strength f_c' not greater than 40 MPa the sufficient reinforcement can be satisfied using the relation as follows:

$$\sum\frac{A_{si}}{bS_i}\sin\alpha_i \geq 0.003 \tag{8}$$

Where, A_{si}, s_i, α_i are the area, spacing and inclination angle of i-th layer of transverse reinforcement crossing the strut axis, respectively, and b is the width of strut. In chapter 11 of ACI

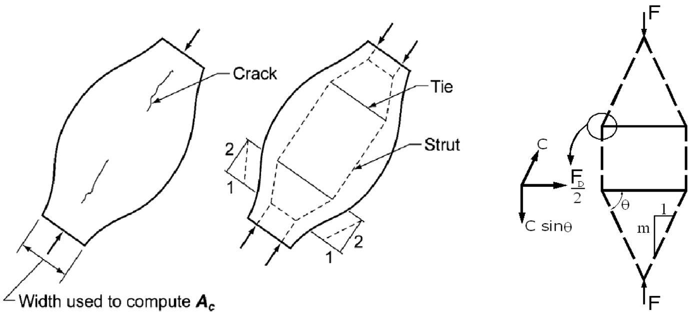


Fig. 9. Bottle-shaped strut: (a) cracking of a bottle-shaped strut; (b) strut-and-tie model of a bottle-shaped strut; (c) Equilibrium of simplified bottle-shaped strut

code a minimum shear reinforcement for deep beams is provided, those requirement ratios are 0.25 and 0.15 percent in the vertical and horizontal directions respectively, on the other hand for members with clear span-to-overall depth ratio smaller than four, which is defined as deep beams, it is provided that, the minimum required reinforcing satisfying section A3.3 shall be used instead of provided minimum vertical and horizontal reinforcing. It means that by removing the shear reinforcement for designing of deep beams, shear strength in deep beams decreases by only 20 percent, compared to those which have sufficient shear reinforcement. Such deep beams are not reliable because they attain failure just near the formation of splitting cracks.

5. Comparison between predicted shear strength with ACI code and actual data

In this section the provisions of ACI 318-08 are used to predict the shear strength of tested specimens. Fig.10 shows a correlation between calculated and experimental ultimate shear strength of specimens. As can be seen, the predicted strengths are higher than actual measured capacities. It is necessary to notice; the requirement reinforcement shall be calculated according to Eq.7, because the specified strength of concrete in all specimens is greater than 40 MPa.

The performed investigation presents that, Eq.7 generally provides heavier shear reinforcement in comparison with Eq.8, therefore based on provisions of ACI 318-08 the sufficient reinforcing cannot be satisfied and β_s or efficiency factor of concrete strength shall be taken as 0.60.

According to measured efficiency factor for tested deep beams, in all specimens except for A-5 and A-6 which by means of the expressions of ACI code predictions are unconservative, the efficiency factor was lower than 0.60. As discussed earlier, assuming the efficiency factor equals to 0.60 for member without any shear reinforcement will be unsafe specifically about concrete with strength greater than 40 MPa. For example, the efficiency factor for specimen A-1 with no shear reinforcement is measured as 0.35 which has a difference about 70 percent in comparison with provided value by ACI code.

The experimental observation proved that, if the web of deep beam is not reinforced or under-reinforced, the usual failing occurs suddenly without any cousin and controlled by splitting strength of concrete strut, but by using web reinforcing the used reinforcement can maintain equilibrium of concrete strut and consequently deep beam can present a ductile behavior as its shear strength will be controlled by crushing of concrete strut. It means that the efficiency factor of concrete strength depends on amount of shear reinforcing which rises by increasing shear reinforcements. Therefore it is proposed that the efficiency factor will be defined based on the levels of used shear reinforcement.

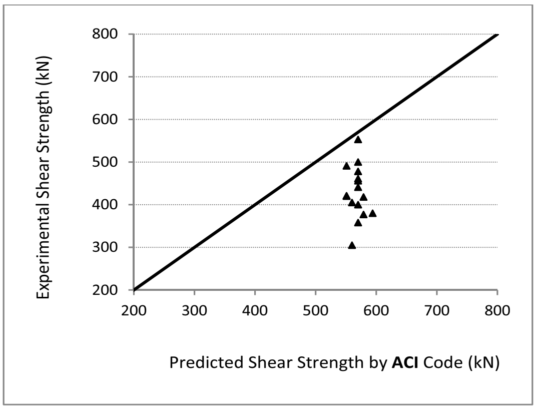


Fig. 10. Correlation between experimental and predicted ultimate strengths

6. Summery and conclusions

The behavior of sixteen tested deep beams was investigated and their ultimate capacities were determined using the provisions of ACI 318-08. According to performed study following conclusion can be made:

a. The shear reinforcement are subjected to variable strains depends on their location in the span of the beams. The steel bars placed in central zone of shear span t are subjected to higher strain than those placed near the support. Consequently it is probable that the shear reinforcement cannot attain their yielding force, it shall be considered in equilibrium conditions of strut-and-tie model.

b. The elastic flexural stiffness is independent of shear reinforcement and all specimens behave nearly linear, but beyond the formation of diagonal cracks a significant softening observed as a decreasing in the slope of load-deflection curve. The shear reinforcement can improve this softening by preventing the crack opening. As explained earlier, the observed decreasing in stiffness of specimens which is under-reinforced or is not reinforced is more significant than decrease in over-reinforced web beams.

c. Amount and arrangement of shear reinforcement are effective on ultimate strength of deep beams and the concentrated bars in central region of shear span have higher efficiency on strengthening of deep beams.

d. The horizontal shear reinforcement can improve the shear strength of reinforced concrete deep beams as well as the vertical reinforcement. The efficiency of horizontal reinforcing decreases due to increasing of shear span-to-depth ratio and can be vanished by setting inclination angle of strut, closer to 25.

e. The inclined shear bars becomes the most efficient when are placed perpendicular to diagonal cracks.

f. The provisions of ACI 318-08 presented scattered predictions. According to appendix A and chapter 11 of ACI 318-08 by decreasing efficiency factor by 20 percent, shear reinforcing can be neglected in deep beams designing. However, in high-strength concrete deep beams, and without sufficient reinforcement, the concrete strut of simplified strut-and-tie model is unable to present a ductile response and fails suddenly due to excessive cracking of concrete. This splitting failure becomes more evident as concrete strength increases.

g. ACI 318-08 defines the efficiency factor of concrete strength based on satisfying the minimum required shear reinforcement and used two levels of shear reinforcement for deep beams. According to experimental and analytical study of deep beams, the efficiency factor depends on amount of shear reinforcement deeply. Therefore the provisions of ACI code requires to be revised and the effective strength of concrete shall be defined as a function of concrete strength and equivalent ratio of shear reinforcement. Also for simplifying of expressions the efficiency factor can be determined based on the different levels of shear reinforcement.

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