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A comparison between lumped and distributed plasticity approaches in the pushover analysis results of a pc frame bridge

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Abstract

The seismic behavior of frame bridges is generally evaluated using nonlinear static analysis with different plasticity models; hence this paper tends to focus on the effectiveness of the two most common nonlinear modeling approaches comprising of concentrated and distributed plasticity models. A three-span prestressed concrete frame bridge in Tehran, Iran, including a pair of independent parallel bridge structures was selected as the model of the study. The parallel bridges were composed of identical decks with the total length of 215 meters supported on different regular and irregular substructures with non-prismatic piers. To calibrate the analytical modeling, a large-scale experimental and analytical seismic study on a two-span reinforced concrete bridge system carried out at the University of Nevada Reno was used. The comparison of the results shows the accuracy of analytical studies. In addition, close correlation between results obtained from two nonlinear modeling methods depicts that the lumped plasticity approach can be decisively considered as the useful tool for the nonlinear modeling of non-prismatic bridge piers with hollow sections due to its simple modeling assumption and less computational time.

Keywords: *Frame bridges, Pushover-based nonlinear static procedures, Discrete finite element models, Lumped plasticity approach, Distributed plasticity approach.*

1. Introduction

The extensive damage of bridges in major earthquakes reveals that older seismic design methods which were based on the equivalent elastic force approach could not efficaciously ensure the expected performance of bridges during moderate and strong ground motions. Therefore, two substitutes comprising nonlinear static and dynamic analysis with different levels of complexity and diverse assumptions have been introduced in current seismic codes. Whereas time history analysis is more accurate and reliable for the seismic evaluation, Pushover-based nonlinear static procedures (NSP) are commonly used in the professional applications due to its efficiency and less complication.

The efficiency of pushover analysis is due to the fact that in bridge structures the first natural mode is almost dominant; hence equivalent lateral force can be considered as an adequate representative for lateral earthquake excitation.

Regarding that NSP results are decisively affected by inelastic modeling aspects, the selection of ppropriate modeling strategies that clearly explain geometric and material nonlinearity are of a great importance. Within this scope, the current research tends to focus on comparing the effectiveness of concentrated and distributed plasticity models, as the two most common nonlinear modeling methods, in the pushover curves of a pre-stressed concrete frame bridge with hollow non-prismatic piers. The model of study was a three-span prestressed concrete frame bridge in Tehran, Iran. This viaduct consisted of a pair of independent parallel bridge decks (each 215 meters long and 12.5 meters wide) supported on non-prismatic wall type piers. Whereas the spans are similar in two bridges, they are composed of regular and irregular substructures based on their piers height. Furthermore the reproduction of large-scale experimental and analytical seismic studies of a two-span reinforced concrete bridge system carried out at the University of Nevada Reno have been employed to evaluate the accuracy of the analytical modeling method [1, 2].

In this research, OpenSees software (OpenSees 2002), an open source finite element program, has been used to develop the bridge models while SAP2000 was employed for the additional controls of some results. OpenSees software allows users to be able to model nonlinear elements with either the lumped or distributed plasticity approach.

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2. Modeling Possibilities

Nonlinear behaviors, generally concentrated in the substructures of bridges, depict the necessity for the nonlinear modeling of such structures in numerical studies. Three categories of modeling with different levels of refinement and complexity can be employed in the nonlinear analysis of concrete structures: (i) Global models, where the nonlinear response of structures is determined in accordance with selected degrees of freedom; (ii) Discrete finite element models, where a structural model is developed as an assembly of interconnected frame elements with either lumped or distributed nonlinearities; (iii) Microscopic FE models, where members and joints are modeled through a large number of FEs connected at a finite number of nodal points [3, 4].

The second class of models that provides the best compromise between simplicity and accuracy for the nonlinear analysis of structures are discussed herein. Global models might be appropriate for the preliminary design phase, as it gives limited information on forces, deformations, and damage distribution in the structure. On the other hand, microscopic FE models are typically limited to detailed study of critical regions such as; beam-column connections in professional use as a result of its complication and high cost computational process. [3, 4]

2.1. Lumped plasticity modeling approach

In the concentrated plasticity philosophy, nonlinear behaviors are assumed at the extremities of the structural element while the body is modeled as an elastic part.[3] Recent experimental and analytical studies show that the concentration of nonlinearity at the both ends of the RC bridge piers allows the application of the lumped plasticity approach for the seismic analysis of the bridges.[1] The plastic hinge modeling assumptions reduce computational cost and requirements for the storage of data in the three dimensional finite element models, although the accurate results cannot be obtained when the knowledge of the users about the calibration of the inelastic element parameters are inadequate. As a result, the plastic hinge length and the characterization of the inelastic section according to the classical plasticity theory in terms of stress-strain results or based on the fiber modeling approach should be given more attention during such modeling process [3-6].

2.2. Distributed inelasticity modeling approach

Numerical model based on the distributed nonlinearity assumptions is undoubtedly known as one of the more accurate methods that clarifies the nonlinear behavior of reinforced concrete structures through simple assumptions as input data. Moreover, compared to the lumped plasticity approach, the nonlinearity in distributed plasticity models can occur at any element section. In this approach, the specification of element behavior by means of weighted integration of the section response, limit possibility of behavior monitoring to the integration points. Similarity to the lumped nonlinearity method, constitutive behavior of section can be described in accordance with either fiber modeling approach or response curves reproducing the element behavior under reversible load. There are also two formulation frameworks namely 1-Stifness formulation resulting from implementation of virtual displacement work theorem in which a cubic shape function maintains a linear distribution for curvature, 2- Flexibility formulation resulting from virtual force work in which a rigorous force function maintains the equilibrium [3, 4].

Recent studies have shown that there are no significant differences in the analytical results of lumped and distributed plasticity models [1,2]; however, the lack of analytical and experimental studies in accordance with discrete FE modeling of hollow non-prismatic wall type bridge piers can be seen.

3. Verification Of Inelastic Modeling Strategies Of PRC Bridges In Opensees Program

As part of a multi-university collaborative project, the extensive experimental and analytical studies were conducted at the University of Nevada, Reno by Johnson et al (2006) and Sadrossadat Zadeh et al (2007) to determine accurately the response of a quarter-scale asymmetric reinforced concrete bridge which was composed of two-spans supported on three, two-column piers. In this paper, the accuracy of the conventional modeling method in simulating the response of a reinforced concrete bridge was evaluated by the analytical model of this two-span frame RC bridge which was tested up to failure on the shake table system at that University. The analytical model of the two-span bridge in OpenSees software, named "verification model", was developed according to the assumptions in the model by Sadrossadat Zadeh et al (2007).

3.1. OpenSees analytical modeling of two-span bridge specimen

The bridge specimen, tasted on the shaking table system, consisted of two spans and three, two-column piers with a monolithic cap-beam and deck connections. As illustrated in Fig.1 and Fig.2, the spans had lengths of 9.14 m and the columns of the three bents had clear heights of 1.83, 2.44, and 1.52m with the tallest bent in the middle. The solid slab of the superstructure was post-tensioned in both longitudinal and transverse directions of the bridge.

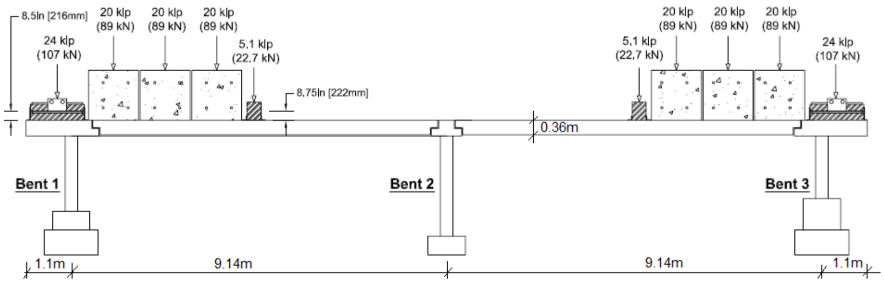

Fig. 1 Elevation and top view of two-span bridge [1]

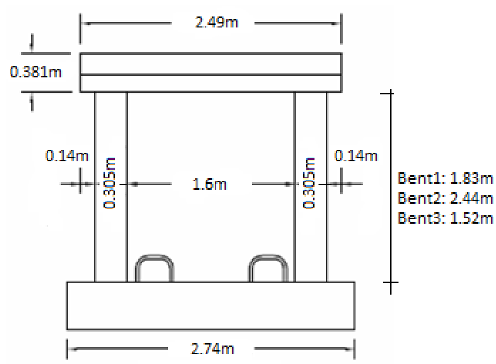

Fig. 2 two-span bridge bents [1]

3D model of bridge specimen was an assemblage of linear and nonlinear elements with the connection at nodes, in which Nodes and elements are defined at centerlines of the bridge components. The columns were characterized to be fully fixed at the base. P-delta effect was included in the computer model. The superstructure's mass was lumped at the tenths of the span lengths and at the thirds of the cantilever lengths based on the tributary area of each node. Superimposed masses were assigned to the nodes which were located at the center of the concrete blocks and lead pallets. The mass nodes of superimposed weights were vertically connected with a rigid beam-column element to the centerline of the superstructure. Rotational inertial masses were excluded.

3.1.1. Computer model of superstructure

It was assumed that all nonlinear behaviors took place in the columns while the superstructure will remain elastic. This assumption was based on two facts: 1) superstructure with post-tensioning was expected to remain without cracked, 2) compared to columns; superstructure and cap-beam were strong and stiff. Hence, the elastic beam-column elements with gross section properties were used to develop superstructure model. The modulus of elasticity in the superstructure was determined using the unconfined concrete compressive strength of 34.5 Mpa.

3.1.2. Computer model of piers

The substructure of the two-span bridge was composed of three, two-circular column piers with the diameters of 0.305 m which had been reinforced with a 1.56 percent longitudinal steel ratio and a spiral reinforcement ratio of 0.9 percent. Since, the columns were expected to experience nonlinear deformations at their extremes; nonlinear elements "BeamWithHinge" in OpenSees software were used to model the two-column piers. The "BeamWithHinge" element tends to present an elastic element with concentrated plasticity at two ends. In the linear part of the columns, cracked section properties and modulus of elasticity based on the unconfined concrete compressive strength of 34.5 Mpa were specified. The inelastic fiber sections were defined in the nonlinear parts located at the two ends of the elements. Fibers for the longitudinal column reinforcement, unconfined concrete, and confined concrete in the inelastic fiber section were characterized through multi-linear stress-strain curves. The constitutive relationships of both the confined and unconfined concrete were defined by the "Concrete01" option in OpenSees which is a uniaxial Kent-Scott-Park concrete material object with degraded linear unloading/reloading stiffness according to the work of Karsan-Jirsa and without tensile strength. The properties of the confined concrete were then specified in reference to Mander's model. [7] Fig.3 and Table.1 have illustrated this concrete model and its relevant data for the bridge specimen. As presented in Fig.4, the bi-linear steel material (Steel01 in OpenSees) with the initial modulus of elasticity (E) of 2×10^5 MPa, the yielding stress (F_y) of 458 MPa, and the strain hardening ratio (b) of 0.2% were used to model the reinforcements. As an important definition, the plastic hinge lengths of the columns at two ends were assumed equal to the half of the column's diameter.

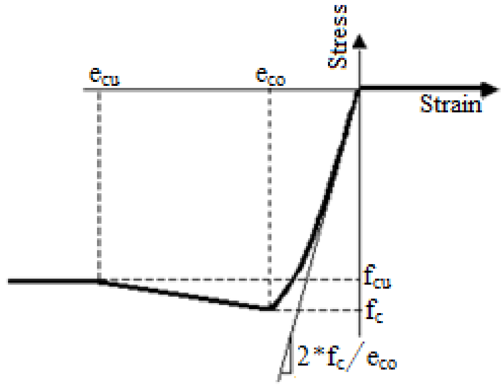

Fig. 3 Concrete01 Material Parameters [7]

Table 1 concrete material properties of two-span Bridge

	Confined concrete	Unconfined concrete
Concrete compressive strength at 28 days (f_c)	45.2	35.4
Concrete crushing strength (f_{cu})	35.1	0
Concrete strain at maximum strength (e_{co})	0.005	0.002
Concrete strain at crushing strength (e_{cu})	0.017	0.006

Constant cracked section properties for both shear and torsion were assigned to the column fiber element sections using 'aggregator' option in OpenSees. The cracked shear stiffness was calculated based on the truss analogy (Eq. 1) [8].

$$K_{v\ cracked} = \frac{\rho_v}{1 + 4n\rho_v} E_S b_w d \geq 0.1 K_{v\ uncracked}$$

Where $K_{v\text{-cracked}}$ and $K_{v\text{-uncracked}}$ are cracked and uncracked stiffness per unit length, respectively. $K_{v\text{-uncracked}}$ is $E_s b_w d/3$, where E_s is elastic modulus of steel, b_w is average column width, and d is effective depth of column or 0.9*column diameter. Moreover, ρ_v is tie steel ratio and n is modular ratio (elastic modulus of steel / elastic modulus of concrete).

Based on above assumption, column failure point was not accurately modeled. This was in part because gradual strength loss at failure was assumed in material models (Fig.3-4) to achieve convergence and computational stability. Moreover, definition of fiber models in the plastic hinge zone made the direct modeling of spiral Rupture impossible.

The minimal number of fibers which resulted in accurate output was determined using the section moment-curvature analysis. No significant changes have been observed with the increment in the number of slices and layers more than 8 and 9, respectively. Fig.5 shows the close correlation between section moment-curvature curves of Verification model and one developed by Sadrossadat Zadeh et al (2007).

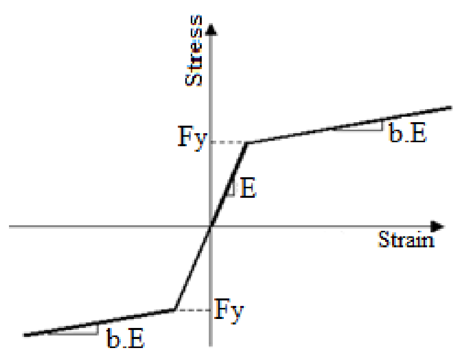

Fig. 4 Steel01 Material Parameters [1]

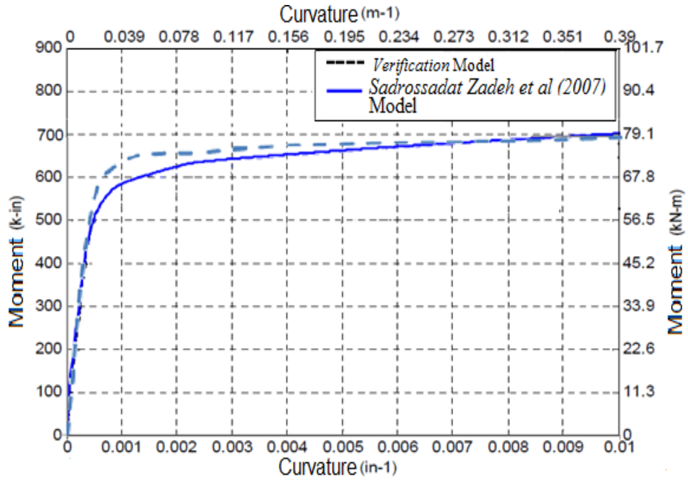

Fig. 5 Moment-Curvature curve for the Column Section of two-span Bridge

3.1.3. Bond-slip modeling

Regarding the Wehbe bond-slip model, bond-slip properties in terms of moment-rotation curve was assigned to zero length elements at the extremities of columns.[9] To reach desirable Moment-rotation curve, the resulting curve of a section moment-curvature analysis was modified based on the steel strain of the extreme section fiber at three points; cracking, yield, and the ultimate. The modified tri-linear moment-rotation curve was then defined in the positive and negative directions using "uniaxialMaterial Hysteretic" option in OpenSees.

3.1.4. Pushover results

To determine the in-plane lateral load response of bridge bents, a pushover analysis was separately performed for each bent. In the analysis, vertical loads resulting from the superstructure weight and superimposed dead load were defined 361 KN for bents 1 and 3, and 351 KN for bent 2. Force-deformation curves obtained from the pushover analysis of the Verification model depicted a difference of less than 5 percents compared to the analytical studies of Sadrossadat Zadeh et al. (Fig.6).

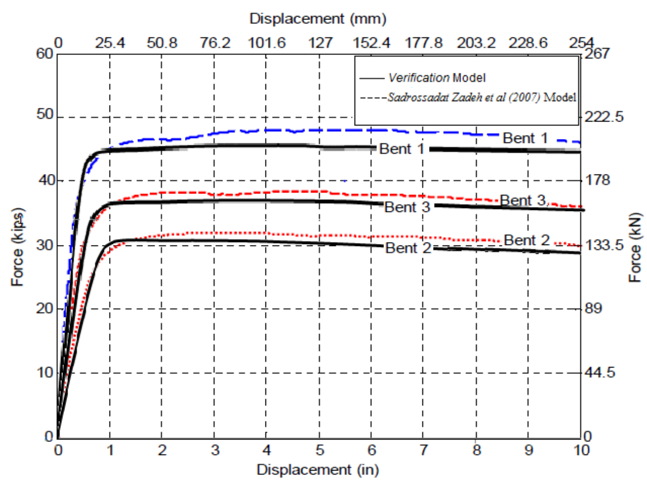

Fig. 6 Capacity curves of two-span Bridge Bents

Extra information on the comparison of the analytical results with the test data, which confirmed the adequacy of the computer model in reproducing two-span RC bridge responses, can be found in the work of Sadrossadat Zadeh et al. (2007).

4. Case Study

The RH Bridge was selected as a case study to be the representative of typical multi-span motorway frame bridges in Iran. This viaduct, located in Tehran North highway, consists of a pair of independent parallel bridge structures. Identical parallel decks are hollow-core pre-stressed concrete girders with 12.5 m wide and 60 m, 95m, and 60 m span lengths (Fig.7 -9). Whereas the spans are similar in the two bridges, they are composed of regular and irregular substructures based on their piers height. The east bridge is composed of similar piers 27m high, while the west bridge has two piers 27.40 m, and 23.40 m high.

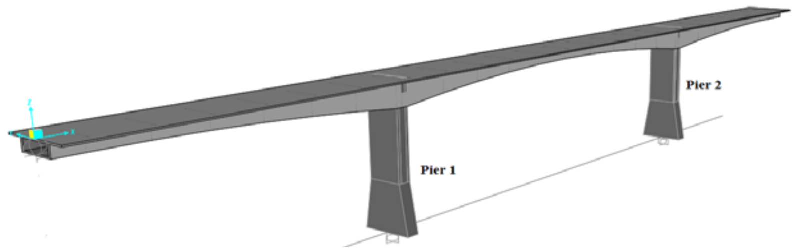

Fig. 7 3D Computer Model of East RH Bridge

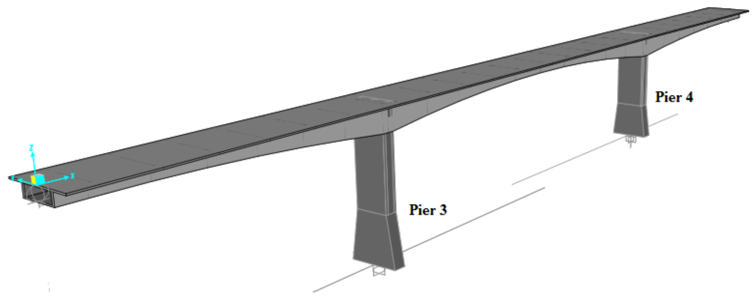

Fig. 8 3D Computer Model of west RH Bridge

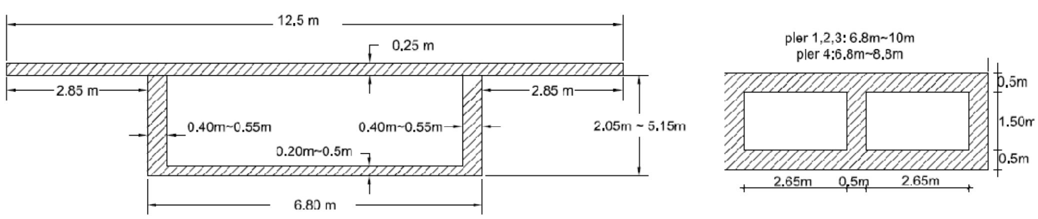

Fig. 9 Member Cross Sections of RH Bridge

4.1. Analytical modeling of RH bridge

To develop the 3D computer model of the RH bridge, nonlinear and linear elements as well as nodes were modeled at the centerlines of the bridge components. The superstructure mass including dead loads of structural components, live loads -based on the Iranian Code of Standard Loads for Bridge-, and pavements were defined to be lumped at the nodes which were located at the tenths of the span lengths. Rigid elements were used to connect the top of the columns to the centerline of the superstructure. P-Δ effects and bond-slip properties were also included in the computer model according to the section 3.1.

4.1.1. Modeling of the bridge deck

As discussed in section 3.1.1, linear elastic beam-column elements were used to develop the superstructure. Since non-prismatic elements cannot defined in OpenSees, equivalent average sections were substituted in the superstructure model. The modulus of the elasticity of the linear elements was specified based on unconfined concrete compressive strength of 40 Mpa.

4.1.2. Modeling of the bridge piers

In order to conduct research on the effects of different nonlinear elements on the force-displacement curve of a RC frame bridge with the hollow section non-prismatic piers, three different types of nonlinear beam-column elements were separately used in piers using OpenSees. The first type, called "nonlinearBeamColumn", is a force based element based on the flexibility formulation which considers the spread of plasticity along the element. A displacement based element based on the classical stiffness method, called "dispBeamColumn", was also employed to construct a nonlinear element with distributed plasticity and linear curvature distribution. The integration along the first type follows the Gauss-Lobatto quadrature rule; however the second type is based on the Gauss-Legendre quadrature rule. To consider the lumped plasticity approach, "beamWithHinge" element object was used in the bridge piers. In this force based element with flexibility formulation, the specification of nonlinear section properties, and the plastic hinge lengths for plastic hinge zones at two ends, as well as the linear section properties for the rest of columns are required as an input. In the 3D model of the RH bridge with lumped nonlinearities, nonlinear fiber based section was defined according to the equivalent average section in the middle of plastic hinge zone. The cracked section properties of columns were also assigned to the elastic part of the "beamWithHinge" elements. The plastic hinge lengths were discussed comprehensively in the section 5.3.

Nonlinear fiber based sections were specified in all nonlinear elements. The moment-curvature analysis was performed to reach the optimized number of fibers. It was then concluded that 0.15m×0.15m mesh for confined concrete core and 0.05m×0.2m mesh for unconfined concrete fibers could accurately reproduce the nonlinear response of bridge piers. Similar to the Verification model, constant cracked section properties for both shear and torsion were assigned to the column fiber element sections. The material properties were defined as the same as the Verification model, however the maximum compressive strength of 30 Mpa was assumed for the unconfined concrete. Furthermore, the peak stress of confined concrete (42MPa) at the strain of 0.006 has linearly reduced to 70% of itself at the strain of 0.017. Longitudinal reinforcement bars were also modeled with the initial modulus of elasticity of 2×10^5 MPa and yielding stress of 400 MPa.

Since the definition of non-prismatic elements is not possible in OpenSees, non-prismatic part of piers in "nonlinearBeamColumn" and "dispBeamColumn" elements was modeled through the equivalent average section. Piers length was discretized with nine elements, with length equal to 5×7.4%, 22.2%, 26% and 2×7.4% of piers length. Fine subdivisions were also employed at the extremities of piers due to the possibility of plastic hinge formation. Through this fine subdivision, the equivalent average section could accurately reproduce the actual response of the bridge piers. The adequate mesh density was confirmed when an increment in the number of subdivisions, had no considerable effects on the pushover curves. It should be noted that, the mesh density in "dispBeamColumn" elements object was increased up to 2 times in order to achieve acceptable transverse capacity curves.

4.1.3. Boundary conditions

The soil-structure interaction was not applied in the computer models. The piers were assumed to be fully fixed at the base, while the role abutment model was specified at the ends of the deck. The role abutment model characterizes a simple boundary condition with vertical constraint as a support, which resulted in the cantilever performance of single column-piers against the transverse lateral load. This uncomplicated boundary condition was taken into account, in view of the fact that the consideration of the soil-structure interaction was not the research object. Nevertheless, it should be argued that the lower-bound estimate of the longitudinal and transverse resistance of the bridge can be expected through the aforementioned method. If a rotational restraint about the superstructure longitudinal axis is assumed for such a model to be representative of the overturning resistance of the abutment, the bridge's overall strength and its ductility can be calculated overestimate and underestimate, respectively. In fact, the actual response of the bridge will be achieved in condition between this restrained and unrestrained rotational degree of freedom. [10]

5. Results

5.1. Eigenvalue analysis results

The preliminary linear Eigenvalue analysis was performed in both OpenSees and SAP2000 to demonstrate the accurate stiffness allocation of structural members and correct mass distribution. SAP2000 was additionally used to verify the modeling aspects in OpenSees, given that the lack of graphic view in OpenSees may conduce to defects in stiffness allocation and mass distribution. To perform the linear Eigenvalue analysis, constant cracked section properties were included in the linear model of piers. Good agreement between natural periods of the basic modes (table.2), derived from the linear Eigenvalue analysis in both software, proved the correct modeling of mass and stiffness in OpenSees analytical models.

Table 2 Modal periods (seconds) of RH Bridge

Mode	east RH Bridge		west RH Bridge	
	sap2000	opensees	sap2000	opensees
1	1.808	1.714	1.802	1.705
2	1.646	1.506	1.617	1.482
3	1.464	1.315	1.358	1.212
4	0.821	0.802	0.814	0.773

5.2. Nonlinear static procedure results

Pushover based nonlinear static procedure was carried out in both transverse and longitudinal directions in order to achieve pushover curves. The reference node was taken as the central point of deck. Additionally, the force pattern was applied based on the ratio of the tributary mass of each column top nodes and superstructure end nodes to the total bridge mass as follows: $F_i = \frac{m_i}{\sum m_i} V$. Resulting pushover curves was illustrated on the Fig.10-13. What should be taken into consideration is that, the distributed plasticity approach with the flexibility formulation results in a more reliable and accurate output (Taucer et al, 1991; Spacone, 2001), since the resulting pushover curves based on the "beamWithHinge" and "dispBeamColumn" elements were compared with the "nonlinearBeamColumn" one. The comparison was made on the subject of the yield force, total initial stiffness and the computational time.

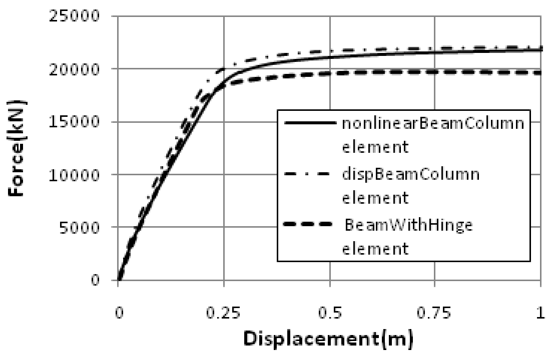

Fig. 10 Longitudinal capacity curve of east Bridge

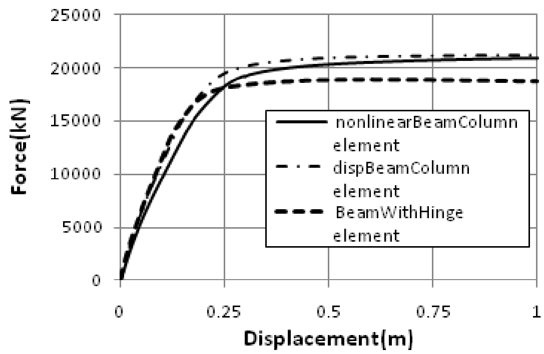

Fig. 11 Longitudinal capacity curve of west Bridge

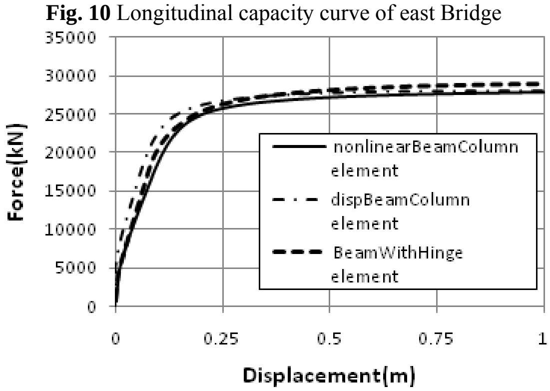

Fig. 12 Transverse capacity curve of east Bridge

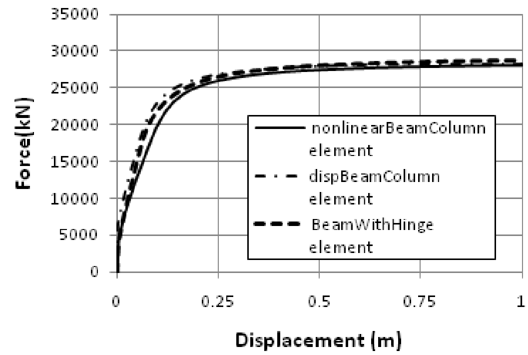

Fig. 13 Transverse capacity curve of west Bridge

Fig.10 and Fig.11 demonstrated nearly the same initial longitudinal strength for both bridge models concerning the lumped nonlinearity model and the distributed plasticity models with the classical stiffness formulation. However, having compared flexibility based distributed plasticity models with these methods; it had an upsurge of up to 13% in the elastic longitudinal stiffness of both the east and west bridges. Whilst, more matching results against longitudinal lateral load seemed to occur beyond yielding for both distributed plasticity approach, computer model with lumped plasticity reached approximately 12% lower yield force. In spite of these slight differences, the lumped plasticity technique can be taken as an efficacious method in the nonlinear modeling of a PC Frame Bridge with hollow non-prismatic piers, due to the considerably reduced time which is spent on the modeling process and computation.

The force-displacement relationship of two frame bridges under the transverse lateral load is illustrated in Fig.12 and Fig.13. Closer matches between different modeling techniques were clearly observed in the transverse pushover curves. Nonetheless, the significant factor of computational time for three types of nonlinear modeling approach demonstrated a substantial difference. The computational time for the model with lumped plasticity decreased approximately by half compared to the model with "nonlinearBeamColumn" elements, while there has been dramatic increment in the computational time for the model with "dispBeamColumn" element. The main reason was that the linear curvature distribution in "dispBeamColumn" element necessitated the use of finer subdivisions in the RH bridge non-prismatic piers with the variation of column depth in transverse orientation. Finer subdivisions can indisputably exacerbate computational time.

For the last comparison, irregularity in bridge structures did not cause a big difference over the outcome of the aforementioned nonlinear modeling approach.

5.3. Plastic hinge length

Integration points in elements with distributed nonlinearity can be advantageously used as monitoring points to identify plastic hinge length in bridge piers. According to this, plastic hinge lengths in the RH bridge piers were roughly equal to 30% of piers length from the contraflexure point to the section with maximum moment. Whereas plastic hinge formed at the both extremities of bridge piers under the lateral load in longitudinal direction, plastic behavior took place at the column-to-footing intersections against transverse lateral load. Cantilever performance of piers under the transverse lateral load was the clear cause of this behavior.

6. Conclusions

By considering the effectiveness of different nonlinear modeling techniques in the pushover curves of the RH bridge (in Tehran, Iran), the following conclusions can be drawn out:

1) The lumped plasticity approach can be decisively considered as the useful tool for the nonlinear modeling of

non-prismatic bridge piers with hollow sections, in view of the fact that acceptable matching results can be obtained with the distributed plasticity approach through a less complicated modeling assumption and considerably less computational time. Within this method, plastic hinge properties can be simply specified by the nonlinear fiber based sections according to the average equivalent section at middle of plastic hinge length.

2) To reach an acceptable agreement between the pushover curves of models with lumped or distributed nonlinearities, plastic hinge lengths of piers in such RH bridge for the Lumped plasticity approach can be assumed equal to 30% of piers length from the contraflexure point to the section with maximum moment.

3) For the distributed plasticity approach, there are no advantages in the use of a displacement based element using the classical stiffness method compared to the force based element with the flexibility formulation. The linear curvature distribution in the first approach necessitated the use of finer subdivisions in the non-prismatic pier, which resulted in a dramatic increment in computational time.

4) The irregularity in bridge structures did not widely affect the outcome of the before mentioned nonlinear modeling approach.

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