Topology optimization of structures using cellular automata with constant strain triangles

E. Sanaei[1,*]**, M. Babaei**[2]

Abstract

Due to the algorithmic simplicity, cellular automata (CA) models are useful and simple methods in structural optimization. In this paper, a cellular-automaton-based algorithm is presented for simultaneous shape and topology optimization of continuum structures, using five-step optimization procedure. Two objective functions are considered and the optimization process is converted to the single objective optimization problem (SOOP) using weighted sum method (WSM). A novel triangle neighborhood is proposed and the design domain is divided into small triangle elements, considering each cell as the finite element. The finite element formulation for constant strain triangles using three-node triangular elements is developed in this article. Topological parameters and shape of the design space are taken as the design variables, which for the purpose of this paper are continuous variables. The paper reports the results of several design experiments, comparing them with the currently available results obtained by CA and genetic algorithm in the literature. The outcomes of the developed scheme show the accuracy and efficiency of the method as well as its timesaving behavior in achieving better results

Keywords: Cellular automaton, Structural optimization, Topology, Shape, Constant Strain triangle (CST)

1. Introduction

In this competitive world, scarcity of the resources and need for efficient structures make challenges for engineers to find cost-effective and efficient solutions and designs. Skill, intuition, and experience of the designers can directly affect the designs. The design of complex and huge structures requires data processing and a large number of calculations. Computer-aided design optimization (CADO), however, has been developed during the last decades. The engineering design and optimization processes benefit vastly from the revolution of calculation using computers. The optimization methods, in the literature, are classified in two different categories; optimality criteria (indirect) methods and search (direct) methods. Optimality criteria are conditions that must be satisfied by a function at its minimum point. Many mathematical (or deterministic) methods and stochastic (or probabilistic) methods are introduced, developed and applied for the optimization of structures, in the literature.

Shape optimization can usually be implemented by varying some typical shape parameters, such as nodal coordinates. But, in topology optimization there are no obvious or simple topological parameters within an optimization process. Some of the variables that can be used to define topology are the number of holes in a structure and the existence or inexistence of material at any particular point in the design space. Therefore, unlike the continuous variables treated in shape problems, the design variables in topology optimization are intrinsically discrete in nature and the problem is essentially a discrete optimization problem of material distribution. This kind of discrete problems are difficult because they require the enumeration of large portions of the solution space, and computational order can grow enormously with the number of discrete variables to be resolved.

A popular strategy is to initially discretize the allowable design space into finite elements (FE) and define the required loading and boundary conditions. The optimization procedure will then be concerned with determining which elements should contain material and which elements are void. Based on this strategy, Bendsoe and Kikuchi [1] defined the problem with a composite material represented by each element having material plus a hole inside. The material properties of each

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element are then dependent on the size and orientation of the void within the element according to a homogenization relationship. A size optimization is then performed to optimize the sizes and orientations of the voids of all the elements for a given objective function. Elements with relatively large holes will represent empty space while those with smaller holes denote that material exists and hence form the structure.

An alternative but conceptually similar approach is to directly use the material density of each element (instead of holes) as the design variable. An empirical formula is then applied to relate this density with the elastic modulus, without the need for a homogenization formulation. The topology designs produced by this material density approach [2] are similar to those obtained with the homogenization method. The Homogenization method is then developed by others. Among them, Hassani and Hinton [3] developed this method theoretically and practically to the structural topology optimization, expanding the software to the method.

Both the homogenization and material density approaches circumvent the difficulty of discrete optimization by using continuous design variables, converting the problem into a continuous optimization problem. However, since the resulting elements can be of intermediate densities ranging from complete holes to completely filled with material, some interpretation by the designer is still needed to determine the final topology and shape of the structure. It is uncertain if the final useful design interpreted from the results is close to the actual optimum point since the criteria for including/excluding any particular element from the structure tends to be arbitrary (for example, based on an arbitrary threshold density value).

There are many types of heuristic methods which have been developed and applied to the engineering problems as well as shape and topology optimization. Among them genetic algorithms [4], ant colony optimization [5-6] and particle swarm optimization [7] are the most famous. There are many studies for optimization of structures using these heuristic methods and results have been showed the robustness of the method but each one has its own shortages [4-7].

For the first time, cellular automaton (CA) was presented by von Neumann [8-9] and Ulam [10], and it has been considered as a discrete simulation scheme in the last four decades. CA, also, has been introduced for more realistic modeling of the behavior of complex systems. Initially, this method was introduced as Automata Networks for modeling of discrete dynamic systems.

On the other hand, CA is known as a special case in which a network graph is uniform and the cells are updated at the same time [11]. In general, in this method, cells are considered as similar square [12] or other shapes, and values of each cell in a special time step is updated using local rules, regarding the status of the cell and its neighbor cells in the previous time step. Kita and Toyoda [12] presented a scheme for optimization of structures by using the concept of a cellular automaton (CA), dividing the design domain into small square cells. To confirm its validity, Kita and Toyoda [12] applied the proposed scheme to a two-dimensional elastic problem. Since these rules are to introduce existing relationships between adjacent and neighboring cells, it is not necessary to know the general rules governing the issue. Thus, CA is a very suitable

method for problems where the accurate information of the general relations is not available.

CA has been used for simulating a variety of problems such as fluid flow and transportation traffic; however, the main idea of applying this method in structural shape optimization for the first time was proposed by Inou et al. [13-14]. The basic idea which is described by Inou et al. is to divide the design domain into small cells and then to obtain von Mises equivalent stress distribution in the cells using the finite element method. Then the amount of stress in each cell is updated using the values of stress in neighboring cells and applying local rules. In this method, Young modulus is considered as a design variable and it is modified in every stage such that the stress of each cell becomes equal to the amount of stress of the cell obtained in the stage. Thus by eliminating cells with relatively small Young's modulus, the goal of shape and geometry optimization of structures are simultaneously implemented.

Local rule applied in these studies is nonlinear relationship between the cell stresses, and the Young modulus has to be considered. The numerical experience shows that there is no reliable connection between the method and mathematical optimization problem. Since the stresses in each cell are updated individually during the optimization process, it is not possible to apply suitable stress constraints.

On the other hand Xie et al. [15-16] introduced evolutionary structural optimization (ESO). In this scheme, the first base value is determined. After analysis using the finite element method, cells with smaller stress than the base amount are removed. In their recent studies, the ESO method of evolutionary structural optimization has been generalized. In this scheme two base values are introduced. Thus, while some cells using the first criterion are removed, another group of cells with regard to other criteria are added. However, the physical concepts of these base quantities are not specified and therefore they should be determined by previous numerical results or previous research experiments. To overcome the above problems, the following algorithm is presented and used. First, the design domain area is divided into small triangle cells and thickness of each cell is considered as a design variable.

In the next step, the whole problem of structural optimization is converted to optimization of each cell using CA constraint condition. Formulation of this method does not involve entering new parameters whose physical nature is not clear, which is considered an advantage of using this method.

2. Cellular automata approach

Cellular Automata (CA) is a mathematical model for systems in which many simple components for complex patterns can work together. CA is made up of a regular network, where each cell can take different amounts. The cells of CA at each step of implementation are updated simultaneously using a local rule in which the value of each cell is determined based on the values of neighboring cells. CA could be divided into various categories. For example, based on the dimension of network criteria, Cellular Automata will be divided into one-dimensional, two-dimensional, or multi-dimensional. Cellular

Automata based on the amount of each cell is divided into two-value Cellular Automata and multi-value.

Cellular Automata based on the network neighbors can be divided into two categories, as CA with periodic boundary and or non-periodic boundary. The most famous neighbors in the two-dimensional Cellular Automata model are known as the Moore neighborhood and Von Newman. In this paper, the design domain is divided into triangles with three-node cells. A variety of neighborhood can be considered for cells. However, in this paper, only the cells that are in common ridge are selected as neighbors, as illustrated in figure 1.

All of these cells are considered as independent components through the finite element analysis and stress distribution in each cell is determined. Usually, in the simulation process using Cellular Automata, value of cells is considered as limited and a finite amount. However, in this paper, these values are considered as continuous quantities. The values of each cell in each step are determined based on the status of the cell and its neighbor cells in the previous step, using the appropriate local rule. Figure 1 shows the neighbor cells of the triangular elements. For the boundary cells or the cells located in the sides, only the adjacent cells are considered as neighboring cells.

CA is definitely a new comer to the field of structural analysis and design. Nevertheless, a number of methods that appear in the structural optimization literature have a basic structure reminiscent of CA algorithms. These methods, especially in the area of topology design, are reviewed in the introduction to the paper by Kita and Toyoda [12]. The work of Kita and Toyoda [12] is the starting point of this review.

CA has been considered as a discrete simulation scheme and also, has been introduced for more realistic modeling of the behavior of complex systems. The topology design objective considered in [12] is to find the optimal thickness distribution of a two-dimensional continuum (plate) under in-plane loads. The basic methodology presented by Kita and Toyoda consists of; 1- finite number of elements are identified as CA cells, 2- the cell neighborhood is identified as the elements sharing a common edge with the cell, for the rectangular FEM mesh used, this is a Moore-neighborhood, and 3- an update rule is devised, based on stresses in the neighborhood, to update the cell thickness.

This work contained some far-reaching features. They formulated the CA design rule, for the first time, as a local optimization problem at the cell (element) level. They based the local update rule on the value of stress resultants in the neighborhood. Moreover, they provided an approximate sensitivity analysis as the basis for selecting the cell (element)
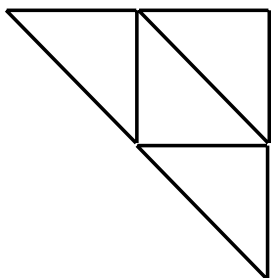
level objective function. The main drawback of their method is that they depended on the evolutionary structural optimization (ESO) method developed by Xie and Steven [15]. In ESO, the von Mises stress is used as a measure to eliminate elements in the domain that are not contributing to the load carrying capacity of the structure. This method is essentially heuristic and was criticized for its lack of mathematical foundations and premature convergence to suboptimal designs in a number of publications [15-17]. Another disadvantage of this CA algorithm is the large number of iterations (in excess of a thousand) required to reach a converged topology.

3. Definition of the optimization problem

3.1. Introduction

It is generally accepted that the proper definition and formulation of a problem takes roughly 50 percent of the total efforts needed to solve it. Therefore, it is critical to follow well defined procedures for formulating design optimization problems. The importance of properly formulating a design optimization problem must be stressed because the optimum solution will only be as good as the formulation. For instance, if we forget to include a critical constraint in the formulation, the optimum solution will most likely violate it because optimization methods tend to exploit deficiencies in design models. Also, if we have too many constraints or if they are inconsistent, they may not be a solution to the design problem.

Results reported by Arora in [18], show that the selection of design variables greatly influences the problem formulation. Once the problem is properly formulated, methods, schemes or algorithms could be applied to solve it. A five step procedure proposed in [18] to formulate design optimization problems, which is applicable for most optimization problems; Project/problem statement, Data and information collection, Identification or definition of design variables, Identification of objective function(s), Identification of constraints.

All optimization problems have at least one optimization criterion that could be used to compare different designs and determine an optimum solution. Most engineering design problems must also satisfy certain equality or inequality (or both) constraints. A standard form of the design optimization model for single objective optimization problem (SOOP) which complies with the literature is as follows:

Find an n-vector $x=(x_1, x_2,...x_n)$ of design variables to minimize (maximize) a cost (profit) function

$$f(x)=f(x_1, x_2,...x_n) \tag{1}$$

subject to the p equality constraints

$$h_j(x)=h_j(x_1, x_2,...x_n)=0; \; j=1 \, tol \tag{2}$$

and the l inequality constraints

$$g_i(x)=g_i(x_1, x_2,...x_n) \leqq 0; \; i=1 \, tol \tag{3}$$

and also q upper and lower limits on the design variables

Fig. 1. The triangular neighborhood

$$x_k^{(L)} \delta x_k \delta x_k^{(U)} \; ; \; k = 1 \text{ to } q \qquad (4)$$

A multi-objective optimization problem has a number of objective functions which are to be minimized or maximized. As in the single-objective optimization problem, here too the problem usually has a number of constraints which any feasible solution (including the optimum solution) must satisfy. In the following, we state the multi-objective optimization problem (MOOP) in its standard form as introduced in [18-19]:

Find an (or set of) n-vector $x=(x_1, x_2,...x_n)$ of design variables to minimize (maximize) cost (profit) functions

$$F(x)=(f_1(x_1, x_2,...x_n),f_2(x_1, x_2,...x_n),...,f_m(x_1, x_2,...x_n); \; m=1 \text{ to } M \quad (5)$$

subject to the p equality constraints

$$h_j(x)=h_j(x_1, x_2,...x_n)=0; \; j=1 \text{ to } p \qquad (6)$$

and the l inequality constraints

$$g_i(x)=g_i(x_1, x_2,...x_n)\leqq 0; \; i=1 \text{ to } l \qquad (7)$$

and also q upper and lower limits on the design variables

$$x_k^{(L)}\leqq x_k \leqq x_k^{(U)} \; ; \; k = 1 \text{ to } q \qquad (8)$$

A solution x that does not satisfy all of the constraints and bounds is called an infeasible solution. Vice-versa, if any solution satisfies all constraints and variable bounds, it is known as a feasible solution.

Multi-objective optimization is sometimes referred to as vector optimization, because a vector of objectives, instead of a single objective, is optimized. In the case of conflicting objectives, usually the set of optimal solutions contains more than one solution. In two-objective optimization problems the solutions trade-off could be obtained; this is called Pareto-optimal solution. In the presence of multiple Pareto-optimal solutions, it is difficult to select one solution over the other without any further information about the problem. If higher level of information is satisfactorily available, this can be used to make biased search. Therefore, in the light of the ideal approach, it is important to find as many Pareto-optimal solutions as possible in a problem. Thus, it can be assumed that there are two goals in a multi-objective optimization: to find a set of solutions as close as possible to the Pareto-optimal front, and to find a set of solutions as diverse as possible.

3.2. Problem statement and data collection

The main purpose of this paper is to develop shape and topology optimization of structures using the concept of cellular automata. To analyze the structure using finite element method, constant strain triangles routine is developed. The objective of the optimization problem developed in this paper is to minimize both the total weight of the structure and the deviation between the yield stress of the materials and the von Mises equivalent stress at the cell. In other words, the problem has two objective functions, called bi-objective optimization problem in the literature.

3.3. Design variables and constraints

Continuous variables are employed for thickness of the cells. To formulate the optimization problem for each element individually a special constraint condition, called CA-constraint condition, is considered. This CA-constraint condition is defined so as to minimize the variation of the equivalent stress of the neighboring cells with respect to the variation of the thickness of the updated cell. The CA-constraint conditions defined in [12] for updating quadrangle elements neighborhood. In this paper, however, it is redefined for triangular elements. These conditions are explained as follows:

$$g_i=(\tilde{\sigma}_i / \tilde{\sigma}^0_i)-1 \equiv \sigma_i -1 = 0 \; , \; (i=1, ...,3) \qquad (9)$$

where σ_i denotes the ratio of equivalent stresses at the neighboring cell i at the present step to the preceding step.

Therefore, this equation ensures that the variation of the equivalent stress at the neighboring cell is small.

3.4. Objective functions

The first objective function of this optimization problem is to minimize the weight of the updated cells. Considering the material and the area of the cells as invariant parameters, the first objective function, which is an explicit function of the design variables, can be defined as follows:

$$f_1(x)=(x_i/t_0)^2 \qquad (10)$$

where t_0 is the initial thickness of the cell.

As implied in the previous part, the second objective function is to minimize the deviation between the yield stress of the material and the von Mises equivalent stress at the cells. This aim is also expressed as follows:

$$f_2(x)=(\sigma_0-1)^2 \qquad (11)$$

where σ_0 is the ratio of the von Mises equivalent stress to the yield stress of the material.

This objective function is an implicit function of the design variables, so it is not possible to formulate the objective function explicitly in terms of the design variables alone. Instead, the intermediate variable, which is a type of stress ratio, is used to formulate the function.

4. Our approach

This article intends to optimize both the objective functions developed in the previous section. This problem is known as multi-objective optimization problem. There are many approaches available to solve multi-objective problems [20-21]. The weighted sum method (WSM) is the simplest and the most common approach to multi-objective optimization problems and is probably the most widely used classical approach [22]. This method, as the name suggests, scalars a set of objective functions into one single objective using pre-multiplying each objective with a user-defined weight. Faced with multiple objectives, this method is the most convenient

one that comes to mind. For example, if one is faced with the two objectives of minimizing the total weight of a structure and minimizing the maximum lateral deflection of each story of a structure, one naturally thinks of minimizing a weighted sum of these two objectives. Although the idea is simple, it introduces a not-so-simple question. The values of the weights one must use could be the question. Of course, there is no unique answer to this question. The answer depends on the importance of each objective in the context of the problem and a scaling factor, which will be addressed in the following section.

The weight of an objective is usually chosen in proportion to the objective's relative importance in the problem. For example, in the above-mentioned two-objective minimization problem, the total weight of the structure may be more important than the maximum lateral deflection of the structure. Thus, the user can set a higher weight for the weight than for the maximum drift. Although there exist ways to quantify the weights from this qualitative information as developed by Parmee et al. [23], the weighted sum approach requires a precise value of the weight for each objective. However, setting up an appropriate weight vector also depends on the scaling of each objective function.

It is likely that different objectives take different orders of magnitude. In the above example again, the total weight of the structure may vary between 100 to 1000 tons, whereas the maximum drift of the structure may vary between 10 to 100 mm. When such objectives are weighted to form a composite objective function, it would be better to scale them appropriately so that each has more or less the same order of magnitude. For example, one may multiply the total weight by 10^{-3} and the maximum drift of the structure by 10^{-2} to make them equally important. This process is called normalization of objectives as introduced by Deb [19].

On the other hand, in order to make objective functions scalar non-dimensional amounts, one may divide each objective function by the initial value of them. For example, in the above mentioned example one may divide the total weight of the structure by the initial constant value for the weight (e.g. initial assumed weight of the structure or initial weight obtained from previous optimization scheme) and divide the maximum lateral deflection of the structure by the initial constant value for the lateral deflection (e.g. the allowable lateral deflection permitted by codes). After the objectives are normalized, a composite objective function $f(x)$ can be formed by summing up the weighted normalized objectives and the MOOP given in equation (5) is then converted to a single-objective optimization problem.

Using the weighted sum method and multiplying the penalty parameter p into the CA-constraint condition and adding it to the linear combination of the two objective functions, the penalty function can be obtained as follows:

$$f(x) = w_1(x_i/t_0)^2 + w_2(\sigma_0-1)^2 + p\sum_{i=1}^{3}(\sigma_i-1)^2 \qquad (12)$$

here, w_1 and w_2 are defined so that the sum of them are equal to one and w_2 could be obtain using the following conditions:

$$w_2 = \begin{cases} \sigma_0 & if\ \sigma_0 < 1 \\ 1 & if\ \sigma_0 \geqq 1 \end{cases} \qquad (13)$$

To obtain the trade-offs between the two objective functions

in multi-objective problems, different amounts for the weight parameters have to be considered. This diagram is obtained in the following section, without consideration of the equation (13). In this work, since simultaneously topology and shape optimization is implemented, unlike topology optimization which considers just existence or inexistence of the material, shape and topological parameters are considered for design variables. After defining the optimization problem, two-dimensional stress and deformation analysis is employed to analyze the solutions. These problems are of plane stress or plain strain type. The finite element formulation for these types of problems using three-node triangular elements is developed in this particle. A three-node triangle in which the displacement is represented as a linear function of the coordinates is called a constant strain triangle (CST). An element of this type is referred to as a CST element. The strain and therefore the stress in these elements are constant. Once the element stiffness is developed, the procedure for global stiffness, boundary condition consideration, and the solution process follow the steps developed by Chandrupatla [24]. The simplicity of the CST element helps us in the development of steps involved in the two-dimensional finite element formulation.

The problem studied in this work is a special case and it is plane strain. Plane stress problems, including problems that can be three dimensional mode and simpler two-dimensional forms are considered [25]. Moreover, domain discretization using three-node triangular elements has been conducted and this is done to investigate studies on the effect of domain discretization on the response of the problems. A new program (subroutine) is developed for the state of the mentioned three-node to perform finite element analysis. Plane eight-node routine which is written in FORTRAN and published by Zienkiewicz et al. [26] is reformed to prepare that subroutine for three-node constant strain triangles. The procedure for optimization is illustrated in figure 2.
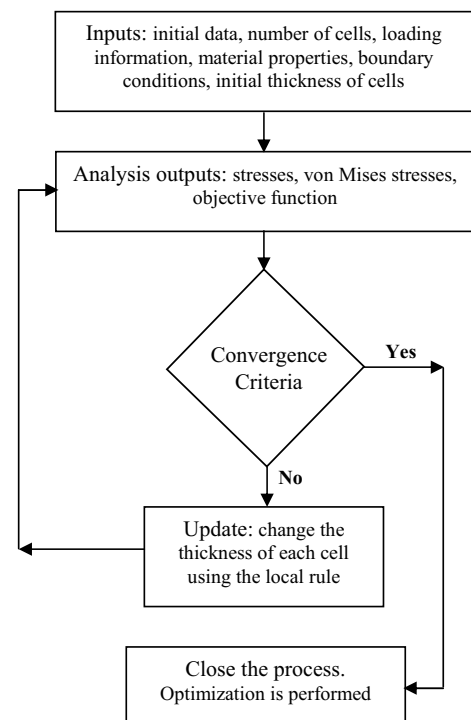

Fig. 2. The flowchart of the developed algorithm

5. Examples

In this paper the developed algorithm is applied in three examples to demonstrate the efficiency and accuracy of the developed method. The considered examples reported in the literature, so to compare the developed algorithm with valid case studies and to discuss the differences between results these examples are tested. The optimized shape and topology of these case studies is obtained after repeating the optimization process. To compare the obtained results with the other publication, specification and initial assumptions of the following examples are similar to the article published by Kita and Toyoda [12]. Results are compared with the reported topologies using quadrangle CA and GA-based models.

5.1. Example 1

Figure 3 shows the design domain, in meter, loading and boundary conditions for this case study. In this instance, a cantilever beam is considered and one point load P is applied at the end of the beam in the mid side. The following design parameters have been assumed during the analysis and the design process. In this consideration, σ_0 refers to the maximum stress at the initial topology. At the initial step, the thicknesses of all cells are considered as equal.

Design Parameters:
Number of cells	$16 \times 24 \times 2$
Penalty parameter	10
Young's modulus	$E = 200(MPa)$
Poisson's ratio	$v = 0.2$
Thicknesses of cells	$t_0 = 1.0m$
Force	$P = 20.2(N)$
Allowable stress	$\sigma_c = 0.8 \times \sigma_0$

There are a number of researches of the genetic algorithm (GA)-based approach to structural optimization in the literature, among them works developed by Jakiela and associated researchers [27-31]. Further details and example problems can be found in these references and the related thesis of Chapman [32] and Duda [33]. Using an evolutionary, survival-of-the-fittest optimization mechanism [34-35], the GA allows designs in a population to compete against one another to serve as parent designs. The example 1 is similar to the case which is investigated in [36]. The results of this work are compared to those reported in [12, 36].
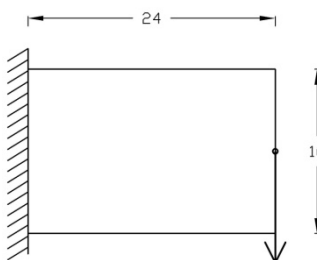
Figure 4(a) displays the optimized distribution of cell thickness, after 100 and 400 iterations obtained using the mentioned scheme in this paper. On the other hand, figure 4(b) represents the profiles at the same iteration as reported by Kita and Toyoda [12]. Figure 4(c), also shows the results reported by Jakiela et al. [36] which achieved using GA. The homogenization-based solution, using a rectangular hole microstructure [37], is also demonstrated in Figure 4(d).It should be notice that both results are obtained using the same
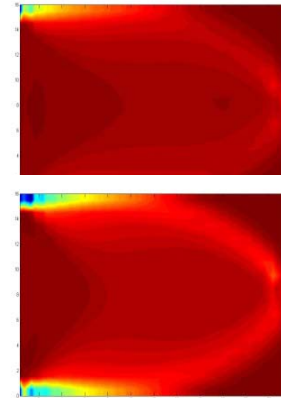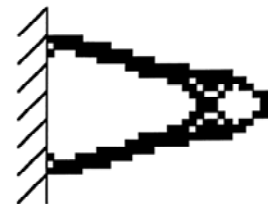

(a): The optimized domain after 100 and 400 iteration
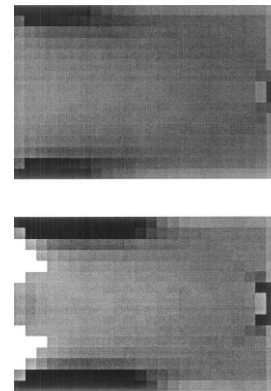
(b): The profiles at the same iterations reported in [8]
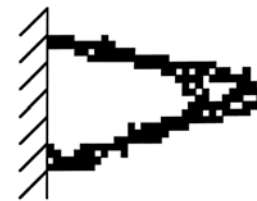
(c): The GA-based solution reported in [32]

(d) The homogenization-based solution reported in [32]

Fig. 4. Thickness distribution of topology optimization of cells

Fig. 3. Design domain and loading of example 1

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material size, properties, and the same loading condition. But the FM solver employed quadrilateral elements, while in this work FM solver utilizes triangular elements. The GA-based solution contains 3% less material than Rodriques's homogenization-based solution. The main advantages of a GA-based solution are that it performs a global search and readily allows a variety of fitness functions and constraints. The GA-based solution, however, required 10-100 times the number of function evaluations as would be required by a homogenization-based solution [36]. While, the developed CA-based approach requires not a number of generations but one solution and requires only about 400 iterations to achieve the optimum solution.

CA-based solution as reported in [12], after 1500 iterations obtained the optimum solution which is comparable with the solution obtained in this work after 400 iterations. Although the final solution in this work has not the accuracy as the profile obtained using Homogenization method reported in [36], but there is a great timesaving behavior in the developed CA-based algorithm. For instance, in this example optimization using GA requires 600 generations with population size 30, as stated in [36]. So, in this case it needs to evaluate the solutions 18000 times. But using the triangle CA-based algorithm a reasonable and near to global optima solution has been obtained after about 400 iterations, and just 400 function evaluations would be required during the optimization process. The reason for this advantage is the shape of the element, which is triangle FM, and the updating rule which vary the thickness of the small triangle cells immediately in any iteration. Since the number of triangle cells is twice the number of quadrangle cells, so the updating rule may affect the result to achieve the optimum solution sooner. These results show the validity, accuracy, and efficiency of the scheme developed in this paper.

5.2. Example 2

In this case study the design domain and the boundary conditions are similar to the previous case study, while the load condition is as illustrated in figure 5. Design parameters are considered similar to the previous case study. The optimized distribution of cell thickness after 100 and 400 iterations are illustrated in figure 6(a).

Figure 6(b) displays the obtained topology at the same iteration reported by Kita and Toyoda [12]. In this case study, also, the topology was obtained using the scheme developed in this paper, showing the accuracy and efficiency of the execution process.
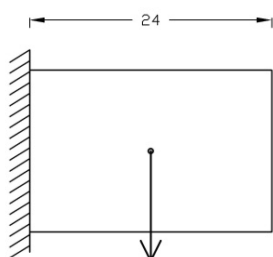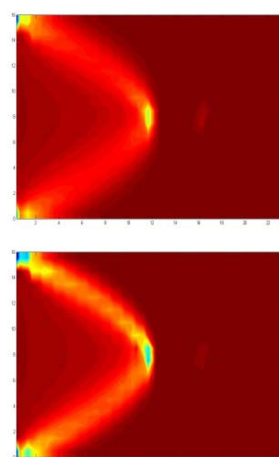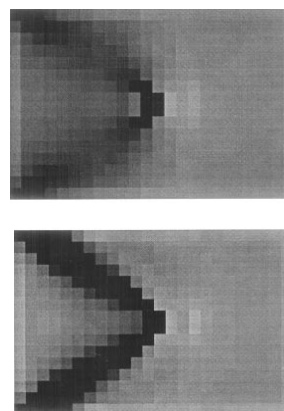

(a): The optimized domain after 100 and 400 iteration

(b): The profiles at the same iterations reported in [8]

Fig. 6. Thickness distribution of topology optimization of cells

5.3. Example 3

In this case study, two concentrated point loads are applied on the cantilever beam, with similar assumptions to the previous case studies, as shown in figure 7. The design domain and parameters of this case are also considered the same as the previous case studies. Figure 8(a) shows the thickness distribution of topology optimization of cells after 100 and 400 iterations. The topology at the same iterations are not reported by Kita and Toyoda, however, they have reported the thickness distribution at final profile after 1500 iteration, as demonstrated in figure 8(b).

Regarding the number of iterations, the results of the thickness distribution which is obtained after 400 iterations
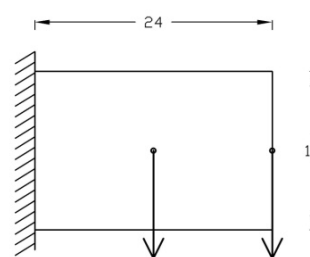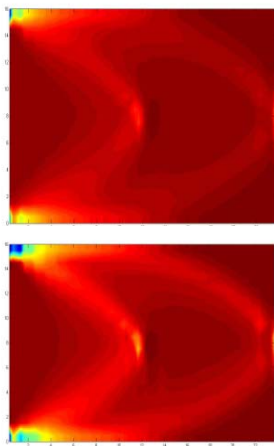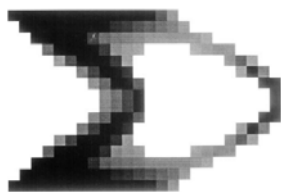

Fig. 5. Design domain and loading of example 2

Fig. 7. Design domain and loading of example 2

(a): The optimized domain after 100 and 400 iteration

(b): The profiles at the 1500$^{\text{th}}$ iteration reported in [8]

Fig. 8. Thickness distribution of topology optimization of cells

using the developed scheme of this paper, appear acceptable. Hence, based on the results of these three case studies and comparing the thickness distributions after some iteration with those reported in the literature, it can be proposed that the method developed in this paper is accurate and valid to apply to other structures. As a future research study, one can apply this scheme to large structure (e.g. tall buildings, dams, etc.) or large water networks.

Employing different value for the weight parameters in the objective function defined in equation (12), Pareto front of the two main objective functions could be obtain. The trade-offs

between the ratio of thicknesses to the initial thickness and the ratio of the von Mises equivalent stress to the yield stress of the material are theoretically demonstrated in figures 9 for the case study 3. The obtained Pareto-front shows that by increasing the thickness of the cells the ratio of the stresses tends to decrease, while decreasing the thicknesses leads to increase the stress ratio. In the other word, when in optimization process the algorithm tends to decrease too much the thicknesses to optimize the weight, the stress ratio increases rapidly and passes the allowable stress constraint. So, in the real procedure, two objectives are converted to a single objective function to form a penalized objective function. When the stress ratio is equal to 1 it means that every element has the maximum stress, so the optimum thickness is achieved, which is about 0.33 and it can be seen in figure 9. Since the weight of the structure is a proportion of the cells thickness, so the optimum weight of the structure is reached.

6. Discussion

An improved topology and shape optimization technique based on the concept of cellular automata is proposed for two-dimensional structures. This research studies particular cases for local rule known as the CA-constraint condition. The method is applied for topology and shape optimization of two-dimensional elastic structures and the design domain is divided into small triangles in order to perform finite elements analysis, which is developed using FORTRAN. The optimized structures illustrated in this article are obtained using finite element analysis considering a novel triangle neighborhood while square cells for design domain has been considered in the literature. Quadrangle cells are divided into two triangles. Numerical case studies indicate the efficiency and accuracy of solutions obtained for optimized topology and shape of the structures. In the other words, the developed scheme in this paper is fast. Optimum shape and topology obtained for the above examples under different loadings, compared with profiles reported in the literature, are more accurate in less
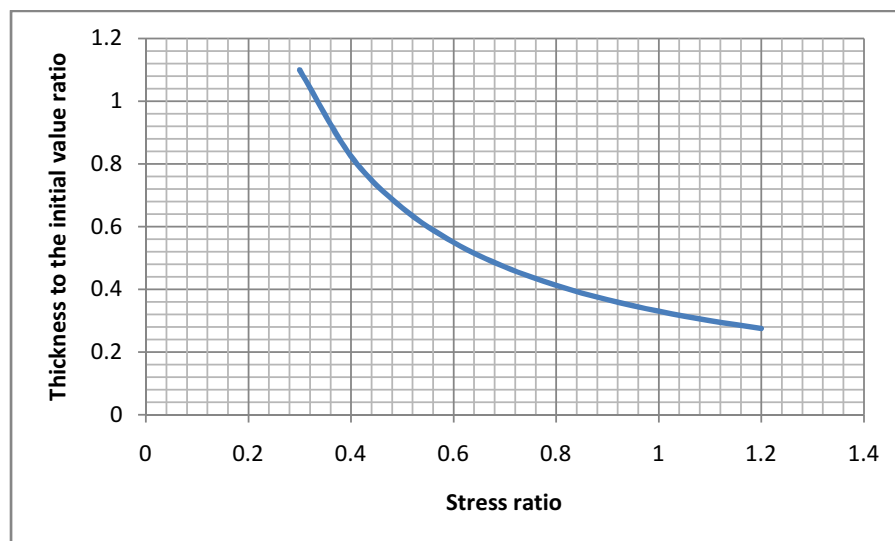

Fig. 8. Thickness distribution of topology optimization of cells

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iteration.

The important features of the algorithm in obtaining better results are: 1- the quadrangle cells are divided into two triangle elements. So, the number of cells is increased, and it will affect the results at the final step. In the other word, in the updating process, cells are affected by smaller neighbors; 2- a novel neighborhood for cells, which is triangle elements, is proposed and applied in the case studies; 3- the nature and characteristic of triangle elements in FEM analysis, comparing quadrangular elements, affects the analysis process; 4- Cells with small thickness have not been deleted during the optimization procedure, because simultaneous shape and topology optimization of the structures are investigated,.

Since, CA meshes the design domain and then updates the cell thicknesses in any iteration using a particular rule; this technique is suitable for continuum structures. While, heuristic methods such as GA, assign the thickness for the cells randomly and evaluate the solutions and then based on the fitness of each solution decide to increase/decrease the thickness of the cells. So, based on the obtained results which compared with the literature reports, the developed CA approach, based on the proposed meshing of the design domain, is suitable for continuum structures.

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