

## Effects of higher modes on vertical distribution of isolated structures under near field earthquakes

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### Abstract

It has been pointed out the static lateral response procedure for a base-isolated structure proposed in International Building Code (IBC) somewhat overestimates the seismic story force. That is why in the current paper, vertical distribution of base shear over the height of isolated structures considering higher mode effects under near field earthquakes is investigated. Nonlinear behavior of isolation systems cause variation of frequencies transmitted to the superstructure and consequently higher modes effects should be considered. In this study base shear distribution obtained from nonlinear dynamic analysis is compared with that achieved from IBC for assessment of the international building code. This investigation has been conducted in two parts, in order to have an appropriate base shear distribution formula for isolated structures under near field earthquakes. In the first part using three first mode shapes of isolated structure and introducing coefficient corresponding to each mode, extracted from nonlinear dynamic analysis under near field earthquakes, a new formula has been derived. In the second part, the mode shape coefficients have been obtained theoretically and consequently a new base shear distribution over the height of isolated structures including the isolation system properties under near field ground motions was proposed.

**Keywords:** Base-isolated structures, Base shear distribution, Near field earthquake, Higher modes effects.

### 1. Introduction

Seismic design of structures is based on increasing the resistance capacity of the structure against earthquakes; however it causes higher accelerations at the floors or increasing of drift in flexible structures. This means during the strong earthquake motions although the structure does not collapse, nonstructural components are damaged significantly. It is not acceptable for buildings equipped with more valuable contents such as telecommunication center or hospitals which need to be operated immediately after the earthquakes [1].

In order to solve this problem, isolation systems are employed in the structures. Using the isolators a shift in the period of system is noticed. By increasing this period acceleration exerted on superstructure due to earthquake is reduced and eventually seismic forces on the superstructure are decreased [2]. Also this prevents the period of system to fall within the range of earthquake dominant frequency,

resulting in the prevention of resonance phenomenon [3].

It is observed that the isolation systems have nonlinear behavior. This characteristic results in major portion of the earthquake energy to dissipate by isolation system. Therefore the structural deformations remain within the elastic range [2]. According to UBC-91[4], the vertical distribution of lateral forces is based on this assumption that motion of the superstructure is similar to a rigid body motion, thus the acceleration in the stories is uniform. Based on this suggestion distribution of the lateral forces is proportional to the mass of story:

$$F_x = \frac{V_s w_x}{\sum_{i=1}^n w_i} \quad (1)$$

$w_x$  and  $w_i$ : the weight of stories at  $x$  and  $i$  levels

$F_x$ : the force at the  $x$  level

Since effects of the higher modes are not considered, it causes non-conservative results. According to UBC-97[5], and IBC [6], base shear distribution ( $V_s$ ) over the height of structure can be obtained by following equation:

$$F_x = \frac{V_s w_x h_x}{\sum_{i=1}^n w_i h_i} \quad (2)$$

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Where  $h_x$  and  $h_i$  represent height of stories at  $x$  and  $i$  levels. This triangular shear distribution for isolated structure is similar to fixed base structures. The previous investigations demonstrate that the triangular distribution is always a conservative estimation comparing to the exact distribution obtained from nonlinear dynamic analysis. In addition the base shear distribution can be indicated as following according to Eurocode [7]:

$$F_x = m_x S_e (T_{eff}) S_{eff} \quad (3)$$

$F_x$  : xth story shear force  
 $m_x$  : mass of the xth story  
 $T_{eff}$  and  $S_{eff}$  : effective period and damping of the isolation system respectively  
 $S_e$  : spectral acceleration

Vertical distribution of lateral forces on structures except for the non-reinforced masonry buildings can be obtained as [8]:

$$F_x = C_{vx} V \quad (4)$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^n w_i h_i^k} \quad (5)$$

$C_{vx}$  is vertical distribution coefficient,  $k=2.0$  for  $T \geq 2.5$  sec,  $k=1.0$  for  $T \leq 0.5$  sec and for  $0.5 \text{ sec} < T < 2.5 \text{ sec}$   $k$  values are obtained by linear interpolation.

Lee and Kim studied vertical distribution of base shear for base-isolated structures [9]. They proposed formula based on the dynamic of a two-mass linear system as follows:

$$F_x = \frac{w_x (1 + \frac{\varepsilon h_x}{\mu h_n})}{\sum w_i (1 + \frac{\varepsilon h_i}{\mu h_n})} V_s \quad (6)$$

Where  $\varepsilon = (\omega_b^2 / \omega_s^2)$  and also  $\omega_b$  and  $\omega_s$  are fixed-base and isolation system frequencies, respectively.  $\mu$  is the effective height coefficient and  $h_n$  is the height of the structure. Donatello, Cardone and Mauro Dolce proposed a formula for vertical distribution of base shear [10]. They Considered nonlinearity, period of isolation system and period of superstructure and proposed equations for first three mode shapes.

$$\Delta_{1i} = \phi_{1i} \quad (7)$$

$$\Delta_{2i} = \phi_{1i} + a_2 \phi_{2i} \quad (8)$$

$$\Delta_{3i} = \phi_{1i} + a_2 \phi_{2i} + a_3 \phi_{3i} \quad (9)$$

Where  $\Delta_{1i}$ ,  $\Delta_{2i}$  and  $\Delta_{3i}$  are profile of displacement,  $\phi_{1i}$ ,  $\phi_{2i}$  and  $\phi_{3i}$  are first three mode shapes,  $a_2$  and  $a_3$  are coefficients related to characteristics of isolation system and superstructure.

Khoshnoudian and Esrafilii proposed a formula for base shear distribution as follows [11]:

$$F_x = \frac{V_s}{n+1} ([A] + \eta[B] + \rho[C]) \quad (10)$$

Where  $[A]$ ,  $[B]$  and  $[C]$  are first three simplified mode shapes of isolated structures.  $\eta$  and  $\rho$  are coefficients that obtained from using nonlinear dynamic analysis and  $n$  is number of story. In addition Khoshnoudian and Mehrparvar proposed a new formula for vertical distribution of base shear over the height of isolated structures as follows [12]:

$$F_x = \mu \frac{w_x}{\sum_{i=1}^n w_i} V_s + (1 - \mu) \frac{w_x h_x}{\sum_{i=1}^n w_i h_i} V_s \quad (11)$$

Where  $\mu$  is a coefficient that obtained from comparison of nonlinear dynamic analysis with proposed formula.

In the previous investigations, simplified formulas were proposed without including isolation properties for equivalent lateral response procedure and the earthquake ground motions were assumed far field earthquakes. That is why in the current paper, the objective is to propose a new formula for vertical distribution of base shear for equivalent lateral response procedure considering near field earthquakes and in addition attempt is made to include the isolation system properties in the proposed formula.

## 2. Modeling of super-structure and isolation system

In this investigation full 3-D models of isolated structures were analyzed by ETABS computer program (based on 3D-basis program) (ETABS, 1999). The super-structures and isolators were modeled using linear and nonlinear behaviors respectively. Nllink elements in ETABS were used for modeling elastomeric nonlinear isolators. The element has coupled plasticity properties for two shear deformations and linear effective properties for the remaining four deformations. The plasticity model is based on the hysteretic behavior proposed by Wen [19], and Park et al [14], and recommended for base isolation analysis by Nagarajaiha et al [18]. The building models under consideration have symmetric plan (Fig.1) and distribution of mass and stiffness is uniform through the height. The structures consist of 2, 4 and 6 stories assuming story height of 3.5m. Ordinary steel moment frames are employed in this investigation. Steel structures have been designed according to American Institute of Steel Construction

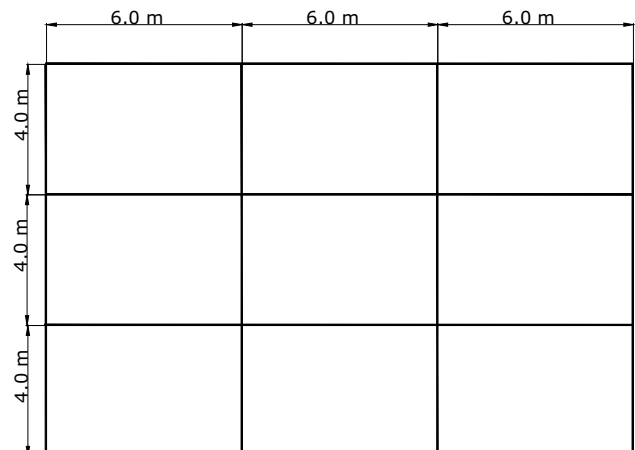


Fig. 1 Plan of structure

(AISC-ASD) [13]. Cross sections of beams and columns are shown in table 1. In addition, the structural properties (periods and effective modal mass of the first 3 modes) are presented in table 2.

Six types of isolation systems have been used in this study. The isolators are supposed elastomeric and are modeled by bi-linear hysteresis behavior having plastic behavior for two shear deformations. Effective linear stiffness for other deformations is assumed constant. Plasticity model is on the basis of hysteretic behavior proposed by Wen et al. [14]. Isolation system parameters involve initial stiffness ( $K_1$ ), secondary stiffness ( $K_2$ ), effective stiffness ( $K_{eff}$ ), intersection of hysteresis cycle and vertical axis ( $Q$ ) and period ( $T$ ) (Fig. 2). Table 3 shows the properties of selected isolation systems, where  $W$  stands for the structure effective weight. Considering Fig.2 the following equations can be derived:

$$D_y = \frac{Q}{K_1 - K_2} \quad (12)$$

$$K_{eff} = K_2 + \frac{Q}{D} \quad (13)$$

**Table 1** Cross sections of beams and columns

No of stories	Column section(cm)	Beam section (X dir.)	Beam section (Y dir.)
2	BOX 20×20 ×1.6	IPE330	IPE200
4	BOX 24×24 ×1.6	IPE360	IPE220
6	BOX 30×30 ×1.6	IPE360	IPE240

**Table 2** Structural properties (periods and effective modal mass of the first 3 modes)

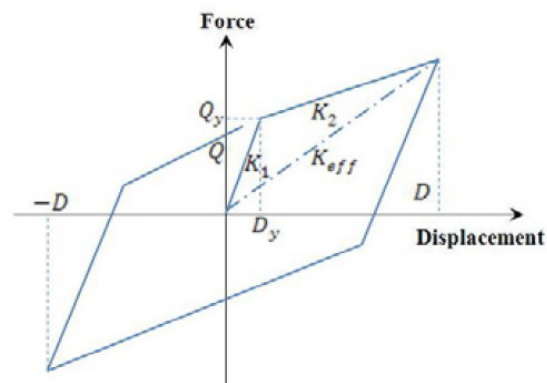
No. of stories	First mode		Second mode		Third mode	
	Period	Modal mass	Period	Modal mass	Period	Modal mass
2	2(s)	99.9801	0.72	0.0193	0.26	0.0006
	2.5(s)	99.9884	0.72	0.0113	0.26	0.0003
	3(s)	99.9945	0.72	0.0054	0.26	0.0001
4	2(s)	99.8151	0.42	0.1771	0.19	0.0069
	2.5(s)	99.9218	0.42	0.0751	0.19	0.0028
	3(s)	99.9624	0.42	0.0361	0.19	0.0013
6	2(s)	99.6251	0.44	0.3527	0.22	0.0195
	2.5(s)	99.8377	0.45	0.1532	0.22	0.008
	3(s)	99.921	0.45	0.0748	0.22	0.0038

**Table 3** Properties of isolation systems

Isolation system type	T(s)	Effective Damping(%)	Q	$K_1$	$\frac{K_1}{K_2}$	$F_y$	$K_{eff}@D_D$	$D_D$ (cm)
1	2	6	0.02W	0.065W	7.2	0.0235W	0.01W	20.5
2	2	16	0.04W	0.065W	8.8	0.046W	0.01W	15.5
3	2	27	0.06W	0.065W	11.8	0.065W	0.01W	13
4	2.5	8	0.02W	0.04W	7.2	0.0235W	0.0064W	23.9
5	2.5	21	0.04W	0.04W	10	0.046W	0.0064W	17.5
6	3	11	0.02W	0.025W	6.8	0.0235W	0.0044W	26.1

### 3. Ground motion

In order to obtain more reliable results, 13 near field ground motions are selected from PEER strong motion database. These records were selected to cover variety of parameters such as PGA, PGV and station distance from fault and also earthquake mechanism. Properties of chosen records are presented in table 4. It is recognized that the characteristics of near-field earthquakes are different from far-field ones [15]. The severity of the earthquake is often measured by the PGA while for the near-field records this is not always true. The near-field ground motions may contain high PGA value that corresponds to a short duration pulse with negligible effect on the structure. On the other hand, a low PGA with long duration pulse may have severe effects on structure. Since the PGA is



**Fig. 2** Bi-linear hysteresis model of isolators

**Table 4** Values of simplified formulation

Record	Station	PGA (g)	PGV (g)	Tp (Sec)
Bam	Bam	0.814	124	1.7
Cape Mendocino	89005 Cape Mendocino	1.497	127.4	0.7
Chi-Chi	TCU068	0.462	263.1	9.4
Duzce	1058 Lamont 1058	0.111	14.2	2
Erzincan	95 Erzincan	0.515	83.9	2.2
Imperial Valley	5115 El Centro Array #2	0.315	31.59	4.2
Kobe	0 KJMA	0.821	81.3	1.3
Mammoth Lakes	54099 Convict Creek	0.442	23.1	0.4
Northridge	24279 Newhall - Fire Sta	0.59	97.2	2
Northridge	77 Rinaldi Receiving Sta	0.838	166.1	1.1
Northridge	24514 Sylmar - Olive View Med FF	0.843	129.6	2.4
San Fernando	279 Pacoima Dam	1.226	112.5	1.1
Tabas	9101 Tabas	0.852	121.4	4.8

not appropriate for quantifying near-field earthquake effects, the velocity pulses (PGV) should be considered. The PGV corresponds to the integration of relatively large pulses in the acceleration time history [16]. In the near-fault region, the short travel distance of the seismic waves does not allow enough time for the high frequency content to be damped. Near-filed earthquakes may contain large amplitude long period pulses. The long period pulses in near-field records may cause strong fundamental mode shapes response of long period structures. In addition, the high frequency content of the same record may coincide with the second or higher modes resulting in severe overall response of the structure [15].

#### 4. Modal shapes of structure and results of nonlinear dynamic analysis

By examining the modal analysis results, it is demonstrated that modal shapes of structures are very similar to each other. This similarity in modal shapes can be utilized to propose a reliable formula for base shear distribution of isolated structures.

Nonlinear time history analysis for structures consist of 2, 4 and 6 stories using 6 type of isolation systems under 13 near field earthquake records have been conducted. It is noted that in Imperial Valley, Kobe, Mammoth Lakes and Northridge-Newhall records due to content of frequencies, the effects of higher modes are more significant. In addition, for 4 and 6 story buildings or for isolation systems with high effective damping the effects of higher modes are more remarkable. The results demonstrate that static procedure for an isolated structure proposed by IBC somewhat overestimates the story force.

#### 5. Proposed formula for base shear distribution without considering isolation system properties

Comparing mode shapes and patterns of base shear distribution shows that besides the first mode, second and third modes have effects on this distribution. Having in mind the similarities in mode shapes by simplifying first three modes (Fig.3) the following formula for base shear distribution can be derived:

$$F_x = \frac{V_s}{n+1} ([A] + \eta[B] + \rho[C]) \tag{14}$$

Where  $n$  is number of stories,  $[A]$ ,  $[B]$  and  $[C]$  are vectors with  $n+1$  rows according to the figure 3. The quantities of  $\eta$  and  $\rho$  are derived from comparison between results of

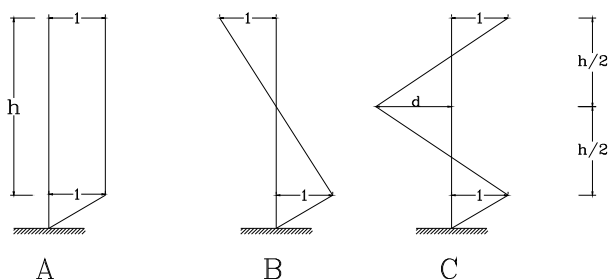


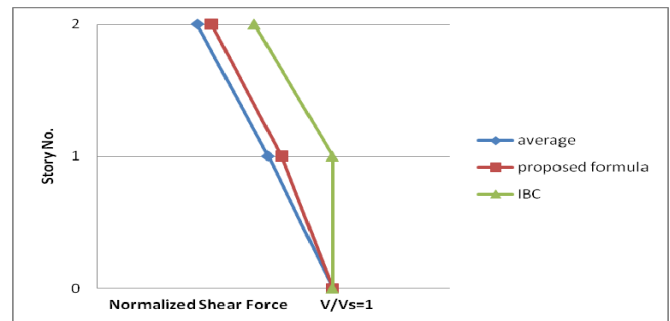
Fig. 3 Simplified mode shapes

nonlinear dynamic analysis and the previous equation as shown in table 5. Fig.4 shows comparison between shear force distributions obtained from average results of nonlinear dynamic analysis, proposed formula and IBC formula for 2, 4 and 6 story structures. It is noted that the proposed formula gives more reasonable distribution of base shear over height of structures with respect to IBC suggestion.

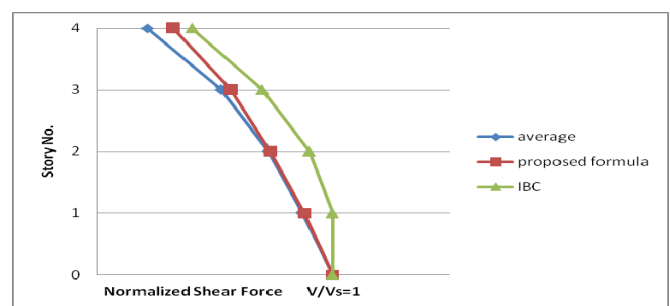
Fig.5 illustrates the bar chart of percentage error of base shear distribution obtained from nonlinear dynamic analysis results, proposed and IBC formula. It was demonstrated that the proposed formula compared to IBC formula is closer to the average results of nonlinear dynamic analysis as an exact

Table 4 Ground motions

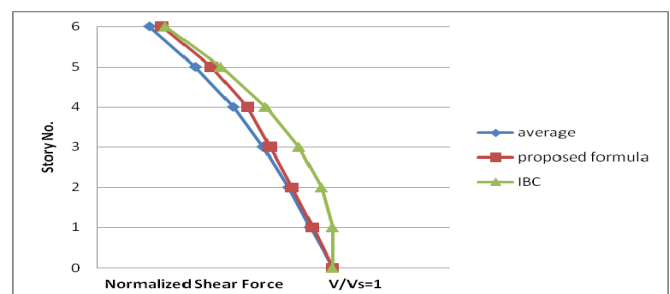
No of story	$\eta$	$\rho$	$d$
2	-0.24	0.03	2
4	-0.45	-0.06	1.5
6	-0.51	0.1	1.33



(a) 2 story building



(b) 4 story building

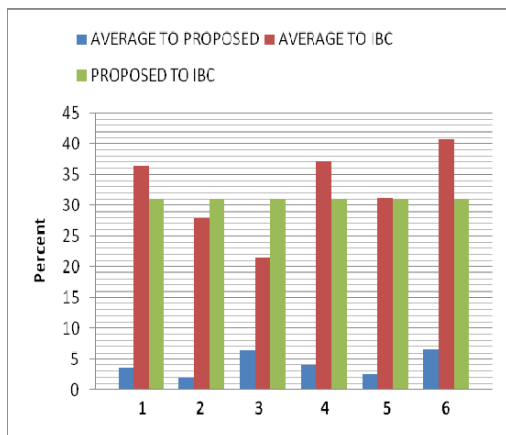


(c) 6 story building

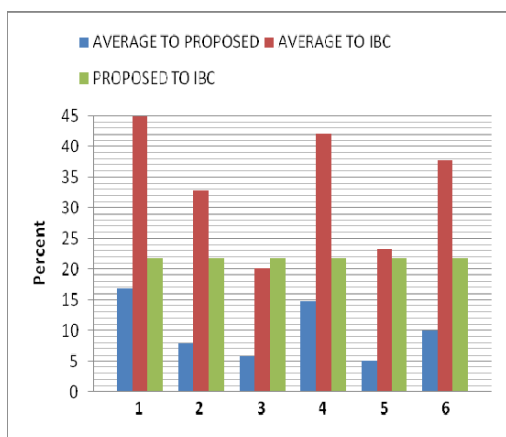
Fig. 4 Comparison between distributions of shear force obtained from nonlinear dynamic analysis, proposed formula and IBC formula

solution; hence proposed formula gives more realistic base shear distribution. In addition, the error of proposed formula in comparing to nonlinear dynamic analysis is up to 17%. This error for 4 and 6 story buildings is more than 2 story buildings. Distribution of base shear of isolated structure with 4 and 6 story building is more function of characteristics of isolation systems. The error of IBC formula in predicting

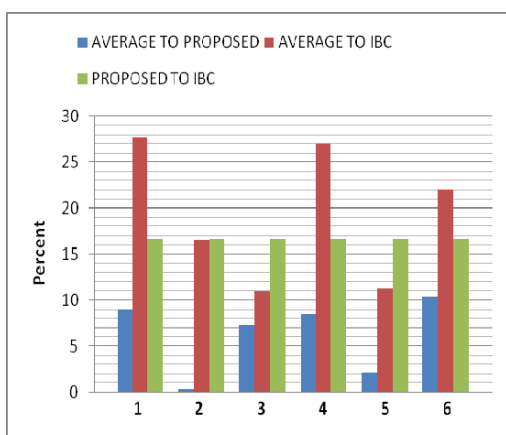
base shear distribution in comparison with nonlinear dynamic analysis is 11% to 45%. The error is more significant for 4 and 6 story buildings comparing to 2 story buildings. Therefore for 4 and 6 story buildings, results of nonlinear dynamic analysis are closer to triangular distribution. In addition, IBC formula overestimates the seismic story force. Difference between proposed formula and IBC suggestion is 31%, 22% and 17% for 2, 4 and 6 story buildings respectively. It is noted that the proposed formula is closer to triangular distribution for buildings with higher height.



(a) 2 story building



(b) 4 story building



(c) 6 story building

Fig. 5 Comparison of proposed formula and IBC suggestion to nonlinear dynamic analysis for various isolation system

## 6. Proposed formula for base shear distribution of isolated structures based on isolators system properties

In the previous section by simplifying the first three modes of isolated structures and allocating coefficients to each mode, a formula for base shear distribution has been proposed. However, the coefficients utilized for each mode is independent on properties of isolation system, i.e. for a 6 story building with any type of isolation system a unique base shear distribution is obtained. In this part by using theory of structural dynamic a new formulation for vertical distribution of base shear over height of isolated structures is proposed including the properties of isolation systems.

### 6.1. Theory and assumptions of the proposed formula

Assuming the isolation systems represent as spring with stiffness  $k_{eff}$  then for an n story building it is possible to present the dynamic equilibrium in the form of the following equation:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{I\}\ddot{x}_g \quad (15)$$

Where,

$$[M] = \begin{bmatrix} m_1 & 0 & & & & \\ 0 & m_2 & & & & \\ & & \ddots & & & \\ & & & & & \\ & & & & & \\ & & & & & m_{n+1} \end{bmatrix} \quad [C] = \begin{bmatrix} c_1 + c_2 & -c_2 & & & & \\ -c_2 & c_2 + c_3 & & & & \\ & & \ddots & & & \\ & & & & & \\ & & & & & \\ & & & & & c_{n+1} \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_{eff} + k_1 & -k_1 & & & & \\ -k_1 & k_1 + k_2 & & & & \\ & & \ddots & & & \\ & & & & & \\ & & & & & k_n \end{bmatrix}$$

Where  $k_{eff}$  is the effective stiffness of isolation system. As a result the governing equation without damping can be expressed as follows:

$$[M]\{\ddot{u}\} + [K]\{u\} = -[M]\{I\}\ddot{x}_g \quad (16)$$

In this study natural frequency and modal shapes have been used and the displacement of the system can be presented using following equation:

$$\{u(t)\} = [\chi]\{y(t)\} \quad (17)$$

$$\text{Modal matrix} = [c] = [\{f1\}, \{f2\}, \dots, \{fn+1\}] \quad (18)$$

Substituting equation (17) in equation (15) and multiplying by  $[\chi]^T$ , the nth row of obtained equation can be written as:

$$M_n \ddot{y}_n(t) + C_n \dot{y}_n(t) + M_n \omega_n^2 y_n(t) = \{\phi_n\}^T \{P_{eff}\} \quad (19)$$

$$\{P_{eff}\} = -[M][I]\ddot{x}_g \quad (20)$$

$$\{f_n(t)\} = \{\phi_n\}^T \{P_{eff}\} = -\{\phi_n\}^T [M][I]\ddot{x}_g = \bar{K}_n \ddot{x}_g \quad (21)$$

$$\bar{K}_n = \{\phi_n\}^T [M][I] \quad (22)$$

$\bar{K}_n$  is earthquake excitation coefficient for nth mode. Equation (19) can be written as following form:

$$\ddot{y}_n(t) + 2\xi_n \omega_n \dot{y}_n + \omega_n^2 y_n = \frac{\bar{K}_n}{M_n} \ddot{x}_g(t) \quad (23)$$

Considering Duhamel's integral,  $\{u(t)\}$  can be shown as follows:

$$y_n(t) = \frac{\bar{K}_n}{\omega M_n} V(t) \quad (24)$$

Where  $V(t)$  is pseudo velocity.

Equation (24) can be expressed as following by substituting equation (24) into equation (17):

$$\{u(t)\} = [\chi] \left\{ \begin{array}{c} \vdots \\ \frac{\bar{K}_n}{M_n \omega_n} V_n(t) \\ \vdots \end{array} \right\} \quad (25)$$

Elastic force vector can be presented as follows:

$$\{f_s(t)\} = [K]\{u(t)\} \quad (26)$$

By substituting equation (25) into (26) elastic force vector becomes:

$$\{f_s(t)\} = [K][\chi] \left\{ \begin{array}{c} \vdots \\ \frac{\bar{K}_n}{M_n \omega_n} V_n(t) \\ \vdots \end{array} \right\} \quad (27)$$

Elastic force vector can be rearranged by using equation 22 as:

$$\{f_s(t)\} = [M][\chi] \left\{ \begin{array}{c} \vdots \\ \omega_n^2 \frac{\bar{K}_n}{M_n} V_n(t) \\ \vdots \end{array} \right\} \quad (28)$$

Since term  $\omega_n^2 \frac{V_n(t)}{M_n}$  is acceleration type, equation (28) can be summarized for nth mode as:

$$\{f_{sn}(t)\} = [M][\phi_n] \frac{\bar{K}_n}{M_n} A_n(t) \quad (29)$$

$$M_n = \{\phi_n\}^T [M]\{\phi_n\} \quad (30)$$

It is noted that by summation of coefficient  $[M] \frac{\bar{K}_n}{M_n} A_n(t)$  in

equation (29) for all necessary modes  $\{f_s(t)\}$  can be obtained. By using equation (22) and (30) the values of  $\bar{K}_n$  and  $M_n$  for each mode can be ascertained. If a relation is set for  $A_s(t)$  an appropriate coefficient for each mode can be presented.

The period of isolated structure is greater than 1 second and consequently  $A(t)$  can be expressed as follows according to building codes:

$$A(t) = \frac{d}{T^\lambda} \quad (31)$$

Where  $T$  is period of structure,  $d$  and  $\lambda$  in various codes have different values. In order to achieve a simple equation, coefficient of each mode should be divided by summation of coefficients of all modes. Herein, the discussion will be limited to the first three modes. Consequently considering appropriate value for  $\lambda$  parameter and using equations (32) to (34) coefficients for each mode are evaluated. Hence, the formula for base shear distribution and the relevant coefficients can be suggested as follows:

$$\alpha = \frac{\frac{\Gamma_1}{T_1^\lambda}}{\frac{\Gamma_1}{T_1^\lambda} + \frac{\Gamma_2}{T_2^\lambda} + \frac{\Gamma_3}{T_3^\lambda}} \quad (32)$$

$$\beta = \frac{\frac{\Gamma_2}{T_2^\lambda}}{\frac{\Gamma_1}{T_1^\lambda} + \frac{\Gamma_2}{T_2^\lambda} + \frac{\Gamma_3}{T_3^\lambda}} \quad (33)$$

$$\gamma = \frac{\frac{\Gamma_3}{T_3^\lambda}}{\frac{\Gamma_1}{T_1^\lambda} + \frac{\Gamma_2}{T_2^\lambda} + \frac{\Gamma_3}{T_3^\lambda}} \quad (34)$$

$$\{F\} = V(\alpha\{\phi_1\} + \beta\{\phi_2\} + \gamma\{\phi_3\}) \quad (35)$$

Where,  $\Gamma_1 = \frac{\bar{K}_1}{M_1}$

Comparison of base shear distribution obtained from nonlinear dynamic analysis and the previous equations shows that for 2 and 4 story buildings parameters  $\Gamma_2/T_2^\lambda$  and  $\Gamma_3/T_3^\lambda$ , for 6 story building parameter  $\Gamma_2/T_2^\lambda$  should be negated to reach a realistic base shear distribution.

In order to simplify the equations, stiffness and mass are assumed to be equal through the stories. Comparing base shear distribution obtained from nonlinear dynamic analysis and equations (32) to (35),  $\lambda=1.0$  for 2 and 4 story buildings and  $\lambda=1.2$  for 6 stories building is appropriate.

Using eigenvalues problem, natural frequencies can be obtained as follow and for 2 story building:

$$\text{Det} \begin{bmatrix} k_{eff} + k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} - \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \omega^2 = 0 \quad (36)$$

Solving above equation results:

$$-m^3 \omega^6 + (4knt^2 + m^2 k_{eff}) \omega^4 - 3(k^2 m + km k_{eff}) \omega^2 + k^2 k_{eff} = 0 \quad (37)$$

Assuming  $\frac{k_{eff}}{k} = \theta$  and  $\frac{\omega^2}{\omega_n^2} = X$  yields:

$$-X^3 + (4 + \theta)X^2 - (3 + 3\theta)X + \theta = 0 \quad (38)$$

Considering various values for  $\theta$  and using the previous equation natural frequencies and modal shapes can be obtained. Utilizing equations (22) and (30) results  $\bar{K}_n$  and  $M_n$  for various  $\theta$  values. Eventually by using equations (32) to (34) the coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  for various  $\theta$  values can be obtained. The results of this method for 2-story building are illustrated in figs. 6-8. Continuing this method for 4 and 6 story buildings provides to calculate  $\alpha$ ,  $\beta$  and  $\gamma$  coefficients.

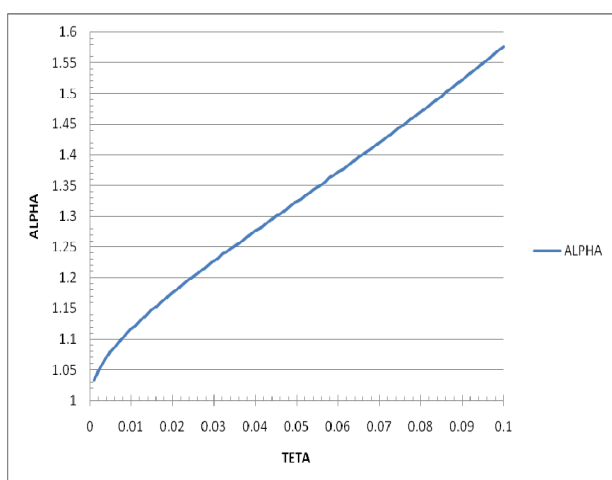


Fig. 6 Coefficient  $\alpha$  versus various  $\theta$  values for 2 story building

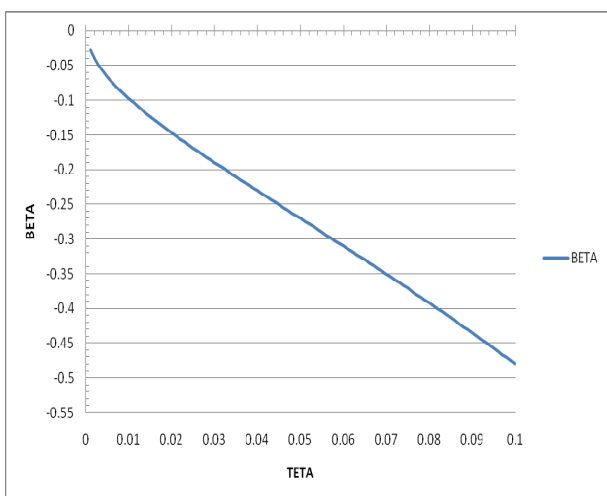


Fig. 7 Coefficient  $\beta$  versus various  $\theta$  values for 2 story building

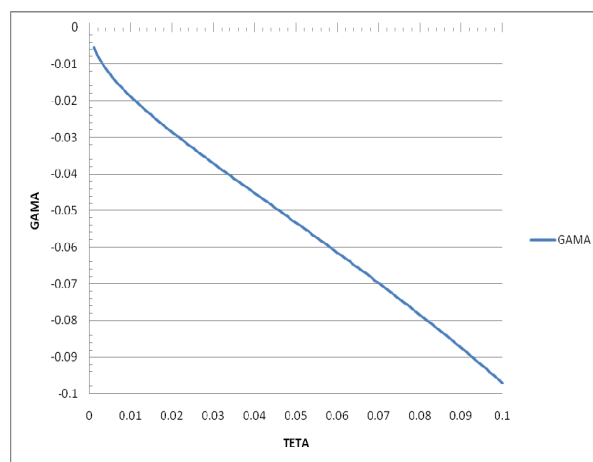


Fig. 8 Coefficient  $\gamma$  versus various  $\theta$  values for 2 story building

### 6.2. Comparison of proposed formula and results of nonlinear dynamic time history analysis

In this section the results of nonlinear dynamic analysis under 13 near field earthquakes are compared with IBC and proposed formula presented in section 5 and 6 of this paper. It is noted that the results of proposed formula presented in section 6 (named proposed formula) is closer to nonlinear dynamic analysis than formula suggested in section 5 (named previous formula) (Figs.9-14).

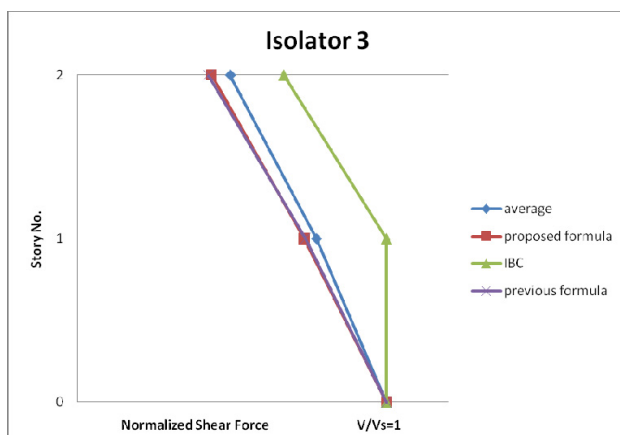
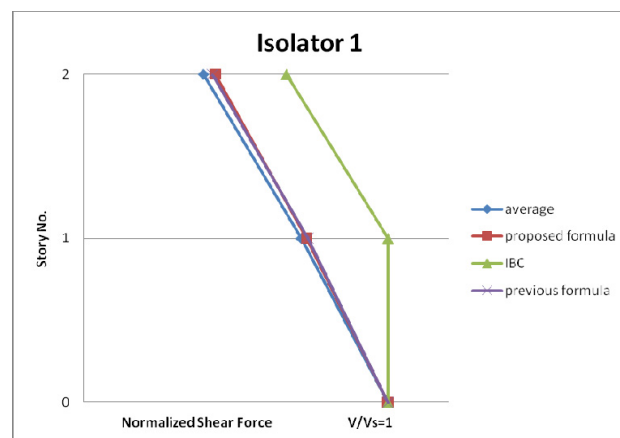
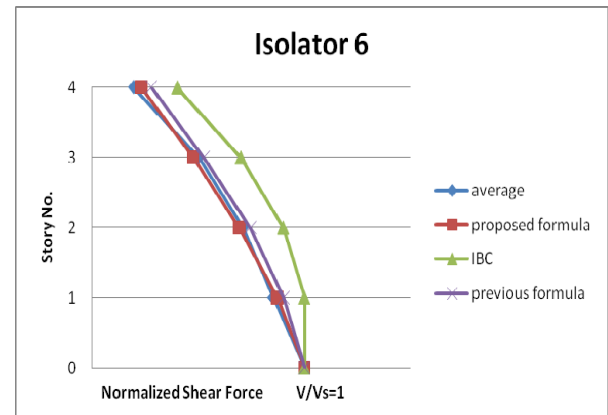
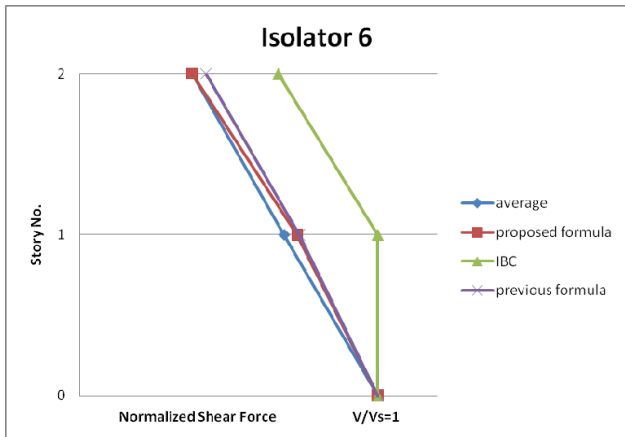
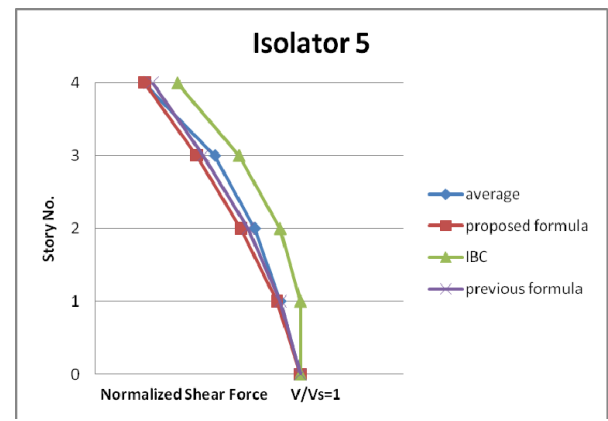
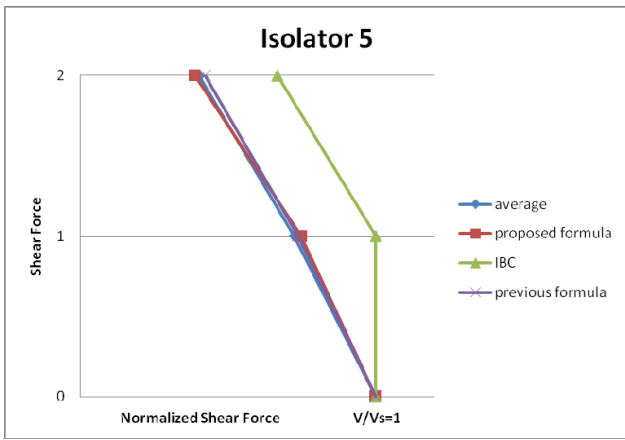


Fig. 9 Comparison of nonlinear dynamic analysis, IBC formula, proposed formula in section 6 and previous formula for 2 story building



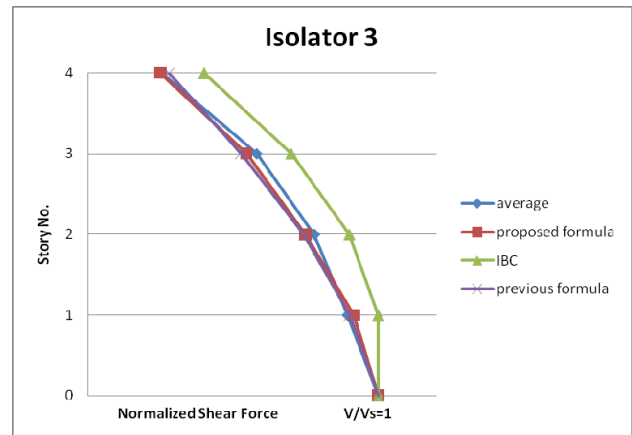
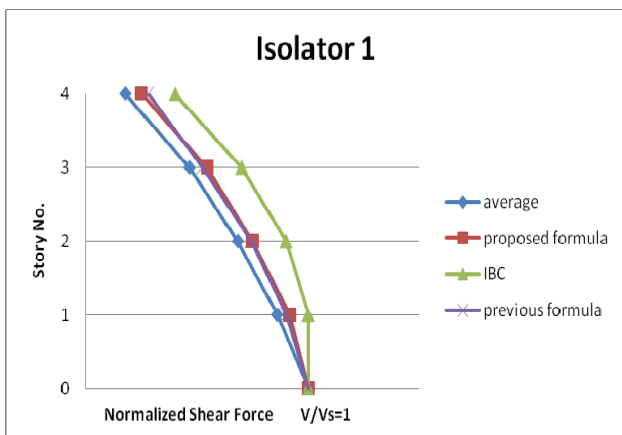
**Fig. 10** Comparison of nonlinear dynamic analysis, IBC formula, proposed formula in section 6 and previous formula for 2 story building

**Fig. 12** Comparison of nonlinear dynamic analysis, IBC formula, proposed formula in section 6 and previous formula for 4 story building

Figs.15 shows the error of base shear distributions obtained from proposed formula in section 6 and 5. Comparing results of suggested formula in section 6 and 5 it has been noted that most of the studied cases have negligible error in comparing to the average results of nonlinear dynamic analysis.

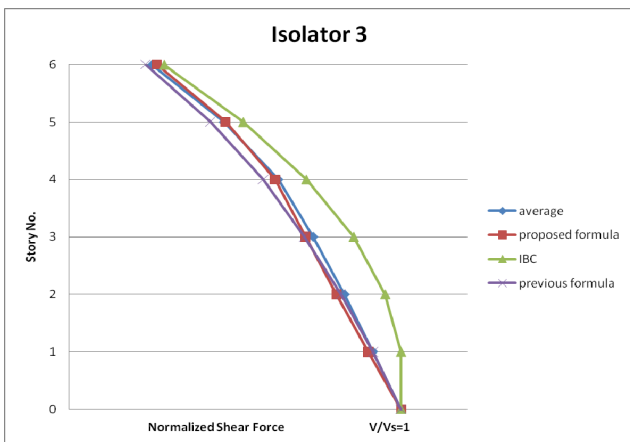
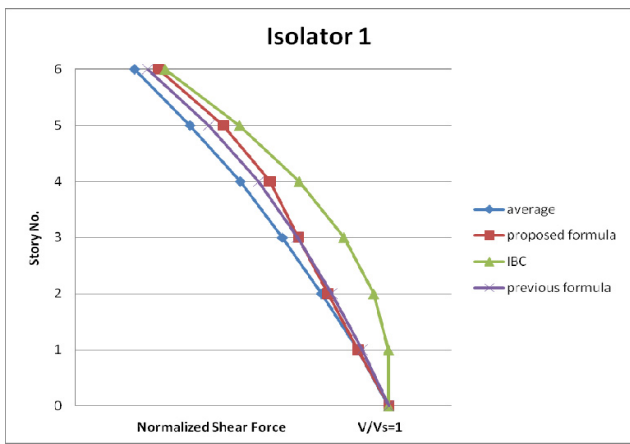
In addition, the previous figure illustrates that for 4 and 6 story buildings the difference between two formulas are more significant than 2 story building. Otherwise, it is observed that maximum percentage error occurs in isolation system type 6

while the isolation period is 3 second. According to the models, isolation systems type 1, 2 and 3 have periods equal to 2s, periods of isolation systems type 4 and 5 are equal to 2.5s and for isolation system type 6 the period is 3s. It is noted that the average results obtained from nonlinear dynamic analysis and proposed formula in section 5 has more error using isolation system type 6. Since the properties of isolation systems have been included in the proposed formula (section 6) the errors become less. It shows the advantage of the proposed formula in section 6.



**Fig. 11** Comparison of nonlinear dynamic analysis, IBC formula, proposed formula in section 6 and previous formula for 4 story building

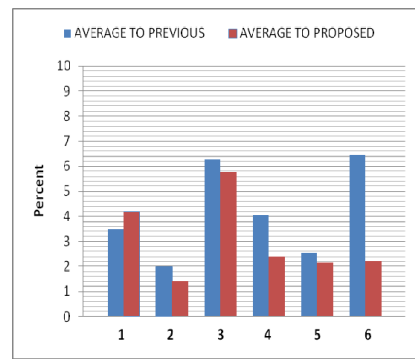




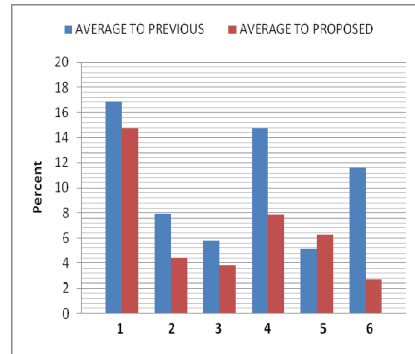
**Fig. 13** Comparison of nonlinear dynamic analysis, IBC formula, proposed formula in section 6 and previous formula for 6 story building

## 7. Conclusion

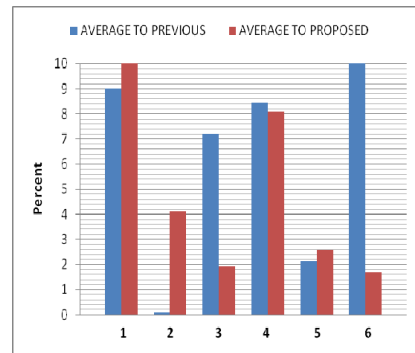
This paper focuses on suggestion a new formula for vertical distribution of base shear over height of isolated structures. For this purpose 2, 4 and 6 story buildings assuming symmetric in plan and height were studied. For verification of proposed formulas for base shear distribution, nonlinear dynamic analysis of isolated structures under thirteen near field earthquakes were considered as an exact solution. The results obtained from nonlinear dynamic analysis, IBC and



(a) 2 story building

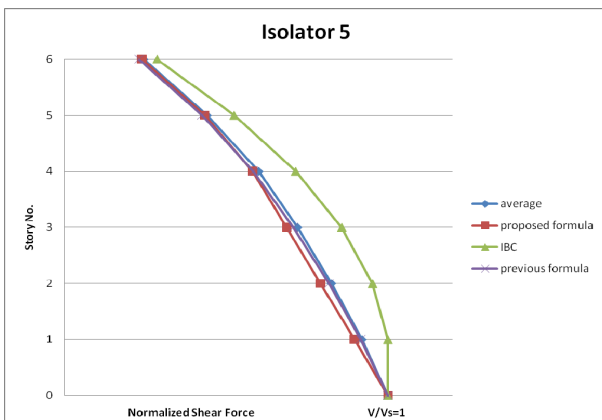


(b) 4 story building



(c) 6 story building

**Fig. 15** Comparison of previous and proposed formula to nonlinear dynamic analysis for various isolation systems



**Fig. 14** Comparison of nonlinear dynamic analysis, IBC formula, proposed formula in section 6 and previous formula for 6 story building

two proposed formula lead to the following results:

1. Base shear distribution of isolated structures falls between uniform and triangular distribution. The suggested formula in IBC overestimates shear story which consequently omits the capability of the isolation system in reducing forces transmits to superstructure.

2. Considering the nonlinear behavior of isolation systems, due to change in stiffness of isolation systems, wide range of periods is transmitted from isolation system to superstructure. As a result higher modes contribute in response of structure. Hence, the effects of higher modes should be considered in response of structure.

3. Proposed formula presented in section 5 is almost near to average results obtained from nonlinear dynamic analysis, however, this formula is independent in properties of isolation system, and i.e. for a building with variety of isolation systems this formula is identical.

4. Another formula was proposed in section 6 and it is function of effective stiffness and period of isolation system. Thus by changing the properties of isolation system the coefficients of first three modes change too. Consequently pattern of base shear distribution will change too.

5. According to the comparison of proposed formulas and nonlinear time history analysis as an exact solution, the accuracy of formulas was demonstrated by illustrating the errors of various cases. In addition, these results confirm the contribution of higher modes on vertical distribution of base shear over height of isolated structures.

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## Notation

$V_s$	calculated base shear from code formula
$w_x, w_i$	weight of stories at x and i levels
$h_x, h_i$	height of stories at x and i levels
$F_x, m_x$	force and mass at x level
$T_{eff}, S_{eff}$	effective period and effective damping of isolation system
$S_e$	spectral acceleration
$C_{vx}$	vertical distribution coefficient
$\varepsilon$	square ratio of isolation system frequency to superstructure frequency
$\mu$	effective height coefficient
$h_n$	height of the structure
$\Delta_{1i}, \Delta_{2i}, \Delta_{3i}$	profile of displacement
$\phi_{1i}, \phi_{2i}, \phi_{3i}$	first three mode shapes
$a_2, a_3$	coefficients related to characteristics of isolation system and superstructure
$[A],[B],[C]$	first three simplified mode shapes of isolated structures
$\eta, \rho$	factors related to second and third mode shapes contribution
$n$	number of stories
$K_1, K_2, K_{eff}$	initial, secondary and effective stiffness respectively
$Q$	intersection of hysteresis cycle and vertical axis
$T$	period of super-structure
$D_y$	yield displacement
$[M],[C],[K]$	mass, damping and stiffness matrix respectively
$\{\dot{u}\}, \{u\}, \{u\}$	acceleration, velocity, and displacement of structure respectively
$\ddot{x}_g$	ground acceleration
$\{y(t)\}$	modal coordinate
$[\chi]$	modal matrix
$K_n$	modal excitation factor for nth mode
$M_n, C_n, K_n$	modal mass, damping, and stiffness for nth mode
$\xi_n$	damping ratio for nth mode
$\dot{V}(t)$	pseudo velocity
$\{f_s(t)\}$	elastic force vector
$\Gamma_1$	participation factor of first mode
$\alpha, \beta, \gamma$	factors corresponding to first, second and third mode shapes contribution
$\theta$	ratio of $K_{eff}$ to stiffness of story