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# An iterative process for pushover analysis of double unsymmetric-plan low- and medium-rise buildings under bi-directional seismic excitations

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***Abstract***

*Double- unsymmetric-plan medium-rise buildings subjected to bi-directional seismic excitation are complex structures where higher-mode effects in plan and elevation are important in estimating the seismic responses using nonlinear static or pushover analysis. Considering two horizontal components of the ground motions makes the problem more intricate. This paper presents a method for nonlinear static analysis of double unsymmetric-plan low- and medium-rise buildings subjected to the two horizontal components of ground motions. To consider bi-directional seismic excitation in pushover analyses, the proposed method utilizes an iterative process until displacements at a control node (centre of mass at the roof level) progressively reach the predefined target displacements in both horizontal directions. In the case of medium-rise buildings, continuous implementation of modal pushover analyses is used to take higher-mode effects into account. To illustrate the applicability and to appraise the accuracy of the proposed method, it is applied to the 4- and 10-storey torsionally-stiff and torsionally-flexible buildings as representative of low- and medium-rise buildings, respectively. For the purpose of comparison, modal pushover analysis (MPA) is also implemented considering the two horizontal components of the ground motions. The results indicate that the proposed method and the MPA procedure can compute the seismic demands of double unsymmetric-plan low- and medium-rise buildings with reasonable accuracy; however, seismic responses resulting from the proposed method deteriorate at the flexible edge of the torsionally-flexible buildings.*

**Keywords:** *An iterative process, Pushover analysis, Modal pushover analysis (MPA), Bi-directional excitation, Low- and medium-rise buildings, Higher-mode effects, Double unsymmetric-plan buildings.*

## 1. Introduction

Conventional pushover analysis suffers from a major drawback that the response of a structure is assumed to be controlled by the fundamental mode. In this pushover analysis, the structure is subjected to monotonically increasing lateral forces with an invariant distribution until a control node reaches a predefined target displacement. Both the invariant load patterns and the target displacement do not take higher-mode contributions into consideration. Therefore, the application of conventional pushover analysis is limited to the cases in which the fundamental mode dominates the response. Theses cases involve low-rise and regular buildings. In recent years, significant research efforts have been undertaken to develop pushover procedures that account for higher-mode influences. These include procedures such as multi-mode pushover (MMP) method [22], modal pushover analysis (MPA) [6], incremental response spectrum analysis (IRSA) [3], upper-bound pushover analysis [12], Adaptive modal combination procedure [13] the extended N2 method [14] and modal spectra combination method [23]. Among these methods, modal pushover analysis (MPA) has attracted more attention due to its elegant concept which is based on structural dynamics theory. Although, the MPA method can predict displacements and storey drifts with acceptable accuracy, plastic rotations of the hinges derived by this method are significantly inaccurate. To overcome this deficiency, the consecutive modal pushover (CMP) procedure [18] was developed in which multi-stage and single-stage pushover analyses were carried out. The response quantities of interest

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(displacements, storey drifts, and hinge plastic rotations) were evaluated by enveloping the peak values computed from the former and latter analyses.

Besides, several attempts have been made to develop pushover procedures for unsymmetric-plan building structures that torsional effects are incorporated into pushover analysis. Modal pushover analysis (MPA) [7] and the consecutive modal pushover (CMP) procedure [19] were extended to the one-way unsymmetric-plan buildings where both torsional and higher-mode effects were taken into account. A comprehensive parametric investigation was done on the seismic response of double unsymmetric-plan single-sotrey and multi-storey buildings subjected to bi-directional excitation [16,17]. The N2 method [11] was also extended to the two-way unsymmetric-plan buildings that it is applicable to low-rise buildings. The MPA [20,21] and N2 [15] methods were also extended to irregular in plan buildings considering the effects of higher modes in plan and elevation.

Double unsymmetric-plan tall buildings are complicated structures wherein torsional and higher-mode effects play a more critical role in evaluation of seismic demands; hence developing a pushover analysis method for these structures is even a challenge. The current article attempts to shed light on this complicated issue and to propose a pushover analysis method for computing the seismic responses of the double unsymmetric-plan medium-rise buildings subjected to bi-directional seismic excitation. In addition to the medium-rise buildings, an attempt is also made to present the proposed method for low-rise buildings. To take bi-directional excitation into consideration for low- and medium-rise buildings, an iterative process is utilized until displacements at a control node (centre of mass at the roof level) almost reach predetermined target displacements in both horizontal directions. The proposed analysis method is explained in detail in the next chapter. To verify the proposed analysis method, double unsymmetric-plan 4 and 10-storey buildings are considered as representative of the low- and medium-rise buildings, respectively, that include torsionally-stiff and torsionally-flexible systems. It is noted that modal pushover analysis (MPA) is also implemented considering the two horizontal components of ground motions for the sake of comparison.

## 2. Principle of the proposed method

In this chapter, an iterative pushover analysis method is developed to compute the seismic responses of the double unsymmetric-plan low- and medium-rise buildings subjected to bidirectional seismic excitation. A single-stage pushover analysis is used for low-rise building structures. In the case of medium-rise building structures, in addition to a single-stage pushover analysis, the method utilizes multi-stage pushover analysis. Relevant lateral loads are applied simultaneously in X and Y-directions. Since displacement of the monitored node is only controlled in one direction, the multi-stage and single-stage pushover analyses are iteratively performed until the predefined displacement is reached for the other direction. For this purpose, one direction is arbitrarily selected as the monitored (first) direction and the load pattern applied in this direction is considered with a scale factor of unity. An arbitrary value of $\gamma$ is assumed for scale factor of the load pattern applied in the other (second) direction. An iterative process is chosen by changing the value of $\gamma$ during some pushover analyses with simultaneous actions of load patterns in X and Y-directions until predefined target displacement is almost reached in the second direction.

To perform the multi-stage (two-stage) pushover analysis for medium-rise buildings, the lateral forces are incrementally applied during the stages. The number of stages in the multi-stage pushover analysis depends on the period (height) of the structure. Linearly-elastic modal properties are employed in the multi-stage pushover analysis. A more detailed discussion of the multi-stage pushover analysis can be found in References [18,19]. The displacement increment at the roof, in each stage of the multi-stage pushover analysis, for each direction is determined as the product of a factor, $\beta_{xi}$ ($\beta_{yi}$) and the relevant total target displacement, $\delta_{xt}$ ($\delta_{yt}$), at the roof in the corresponding direction. The factor, $\beta_{xi}$ ($\beta_{yi}$) is calculated from the initial modal properties of the linearly-elastic structure. The displacement increments in X and Y-directions, $u_{xir}$ and $u_{yir}$, at the center of mass (CM) at the roof for the ith stage of the multi-stage pushover analysis, are determined as follows:

$$u_{xir} = \beta_{xi}\,\delta_{xt} \qquad u_{yir} = \beta_{yi}\,\delta_{yt} \tag{1}$$

in which

$$\beta_{xi} = \alpha_{xi} \qquad \beta_{yi} = \alpha_{yi} \quad \text{for the stages before the last stage} \tag{2}$$

and

$$\beta_{xi} = 1 - \sum_{j=1}^{N_s-1} \alpha_{xj} \;,\; \beta_{yi} = 1 - \sum_{j=1}^{N_s-1} \alpha_{yj} \quad \text{for the last stage} \tag{3}$$

where $N_s$ is the number of stages in the multi-stage pushover analysis and $\alpha_{xi}$ and $\alpha_{yi}$ are the effective modal mass ratios for the $i$th mode in X and Y-directions, respectively that can be defined as

$$\alpha_{xn} = \frac{M^*_{xn}}{\sum_{j=1}^{N} m_j} \qquad \alpha_{yn} = \frac{M^*_{yn}}{\sum_{j=1}^{N} m_j} \tag{4}$$

in which $M^*_{xn}$ and $M^*_{yn}$ are the effective modal masses in X and Y-directions and $m_j$ is the lumped mass at $j$th floor level. Also, $N$ is the number of storeys. It can be easily shown that, in the case of double unsymmetric-plan buildings, the sum of the effective modal participating mass ratios over all modes, in each direction, is equal to unity.

$$\sum_{n=1}^{3N} \alpha_{xn} = 1 \qquad \sum_{n=1}^{3N} \alpha_{yn} = 1 \tag{5}$$

Owing to the fact that the structures considered in this investigation are double unsymmetric-plan buildings, the lateral forces ($s^*_n = M\phi_n$) in the multi-stage pushover analysis include two lateral forces and one torque at each floor level as follows:

$$\mathbf{s}_n^* = \mathbf{M}\Phi_n = \begin{bmatrix} \mathbf{m} & 0 & 0 \\ 0 & \mathbf{m} & 0 \\ 0 & 0 & \mathbf{I_o} \end{bmatrix} \begin{Bmatrix} \Phi_{xn} \\ \Phi_{yn} \\ \Phi_{\theta n} \end{Bmatrix} = \begin{Bmatrix} \mathbf{m}\Phi_{xn} \\ \mathbf{m}\Phi_{yn} \\ \mathbf{I_o}\Phi_{\theta n} \end{Bmatrix} \quad (6)$$

The matrix, $M$, includes three diagonal sub-matrices $m$, $m$ and $I_o$, each of order $N$; $m$ is a diagonal matrix with , the mass lumped at the jth floor diaphragm. $m_{jj} = m_j$ is a diagonal matrix with $I_{jj} = I_{oj}$, the polar moment of inertia of the $j$th floor diaphragm about a vertical axis through the centre of mass (CM).

Pushover analyses in each stage of the multi-stage pushover analysis will be implemented simultaneously in X and Y-directions in order to consider bi-directional excitation. To perform the first stage of the multi-stage pushover analysis, a force distribution obtained by using the first dominant mode in X-direction is simultaneously considered with that produced from the first dominant mode in Y-direction. An iterative process is required to be performed by changing the value of $\gamma$, until displacements at the roof in X and Y-directions reach the predetermined incremental displacements which were defined in Eqn. 1. Then, the second stage is continuously performed in an iterative process until the target displacements in X and Y-directions are reached. For the second stage, a force distribution derived from the second dominant mode in X-direction is simultaneously applied with that resulting from the second dominant mode in Y-direction. It is of great importance to note that each iteration analysis in the second stage starts with an initial structural condition that is the same as the state at the end of the previous stage. Therefore, the force distribution in the second stage is incrementally added to that in the first stage. It is also noted that a mode is called as dominant in X-direction when the effective modal participating mass ratio for this mode in X-direction is considerably larger than that in Y-direction. A dominant mode in Y-direction can be defined in a similar way. It's noted that a mode which is dominant in a direction has one rotational component and two translational components since the building is unsymmetric about both X and Y-axes.

In addition to the two-stage pushover analysis, which is carried out for torsionally-stiff medium-rise buildings, a single-stage pushover analysis is performed by using triangular force distributions in X and Y-directions. For torsionally-flexible low- and medium-rise buildings, the single-stage pushover analysis is carried out by using the force distributions [Eqn. (6)] which are derived from the first dominant modes in X and Y-directions. As discussed in Reference [19], dynamic behavior of torsionally-flexible buildings can not be properly taken into account by inverted triangular force distribution. On the other hand, the force distribution produced from the first dominant mode can consider the dynamic behavior of torsionally-flexible buildings.

At the end, the seismic responses of the unsymmetric-plan medium-rise buildings are computed by enveloping the peak responses derived from the multi-stage and single-stage pushover analyses. The proposed method for low-rise buildings is similar to the single-stage pushover analysis which was described for medium-rise buildings.

## 3. Fulfillment steps of the proposed method

The proposed method for low- and medium-rise buildings is summarized in the following steps:

*a) Low-rise buildings*

1. Determine the target displacements for X- and Y-directions, $\delta_x$ and $\delta_y$ , respectively.
2. In the case of torsionally-stiff buildings, perform a pushover analysis by using triangular force distributions applied simultaneously in X and Y-directions. For this purpose, apply a triangular force distribution with scale factor of 1 in one direction and monitor the displacement in this direction. In the other (second) direction, apply simultaneously a triangular force distribution with a scale factor of $\gamma$. If displacement, at the roof, for the second direction is not nearly identical to the predetermined target displacement, change the value of $\gamma$ and perform pushover analysis again until displacement in the second direction reaches the relevant predetermined target displacement. Hence, an iterative process has to be used to repeat pushover analyses by changing the scale factor of $\gamma$ for the force distribution applied in the second direction until displacement of the centre of mass, at the roof, and predefined target displacements are almost close together within the required accuracy. In the case of torsionally-flexible buildings, perform this step by using force distributions obtained from the first dominant modes in the X- and Y-directions.
3. Calculate the peak values of the desired seismic responses for the last iteration of pushover analysis.

*b) Medium-rise buildings*

1. Calculate the mode-shapes, $\phi_n$. For dominant mode in a direction, the mode-shape is normalized so that the lateral component of $\phi_n$, at the roof, in that direction, equals unity ($\phi_{rxn}$=1 or $\phi_{ryn}$=1).
2. Compute the incremental lateral force distribution $s^*_n = M\phi_n$ (Eqn. (6)) by using dominant modes in X and Y-directions over the height of the structure for different stages of the multi-stage pushover analysis. Note that the force distribution which is dominant in one direction has a torsional component and a translational component in the other direction.
3. Compute the target displacements, $\delta_x$ and $\delta_y$, at the roof for X and Y-directions, respectively. The displacement increments in X and Y-directions for each stage of the multi-stage pushover analysis are obtained by using Eqns. (1) to (3).
4. Apply the gravity loads and then perform the single-stage and multi-stage (two-stage) pushover analyses as follows:

4.1. Perform the single-stage pushover analysis by using appropriate load distributions considered simultaneously in X and Y-directions for medium-rise buildings similarly to that described for unsymmetric-plan low-rise buildings.

4.2. Perform the two-stage pushover analysis. At the First stage, perform this pushover analysis using simultaneous actions of force distributions $s^*_{x1} = M\phi_{x1}$ and $s^*_{y1} = M\phi_{y1}$ with scale factors of $\gamma$ (an arbitrary assumed value) and 1, respectively, until the incremental displacement for the monitored direction (herein Y-direction) reaches $u_{yr1}= \beta_{yi}\delta_{yt}$ (Eqn. (1); $i=1$ ). $\phi_{x1}$ and $\phi_{y1}$ are the first dominant modes in X and Y-directions which were described earlier. If the incremental displacement in the direction else than the monitored direction (herein X-direction) is not almost identical to $u_{xr1}= \beta_{xi}\delta_{xt}$ (Eqn. (1); $i=1$) , change the value of $\gamma$ and repeat pushover analysis. Perform this task during some iteration steps until the predefined incremental displacement in the direction else than the monitored direction is nearly reached. It is noted that $\beta_{xi}=\alpha_{xt}$ (Eqn. (2); $i=1$) and $\beta_{yi}=\alpha_{yt}$ (Eqn. (2); $i=1$). $\alpha_{x1}$ and $\alpha_{y1}$ are the effective modal participating mass ratios for the first dominant modes in X and Y-directions, respectively. Continue the analysis with incremental lateral forces $s^*_{x2}=M\phi_{x2}$ and $s^*_{y2}=M\phi_{y2}$ (Eqn. (6); $n=2$) until the displacement increment at the roof in the monitored direction (herein Y-direction) equals the predetermined value ($u_{yr2}= \beta_{y2}\delta_{yt}$ (Eqn. (1); $i=2$) where $\beta_2=1-\alpha_1$ (Eqn. (3); $i=3$)). If the incremental displacement in the direction else than the monitored direction (i.e. X-direction) is not almost identical to $u_{xr2}= \beta_{x2}\delta_{xt}$ (Eqn. (1); $i=2$), assume another value for $\gamma$ and repeat pushover analysis during some iteration steps until the predefined incremental displacement for the second stage in the direction else than the monitored direction is nearly reached. Note that the initial condition at the second stage of the two-stage analysis, in each iteration, is the same as the condition at the end of the previous stage.

5. Calculate the peak values of the seismic responses for the single- and multi-stage (two-stage) pushover analyses. The peak values resulting from these analyses are denoted by $r_i$. Index $i$ denotes the number of stage(s).

6. Calculate the envelope, $r$ , of the peak responses as follows:

$$r=Max\{r_1,r_2\}$$

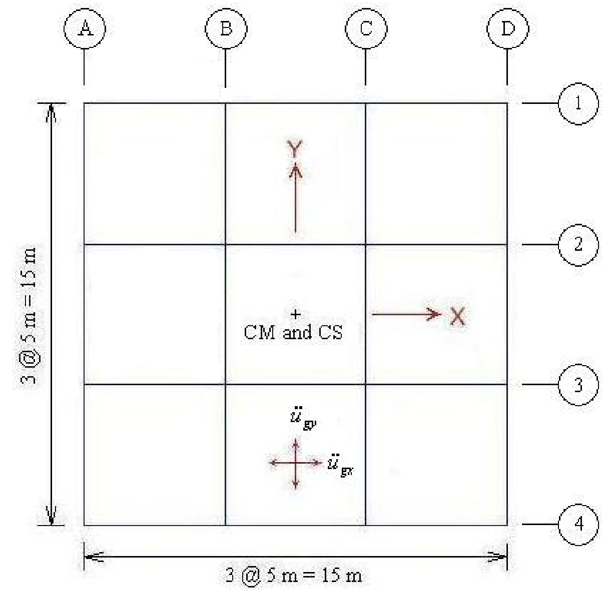
## 4. Modal pushover analysis (MPA)

Modal pushover analysis (MPA) was extended to unsymmetric-plan buildings subjected to simultaneous actions of two horizontal components of the ground motions [20]. Dynamic responses are individually computed for different modes due to the X and Y-components of the ground motions and correspondingly combined using the CQC combination rule. The obtained seismic responses for the X and Y-components of the ground motions are combined by the SRSS multi-component combination scheme. Details of the fulfillment steps of the method are not given in this paper for brevity proposes and they can be found in Reference [20].

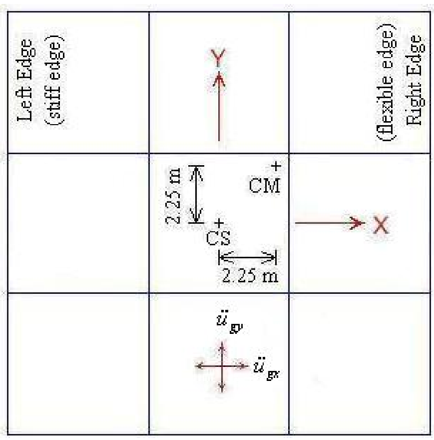
## 5. Analytical models

Unsymmetric-plan structural models were created from original symmetric-plan models. The original symmetric-plan buildings were 4 and 10-storey buildings which they were considered as representative of low- and medium-rise buildings, respectively. As shown in Fig. 1(a), both buildings had the same floor plan with three longitudinal bays by three transverse bays. The lateral load-resisting system of the buildings was a special steel moment-resisting frame (SMRF) in both directions. The seismic effects were determined according to the requirements of Iranian code of practice for the seismic resistant design of buildings [24]. The buildings were designed according to the allowable stress design method [1]. The structures satisfied the detail requirements of the Iranian seismic code. More details of the assumptions are given in Reference [19]. A detailed description of the beams and columns sections for the 4-storey building is presented in Appendix A. Details of the 10-storey building are available in Reference [19].

Unsymmetric-plan building models were assumed to be mass-eccentric and unsymmetric about both the X and Y-directions. In order to create the mass-eccentric buildings, symmetric-plan buildings were modified. To achieve this goal, the stiffness properties of each symmetric-plan building were preserved and the center of mass (CM) was specified




a)




b)

**Fig. 1** (a) Plan of the original symmetric-plan buildings [19]; (b) Plan of the created double unsymmetric-plan 4 and 10-storey buildings eccentric relative to the center of stiffness (CS) along both X and Y-axes [see Fig. 1(b)]. The eccentricity between the CM and CS in each direction was assumed to be equal to 15% of the plan dimension. Torsionally-stiff (TS) and torsionally-flexible (TF) buildings were created corresponding to each symmetric-plan building by modifying the ratio of the floor moment of inertia ($I_{oj}$) to the floor mass ($m_j$) [7]. The ratios of the floor moment of inertia to the floor mass between the unsymmetric-plan buildings and their counterpart symmetric-plan building, the first four periods of linearly-elastic structures are given in Table 1. It is noted that stiff and flexible edges in unsymmetric-plan buildings can be recognized from static analysis in which lateral load is applied at the center of mass in different floor levels. It is obvious that displacement at the flexible edge of the building is larger than that at the stiff edge.

## 6. Description of analyses

The proposed iterative pushover analysis method and nonlinear response history analysis (NL-RHA), as a benchmark solution, were fulfilled for the 4- and 10-storey buildings. In the case of medium-rise buildings, in addition to the proposed pushover analysis method, pushover analysis with triangular load patterns (TLP) was performed in which triangular load distributions were applied simultaneously in X and Y-directions. *Wilson-θ* time integration scheme with a time step of 0.02 s was used throughout the nonlinear response history analyses. Seven ground motion records including the Imperial valley(1979), Victoria(1980), Morgan hill(1984), Hollister(1986), Landers(1992), Northridge(1994) and Duzce(1999) were used in the NL-RHA. For each record, the component with the larger peak ground acceleration (PGA) was scaled to $a_g$ and applied in Y-direction. In order to produce nonlinear responses, the value of $a_g$ was assumed to be equal to 0.45g and 0.9g for the 4- and 10-storey buildings, respectively. The other horizontal component of each record (the weaker component) was applied in X-direction and scaled such that the ratio between the peak ground accelerations of the two horizontal components remains constant. This set of ground motions was denoted by (ax , ay).

The second set ($-a_x$, $a_y$) includes the same seven ground motion records. The $-a_x$ means that the weaker component was multiplied by -1. The two other sets of ground motions can be denoted by ($a_x$ ,- $a_y$) and ($-a_x$ ,- $a_y$). The two latter sets produce results which are relatively similar to those obtained from the former sets. Nonlinear response history analyses were therefore carried out by using the two former sets of ground motion records. The results of NL-RHA were determined as the mean values of maximum seismic responses obtained from nonlinear response history analyses (NL-RHAs) by using the two sets of ground motions described above. It's noted that the influence of angle of incidence of the ground motions on seismic responses is ignored. Rayleigh damping was used with 5% damping for the first dominant modes in X and Y-directions to represent the viscous damping. The second order ($P$-$\Delta$) effects were considered in the calculations. In this investigation, the target displacements (displacements of the CM at the roof) for pushover analyses in X and Y-directions (i.e. $\delta_x$ and $\delta_y$, respectively) were determined as the mean values of the maximum top floor displacements resulting from the NL-RHAs. It is noted that target displacement, at the roof, can be obtained by using approximate methods described in the guidelines, i.e., the capacity spectrum method [2], the displacement coefficient approach [4], or the N2 method [5]. Note that these methods may result in some errors, but the errors are expected not to be large. The pushover analyses were carried out until the target displacements nearly reach $\delta_x$ and $\delta_y$ in X and Y-directions, respectively. Due to unsymmetry of the buildings plan, the pushover analyses were once again implemented until the target displacements reach $-\delta_x$ and $\delta_y$ in X and Y-directions, respectively. The former pushover analyses indicate that the structure is pushed in the positive direction of X and Y-axes. On the other hand, the latter pushover analyses imply that the structure is pushed in the positive direction of the Y-axis and in the negative direction of the X-axis. Modal pushover analysis was performed using three pairs of modes.

Three dominant modes in X-direction and three dominant modes in Y-direction were used to consider X and Y components of ground motions, respectively. It is noted that the second and third pairs of modes are important in estimating storey drifts for the 10-storey buildings. All the nonlinear static and dynamic analyses were performed by using the nonlinear version of computer program SAP2000 [9]. The nonlinear behavior in the static and dynamic analyses was represented with rigid-plastic hinges. It was assumed that the nonlinear deformations take place at the end of members. Plastic hinges were therefore defined at the ends of the beams and columns. Modeling parameters of the plastic hinges were defined in accordance with the FEMA-273 [4]. The hysteretic behavior of the hinges is bilinear with 3% post-yield stiffness. Stiffness degradation was ignored in the NL-RHA.

**Table 1** Details of the analyzed building structures

| Number of storeys | Total height (m) | Type of buildings | $\frac{(I_{oj}/m_j)_{unsymmetric}}{(I_{oj}/m_j)_{symmetric}}$ | Periods (seconds) | | | |
|---|---|---|---|---|---|---|---|
| | | | | $T_1$ | $T_2$ | $T_3$ | $T_4$ |
| 4-storey | 12.8 | Torsionally-stiff | 0.28 | 1.009 | 0.755 | 0.327 | 0.285 |
| | | Torsionally-flexible | 5.67 | 1.689 | 0.950 | 0.699 | 0.519 |
| 10-storey | 32 | Torsionally-stiff | 0.28 | 1.626 | 0.687 | 0.545 | 0.307 |
| | | Torsionally-flexible | 5.67 | 3.328 | 1.502 | 1.135 | 0.632 |

## 7. Comparison of the results

Results derived from the approximate proposed pushover analysis method, modal pushover analysis and pushover analysis with triangular load pattern (TLP) are presented in this chapter. The mean values obtained through the NL-RHAs together with the mean plus standard deviation values are also presented. Shown in Figures 2 to 5 are height-wise variation of displacements and storey drifts at the centre of mass (CM) and at the flexible and stiff edges in X and Y-directions for the torsionally-stiff 4 and 10-storey buildings. The results obtained by the proposed method are coincident to those produced by the TLP for the torsionally-stiff 4-storey building and for the lower storeys of the torsionally-stiff 10-storey building. In the case of low-rise building (i.e. 4 storey building), the results obtained from pushover analyses are very

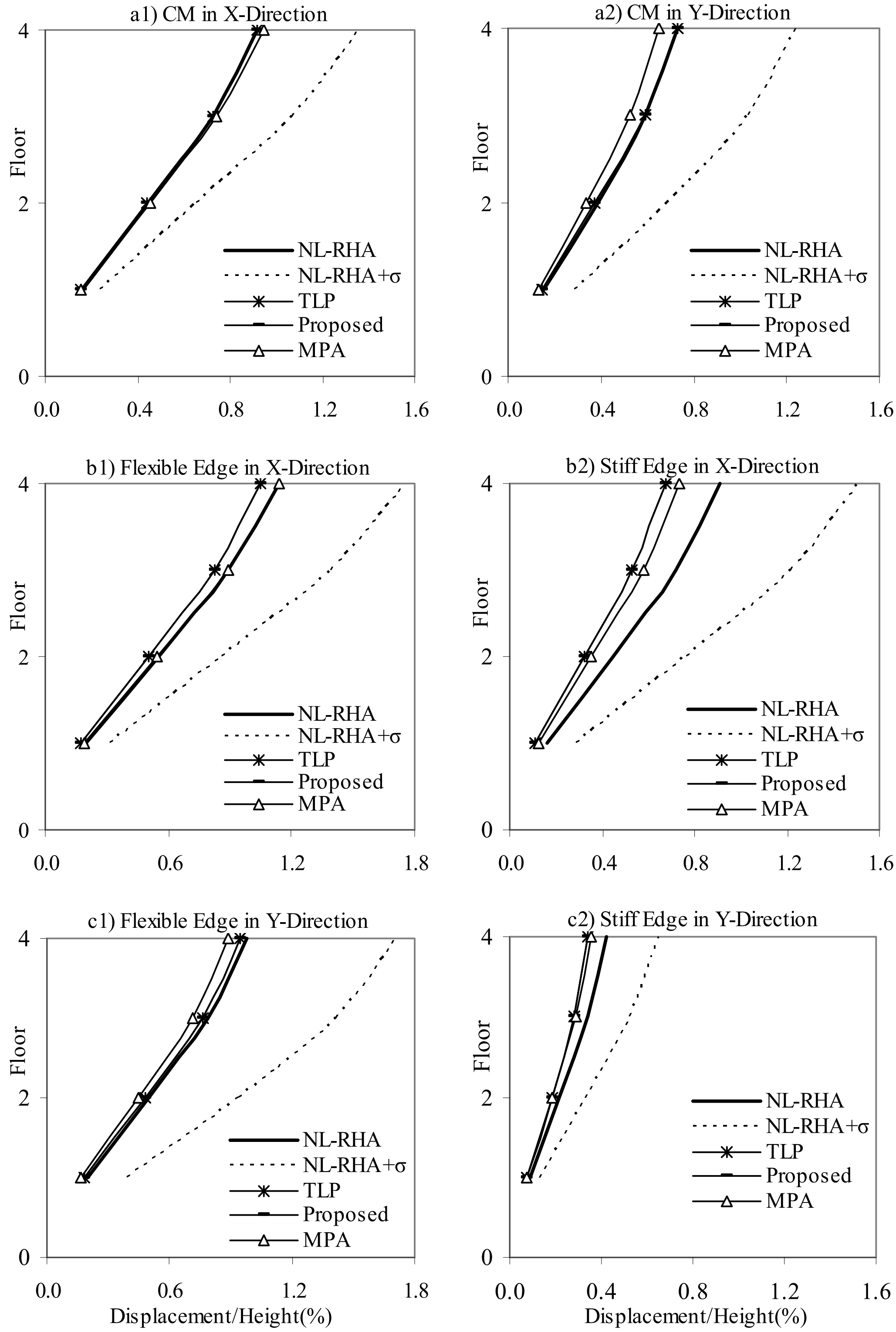



**Fig. 2** (a) Height-wise variation of the displacements for the torsionally-stiff 4-storey building: a) at the CM in X and Y-directions; b) at flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

close to those from the NL-RHA at the CM when compared to those at the flexible and stiff edges because torsional influences at the edges are larger than those at the CM. As seen from Figure 5, estimates of the storey drifts obtained by TLP are not accurate enough at the upper storeys of the torsionally-stiff 10-storey building subjected to the bi-directional excitation, even at the CM. On the other hand, the figure demonstrates that the MPA and proposed method gives better estimates of the storey drifts for the torsinally-stiff buildings than the TLP. The improvement by the proposed method over the TLP is therefore noticeable at the upper storeys of this building in which higher-mode effects are significant (see Figure 5). As a result, the proposed method can mostly compute displacements and storey drifts with reasonable accuracy when two horizontal components are taken into consideration. Also, results show that the proposed method can

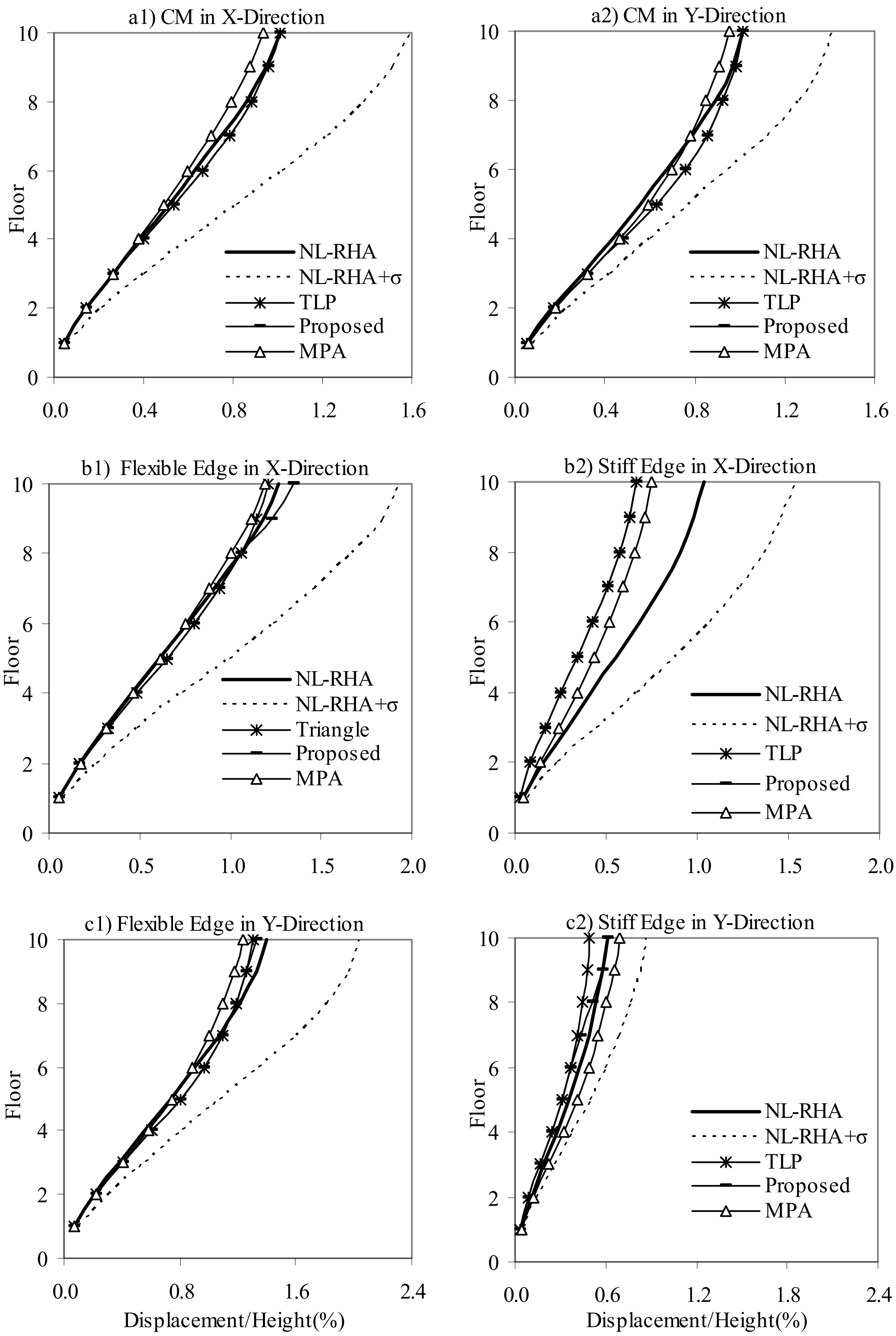



**Fig. 3** Height-wise variation of the displacements for the torsionally-stiff 10-storey building: a) at the CM; b) at the flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

estimate storey drifts more accurately than the MPA at the CM and at the flexible edge of the torsionally-stiff 10-storey building (See Figure 5 (a1, a2, b1 and c1)). It's noted that storey drifts obtained by different methods may deteriorate at the stiff edge of the torsionally-stiff 10-storey building (see Figure 5(b2)).

Displayed in Figures 6 to 9 are displacements and storey drift ratios for the torsionally-flexible 4 and 10-storey buildings. The figures illustrate that the MPA and proposed methods are able to accurately estimate the above-mentioned inelastic responses. Figures 6(b2 and c2) to 9(b2 and c2) demonstrate that the seismic responses resulting from TLP are considerably underestimated at the stiff edges of torsionally-flexible low- and medium-rise buildings in both X and Y-directions. For instance, storey drifts are underestimated by up to 66% and 75% at the stiff edges of

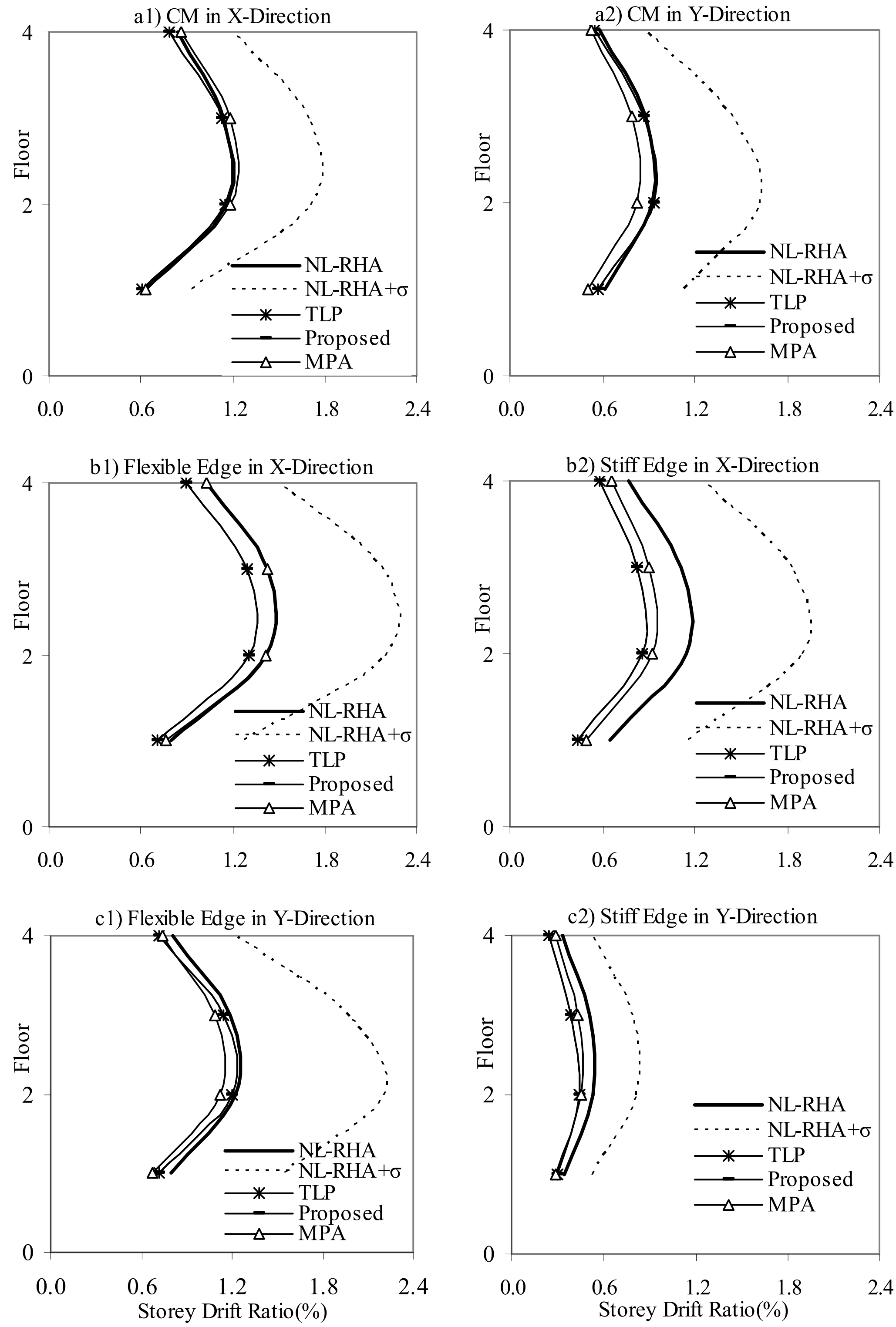



**Fig. 4** Height-wise variation of the storey drifts for the torsionally-stiff 4-storey building: a) at the CM; b) at the flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

the torsionally-flexible 4 and 10-storey buildings, respectively (see Figures 8(b2), 8(c2), 9(b2) and 9(c2)). On the other hand, the proposed iterative pushover analysis method and the MPA provide an important improvement in predicting the seismic responses at the stiff edges of these buildings subjected to bi-directional seismic excitation in comparison with the TLP. The achievement by the proposed method and the MPA at the stiff edge of the torsionally-flexible buildings (both low- and medium-rise buildings) is due to the use of modal properties in pushover analysis of the torsionally-flexible structures. It's noted that the estimation of seismic responses has been found difficult at the stiff edge of torsionally-flexible buildings in the previous investigations. Figure 9(a) shows storey drift ratios at the CM for the torsionally-flexible 10-storey building. The figure obviously provides evidence that the proposed method

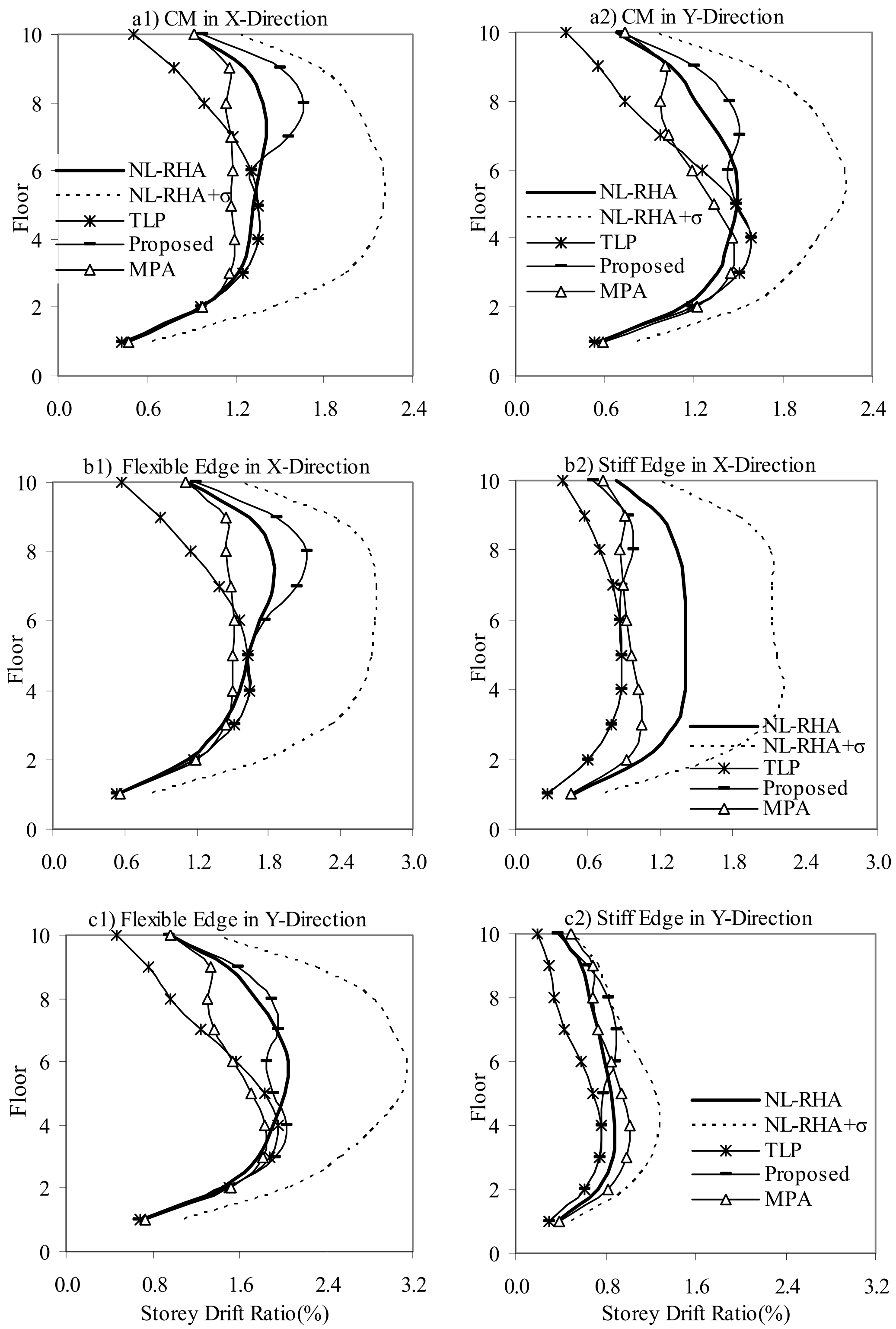



**Fig. 5** Height-wise variation of the storey drifts for the torsionally-stiff 10-storey building: a) at the CM; b) at the flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

can appropriately consider the higher mode-effects for the torsionally-flexible 10-storey building. As a result, not only does the proposed method in this paper take the higher-mode effects into account for medium-rise buildings, but it also can appropriately consider bi-directional excitation for unsymmetric-plan medium-rise buildings. Figures 6 (b1 and c1) to 9 (b1 and c1) show that displacements and storey drifts resulting from the proposed method may be underestimated at the flexible edges of torsionally-flexible buildings while the MPA computes the seismic responses more accurately than the proposed method at these edges. It is noted that errors in seismic responses from the proposed method are not large in this case. As seen from Figures 6 through 9, the seismic responses at the stiff edges of torsionally-flexible buildings, in X and Y-directions, may be overestimated by the proposed method, but the responses are between the

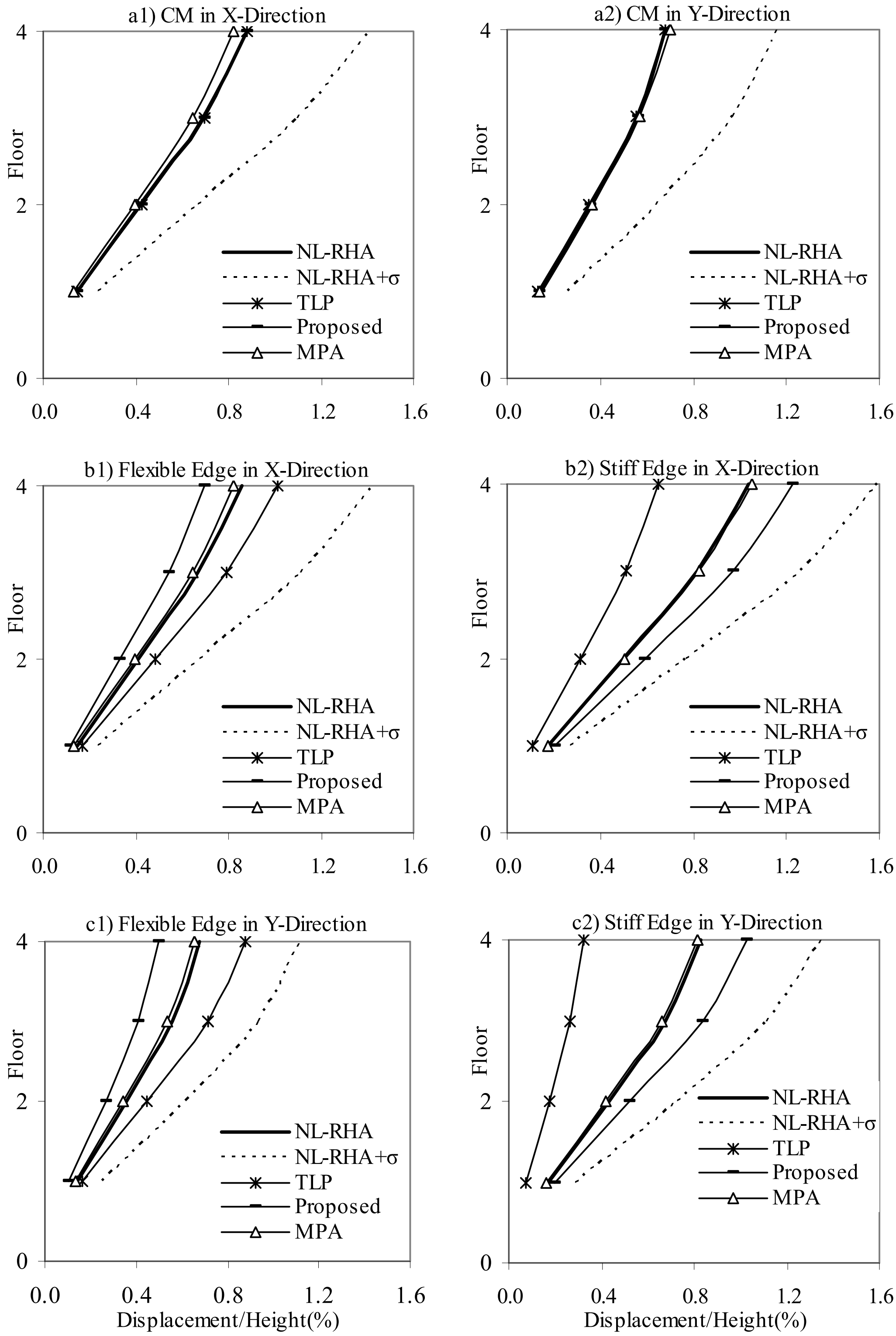



**Fig. 6** Height-wise variation of the displacements for the torsionally-flexible 4-storey building: a) at the CM; b) at the flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

mean values of the NL-RHA and mean plus standard deviation values. Estimates derived from the MPA are mostly closer to the NL-RHA than the proposed method in this case.

It is noted that the convergence is obtained after some iterations in the proposed method. Also, if the monitored direction is changed, the change in the results is negligible. It is worthwhile to mention that the proposed method has a limitation and tends to be associated with computational cost, particularly when the building is tall. Then, in the case of tall building structures, the number of required stages (modes) in the multi-stage pushover analysis will grow. Therefore, it will require a time-consuming iterative process in implementing pushover analyses to reach predefined displacements with an acceptable accuracy. Research in this area continues.

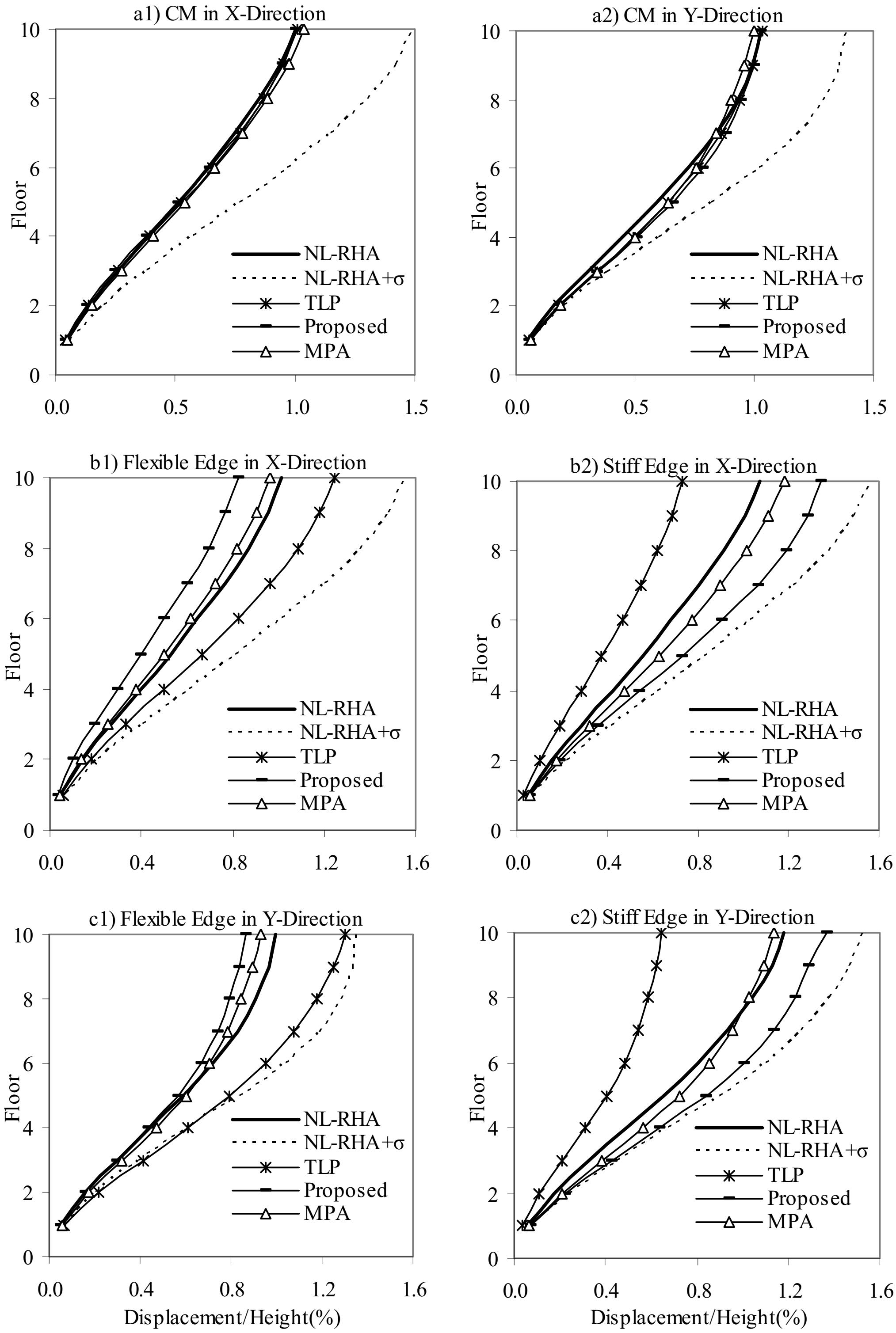



**Fig. 7** Height-wise variation of the displacements for the torsionally-flexible 10-storey building: a) at the CM; b) at the flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

## 8.Conclusion

An iterative pushover analysis method was proposed to take into account the bidirectional seismic excitation in seismic evaluation of double unsymmetric-plan building structures. The proposed method uses an iterative approach in performing pushover analyses until predefined target displacements are almost reached in both horizontal directions. The method was developed for low- and medium-rise buildings. In the case of medium-rise buildings, the effects of higher-modes were taken into account by continuous implementation of the modal pushover analyses. In this case, the seismic responses were obtained by enveloping the peak responses resulting from single-stage and multi-stage pushover analyses. The results demonstrated that the proposed method can consider bi-directional seismic excitation and estimate the seismic responses with reasonable accuracy in X and

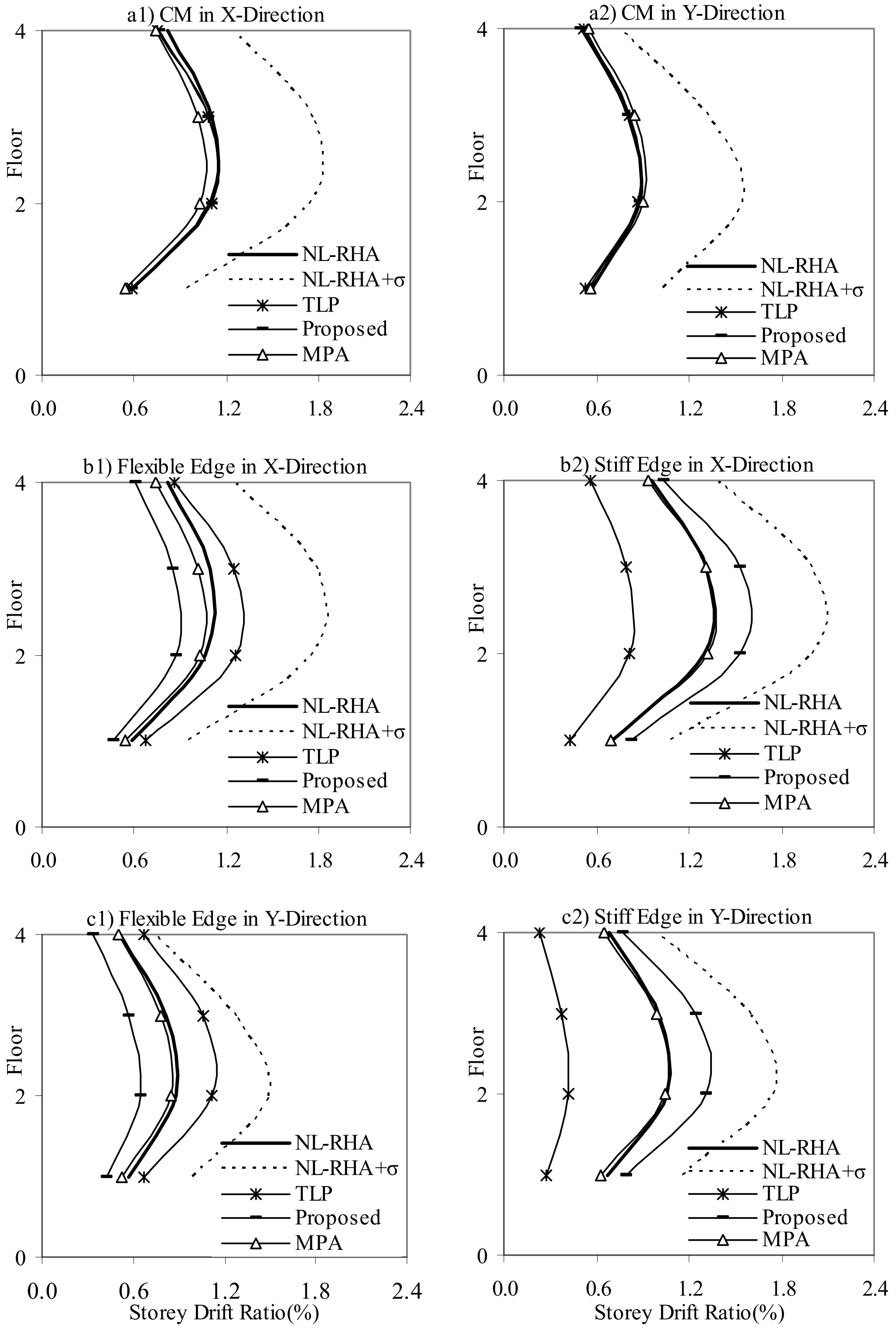



**Fig. 8** Height-wise variation of the storey drifts for the torsionally-flexible 4-storey building: a) at the CM; b) at the flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

Y-directions. Furthermore, the proposed method can properly take higher-mode effects into account for both torsionally-stiff and torsionally-flexible medium-rise buildings. The improvement in estimating the seismic responses by the proposed method and the MPA procedure was pronounced at the upper storeys of medium-rise building. In the case of torsionally-flexible low- and mid-rise buildings, seismic responses derived from simultaneous application of the triangular force distributions in X and Y-directions were significantly underestimated at the stiff side of torsionally-flexible buildings. On the other hand, in this case the results produced by the MPA and proposed methods were greatly improved and they are considerably better than those obtained by TLP, when compared to nonlinear response history analysis because modal properties were used in computing the force distributions for torsionally-flexible buildings. Modal pushover analysis gives better estimates

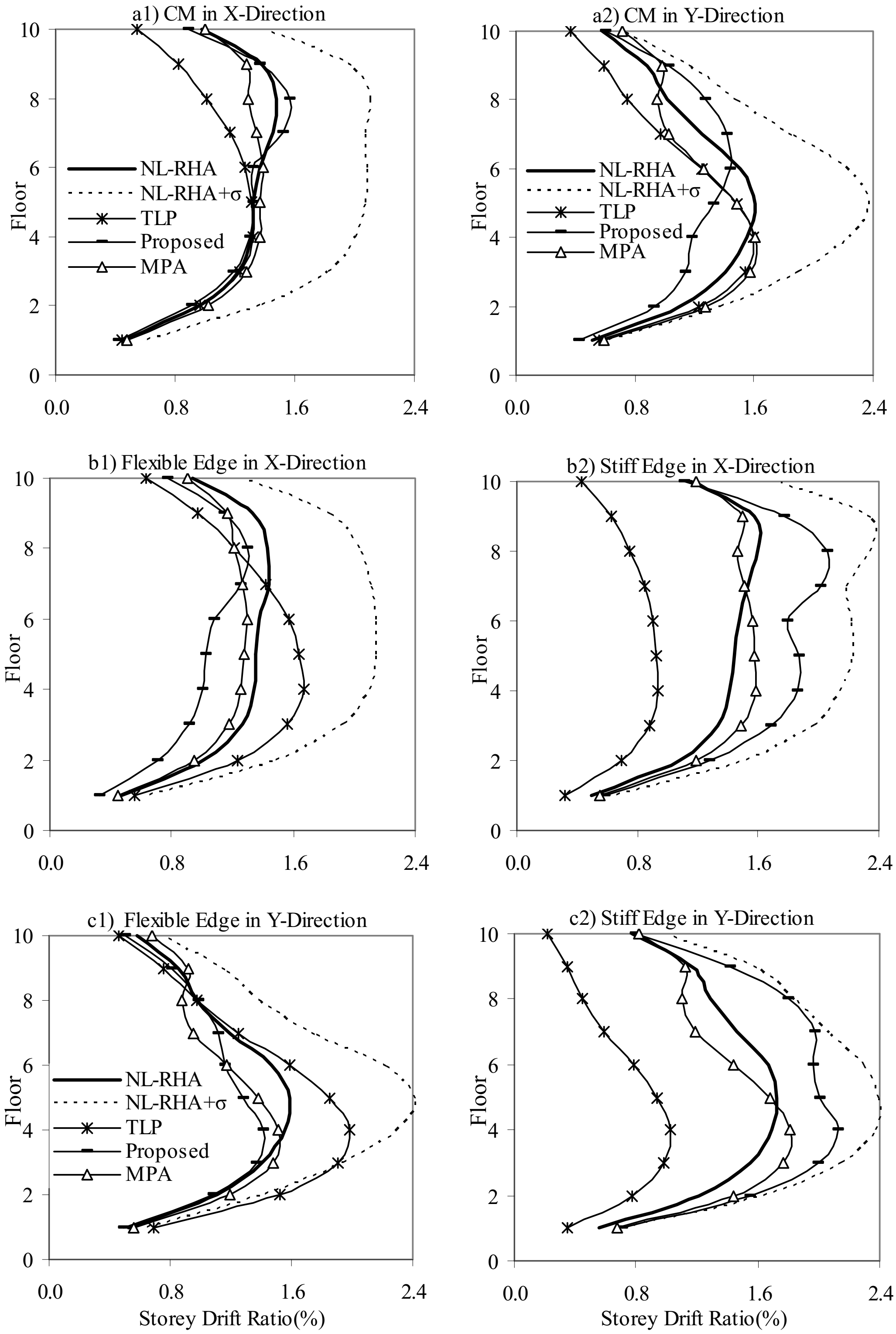



**Fig. 9** Height-wise variation of the storey drift ratios for the torsionally-flexible 10-storey building: a) at the CM; b) at the flexible and stiff edges in X-direction; and c) at the flexible and stiff edges in Y-direction

of seismic responses at the flexible edges of the torsionally-flexible buildings than the proposed method whereas the proposed method provides better estimates of storey drifts at the flexible edges of the medium-rise torsionally-stiff buildings than the MPA procedure.

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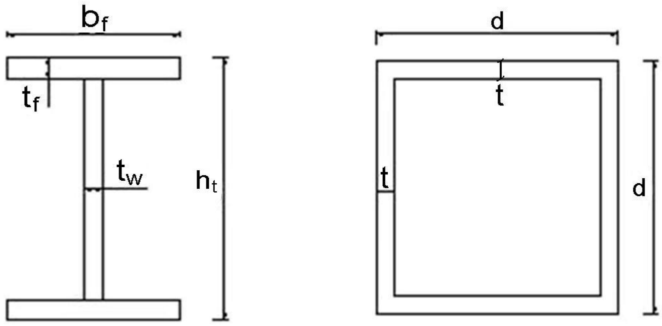
## Appendix A

Beams and columns sections for the 4-storey building are shown in Fig. A1. Specifications of the sections of the members for the 4-storey building are presented in Tables A1 to A4. Axes A to D and 1 to 4 have been shown in Figure 1(a).




a) Beam section    b) Column section

**Fig. A1** Sections of the beams and columns

**Table A1** Details of the sections of the columns

| Section | d (cm) | t (cm) |
|---|---|---|
| SC1 | 25 | 1.5 |
| SC2 | 30 | 2 |
| SC3 | 35 | 2.5 |

**Table A2** Details of the sections of the columns

| Section | $h_t$ (cm) | $t_w$ (cm) | $b_f$ (cm) | $t_f$ (cm) |
|---|---|---|---|---|
| SB1 | 25 | 0.6 | 17.5 | 1.2 |
| SB2 | 25 | 0.6 | 17.5 | 1.5 |
| SB3 | 30 | 0.8 | 20 | 1.5 |
| SB4 | 35 | 0.8 | 22.5 | 2 |

**Table A3** Sections of the columns for the 10-storey building

| Position | Storey | Section |
|---|---|---|
| B1, C1, B4 and C4 | 1-4 | SC2 |
| B2, C2, B3 and C3 | 1-4 | SC3 |
| A1, D1, A4, D4, | 1 and 2 | SC2 |
| A2, D2, A3 and D3 | 3 and 4 | SC1 |

**Table A4** Sections of the beams for the 10-storey building

| Axis | Floor | Section |
|---|---|---|
| 1 and 4 | 1-4 | SB1 |
| 2 and 3 | 1-4 | SB2 |
| A and D | 1 and 2 | SB3 |
| | 3 and 4 | SB2 |
| B and C | 1-4 | SB4 |