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Optimal assignment of seismic vibration control actuators using genetic algorithm

M. Abbasi[1], A.H.D. Markazi[2,]*

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Abstract

An important factor in the design and implementation of structural control strategies is the number and placement of actuators. By employing optimally-located actuators, the effectiveness of control system increases, while with an optimal number of actuators, an acceptable level of performance can be achieved with fewer actuators. The method proposed in this paper, simultaneously determines the number and location of actuators, installed in a building, in an optimal sense. In particular, a genetic algorithm which minimizes a suitably defined structural damage index is introduced and applied to a well-known nonlinear model of a 20-story benchmark building. It is shown in the paper that an equal damage protection, compared to the work of other researchers, can be achieved with fewer numbers of optimally placed actuators. This result can be important from economic point of view. However, the attempt to minimize one performance index has negative effect on the others. To cope with this problem to some extent, the proposed genetic methodology has been modified to be applied in a multi-objective optimization problem.

Keywords: *Structural control, Active control, Optimization, Methodology, Evolutionary algorithms, Benchmarks.*

1. Introduction

The threat of earthquakes is an inevitable fact in many regions around the world. The loss of lives and damages to buildings and premises, resulting from severe seismic excitations has motivated many designers and researchers to devise protective measures for controlling the vibration of various kinds of structures. The significance of such measures seems even more crucial in high risk areas, with lower level of applied building technologies. For instance, in a recent earthquake in a southern city of Iran, Bam, more than 43200 people were killed and more than 30000 were injured. On the other hand, the problem of re-usability of buildings and structures under severe earthquakes, as well as the human comfort, is an increasingly important factor even in developed countries.

The main purpose of structural control strategies, which are divided into active, passive, and semi-active types, is to either dissipate the energy of severe dynamic loadings or use externally controlled actuators to apply real-time counter-acting forces to the structure.

A passive control system does not require external power source, which is a merit from practical point of view, while the effectiveness of such systems is limited. In an active control system, an external power source, control actuator(s) that apply forces to the structure in a prescribed manner [1]. Semi-active control systems are a class of active control systems in which the controlling actuators are, in fact, some controllable passive devices, e.g., Magneto-Rheological (MR) dampers. According to recent studies, the effectiveness of these control strategies is promising, while the amount of energy they require from external sources is very small [2].

One of the important challenges in implementation of active control systems is the decision made on the number and placement of actuators or semi-active dampers. Lopez and Soong [3] proposed the sequential search methods for determining the optimal placement of dampers. Although, these algorithms are computationally efficient, they may get trapped in local optima [4]. As an alternative, global optimization approaches based on genetic algorithms (GAs) were employed to place such devices. These approaches have shown significant promise in their ability to solve problems where the objective function is not a continuous function of the design variables and/or the variable space is discrete [4]. Abdullah et al. [5] combined genetic algorithms with a gradient-based optimization technique to design the optimal position of direct velocity feedback control controllers in buildings. Furthermore, Wongprasert and Symans [6] employed GA for identifying the optimal damper distribution to control the nonlinear seismic response of a 20-story benchmark building.

* *Corresponding author: markazi@iust.ac.ir*

1 MSc. graduate, Department of Mechanical Engineering, Iran University of Science and Technology (IUST)

2 Professor, Department of Mechanical Engineering, Iran University of Science and Technology (IUST), Narmak, Tehran, Iran

Furthermore, Tan et al [7] proposed methodology for integrating device placement and control design in civil structures via GA.

Although many researches have been reported on the issue of optimum placement of predetermined number of actuators, in the literature, the problem of *optimal number* of actuators for a given level of suitably defined objective indices, has received less attention. In a recent study, Amini and Tavassoli [8] have found the minimum number of optimally placed controllers ,with maximum force limitation, such that the maximum displacement response of the structure would be restricted to a predetermined value. In this paper the optimum number and location of actuators are determined, by minimizing certain objective indices, related to the safety and survivability of structures. For the purpose of illustration, the proposed method is applied to a well-known nonlinear model of a 20-story benchmark building, proposed by Ohtori et al. [9]. In particular, it is shown that with an *LQG* control strategy, an equal level of safety, as in the work of Ohtori et al. [9] can be achieved while the number of actuators is reduced to less than 15, instead of 25. However, minimizing the numbers and places of actuators based on safety, will increase the accelerations and control forces in the building. This problem can be partly solved by modifying the approach to be used for multi-objective optimization. The proposed methodology is made simple enough so that it can be used by professional engineers, without a need for an excessive amount of computational effort.

2. Structural Modeling and Controller Design

The nonlinear evaluation of the building model requires a *MATLAB*-based program, implemented as a *SIMULINK* system function (S function), to calculate the nonlinear response of the model. This S function performs the nonlinear dynamic analysis using Newmark-β method [10]. The simulator for the nonlinear evaluation model is illustrated in Fig. 1. To run the simulation the researcher/designer must include *SIMULINK* blocks for their sensor(s) and controller/control device(s) [9].

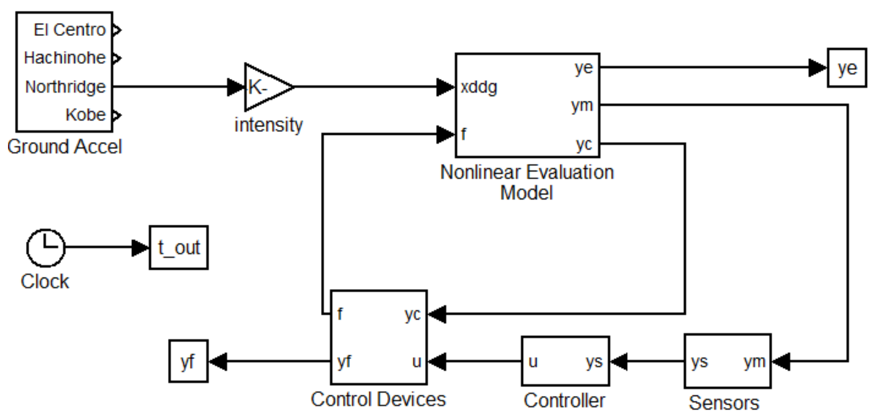

Fig. 1 *SIMULINK* block diagram for vibration control simulator [9]

In this study, we assume that the controlled actuating devices are able to provide enough force to keep the structure in the linear range; therefore, the dynamic equation used to describe the motion of a building can be written as;

$$M\ddot{x} + C\dot{x} + Kx = -M\,\Gamma\ddot{x}_g + bu, \tag{1}$$

where, M, C, and K are mass, damping, and stiffness matrices of the system, respectively; x is response vector; u is control input vector and $\ddot{\mathbf{x}}_\mathbf{g}$ is ground acceleration vector; b is participation matrix for the control input which is determined based on location of the controllers; and Γ is a vector defining the loading of the ground acceleration to the structure. The state space representation of this equation is as;

$$\dot{x}_s = Ax_s + Bu + E\ddot{x}_g \tag{2}$$

$$y_m = C_m x_s + D_m u + F_m \ddot{x}_g + v \tag{1}$$

$$y_e = C_e x_s + D_e u + F_e \ddot{x}_g \tag{2}$$

where, A and B are system matrices; $\mathbf{x_s} = [\mathbf{x}, \dot{\mathbf{x}}]^T$ is the state vector; $\mathbf{y_m}$ is the measured output vector; v is measurement noise vector; $\mathbf{y_e}$ is the regulated output vector. $\mathbf{C_m, C_e, D_m, D_e, F_m}$, and $\mathbf{F_e}$ are appropriately chosen matrices corresponding to the associated output vectors defined by designers. In this paper the reduced order linear model of a benchmark building [9] is employed.

An LQG controller is used to mitigate the vibration of the structure. The first design step seeks a state feedback law $u = -k\hat{x}$ that minimizes the cost function *J (u)* defined as;

$$J(u) = \int_0^\infty (\hat{x}^T Q\hat{x} + u^T Ru)dt \tag{5}$$

The weighting matrices Q and R are specified by the user, representing the relative importance of the regulation performance, i.e., how fast *x(t)* goes to zero, versus the required control effort. Here, K is the feedback gain matrix of applied to the estimated states and $\hat{\mathbf{x}}$ is the estimated state vector obtained by a Kalman filtering as an optimal estimator, in the form of where L is the observer gain matrix of the Kalman filter.

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C_m\hat{x} - D_m u), \tag{6}$$

In this paper the sample controller proposed by Ohtori et al. [9] is employed. The values for R and Q are chosen to be (1/16) [I] and $3\times10^9[\mathrm{I}]$, respectively. In addition, measurement noises are modeled as Gaussian rectangular pulse processes with a pulse width of 0.01 s. Each of the measured responses contains a (root-mean square) noise of 0.03 V. The A/D converters used in the digitally implemented controller have a span of ±10 V. So the measurement noise is approximately 0.3% of the full span of the A/D converters. Also the ratio of the process noise covariance ($S_{\ddot{x}_g\ddot{x}_g}$) to the measurement noise covariance ($S_{v_iv_i}$) is assumed as $\gamma_g = S_{\ddot{x}_g\ddot{x}_g}/S_{v_iv_i}$ =25.

3. Proposed Genetic Algorithm Description

Genetic algorithm is a heuristic random search technique based on the survival of the fittest concept in the nature. To employ this optimization method, first we have to set up a bridge between the real world and the GA framework. This step is commonly called *representation,* which involves coding the problem parameters into genetic strings known as *chromosomes* or *individuals*. Then, a genetic population which is composed of different chromosomes is prepared. In the next step a fitness value is assigned to each of these chromosomes according to the value of the *cost function* which is to be minimized or maximized. The fitness value indicates the chromosome goodness in the *population*, the higher the fitness value of the chromosome, the more chance it would have to survive and reproduce in the next *generation*. Genetic algorithm goes on by taking one population, and generating a successive population with higher fitness valued chromosomes. This is done by employing GA operators; namely, *selection*, *crossover*, and *mutation* to combine features of the chromosomes with the highest fitness.

In the method proposed here, a genetic algorithm is employed to determine the optimum number and placement of the actuators. For this purpose, an initial value for the required number of actuators is assumed, first. Next, the genetic algorithm is employed to minimize some suitably defined objective indices and find the optimal location of the selected actuators to minimize those indices. In order to determine the minimum number of required actuators for the given acceptable level of performance, the minimized value of the selected indices is compared with the maximum allowed limits. If the obtained values of the indices are higher (lower) than the allowed limit, the number of selected actuators is increased (decreased), and the optimization procedure is repeated, until the minimized objective index is almost equal to the specified limit. Fig. 2, illustrates a functional flowchart of the proposed algorithm.

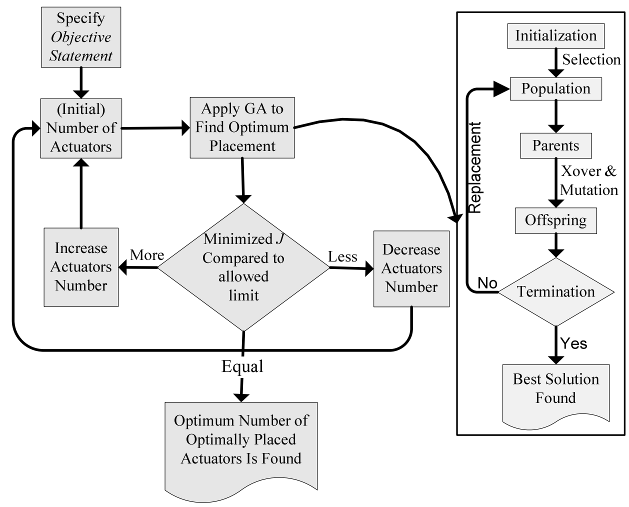

Fig. 2 Proposed optimization algorithm

4. Application to Benchmark Problem

The proposed methodology is applied to a well-known benchmark building [9]. The objective here is the optimum number and position for the actuators. Although the sample control strategy proposed in [9] is not intended to be competitive, it can be used to illustrate the effectiveness of the proposed optimization procedure, to some extent. The building response is sensed by a set of accelerometers located at the 4th, 8th, 12th, 16th, and 20th floors, as in [9], and every actuator is installed within a chevron bracing arrangement; i.e., one end of the actuator is attached to the ceiling and the other end to the top of a stiff chevron brace.

4.1. Objective indices

Minimizing the damage to a building under seismic excitations is a critical issue. The evaluating indices, defined for the benchmark building of Ohtori et al. [9], are divided into four categories: 1) building response indices (J_1-J_6), 2) building damage indices (J_7-J_{10}), 3) actuators action (J_{11}-J_{14}), and 4) the control strategy requirements (J_{15}-J_{17}). These indices are defined such that smaller values represent a better performance. Each of these indices, or any weighted combination of those, can be used as a single objective index for the purpose of optimization. In order to simplify the presentation of the proposed method, one of the most important damage indices, namely, the ductility index, J_7, is selected as the objective index for the optimization problem. The normalized ductility index is defined as;

$$J_7 = \max_{\substack{\text{ElCentro}\\ \text{Hachinohe}\\ \text{Northridge}\\ \text{Kobe}}} \left\{ \frac{\max\limits_{t,j} \dfrac{|\phi_j(t)|}{\phi_{yj}}}{\phi^{\max}} \right\} \tag{7}$$

where, ϕ_j is the curvature at the ends of the jth member (beam or column of structure); ϕ_{yj} is the yield curvature; and $\phi^{\max}$ is the maximum curvature of uncontrolled structure.

Therefore, the selected *objective statement* is something like "*J_7 should be less than a certain level*". The *certain level* in the objective statement can be defined by designers, based on the desired level of safety of the building. The value of J_7 achieved in Ohtori et al. [9] was about 0.978. In what follows, we try to achieve the same or somewhat better performance with a less number of actuators.

4.2. Problem coding and GA parameter study

The proposed algorithm starts with an initial guess on the number of required actuators, say 25, and proceeds by employing the genetic algorithm to minimize the selected objective index, J_7, and find the optimal location of actuators for the selected number of actuators. Next, the number of actuators is successively reduced and the effect on the performance degradation is evaluated. In order to encode the problem parameters, an *integer* representation is used. The length of a chromosome is selected to be equal to the number of actuators. Each gene, an integer number in the interval of [1 20], represents the floor number; e.g. the chromosome [1 4 5 20] is decoded as; 4 actuators acting on the 1st, 4th, 5th, and 20th floors, respectively.

A parameter study is carried out to determine the GA parameters. After running the problem with various values of the parameters, the initial population is generated randomly. The size of this population is selected to be 120. By choosing an appropriately sized population, a good convergence speed can be obtained without *loss of diversity* in the GA population. The tuned parameters of mutation and crossover are determined based on a trade-off between convergence speed and avoidance of trapping in the local optimum. That is, if the amount of mutation (crossover) decreases (increases) the convergence speed increases; however, decreasing (increasing) the mutation (crossover) value has the risk of losing the global optimum and trapping in the local optimum.

4.3. Optimization procedure

In order to evaluate the effectiveness of each individual (chromosome), representing a possible actuator configuration, the matrices *B* and *K* of the controlled structure model should be modified accordingly. In order to assign a goodness value, without excessive computational burden, among all the earthquake records depicted in Fig. 3, the most *important part of* the recorded Northridge earthquake signal, in which the need for control effort is higher, is applied to each GA chromosome,. For this purpose, the seismic response of the controlled building is simulated for *12* seconds, i.e., the more active duration of the earthquake. The GA algorithm is conducted by successive application of the standard genetic operations until a local minimum for J_7 is achieved. Fig. 4 shows a typical convergence curve obtained by the optimization process. It must be noted that, while the value of J_7 obtained in this step is used for evaluation of the fitness of the individuals, the more exact value of J_7, calculated using the whole duration time response, is considered for the benchmarking structure.

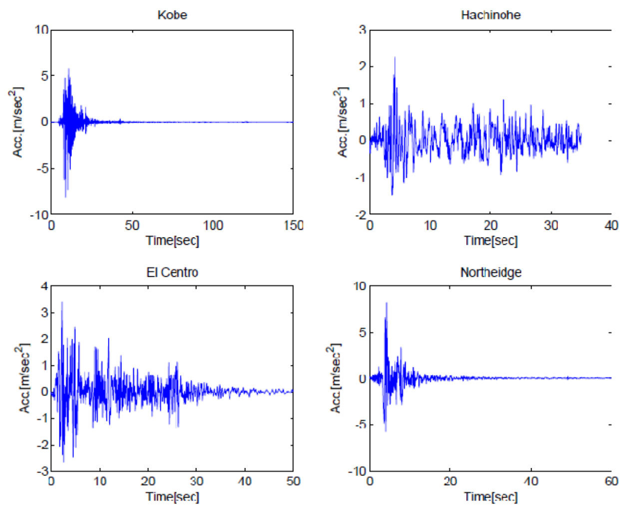

Fig. 3 Earthquake records used in the benchmark study

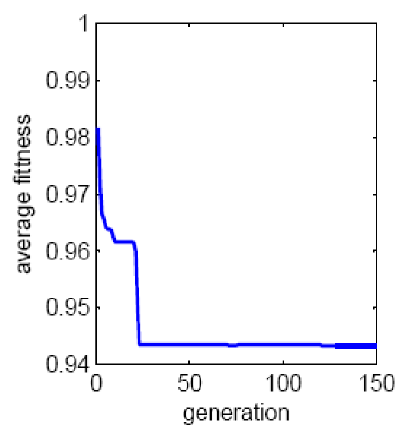

Fig. 4 Typical convergence curve of the GA used in the optimization process

For the purpose of comparison, the controlled building with the final actuator configuration is excited by 10 different well-known earthquakes in their full time records, as in Ohtori et al. [9]. The maximum value of J_7 among these 10 runs is considered as the damage index of the building.

As a parameter study, the above procedure is repeated for five different numbers of actuators, i.e., 25, 20, 15, and 10. The values of J_7 associated with every case are depicted in Fig. **5** and the resulting optimal configurations of the actuators are depicted in Fig. 6.

For the sake of comparison the detailed results of objective indices for 25 non-optimal and 15 optimal actuators are presented in Table 1. To have a better sense, the value of story drift and acceleration, as well as maximum actuator force are calculated from the corresponding indices and listed in Table 2. As can be seen, the optimization procedure has some drawbacks on the efficiency of the control system such as increase on the required control forces and the story accelerations. To reduce these negative effects a designer may decide to perform multi-objective optimization to compromise between merits and demerits of optimization procedure.

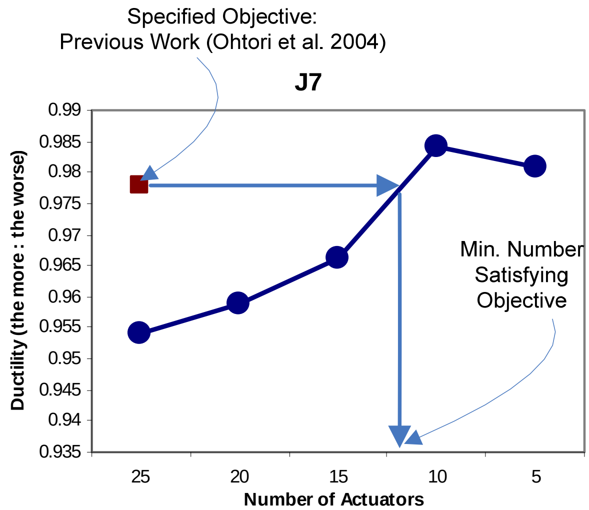

Fig. 5 Result of optimization procedure

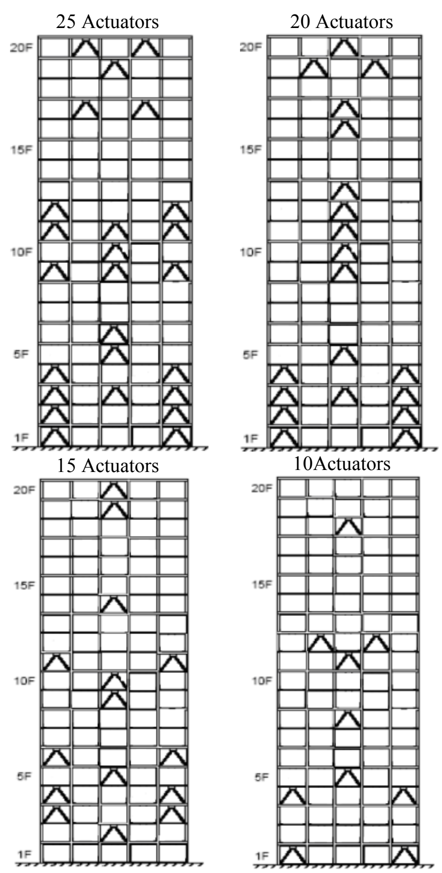

Fig. 6 Configurations for different number of optimally placed actuators

Table 1 Comparison of indices for optimal and non-optimal designs

Earthquake:		El Centro			Hachinohe			Northridge		Kobe		Max. Value
Intensity:		0.5	1	1.5	0.5	1	1.5	0.5	1	0.5	1	
J_1	Opt.	0.760	0.762	0.762	0.889	0.891	0.908	0.894	0.928	0.847	0.695	0.928
	Non	0.747	0.748	0.748	0.883	0.887	0.907	0.859	0.942	0.816	0.728	0.942
J_2	Opt.	0.686	0.680	0.697	0.777	0.776	0.828	0.847	0.868	0.728	0.939	0.939
	Non	0.648	0.646	0.664	0.746	0.743	0.833	0.807	0.904	0.702	0.839	0.904
J_3	Opt.	0.805	0.806	0.928	1.009	1.011	1.037	0.912	0.995	0.974	1.055	1.055
	Non	0.780	0.782	0.909	0.977	0.982	1.009	0.885	0.969	0.925	1.066	1.066
J_4	Opt.	0.731	0.731	0.738	0.892	0.892	0.915	0.891	0.911	0.709	0.378	0.915
	Non	0.662	0.663	0.670	0.885	0.884	0.903	0.724	0.929	0.648	0.23	0.929
J_5	Opt.	0.627	0.626	0.645	0.704	0.700	0.707	0.692	0.712	0.679	0.793	0.793
	Non	0.563	0.560	0.578	0.658	0.652	0.661	0.592	0.637	0.579	0.713	0.713
J_6	Opt.	0.798	0.798	0.805	0.881	0.881	0.891	0.886	0.912	0.799	0.947	0.974
	Non	0.724	0.723	9.729	0.849	0.848	0.858	0.776	0.841	0.789	0.840	0.858
J_7	Opt.	0.795	0.795	0.754	0.963	0.965	0.937	0.879	0.966	0.833	0.759	0.966
	Non	0.772	0.773	0.722	0.955	0.959	0.943	0.728	0.978	0.688	0.688	0.978
J_8	Opt.	-	-	0.158	-	-	0.624	0.373	0.592	0.438	0.466	0.642
	Non	-	-	0.078	-	-	0.714	0.220	0.548	0.144	0.323	0.714
J_9	Opt.	-	-	0.535	-	-	0.837	0.708	0.896	0.385	0.905	0.905
	Non	-	-	0.372	-	-	0.791	0.542	0.906	0.308	0.810	0.906
J_{10}	Opt.	0.809	0.809	0.726	0.889	0.890	0.914	0.869	0.911	0.784	0.429	0.914
	Non	0.733	0.733	0.656	0.847	0.847	0.890	0.632	0.944	0.777	0.227	0.944
J_{11}	Opt.	3.24	6.43	9.20	1.70	3.34	4.56	7.13	8.41	8.15	9.20	9.20
$\times 10^{-3}$	Non	1.68	3.35	5.01	1.92	3.55	5.06	6.71	8.18	4.97	8.91	8.91
J_{12}	Opt.	0.079	0.080	0.080	0.078	0.078	0.083	0.089	0.102	0.143	0.111	0.143
	Non	0.072	0.072	0.072	0.075	0.076	0.080	0.0781	0.103	0.126	0.114	0.126
J_{13}	Opt.	1.67	3.27	5.07	0.52	0.97	1.41	2.52	2.97	3.29	6.12	6.12
$\times 10^{-3}$	Non	1.26	2.47	3.88	0.83	1.69	2.60	4.03	5.12	4.03	8.81	8.81
J_{14}	Opt.	0.038	0.074	0.116	0.024	0.045	0.069	0.053	0.079	0.059	0.103	0.116
$\times 10^{-3}$	Non	0.055	0.103	0.161	0.041	0.073	0.113	0.079	0.122	0.077	0.145	0.161
J_{15}	Opt						15					
	Non						25					
J_{16}							5					
J_{17}							20					

5. Multi-Objective Optimization

In order to compromise between different conflicting indices, the proposed genetic methodology can be modified to be applied as a multi-objective optimization problem. According to Table **2**, the optimally placed actuators have to work under saturation condition. To reduce the load on the actuators, the control force index, J_{11}, can be considered in the optimization procedure as well. For this purpose, the compound objective index can be defined as;

$$J_{mo} = w_1 \frac{\left|J_7 - J_7^0\right|}{\left|J_7^0\right|} + w_2 \frac{\left|J_{11} - J_{11}^0\right|}{\left|J_{11}^0\right|} \tag{8}$$

Table 2 Building responses for uncontrolled and different controlled designs

	Drift^{max} (mm)	Acceleration^{max} (m/s^2)	Force^{max} (kN)
Uncontrolled structure	74.4	8.51	-
Ohtori et al. [9]	70.1	7.69	970.2
Optimal design with 25 actuators	68.6	7.47	1000 (saturation value)
Optimal design with 15 actuators	69.1	9.36	1000 (saturation value)

where, J_i^0 is the minimum of J_i index, regardless of other objective(s); and w_j is the weighting scalar defining the relative importance of every index. The control force index, J_{11}, is defined as

$$J_{11} = \max_{\substack{\text{ElCentro} \\ \text{Hachinohe} \\ \text{Northridge} \\ \text{Kobe}}} \left\{ \frac{\max_{t,l} |f_l(t)|}{W} \right\} \quad (9)$$

where, $f_l(t)$ is the force generated by the l-th control device over the time history of every applied earthquake excitations, and W is the weight of the building which is 1.11×10^8 N for 20-story benchmark building.

The bar charts shown in Fig. **7** and Fig. **8** show the result of optimization for *25* actuators, based on J_7, J_{11}, and J_{mo} objective indices. When the optimization process is carried out based on J_7, the value of J_{11} becomes even worse than the non-optimal case and vice versa. However, a trade off between these two conflicting objective indices can be made by defining the compound objective index, J_{mo}.

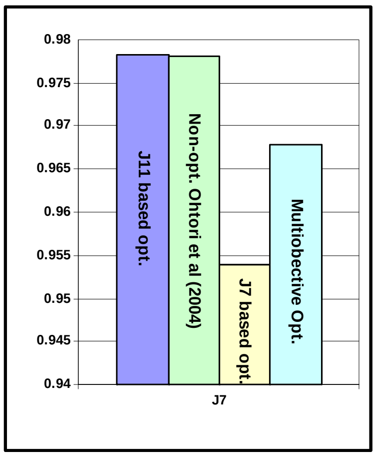

Fig. 7 J_7 index value in the optimization process with different objectives

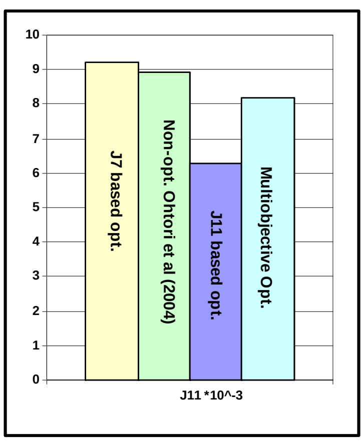

Fig. 8 J_{11} index value in the optimization process with different objectives

6. Discussion

Although the results of the sample controller, proposed by Ohtori et al. [9], was not intended to be competitive, what justifies the comparison of our method with their sample controller is to make the advantage of the proposed method for a given control algorithm more clear.

As depicted in Fig. **5**, using the optimum configuration for actuators, the value of J_7 is reduced from 0.978, in Ohtori et al. [9], to 0.953, i.e., a 2.5 percent reduction, which is a significant improvement for the large structure of the Benchmark Building. Another observation from this figure is that the amount of J_7 achieved by Ohtori et al. [9] can be matched by our proposed method, by using less than 15 actuators instead of 25. Since the optimization procedure is performed using the single objective index, J_7, the effect on other indices must be studied as well. For this reason, the value of all the indices with only 15 optimally placed actuators are compared with the 25 non-optimally placed actuators, in Table 1. It can be seen that, despite a 60 percent decrease in the number of required actuators, the values of indices related to drift, i.e., J_1 to J_4, and the indices related to the structural damage, i.e., J_7 to J_{10} are also improved. The optimization based on J_7, however, has undesired effect on indices related to the acceleration, J_2, J_3, J_5, J_6, as well as, the actuator control force, J_{11}, and the control device stroke, J_{12}. Indeed, higher values for the actuator related indices is quite expected, because reducing the number of actuators, naturally leads to more required action from each of the remaining ones.

7. Conclusion

The proposed methodology is intended to offer a systematic way for finding an optimum number and location of actuators acting on the structure. For this purpose, the optimum location of predetermined number of actuators is found by genetic algorithm, and the optimum number of actuators is determined by repetitive application of the algorithm with different number of actuators. For the purpose of illustration, the methodology was applied to a well-known benchmark 20-story building. It is observed that the results for the optimum number and placement of actuators depend on the selected objective indices, which can be either a single criterion or some compound criteria defined based on the designers objective(s). In particular, the study of this research shows that the number of optimally placed actuators, for a certain value of damage, is significantly less than the required actuators in a non-optimal design However; performing optimization based on damage index, change the required control forces and the stroke of the controlling actuators in a negative way. To compromise between damage and control force, it is possible to put more constraints on the control force, by introducing a suitably defined compound criterion for the optimization procedure.

Moreover, an LQG controller was used to suppress the vibration of the structure. This control algorithm can also be employed as a nominal controller in the design procedure of semi-active controllers, using methods such as clipped-optimal controller [11]. Therefore, the optimization approach presented in this paper can be applied to active, as well as, semi-active control devices.

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