A multi-mode resource-constrained project scheduling model with bi-random coefficients for drilling grouting construction project

Zh. Zhang[1,*], **J. Xu**[2,3]

Abstract

To improve the construction efficiency of the Longtan Hydropower Project, this paper studies the multi-mode resource-constrained project scheduling problem in its Drilling Grouting Construction Project. A multiple objective decision making model with bi-random coefficients is first proposed for this practical problem to cope with hybrid uncertain environment where twofold randomness exists. Subsequently, to deal with the uncertainties, the chance constraint operator is introduced and the equivalent crisp model is derived. Furthermore, the particular nature of our model motivates us to develop particle swarm ptimization algorithm for the equivalent crisp model. Finally, the results generated by computer highlight the performances of the proposed model and algorithm in solving large-scale practical problems.

Keywords: Optimization model, Large scale scheduling, Resource-constrained, Bi-random coefficients, Multi-mode.

1. Introduction

In recent years, project scheduling has attracted growing attention from both the fields of theory and practice. Applications of project scheduling can be found in construction engineering, aircraft landing, co-ordination navigation and so on. Project scheduling problems consist of resources, activities, performance measures and precedence constraints [2]. The problems are known as resource-constrained project scheduling problems (rc-PSP) when the capacity of resources is limited. The activities of a project in the rc-PSP must be scheduled to some objectives, meanwhile subject to precedence relations of all activities and the limited resource availabilities. There have been numerous project optimization scheduling models in project management theory, commonly referred to rc-PSP, which have been reviewed by Brucker et al. [1], Demeulemeester and Herroelen [3], Kolisch and Padman [4].

In many practical projects, such as in the process of construction, it is possible to perform the individual activities in alternative ways (modes). For example, considering two modes of the activity "Drilled drain hole" that belongs to the project "Drilling Grouting". If the activity is performed in the first mode, three thousands skilled workers and one thousand unskilled ones are needed, and the processing time equals to 34 months. If it is performed in the second mode, only two thousands skilled workers are needed, but its duration equals to 37 months. The modes in rc-PSP differ in processing time, time lags to other activities, and resource requirements [5]. Therefore, as a generalization of the rc-PSP, resource-constrained project scheduling problems with multi-mode (rc-PSP/mM) aims at finding the order of activities (the start time of each activity) and the execution modes for all activities in a project while verifying a set of resource constraints. Following the research of Drexl and Grunewald [6], Herroelen et al. [7] studied generalized precedence relations, then Heilmann [8] considered a branch-and-bound procedure with minimum and maximum time lags, and Mika et al. [9] used simulated annealing and tabu search with positive discounted cash flows and different payment models. Subsequently, in 2006, Buddhakulsomsiri and Kim [10] proposed a priority rule-based heuristic succeeded priority with resource vacations and activity splitting. Lorenzoni et al. [11] discussed the problem of attending ships within agreed upon time limits at a port under the condition of the "first come, first served" order. In 2008, He and Xu [12] studied multi-mode project payment scheduling problems with bonus-penalty structure. Currently, deeper research in rc-PSP/mM is being addressed

* *Corresponding Author: zhangzhe@njust.edu.cn*
1 School Economics & Management, Nanjing University of Science and Technology, Nanjing 210094, P. R. CHINA
2 State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu 610064, P. R. CHINA
3 Uncertainty Decision-Making Laboratory, Sichuan University, Chengdu 610064, P. R. CHINA

by a number of researchers, such as [13, 14]. In this study, motivated by the practical application in Drilling Grouting Project of Longtan Dam, we first propose a bi-random multiple objective decision making model to resolve rc-PSP/mM in this large-scale water conservancy and hydropower construction project.

In practice, however, uncertainty is still unavoidable in rc-PSP/mM because of the vague variables, such as processing time, sufficient quantities of resources, the due date of the project, the maximum-limited resources, and so on [15-19]. In above literatures, the uncertainty is traditionally assumed to be random. Random variables, however, are sometimes not able to cope with the complicated situation in rc-PSP/mM to obtain more suitable scheduling. For instance, the quantity of "∅ 25 type grouting steel pipe" is a random variable at the first place, but it may change in the construction period because steel pipe's price is too high. To deal with this change, the mean of the quantity is also a random variable, which means that the quantity of "∅ 25 type grouting steel pipe" is a random variable taking a random parameter. In this case, a bi-random variable, which first proposed by Peng and Liu [20] in 2007, can be a useful tool for this hybrid uncertainty. So far, no attempt has been made in considering the quantity of resources as a bi-random variable in a practical large-scale water conservancy and hydropower construction project. Hence, it is a strong motivation and justification for this study.

The remainder of the paper is organized as follows. The problem statement of Drilling Grouting Project in Longtan Hydropower Station is presented in Section 2. Subsequently, a multiple objective decision making model with bi-random coefficients is proposed for this practical project scheduling problem in section 3, including the explanation of motivation and justification for employing bi-random variables. In section 4, an equivalent crisp model is obtained by bi-random chance constrained operator. Then, particle swarm optimization (PSO) algorithm is utilized to resolve the practical rc-PSP/mM in section 5. The effectiveness of the proposed model and algorithm is proven by the practical application in section 6. Concluding remarks are made in section 7, along with discussion about further research.

2. Problem statement

The problem considered in this paper is from the project of Longtan Hydropower Station, which is a large-scale water conservancy and hydropower construction project in the southwest region of China. The Longtan hydropower station, which locates in Tian'e County of Guangxi Zhuang Autonomous Region, is one of ten major indicative projects of the national strategies of the Great Western Development in China. The primary mission of the project is flood control and navigation. Both the level and scale of Longtan Hydropower Station are I, and the layout is as follows: roller compacted concrete gravity dam, flood building with seven outlets and two bottom outlets are arranged in the river bed, and power capacity of stream systems with nine installations are in the left bank. Navigation structures are arranged in right bank, and 2-stage vertical ship lift is used for navigation (see Fig. 1). The project is designed at 400 m in accordance with the normal

pool level, and the installed plant capacity is 5400 MW. Tunnel diversion is applied in river diversion during construction, and two diversion openings are arranged in left bank and right bank, respectively. The standard of diversion is ten-years flood, and the corresponding flow is 27134 m³/s.

In the whole project, GD Electric Power Design Institute is in charge of the exploration, design, project scheduling for Drilling Grouting Project. Before planning overall progress in the preparation, GD completes the preliminary design. The project is composed by a series of interrelated activities with different modes. Each mode is characterized by a certain known processing time and uncertain resource requirements. In the preliminary design report, duration of the Drilling Grouting Project is 5 years and 8 months (68 months). Experts recommend that both the design of construction organization and construction progress are feasible. However, in view of the current large-scale emergence of the national electricity supply and demand tension, we still need to shorten the duration for power generation. In this situation, three objectives--- duration, cost and quality are taken into account synchronously. Therefore, GD electric power design institute plan to design a model of rc-PSP/mM to determine not only the overall duration of the project, but also the cost and quality of the project.

3. Mathematical model

In this section, before presenting the mathematical formulation model for rc-PSP/mM in Drilling Grouting Project of Longtan Dam, we explain the motivation and justification for employing bi-random variables in a practical rc-PSP/mM model first. Subsequently, assumptions and notations are presented in turn.

3.1 Motivation and justification for employing bi-random variables in the Drilling Grouting Project

To cope with the hybrid uncertainty in rc-PSP/mM, we employ the bi-random variables in this study. Actually, the bi-random variable has been successfully applied in many areas, such as flow shop scheduling problem [21], portfolio selection [22], vendor selection [23], navigation coordinated scheduling [24], etc. These studies show the efficiency of the bi-random

Fig. 1 Longtan hydropower station

variables in dealing with a hybrid uncertain environment where twofold randomness exists.

As we know, it is hard to describe random parameters as crisp ones in a practical construction project. For instance, in the Drilling Grouting Project, $\varnothing$ 25 type grouting steel pipe is one of resources in activity "Embedded $\varnothing$ 25 type grouting steel pipe", and its quantity is a random variable at the first place, i.e., $\tilde{r} \sim N(14200,1)$ (unit: ton). However, at present, the quantity may be changed in the construction period because the price of steel is too high and it is supposed to be a random variable in the international futures market. In this situation, in order to reduce the cost, the project managers want to reduce the quantity of "$\varnothing$ 25 type grouting steel pipe" in a small range. Meanwhile, the project managers also hope that this reduction can be modified following the steel's price that is a random variable in the international futures market. Hence, managers revise the mean of the quantity of "$\varnothing$ 25 type grouting steel pipe" to a random variable, i.e., $\tilde{\tilde{r}} \sim N(\tilde{\mu},1)$ with $\tilde{\mu} \sim N(14075,0.64)$. It means that the quantity of "$\varnothing$ 25 type grouting steel pipe" is a random variable taking a random parameter, i.e, a bi-random variable (see Fig. 2 and Fig. 3).

In fact, there are additional uncertainties in rc-PSP/mM, such as the complexity of construction environment, treacherous weather, equipment failure, and so on. Therefore, the managers of Drilling Grouting Project would like to employ the bi-random variable to cope with the hybrid uncertainty and obtain more feasible schedule.

3.2 Assumptions

Rc-PSP/mM in this study could be stated as follows. A single project consists $i=0,1,...,e,...,I+1$ activities with the resource constraints and the precedence constraints. Activities 0 and $I+1$ are dummies, they have no duration and just represent the initial and final activity, (i.e., $p_{0j}=p_{(I+1)j}=0$, where p represents the duration). Meanwhile, define the modes of activities 0 and $I+1$ are both 1 (i.e., $m_0=m_{(I+1)}=1$ where m_0 and m_{I+1} represent the executed modes of activities 0 and $I+1$). Resources involved in a project can be renewable (i.e., recoverable after serving an activity, such as equipment and manpower) or nonrenewable (i.e., limited in amount over project process and not recoverable, such as money and materials). There is a set of renewable and nonrenewable resources $k=\{1,2,...,K\}$ and $n=\{1,2,...,N\}$ in rc-PSP/mM, respectively. No activity may be

started before all its predecessors are finished, and each activity can be executed in one of m_i possible modes. Each activity-mode combination has a fixed duration and requires one or more types of renewable and nonrenewable resources.

To model the rc-PSP of Drill Grouting Project in Longtan Dam with the twofold random environment in this paper, assumptions are as follows:

(1) A single project consists of a number of activities with several execution modes;

(2) The start time of each activity is dependent upon the completion of some other activities (precedence constraints of activities);

(3) Activities cannot be interrupted, and each activity must be performed just in one mode;

(4) Mode switching is not allowed;

(5) Resources are available in certain limited quantities, and the consumption of each resource is a bi-random variable;

(6) The objectives are to minimize the duration of the whole project, minimize the total tardiness penalty and maximize the quality of the project.

3.3 Modeling

In order to formulate the model, indices, variables, certain parameters, bi-random coefficients, and the decision variable are introduced in Appendix A. Based on the assumptions and notations, we will propose the rc-PSP/mM model with bi-random coefficients for Drilling Grouting Project of Longtan Dam as follows:
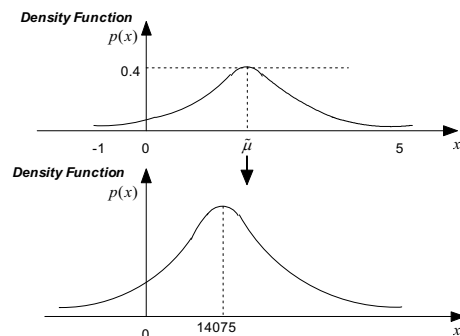

Fig. 3 Employing a bi-random variable to express the quantity of "$\varnothing$ 25 type grouting steel pipe"
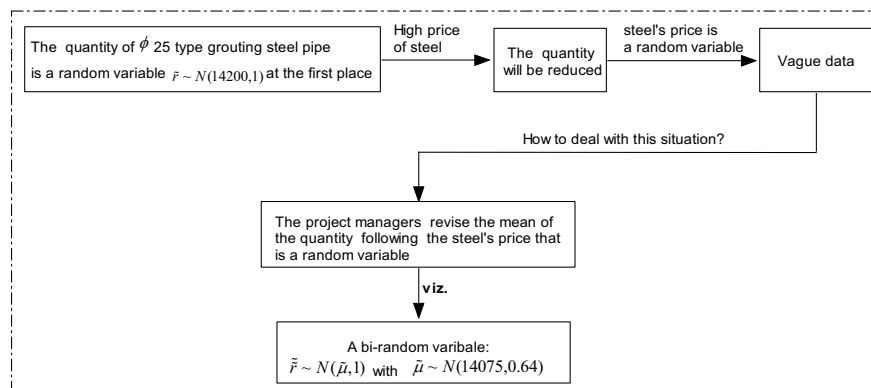

Fig. 2 Flowchart of the reason for the quantity of "$\varnothing$ 25 type grouting steel pipe"as a bi-random variable

Objective functions

The first objective is to minimize the project duration T. In this paper, we use the finish time of the last activity in the project, considering its entire possible executed mode to describe it as:

$$T = \sum_{j=1}^{m_I} \sum_{t=t_I^{EF}}^{t_I^{LF}} tx_{Ijt} \tag{1}$$

Furthermore, the second objective is to measure and minimize the total tardiness penalty.

Let $\sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} tx_{ijt}$ be denoted as the actual finish time of activity i and t_i^E be the expected finish time of activity i, thus:

$$C = \sum_{i=0}^{I+1} c_i^{TP} \left| \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} tx_{ijt} - t_i^E \right| \tag{2}$$

where $|.|$ let the value of C be non-negative.

Finally, as one of the key objectives in construction project management, the quality of the project is the precondition of the duration and cost. Denote α_i as the slope of duration-quality, and

$\alpha_i = \dfrac{q_i^L - q_i^S}{p_{ij}^L - p_{ij}^S}$. Thus, we obtain that $q_i = q_i^S + \alpha_i(p_{ij} - p_{ij}^S)$. Then, the whole quality of the project, i.e., the third objective is stated in the following equation:

$$Q = \sum_{i=0}^{I+1} \omega_i \left[q_i^S + \alpha_i(p_{ij} - p_{ij}^S) \right] \tag{3}$$

where $\sum_{i=0}^{I+1} \omega_i = 1, \omega_i > 0$.

Precedence constraint

The constraints of rc-PSP/mM are divided into time constraints and resource constraints. Since a specific activity must be finished before changing to another activity, therefore successive activities must and only be scheduled after all the predecessors have been completed already. For ensuring that each activity will be completed within $[t_i^{EF}, t_i^{LF}]$, we use the relationship:

$$\sum_{j=1}^{m_e} \sum_{t=t_e^{EF}}^{t_e^{LF}} tx_{ejt} + \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} p_{ij}x_{ijt} \leq \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} tx_{ijt} \tag{4}$$

where $i=0,1,...,e,...,I+1$, $e \in \mathrm{Pre}i$ ($\mathrm{Pre}i$ is the set of immediate predecessors of activity i), $\sum_{j=1}^{m_e} \sum_{t=t_e^{EF}}^{t_e^{LF}} tx_{ejt}$ is the actual finish time of all immediate predecessors e of activity i, $\sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} p_{ij}x_{ijt}$ is the process time and $\sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} tx_{ijt}$ is the actual finish time of activity i, respectively.

Resources constraint

It is also important to limit the total consumption of resources used by all activities. In rc-PSP/mM, the amount of renewable resource k used by all activities could not exceed its limited quantity r_k^M during each period. Besides, the total resource consumption of nonrenewable resource n also limit to the maximum available amount r_n^M. Thus, the constraints

$$\sum_{i=1}^{I} \sum_{j=1}^{m_i} \tilde{\tilde{r}}_{ijk} \sum_{\tau=t}^{t+p_{ij}-1} x_{ij\tau} \prec r_k^M \tag{5}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{m_i} \tilde{\tilde{r}}_{ijn} \sum_{t=t_i^{EF}}^{t_i^{LF}} x_{ijt} \prec r_n^M \tag{6}$$

can be employed, where $k=\{1,2,...K\}$, $n=\{1,2,...N\}$.

Maturity constraint

In order to schedule all activities in the project, the finish time of each activity must be in $[t_i^{EF}, t_i^{LF}]$. Therefore, we use the constraint

$$\sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} x_{ijt} = 1, i = 0, 1, 2, \cdots, I+1 \tag{7}$$

Logical constraint

For the practical sense, we employ following mathematical formulas for describing some non-negative variables and 0-1 variable in the model:

$$\begin{aligned}
&p_{ij}^L \geq p_{ij} \geq p_{ij}^S > 0, q_i^L \geq q_i \geq q_i^S > 0 \\
&t_{ij}^F \geq 0, t_{ij}^{EF} \geq 0, t_{ij}^{LF} \geq 0 \\
&\omega_i > 0, \sum_{i=0}^{I+1} \omega_i = 1 \\
&x_{ijt} = 0 \text{ or } 1
\end{aligned} \tag{8}$$

where $i=0,1,...,e,...,I+1$, $j=1,2,...,m_i$.

From the discussions above, the bi-random rc-PSP/mM model for Drilling Grouting Construction Project in Longtan Dam can be stated as:

$$(P1)\begin{cases}
\min T = \sum_{j=1}^{m_I} \sum_{t=t_I^{EF}}^{t_I^{LF}} tx_{Ijt} \\
\min C = \sum_{i=0}^{I+1} c_i^{TP} \left| \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} tx_{ijt} - t_i^E \right| \\
\max Q = \sum_{i=0}^{I+1} \omega_i \left[q_i^S + \alpha_i(p_{ij} - p_{ij}^S) \right] \\
s.t.\begin{cases}
\sum_{j=1}^{m_e} \sum_{t=t_e^{EF}}^{t_e^{LF}} tx_{ejt} + \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} p_{ij}x_{ijt} \leq \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} tx_{iji} \\
\sum_{i=1}^{I} \sum_{j=1}^{m_i} \tilde{\tilde{r}}_{ijk} \sum_{\tau=t}^{t+p_{ij}-1} x_{ij\tau} \prec r_k^M \\
\sum_{i=1}^{I} \sum_{j=1}^{m_i} \tilde{\tilde{r}}_{ijn} \sum_{t=t_i^{EF}}^{t_i^{LF}} x_{ijt} \prec r_n^M \\
\sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} x_{ijt} = 1 \\
p_{ij}^L \geq p_{ij} \geq p_{ij}^S > 0 \\
q_i^L \geq q_i \geq q_i^S > 0 \\
t_{ij}^F \geq 0 \\
t_{ij}^{EF} \geq 0 \\
t_{ij}^{LF} \geq 0 \\
\omega_i > 0 \\
\sum_{i=0}^{I+1} \omega_i = 1 \\
x_{ijt} = 0 \text{ or } 1
\end{cases}
\end{cases}$$

where $i=0,1,...,e,...,I+1$, $j=1,2,...,m_i$, $k=\{1,2,...K\}$ and $n=\{1,2,...N\}$.

In order to solve model ($P1$) with bi-random constrains, we discuss the equivalent crisp model in the next section.

Zh. Zhang, J. Xu

4. Equivalent crisp model

In this paper, we consider the bi-random variable defined by Peng and Liu [20]. For understanding the following text better, the basic properties of bi-random variables are reviewed first.

Definition 1 [20] A bi-random variable $\tilde{\tilde{\xi}}$ is a mapping from a probability space $(\Omega,\ A,\ Pr)$ to a collection of random variables S such that for any Borel subset B of the real line R, the induced function $Pr\{\tilde{\tilde{\xi}}(\omega){\in}B\}$ is a measurable function with respect to ω, see Fig. 4 [25].

Definition 1 suggests that the bi-random variable is a measurable function from a probability space to a collection of random variables. Roughly speaking, therefore, a bi-random variable is a random variable taking random values. In exceptional circumstance, if Ω consists of a single element or S is a collection of real numbers, then the bi-random variable degenerates to a random variable.

Example 1. [25] Let $\Omega=\{\omega_1,\omega_2,\dots\omega_n\}$, and $\{\omega_1\}=Pr\{\omega_2\}=Pr\{\omega_n\}=1/n$. Assume that $\tilde{\tilde{\xi}}$ is a function on $(\Omega,\ A,\ Pr)$ as follows,

$$\tilde{\tilde{\xi}}(\omega) = \begin{cases} \tilde{\xi}_1, & \text{if } \omega=\omega_1 \\ \tilde{\xi}_2, & \text{if } \omega=\omega_2 \\ \vdots & \\ \tilde{\xi}_n, & \text{if } \omega=\omega_n \end{cases}$$

where $\tilde{\xi}_1$ is a random variable uniformly distributed on [0,1], $\tilde{\xi}_2$ is a normally distributed random variable with mean 1 and
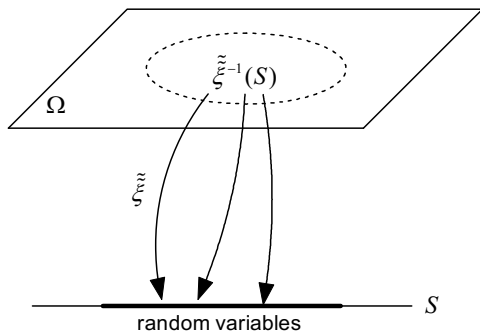
standard variance 0.5, and $\tilde{\xi}_3,\dots,\tilde{\xi}_n$ are standard normally distributed random variables with mean 0 and standard variance 1, i.e., $\tilde{\xi}_1 \sim U[0,1]$, $\tilde{\xi}_2 \sim N(1,0.5)$ and $\tilde{\xi}_3,\dots,\tilde{\xi}_n \sim N(1,0.5)$. According to Definition 1, $\tilde{\tilde{\xi}}$ is clearly a bi-random variable (see Fig. 5).

Example 2. [26] Assume that $\tilde{\tilde{\xi}} \sim U[\tilde{a}(\omega),\ \tilde{b}(\omega)]\forall\omega{\in}\Omega$. $\tilde{a}(\omega)$ and $\tilde{b}(\omega)$ are random variables in $(\Omega,\ A,\ Pr)$ and $\tilde{a}(\omega){\sim}N(1,1)$and $\tilde{b}(\omega){\sim}U[1,3]$, respectively. Then, $\tilde{\tilde{\xi}}$ is a bi-random variable (see Fig. 6).

Definition 2 [27] Let $\tilde{\tilde{\xi}}_i$ be a bi-random variable defined in $(\Omega,\ A,\ Pr_i)$, $i=1,2,\dots,n$ respectively, and then $\tilde{\tilde{\xi}}_i=(\tilde{\tilde{\xi}}_1,\tilde{\tilde{\xi}}_2,\dots,\tilde{\tilde{\xi}}_n)$ is a bi-random vector.

Lemma 1 [27] Let $\tilde{\tilde{\xi}}_i$ be a bi-random vector, and f be a Borel measurable function from R^n to R, and then $f(\tilde{\tilde{\xi}})$ is a bi-random variable.

Due to the existence of bi-random parameters, we usually cannot find a precise decision for complicated real-life problems. However, the chance-constraint operator of the bi-random variable can be employed to obtain the equivalent crisp model. Chance-constrained programming (CCP), proposed by Charnes and Cooper [28] in 1959, is a means of dealing with randomness by specifying a confidence level at which the stochastic constraints hold. Based on CCP, Peng and Liu defined the primitive chance of bi-random event as follows.

Definition 3 [20] Let $\tilde{\tilde{\xi}}=(\tilde{\tilde{\xi}}_1,\tilde{\tilde{\xi}}_2,\dots,\tilde{\tilde{\xi}}_n)$ be a bi-random vector on $(\Omega,\ A,\ Pr)$, and $f{:}R^n{\to}R^m$ be a vector-valued Borel measurable function. Then the primitive chance of bi-random event characterized by $f(\tilde{\tilde{\xi}}){\leq}0$ is a function from (0, 1] to [0, 1], defined as

$$Ch\left\{f\left(\tilde{\tilde{\xi}}\right)\leq 0\right\}(\alpha)=\sup\left\{\beta\ |\ Pr\left\{\omega\in\Omega\ |\ Pr\left\{f\left(\tilde{\tilde{\xi}}\right)\leq 0\right\}\geq\beta\right\}\geq\alpha\right\} \quad (9)$$

where $\alpha,\beta{\in}[0,1]$ are prescribed probability levels, and $Pr\{.\}$ is the probability of a random event.

Remark 1. The primitive chance represents that the bi-random event holds with probability β at probability α.

Thus, by Definition 3, the chance constraints of (5) and (6) in $(P1)$ are as follows:

$$Ch\left\{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}\leq r_k^M\right\}(\varsigma)\geq\eta \Leftrightarrow Pr\left\{\omega\in\Omega\ |\ Pr\left\{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{r}_{ijk}(\omega)\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}\leq r_k^M\right\}\geq\eta\right\}\geq\varsigma$$

$$\tag{10}$$

$$Ch\left\{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}\leq r_k^M\right\}(\varsigma)\geq\eta \Leftrightarrow Pr\left\{\omega\in\Omega\ |\ Pr\left\{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{r}_{ijk}(\omega)\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}\leq r_k^M\right\}\geq\eta\right\}\geq\varsigma$$

$$\tag{11}$$

Fig. 4 Bi-random variable [25]

Fig. 5 Representation of the bi-random variable in Example 1 [25]
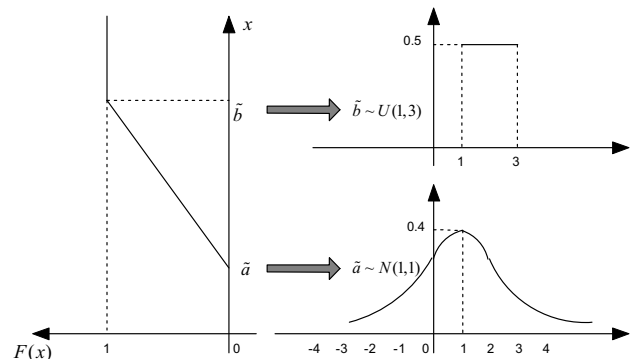
Fig. 6 Representation of the bi-random variable in Example 2 [26]

[Downloaded from www.iust.ac.ir on 2024-10-04]

where $\varsigma, \eta, \varepsilon, \rho \in [0,1]$ are prescribed probability levels.

Theorem 1. Let $\tilde{\tilde{r}}_{ijk} \sim N(\tilde{\mu}_{r_{ijk}}, \sigma_{r_{ijk}}^2)$ with $\tilde{\mu}_{r_{ijk}} \sim N(\mu', \sigma'^2)$ and $\tilde{\tilde{r}}_{ijn} \sim N(\tilde{\mu}_{r_{ijn}}, \sigma_{r_{ijn}}^2)$ with $\tilde{\mu}_{r_{ijn}} \sim N(\mu'', \sigma''^2)$. Then

$$\Pr\left\{\omega \in \Omega \mid \Pr\left\{\sum_{i=1}^I \sum_{j=1}^{m_i} \tilde{\tilde{r}}_{ijk}(\omega) \sum_{\tau=t}^{t+p_{ij}-1} x_{ij\tau} \le r_k^M\right\} \ge \eta\right\} \ge \varsigma \qquad (12)$$

$$\Leftrightarrow \Phi^{-1}(\eta)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma_{r_{ijk}}^2 \sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + $$
$$\Phi^{-1}(1-\varsigma)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma'^2 \sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + $$
$$\sum_{i=1}^I \sum_{j=1}^{m_i} \mu' \sum_{\tau=t}^{t+p_{ij}-1} x_{ij\tau} \ge r_k^M$$

and

$$\Pr\left\{\omega \in \Omega \mid \Pr\left\{\sum_{i=1}^I \sum_{j=1}^{m_i} \tilde{\tilde{r}}_{ijn}(\omega) \sum_{t=t_i^{EF}}^{t_i^{LF}} x_{ijt} \le r_n^M\right\} \ge \rho\right\} \ge \varepsilon \qquad (13)$$

$$\Leftrightarrow \Phi^{-1}(\rho)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma_{r_{ijn}}^2 \sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} + $$
$$\Phi^{-1}(1-\varepsilon)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma''^2 \sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} + $$
$$\sum_{i=1}^I \sum_{j=1}^{m_i} \mu'' \sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2 \ge r_n^M$$

Proof. See Appendix B.

Thus, by the chance constraint operator of bi-random variable, we transform the model (P1) into a deterministic one. In addition, the weighted value λ_1, λ_2, λ_0, which reflect the importance of the overall project considering the three objective functions based on experiential value, have been given by the decision managers in a preliminary design report. Now, we get obtain model (P2),

$$(P2)\begin{cases} \min F = \lambda_T \sum_{i=1}^I \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} t x_{ijt} + \lambda_C \sum_{i=0}^{I+1} c_i^{TP}\left|\sum_{i=1}^I \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} t x_{ijt} - t_i^E\right| - \lambda_Q \sum_{i=0}^{I+1} \omega_i[q_i^S + \alpha_i(p_{ij} - p_{ij}^S)] \\[6pt]
s.t. \begin{cases}
\sum_{e=1}^{m_e} \sum_{t=t_i^{EF}}^{t_i^{LF}} t x_{ejt} + \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} p_{ij} x_{ijt} \le \sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} t x_{ijt} \\
\Phi^{-1}(\eta)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma_{r_{ijk}}^2 \sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \Phi^{-1}(1-\varsigma)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma'^2 \sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} \\
\quad + \sum_{i=1}^I \sum_{j=1}^{m_i} \mu' \sum_{\tau=t}^{t+p_{ij}-1} x_{ij\tau} \ge r_k^M \\
\Phi^{-1}(\rho)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma_{r_{ijn}}^2 \sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} + \Phi^{-1}(1-\varepsilon)\sqrt{\sum_{i=1}^I \sum_{j=1}^{m_i} \sigma''^2 \sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} \\
\quad + \sum_{i=1}^I \sum_{j=1}^{m_i} \mu'' \sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2 \ge r_n^M \\
\sum_{j=1}^{m_i} \sum_{t=t_i^{EF}}^{t_i^{LF}} x_{ijt} = 1 \\
p_{ij}^L \ge p_{ij} \ge p_{ij}^S > 0 \\
q_i^L \ge q_i \ge q_i^S > 0 \\
t_{ij}^F \ge 0 \\
t_{ij}^{EF} \ge 0 \\
t_{ij}^{LF} \ge 0 \\
\omega_i > 0 \\
\sum_{i=0}^{I+1} \omega_i = 1 \\
x_{ijt} = 0 \text{ or } 1
\end{cases}
\end{cases}$$

where $i = 0,1,...,e,...,I+1$, $j = 1,2,...,m_i$, $k = \{1,2,...K\}$ and $n = \{1,2,...N\}$.

Note: In (P2), the units in the objective function have been normalized.

5. Particle swarm optimization (PSO) for rc-PSP/mM

As a generalization of the classical job shop scheduling problem, rc-PSP/mM belongs to the class of NP-hard optimization problems ([3, 29, 30]). As shown by [31, 32], exact methods are unable to find optimal solutions for rc-PSP/mM. In this case, several heuristic procedures have been proposed to solve rc-PSP/mM, such as simulated annealing algorithm [2],

local search procedure [33], genetic algorithm [14, 30, 34], particle swarm optimization [35], ant colony algorithms [36], and so on. In this paper, the particular nature of our model motivates us to develop particle swarm optimization algorithm (PSO) to obtain the order of activities (the start time of every activity) and execution modes. Furthermore, the software development of PSO algorithm is easy-to-implement for other rc-PSP/mM in Longtan Hydropower Station.

PSO was first proposed by Kennedy and Eberhart in 1995 [37], and had become one of the most important swarm intelligence paradigms. PSO has superior search performance for numerous difficult optimization problems with faster and more stable convergence rates compared with other population-based stochastic optimization methods. In PSO, an n-dimensional position of a particle (called solution), initialized with a random position in a multidimensional search space, represents a solution to the problem [38]. The particles, which are characterized by their positions and velocities, fly through the problem space following the current optimum particles [39, 40]. Unlike other population-based algorithms, the velocity and position of each particle are dynamically adjusted according to the flying experiences or discoveries of its own and those of its companions.

Since PSO can be implemented easily and effectively, it has been rapidly applied in solving real-world optimization problems in recent years, such as [37, 41, 42]. In PSO, the following formulas are applied to update the position and velocity of each particle [37]:

$$v_d^l(\tau+1) = \omega(\tau)v_d^l(\tau) + c_p r_1[p_d^{l,best}(\tau) - p_d^l(\tau)] + c_g r_2[g_d^{best}(\tau) - p_d^l(\tau)] \qquad (14)$$

$$p_d^l(\tau+1) = p_d^l(\tau) + v_d^l(\tau+1) \qquad (15)$$

where $v_d^l(\tau)$ is the velocity of l particle at the d th dimension in the iteration, $p_d^l(\tau)$ is an inertia weight, l is the position of particle at the d th dimension, r_1 and r_2 are random numbers in the range [0,1], c_p and c_g are personal and global best position acceleration constant respectively, meanwhile, $p_d^{l,best}$ is personal best position of l particle at the d th dimension and g_d^{best} is global best position at the d th dimension.

5.1 Notations

The notations in PSO are shown in Appendix C.

5.2 Framework of PSO for rc-PSP/mM

Now, we will present the framework of PSO to solve rc-PSP/mM in Drilling Grouting Project as below:

Step 1. Initialize particles. Set iteration $\tau = 1$. For $l = 1,2,...,L$, generate the position of the l th particle for the i th activity with integer random position

$$P_i^l(1) = \left[p_{i1t_i^{EF}}^l(1), p_{i1(t_i^{EF}+1)}^l(1), \cdots, p_{i1t_i^{LF}}^l(1); p_{i2t_i^{EF}}^l(1), p_{i2(t_i^{EF}+1)}^l(1), \cdots, p_{i2t_i^{LF}}^l(1); \cdots; p_{im_it_i^{EF}}^l(1), p_{im_i(t_i^{EF}+1)}^l(1), \cdots, p_{im_it_i^{LF}}^l(1)\right]$$

and the value is 0 or 1. In addition,

$$V_i^l(1) = \left[v_{i1}^l(1), v_{i2}^l(1), \cdots, v_{im_i(t_i^{LF}-t_i^{EF}+1)}^l(1)\right]$$

Step 2. Check the feasibility of solutions. For $l = 1,2,...,L$, if the feasibility criterion is met by all particles, i.e., all particles

Zh. Zhang, J. Xu

satisfied the constraints of the model (*P2*). Then, the particles are feasible.

Remark 2: The nonrenewable resource constraints,

$$\Phi^{-1}(\rho)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma^2{}_{r_{ijn}}\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} + \Phi^{-1}(1-\varepsilon)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma''^2\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2}$$
$$+ \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu''\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2 \geq r_n^M$$

may be infeasible. After serving some activities, the requirement of nonrenewable resources would be greater than the total quantities. Therefore, it is necessary to check and adjust of particle-represented solutions to avoid nonrenewable resource infeasibility.

Step 2.1 Let *Inf* denote the infeasibility of nonrenewable resources, according to the particle-represented in the *l* th particle, compute

$$Inf = \begin{cases} 1, & \Phi^{-1}(\rho)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma^2{}_{r_{ijn}}\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} + \Phi^{-1}(1-\varepsilon)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma''^2\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} \\ & + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu''\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2 - r_n^M \leq 0 \\ 0, & otherwise \end{cases}$$

Step 2.2 If *Inf*=0 for all nonrenewable resources, go to step 2.6. Otherwise, go to step 2.3.

Step 2.3 Select an activity *i* with multiple execution modes, i.e. $m_i>1$.

Step 2.4 Select a new mode $\forall j' \neq j(\tau)$ and within $1,2,...,m_i$ for activity *i* randomly, then compute *Inf* again.

Step 2.5 If *Inf* = 0, replace j' with *j* and then go to step 2.6. Otherwise, repeat step 2.4 until all modes of activity *i* have been iterated.

Step 2.6 Stop.

The procedure is presented in Fig. 7.

Step 3. Evaluate each particle. The fitness value used to evaluate the particle is the objective function of (P2),

$$Fitness_i(P_i^l(\tau)) = \lambda_T \sum_{j=1}^{m_i}\sum_{t=t_i^{EF}}^{t_i^{LF}} t x_{ijt} + \lambda_C \sum_{i=0}^{I+1} c_i^{TP}\left|\sum_{j=1}^{m_i}\sum_{t=t_i^{EF}}^{t_i^{LF}} t x_{ijt} - t_i^E\right|$$
$$- \lambda_Q\left(\sum_{i=0}^{I+1}\omega_i\left[q_i^S + \alpha_i(p_{ij} - p_{ij}^S)\right]\right)$$

Thus, for *l*=1,2,...,*L*, compute the performance measurement by the fitness value, and identify the personal best of each particle and the global best in the swarm.

Step 4. Update the particles. If the maximum number of cycles has not been reached, then:

Step 4.1 Update the velocity of particles. Compute the speed of each particle using the following formula:
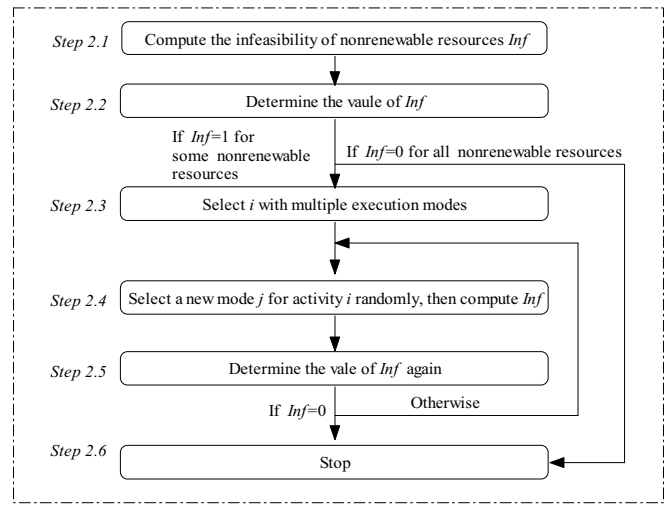

Fig. 7 Procedure of checking and adjusting nonrenewable resource constraints

$$v_{id}^l(\tau+1) = \omega(\tau)v_{id}^l(\tau) + c_p r_1[p_{id}^{l,best}(\tau) - p_{id}^l(\tau)] + c_g r_2[g_{id}^{l,best}(\tau) - p_{id}^l(\tau)] \tag{16}$$

where the inertia weight $\omega(\tau) = \omega(T) + \frac{\tau-1}{1-T}[\omega(1) - \omega(T)]$ [43].

Step 4.2 Update the positions of particles. Adding the speed produced from the previous step, compute the new positions of the particles

$$p_{id}^l(\tau+1) = p_{id}^l(\tau) + v_{id}^l(\tau+1) \tag{17}$$

Step 5. Decode particles into solutions. For *l*=1,2,...,*L*, decode $p_i^l(\tau)$ to a solution as:

$$x_{ijt}(\tau) = p_{ijt}^l(\tau)$$

If $p_{ij(t_i^{EF}+t_{i-order})}^l(\tau) = 1$, then the th activity, which is executed by mode *j* th, will start at $t_i^{EF} + t_{i-order}$, $t_{i-order} \in [0,1,\cdots,t_i^{LF} - t_i^{EF}]$.

Mapping between one potential solution of rc-PSP/mM and particle representation is shown in Fig. 8.

Step 6. Stop. Output the fitness value and the global best particle-represented solution.

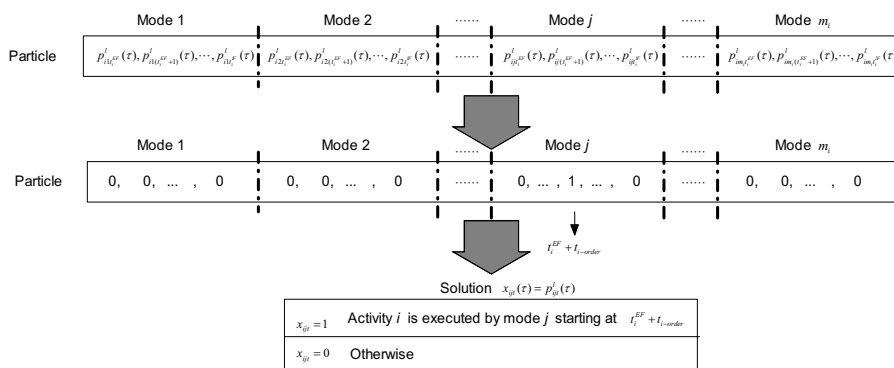Based on above, we can get the framework of PSO for rc-PSP/mM in Fig. 9.

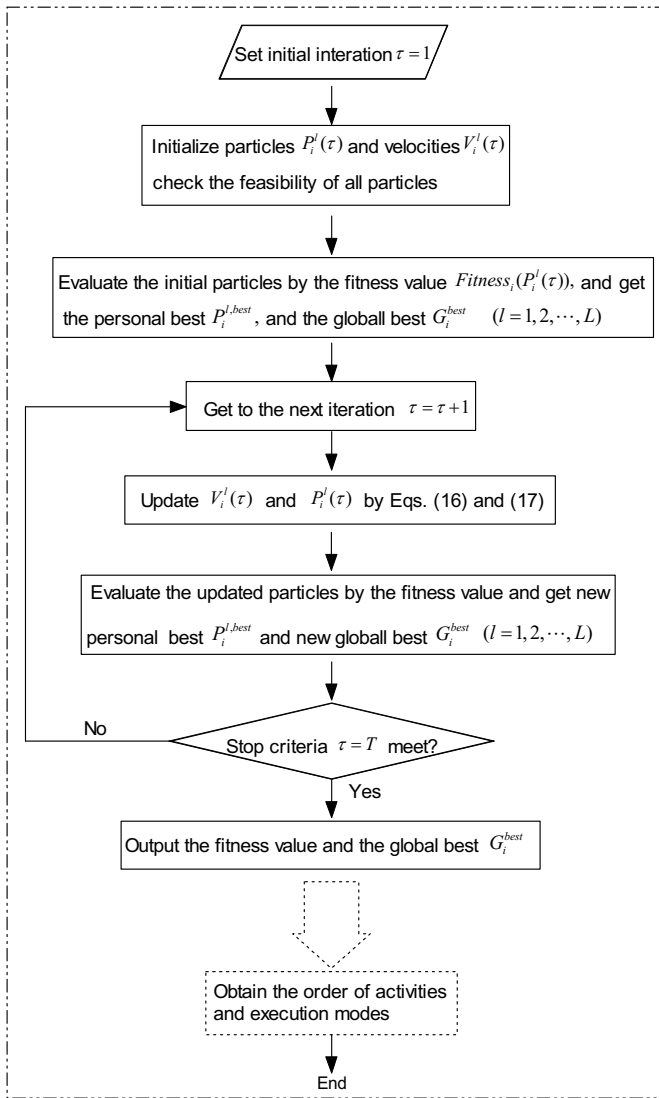**Fig. 8** Decoding method and mapping between PSO particles and solutions to (P2)

Fig. 9 Procedure of the PSO framework for rc-PSP/mM

6. A case study

In this section, we perform the practical application of Drilling Grouting Construction Project in the Longtan Dam. At present, there are 13 activities in the Drilling Grouting Project:

A1: Backfill grouting

A2: Closure grouting

A3: Embedded $\varnothing$ 25 type grouting steel pipe

A4: Contact grouting

A5: Embedded $\varnothing$ 50 type grouting PVC pipe

A6: Consolidation grouting (including drilling)

A7: Inclinometer hole drilling

A8: Drilled drain hole (including flexible drain)

A9: Rock multi-point bore-hole of deformation

A10: Osmometer hole drilling

A11: Bolt stress meters hole drilling

A12: Underground water level observation hole drilling

A13: Consolidation grouting check hole drilling and water pressure

---

Each activity has certain executed modes, the successors and the expected finish time according to the decision managers. Besides, there are three types of resources, i.e., manpower (r_1, renewable resources), equipment (r_2, renewable resources) and materials (r_3, nonrenewable resources) in this project scheduling problem.

The successors, detailed data of process time, resource consumption of each activity are shown in Fig. 10 in details (S and T are dummy activities). In addition, for calculating various kinds of resources expediently, we measure all the resources to the consumption amount and unify dimensionless units into a cash value (ten thousands CNY per unit).

6.1 Related data

Each activity has certain maximal multiple unit requirements of three different of resources, (Manpower, Equipment, Materials) = (r_1^M, r_2^M, r_3^M) = (750, 830, 75) (Units: ten thousands CNY). Besides, since more particles require more evaluation runs and leading to more optimization costs [44], we select 50 particles as the population size and 100 as the iteration number in this study. Hence, the parameters of problem are set as follows: Population size L=50, Iteration number T=100, Acceleration constant c_p=2.0 and c_g=2.0, Inertia weight $w(1)$=0.9 and $w(T)$=0.1, respectively. In addition, λ_T=0.45, λ_C=0.20, λ_Q=0.35, η= ζ =ρ =σ = 0.9. Other main certain variables are given in Table 1, and the bi-random coefficients are given in Appendix D.

6.2 Results

In this practical application, we use Matlab 7.0 and Visual C++ language on an Inter Core I3 M370, 2.40 GHz, with 2048 MB memory, running Microsoft Windows 7. After re-run 10 times of PSO computer program, we obtain the project schedule of Longtan case in Fig. 11, and the detailed optimal results in Table 2, respectively.

Following the results, we can notice that the total duration is 5 years and 3 months (63 months). Compared with the initial design, we actually decrease 5 months from the total duration. In ensuring the quality of the project, total tardiness penalty is within an acceptable range (40.02 ten thousands CNY). The results are very satisfactory for the project decision managers.

6.3. Comparison analysis for the algorithm

To prove efficiency and effectiveness of the PSO for solving rc-PSP/mM, we compare PSO with genetic algorithm (GA) that is proved to solve rc-PSP/mM successfully [45-48]. Some parameters for the GA are selected as follows: population size is the same as for the PSO approach, i.e, popsize=50, the rate of crossover p_c=0.6, the rate of mutation p_m= 0.6 and maximum generation maxGen=100, respectively. Fig. 12 depicts the computational results for the PSO and GA.

The run-time required by PSO and GA is given in Table 3. It presents minimum, mean, maximum run-time in
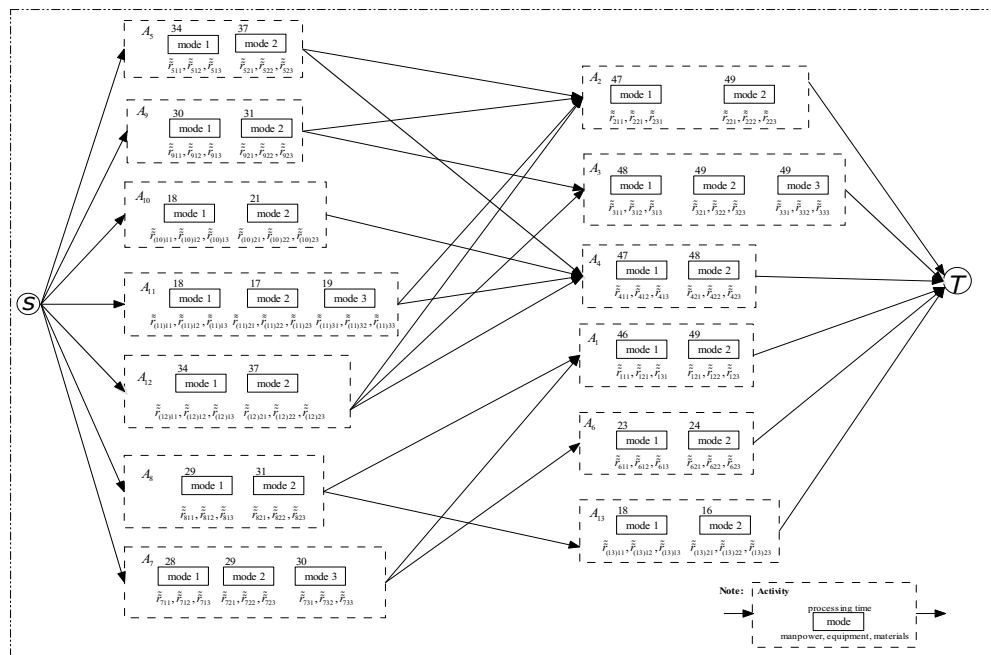
Fig. 10 Configuration of 13 activities in Drilling Grouting Project

Table 1 Main certain variables

Variables	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}
ω_i	0.115	0.115	0.115	0.115	0.086	0.058	0.072	0.072	0.072	0.048	0.044	0.044	0.044
α	0.58	0.60	0.60	0.56	0.45	0.30	0.39	0.41	0.40	0.25	0.23	0.20	0.22
q^s	2.31	2.31	2.31	2.31	1.73	1.15	1.44	1.44	1.44	0.96	0.87	0.87	0.87
t_i^E	48	48	48	48	36	24	30	30	30	20	18	18	18
c_i^{TP}	5.25	5.13	5.09	4.27	2.83	3.32	3.60	3.45	2.46	2.32	2.89	3.69	2.28

minutes (MIN, MEAN, MAX) to obtain the optimal solutions. Table 3 shows that the mean time of PSO is faster than GA for small number of activities (13 activities). Additionally, the convergence histories, which are given in Fig. 12, indicate that PSO converges faster to find the optimal solutions (the optimal fitness value of PSO and GA are both 30.0412). All above can prove that for small number of activities in rc-PSP/mM, PSO is a very efficient and effective algorithm.

Fig. 11 Project schedule of Longtan Case

7. Conclusions and future research

In this paper, we first proposed a multi-mode resource-constrained project scheduling model under a bi-random phenomenon for a practical Drilling Grouting Project in Longtan Large-Scale Water Conservancy and Hydropower Construction Project. In the proposed model, we considered three objectives--- the overall duration of the project, the total tardiness penalty, and the quality of the project. The model employed the bi-random variables to characterize the hybrid
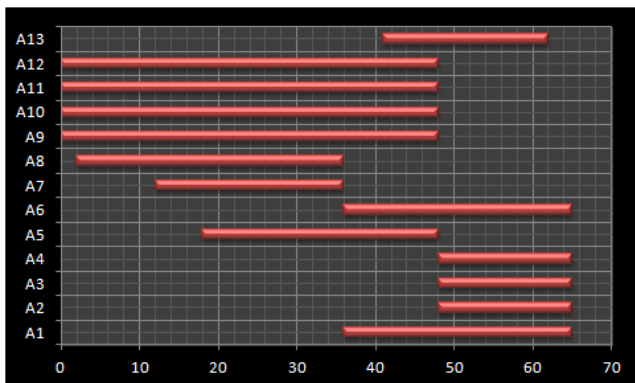
Table 2 Results of the Longtan case

Activity	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}
Mode	1	1	1	2	1	2	3	1	1	1	2	1	2

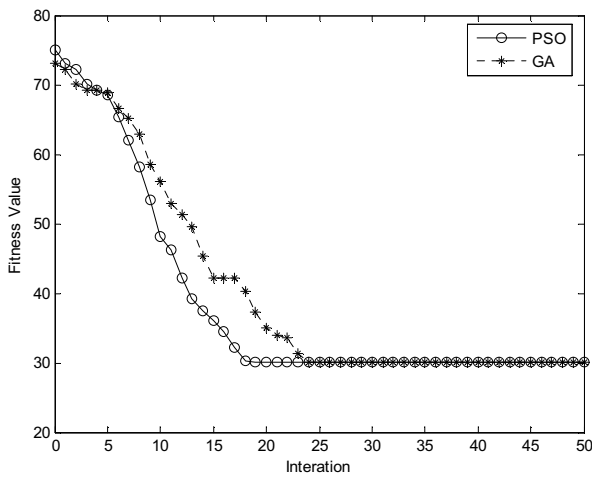|	
Resource consumptions	Manpower: 721.43
	Equipment: 809.31
	Materials: 69.76
Project duration	63 months
Tardiness penalty	40.02
Quality	2.59
Fitness value	30.0412

Fig. 12 Convergence histories of PSO and GA

Table 3 Run-time of GA and PSO in minutes

	pop-size	max-Gen	MIN	MEAN	MAX
GA	50	100	10.26	12.34	27.44
PSO	50	100	6.48	7.86	12.76

uncertain environment where two-fold randomness exists. We first considered the bi-random phenomenon in a practical construction project, and this work was original. For handling bi-random variables, we employed the chance constraint operator to the constraints with bi-random coefficients. Subsequently, we applied particle swarm optimization (PSO) to resolve the rc-PSP/mM that was well known as a NP-hard problem.

One of the most important follow-up researches is the application of the proposed model and algorithm in this study to other practical construction projects in Longtan case. Besides, multi-mode multi-project resource-constrained scheduling problem should be concerned in the continued research because of its practical significance in large-scale practical problem. Another area for continued research should be focus on the software development that was based on the proposed model and algorithm in this study. All of these areas are very important and worth of an equal concern.

References

[1] Brucker, P., Drexl, A. and et al., Resource-constrained project scheduling: notation, classification, models, and methods, European Journal of Operational Research, 1999; 112(1), 3-41.

[2] Slowinski, R., Soniewicki, B., Weglarz, J., Dss for multiobjective project scheduling, European Journal of Operational Research, 1994; 79, 220-229.

[3] Demeulemeester, E., Herroelen, W., Project scheduling: A research handbook, Boston: Kluwer Academic Publishers (2002).

[4] Kolisch, R. and Padman, R., An integrated survey of deterministic project scheduling, Omega, 2001; 29, 249-272.

[5] Nudtasomboon, N., Randhawa, S., Resource-constrained project scheduling with renewable and non-renewable resources and time-resource tradeoffs, Computers & Industrial Engineering, 1997; 32(1), 227-242.

[6] Drexl, A., Grunewald, J., Non-preemptive multi-mode resource-constrained project scheduling, IIE Transactions, 1993; 25, 74-81.

[7] Herroelen, W., Reyck, B., and Demeulemeester, E., A classification scheme for project scheduling, Handbook of Recent Advances in Project Scheduling, Kluwer Academic Publishers (1999).

[8] Heilmann, R., A branch-and-bound procedure for the multi-mode resource-constrained project scheduling problem with minimum and maximum time lags, European Journal of Operational Research, 2003; 144, 348-365.

[9] Mika, M., Waligora, G., and Weglarz, J., Simulated annealing and tabu search for multi-mode resource-constrained project scheduling with positive discounted cash flows and different payment models, European Journal of Operational Research, 2005; 164, 639-668.

[10] Buddhakulsomsiri, J., Kim, D., Properties of multi-mode resource-constrained project scheduling problems with resource vacations and activity splitting, European Journal of Operational Research, 2006; 175, 279-295.

[11] Lorenzoni, L., Ahonen, H. and Alvarenga, A., A multi-mode resource-constrained scheduling problem in the context of port operations, Computers & Industrial Engineering, 2006; 50, 55-65.

[12] He, Z. and Xu, Y., Multi-mode project payment scheduling problems with bonus penalty structure, European Journal of Operational Research, 2008; 189, 1191-1207.

[13] Jarboui, B., Damak, N., Siarry, P., Rebai, A., A combinatorial particle swarm optimization for solving multi-mode resource-constrained project scheduling problems, Applied Mathematics and Computation, 2008; 195, 299-308.

[14] Peteghem, V. and Vanhoucke, M., A genetic algorithm for the preemptive and non-preemptive multi-mode resource-constrained project scheduling problem, European Journal of Operational Research, 2010; 201, 409-418.

[15] Aytug, H., Lawley M., and et al., Executing production schedules in the face of uncertainties: A review and some future directions, European Journal of Operational Research, 2005; 161, 86-110.

[16] Bidot, J., Thierry, V. and et al., A theoretic and practical framework for scheduling in a stochastic environment, Journal of Scheduling, 2008; 10, 41-65.

[17] Herroelen, W., Leus, R., Project scheduling under uncertainty: Survey and research potentials, European Journal of Operational Research, 2005; 165, 289-306.

[18] Vonder, V., Demeulemeester, S., and et al., The use of buffers in project management: The trade-off between stability and makespan, International Journal of Production Economics, 2005; 97, 227-240.

[19] Yagi, J., Arai, E., and et al., Action-based union of the temporal opposites in scheduling: non-deterministic approach, Automation in Construction, 2003; 12, 321-329.

[20] Peng, J. and Liu, B., Birandom variables and birandom programming, Computers and Industrial Engineering, 2007; 53, 433-453.

[21] Xu, J., Zhou X., A class of multi-objective expected value decision-making model with bi-random coefficients and its application to flow shop scheduling problem, Information Sciences, 2009; 179, 2997-3017.

[22] Yan L., Chance-constrained Portfolio Selection with Bi-random Returns, Modern Applied Science, 2009; 3(4), 161-165.

[23] Xu, J., Ding C., A class of chance constrained multiobjective linear programming with birandom coefficients and its application to vendors selection, International Journal of

Production Economics, 2011; 131(2), 709-720.

[24] Xu, J., Zhang, Z., and Mookerjee, V., Applying Bi-Random MODM Model to Navigation Coordinated Scheduling: A Case Study of Three Gorges Project, Transport, 2011; Inpress.

[25] Zhang, Z., Applying improved particle swarm optimization algorithm to bi-level linear programming problem with bi-random coefficients, Proceedings of The Fifth International Conference on Management Science and Engineering Management, 2011; 147-152.

[26] Zhang, Z., Bi-Level Multi-Objective Resource-Constrained Project Scheduling Models under complex random phenomena and the Application, Doctoral Dissertation, Sichuan University (In Chinese), 2011.

[27] Liu, B., Theory and Practice of Uncertain Programming, New York: Physica Verlag (2002).

[28] Charnes, A., Cooper, W., Chance-constrained programming, Management Science, 1959; 6, 73-79.

[29] Boctor, F., A new and effcient heuristic for scheduling projects with resource restrictions and multiple execution modes, European Journal of Operational Research, 1996; 90, 349-361.

[30] Ozdamar, L, A genetic algorithm approach to a general category project scheduling problem, IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews, 1999; 29, 44-59.

[31] Sprecher, A., Resource-constrained project scheduling: Exact methods for the multi-mode case, Lecture Notes in Economics and Mathematical Systems, Berlin: Springer (1994).

[32] Sprecher, A. and Drexl, A., Multi-mode resource-constrained project scheduling by a simple, general and powerful sequencing algorithm, European Journal of Operational Research, 1998; 107, 431-450.

[33] Kolisch, R. and Drexl, A., Local search for nonpreemptive multi-mode resource-constrained project scheduling, IIE Transactions, 1997; 29, 987-999.

[34] Mori, M. and Tseng, C., A genetic algorithm for multi-mode resource constrained project scheduling problem, European Journal of Operational Research, 1997; 100, 134-141.

[35] Zhang, H., Tam, C., and Li, H., Multimode project scheduling based on particle swarm optimization, Computer-Aided Civil and Infrastructure Engineering, 2005; 21(2), 93-103.

[36] [36] Shan, M., Wu, J., and Peng, D., Particle swarm and ant colony algorithms hybridized for multi-mode resource-constrained project scheduling problem with minimum time lag, Wireless Communications, Networking and Mobile Computing, 2007; 5898--5902.

[37] [37] Kennedy, J., Eberhart, R., Particle swarm optimization, Proceedings of the IEEE Conference on Neural Networks, Piscataway: IEEE Service Center, 1995; 1942--1948.

[38] [38] Robinson, J., Sinton, S., Rahmat-Samii, Y., Particle swarm, genetic algorithm, and their hybrids: Optimization of a profiled corrugated horn antenna, IEEE Antennas and Propagation Society International Symposium and URSI National Radio Science Meeting, San Antonio, 2002; 168--175.

[39] [39] Cervantes, A., Galvan, I., Isasi, P., Ampso: A new particle swarm method for nearest neighborhood classification, IEEE Trans. Syst., Man, Cybern. B, Cybern, 2009; 39, 1082-1091.

[40] [40] Kennedy, J., Eberhart, R., Swarm Intelligence, Morgan Kaufmann (2001).

[41] [41] Ling, S., Iu, H., and et al., Hybrid particle swarm optimization with wavelet mutation and its industrial applications, IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 2008; 38, 743-763.

[42] Sha, D., Cheng-Yu Hsu., A hybrid particle swarm optimization for job shop scheduling problem, Computers & Industrial Engineering, 2006; 51, 791-808.

[43] Shi, Y., Eberhart, R., Particle swarm optimization, Proc. IEEE Int. Conf. on Neural Networks, 1998; 69-73.

[44] Trelea, I., The particle swarm optimization algorithm: Convergence analysis and parameter selection, Information Processing Letters, 2003; 85(6), 317-325.

[45] Alcaraz, J., Maroto, C., Ruiz, R. Solving the Multi-Mode Resource-Constrained Project Scheduling Problem with Genetic Algorithms, The Journal of the Operational Research Society, 2006; 54, 614-626.

[46] Elloumi, S. and Fortemps, P., A hybrid rank-based evolutionary algorithm applied to multi-mode resource-constrained project scheduling problem, European Journal of Operational Research, 2010; 205, 31-44.

[47] Lova, A., Tormosa, P. and et al., An efficient hybrid genetic algorithm for scheduling projects with resource constraints and multiple execution modes, International Journal of Production Economics, 2009; 117, 302-316.

[48] Peng, W., and Wang, C., A multi-mode resource-constrained discrete time-cost trade off problem and its genetic algorithm based solution, International Journal of Project Management, 2009; 27, 600-609.

Appendix A. Notations in mathematical model

Indices:

i activity index, $i=0,1,...,e,...,I+1$

j mode index, $j=1,2,...,m_i$ (is the number of possible modes of activity i)

k renewable resource type index, $k=1,2,...,K$

n non-renewable resource type index, $n=1,2,...,N$

t period index

Objectives:

T total project duration

C *total cost of the project*

Q *total quality of the project*

Certain parameters:

Pre_i set of immediate predecessors of activity i

c_i^{TP} the penalty cost of activity i

p_{ij} processing time of activity i selected mode j

p_{ij}^S the earliest processing time of activity i selected mode j

p_{ij}^L the latest processing time of activity i selected mode j

q_i the quality of activity i

q_i^S the smallest acceptable quality of activity i

q_i^L the best acceptable quality of activity i

ω_i the weight of the quality of activities i for the quality

th   of the whole project

t_i^E expected finish time of activity i

r_k^M maximum-limited renewable resource k

r_n^M maximum-limited non-renewable resource n

t_i^{EF} early finish time of activity i

t_i^{LF} lately finish time of activity i

Bi-random coefficients:

$\tilde{\tilde{r}}_{ijk}$ *renewa*ble resource k required to execute activity i using mode j

$\tilde{\tilde{r}}_{ijn}$ non-renewable resource n required to execute activity i using mode j

Decision Variables:

$$x_{ijt} = \begin{cases} 1, & \text{if activity i} \\ 0, & \end{cases}$$

Note: x_{ijt} confirms the finish time of the current activity regardless of whether the certain executed mode is scheduled during this certain time or not.

Appendix B. The proof of Theorem 1.

Let $\tilde{\tilde{\theta}} = \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M$, then $\tilde{\tilde{\theta}}$ is a normally distributed random variable with the following expected value and variance (the mean of bi-random variable $\tilde{\tilde{r}}_{ijk}$ is not considered as a random variable temporarily):

$$E\left[\tilde{\tilde{\theta}}\right] = \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M$$

$$D\left[\tilde{\tilde{\theta}}\right] = \sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2$$

Thus,

$$\frac{\tilde{\tilde{\theta}} - E\left[\tilde{\tilde{\theta}}\right]}{\sqrt{D\left[\tilde{\tilde{\theta}}\right]}} = \frac{\left(\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M\right) - \left(\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M\right)}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}$$

is standardized normally distributed random variable. Since the inequality

$$\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}(\omega)\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} \leq r_k^M$$

is equivalent to

$$\frac{\left(\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M\right) - \left(\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M\right)}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}} \leq \frac{-\left(\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M\right)}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}$$

Let $\gamma = \frac{\left(\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M\right) - \left(\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M\right)}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}$

we have

$$\Pr\left\{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}(\omega)\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} \leq r_k^M\right\} \geq \eta$$

$$\Leftrightarrow \Pr\left\{\gamma \leq \frac{r_k^M - \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}\right\} \leq \eta$$

$$\Leftrightarrow \Phi\left\{\frac{r_k^M - \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}\right\} \leq \eta$$

$$\Leftrightarrow \frac{r_k^M - \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}} \leq \Phi^{-1}(\eta)$$

$$\Leftrightarrow \Phi^{-1}\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} \geq r_k^M$$

Now, let

$\tilde{\chi} = \Phi^{-1}\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M$,

then $\tilde{\chi}$ is a normally distributed random variable with the following expected value and variance

$$E[\tilde{\chi}] = \Phi^{-1}\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} - r_k^M$$

$$D[\tilde{\chi}] = \sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2$$

Let $M = \Phi^{-1}\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} - r_k^M$, and

$$N = \frac{(M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau}) - (M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}$$

is a standardized normally distributed random variable. Then, the inequality

$$\Phi^{-1}\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} \geq r_k^M$$

$$\Leftrightarrow N \geq \frac{-(M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}$$

Thus,

$$\Pr\left\{\omega \in \Omega \mid \Pr\left\{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijk}(\omega)\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} \leq r_k^M\right\} \geq \eta\right\} \geq \varsigma$$

$$\Leftrightarrow \Pr\left\{\Phi^{-1}(\eta)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{ijk}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\mu}_{ijk}\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} \geq r_k^M\right\} \geq \varsigma$$

$$\Leftrightarrow \Pr\left\{N \geq \frac{-(M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}\right\} \geq \varsigma$$

$$\Leftrightarrow 1 - \Pr\left\{N \leq \frac{-(M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}\right\} \geq \varsigma$$

$$\Leftrightarrow \Pr\left\{N \leq \frac{-(M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}\right\} \leq 1 - \varsigma$$

$$\Leftrightarrow \Phi\left(\frac{-(M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}}\right) \leq 1 - \varsigma$$

$$\Leftrightarrow \frac{-(M + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}} \leq \Phi^{-1}(1-\varsigma)$$

$$\Leftrightarrow \Phi^{-1}(1-\varsigma)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} + M \geq 0$$

$$\Leftrightarrow \Phi^{-1}(\eta)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{r_{jk}}^{2}\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2} + \Phi^{-1}(1-\varsigma)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma'^2\sum_{\tau=t}^{t+p_{ij}-1}(x_{ij\tau})^2}$$
$$+ \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu'\sum_{\tau=t}^{t+p_{ij}-1}x_{ij\tau} \geq r_k^M$$

Similarly, we can get
$$\Pr\left\{\omega \in \Omega \mid \Pr\left\{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\tilde{\tilde{r}}_{ijn}(\omega)\sum_{t=t_i^{EF}}^{t_i^{LF}}x_{ijt} \leq r_n^M\right\} \geq \rho\right\} \geq \varepsilon$$

$$\Leftrightarrow \Phi^{-1}(\rho)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma_{r_{jn}}^{2}\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2} + \Phi^{-1}(1-\varepsilon)\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{m_i}\sigma''^2\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2}$$
$$+ \sum_{i=1}^{I}\sum_{j=1}^{m_i}\mu''\sum_{t=t_i^{EF}}^{t_i^{LF}}(x_{ijt})^2 \geq r_n^M$$

The proof of Theorem 1 is completed. □

Appendix C. Notations in PSO

τ iteration index, $\tau=1,...,T$

l particle index, $l=1,...,T$

d dimension index, $d=1,...,m_i(t_i^{LF}-t_i^{LF}+1)$

i index of activity, $i=0,1,...,I+1$

j index of mode, $i=1,...,m_i$

r_1, r_2 uniform distributed random number within [0, 1]

$w(\tau)$ inertia weight in the τ iteration

$v_{id}^l(\tau)$ velocity of the ith activity of the lth particle at the ith dimension in the τth iteration

$p_{id}^l(\tau)$ Position of the ith activity of the lth particle at the ith dimension in the τth iteration

$P_{id}^{l,best}$ personal best position of the ith activity of the lth particle at the dth dimension

g_{id}^{best}	global best position of the ith activity at the dth dimension
c_p	personal best position acceleration constant
c_g	global best position acceleration constant
$V_i^l(\tau)$	vector velocity of the ith activity of the lth particle in the τth iteration
$P_i^l(\tau)$	vector position of the ith activity of the lth particle in the τth iteration
$P_i^{l,best}$	vector personal best position of the ith activity of the lth particle,
G_i^{best}	vector global best position of the ith activity,
$Fitness_i(P_i^l(\tau))$	Fitness value of $P_i^l(\tau)$

Appendix D. Resource with bi-random coefficients

The first renewable resource Manpower:

$\tilde{\tilde{r}}_{111}\sim N(\mu,0.04)\ with\ \tilde{\mu}\sim N(65.0,1.00)$
$\tilde{\tilde{r}}_{211}\sim N(\mu,0.01)\ with\ \tilde{\mu}\sim N(62.8,0.04)$
$\tilde{\tilde{r}}_{311}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(68.4,0.49)$
$\tilde{\tilde{r}}_{331}\sim N(\mu,0.01)\ with\ \tilde{\mu}\sim N(61.3,0.16)$
$\tilde{\tilde{r}}_{421}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(67.1,0.01)$
$\tilde{\tilde{r}}_{521}\sim N(\mu,0.49)\ with\ \tilde{\mu}\sim N(50.6,0.81)$
$\tilde{\tilde{r}}_{621}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(29.8,1.00)$
$\tilde{\tilde{r}}_{721}\sim N(\mu,0.01)\ with\ \tilde{\mu}\sim N(43.1,0.49)$
$\tilde{\tilde{r}}_{811}\sim N(\mu,0.64)\ with\ \tilde{\mu}\sim N(68.5,0.09)$
$\tilde{\tilde{r}}_{911}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(71.2,4.00)$
$\tilde{\tilde{r}}_{(10)11}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(48.1,1.00)$
$\tilde{\tilde{r}}_{(11)11}\sim N(\mu,0.09)\ with\ \tilde{\mu}\sim N(51.3,0.64)$
$\tilde{\tilde{r}}_{(11)31}\sim N(\mu,0.64)\ with\ \tilde{\mu}\sim N(52.2,0.49)$
$\tilde{\tilde{r}}_{(12)21}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(53.8,0.04)$
$\tilde{\tilde{r}}_{(13)21}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(52.5,1.00)$
$\tilde{\tilde{r}}_{121}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(62.2,0.01)$
$\tilde{\tilde{r}}_{221}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(63.0,1.00)$
$\tilde{\tilde{r}}_{321}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(65.0,1.00)$
$\tilde{\tilde{r}}_{411}\sim N(\mu,9.00)\ with\ \tilde{\mu}\sim N(61.8,4.00)$
$\tilde{\tilde{r}}_{511}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(48.4,1.00)$
$\tilde{\tilde{r}}_{611}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(33.7,0.16)$
$\tilde{\tilde{r}}_{711}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(40.4,0.25)$
$\tilde{\tilde{r}}_{731}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(41.2,1.00)$
$\tilde{\tilde{r}}_{821}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(64.6,0.04)$
$\tilde{\tilde{r}}_{921}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(63.9,9.00)$
$\tilde{\tilde{r}}_{(10)21}\sim N(\mu,0.01)\ with\ \tilde{\mu}\sim N(43.6,0.36)$
$\tilde{\tilde{r}}_{(11)21}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(50.9,4.00)$
$\tilde{\tilde{r}}_{(12)11}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(54.1,1.00)$
$\tilde{\tilde{r}}_{(13)11}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(55.7,0.49)$

The second renewable resource Equipment:

$\tilde{\tilde{r}}_{112}\sim N(\mu,0.09)\ with\ \tilde{\mu}\sim N(94.2,4.81)$
$\tilde{\tilde{r}}_{212}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(92.3,1.00)$
$\tilde{\tilde{r}}_{312}\sim N(\mu,0.16)\ with\ \tilde{\mu}\sim N(90.5,4.00)$
$\tilde{\tilde{r}}_{332}\sim N(\mu,0.64)\ with\ \tilde{\mu}\sim N(93.2,0.49)$
$\tilde{\tilde{r}}_{422}\sim N(\mu,0.49)\ with\ \tilde{\mu}\sim N(95.5,0.81)$
$\tilde{\tilde{r}}_{522}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(96.7,0.01)$

$\tilde{\tilde{r}}_{622}\sim N(\mu,0.16)\ with\ \tilde{\mu}\sim N(43.4,1.00)$
$\tilde{\tilde{r}}_{722}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(56.4,4.00)$
$\tilde{\tilde{r}}_{812}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(55.2,0.64)$
$\tilde{\tilde{r}}_{912}\sim N(\mu,0.49)\ with\ \tilde{\mu}\sim N(55.5,0.09)$
$\tilde{\tilde{r}}_{(10)12}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(38.5,4.00)$
$\tilde{\tilde{r}}_{(11)12}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(33.2,0.49)$
$\tilde{\tilde{r}}_{(11)32}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(34.2,0.64)$
$\tilde{\tilde{r}}_{(12)22}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(32.2,0.49)$
$\tilde{\tilde{r}}_{(13)21}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(37.1,0.09)$
$\tilde{\tilde{r}}_{122}\sim N(\mu,0.64)\ with\ \tilde{\mu}\sim N(90.6,4.00)$
$\tilde{\tilde{r}}_{222}\sim N(\mu,0.49)\ with\ \tilde{\mu}\sim N(96.5,0.09)$
$\tilde{\tilde{r}}_{322}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(91.9,1.00)$
$\tilde{\tilde{r}}_{412}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(98.6,4.00)$
$\tilde{\tilde{r}}_{512}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(68.2,0.09)$
$\tilde{\tilde{r}}_{612}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(45.2,0.64)$
$\tilde{\tilde{r}}_{712}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(58.3,0.49)$
$\tilde{\tilde{r}}_{732}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(57.6,1.00)$
$\tilde{\tilde{r}}_{822}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(58.4,0.01)$
$\tilde{\tilde{r}}_{922}\sim N(\mu,0.16)\ with\ \tilde{\mu}\sim N(54.9,1.00)$
$\tilde{\tilde{r}}_{(10)22}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(36.9,0.09)$
$\tilde{\tilde{r}}_{(11)22}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(31.5,4.00)$
$\tilde{\tilde{r}}_{(12)12}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(33.7,1.00)$
$\tilde{\tilde{r}}_{(13)12}\sim N(\mu,0.64)\ with\ \tilde{\mu}\sim N(34.4,0.81)$

The final nonrenewable resource Materials:

$\tilde{\tilde{r}}_{113}\sim N(\mu,0.09)\ with\ \tilde{\mu}\sim N(7.2,1.00)$
$\tilde{\tilde{r}}_{213}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(7.7,0.25)$
$\tilde{\tilde{r}}_{313}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(7.6,4.00)$
$\tilde{\tilde{r}}_{333}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(7.4,0.81)$
$\tilde{\tilde{r}}_{423}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(8.2,0.09)$
$\tilde{\tilde{r}}_{523}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(5.7,1.00)$
$\tilde{\tilde{r}}_{623}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(4.7,0.49)$
$\tilde{\tilde{r}}_{723}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(4.9,0.36)$
$\tilde{\tilde{r}}_{813}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(5.3,1.00)$
$\tilde{\tilde{r}}_{913}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(4.9,0.01)$
$\tilde{\tilde{r}}_{(10)13}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(3.3,1.00)$
$\tilde{\tilde{r}}_{(11)13}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(3.1,0.36)$
$\tilde{\tilde{r}}_{(11)33}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(2.8,1.00)$
$\tilde{\tilde{r}}_{(12)23}\sim N(\mu,0.09)\ with\ \tilde{\mu}\sim N(3.0,0.64)$
$\tilde{\tilde{r}}_{(13)23}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(3.8,0.49)$
$\tilde{\tilde{r}}_{123}\sim N(\mu,0.064)\ with\ \tilde{\mu}\sim N(6.9,0.81)$
$\tilde{\tilde{r}}_{223}\sim N(\mu,0.64)\ with\ \tilde{\mu}\sim N(8.0,0.49)$
$\tilde{\tilde{r}}_{323}\sim N(\mu,4.00)\ with\ \tilde{\mu}\sim N(7.5,4.00)$
$\tilde{\tilde{r}}_{413}\sim N(\mu,0.48)\ with\ \tilde{\mu}\sim N(8.5,1.00)$
$\tilde{\tilde{r}}_{513}\sim N(\mu,1.00)\ with\ \tilde{\mu}\sim N(5.5,400)$
$\tilde{\tilde{r}}_{613}\sim N(\mu,0.49)\ with\ \tilde{\mu}\sim N(4.4,0.81)$
$\tilde{\tilde{r}}_{713}\sim N(\mu,0.36)\ with\ \tilde{\mu}\sim N(5.2,0.09)$
$\tilde{\tilde{r}}_{733}\sim N(\mu,0.09)\ with\ \tilde{\mu}\sim N(5.7,4.00)$
$\tilde{\tilde{r}}_{823}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(5.5,0.49)$
$\tilde{\tilde{r}}_{923}\sim N(\mu,0.64)\ with\ \tilde{\mu}\sim N(5.0,9.00)$
$\tilde{\tilde{r}}_{(10)23}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(2.9,0.49)$
$\tilde{\tilde{r}}_{(11)23}\sim N(\mu,0.81)\ with\ \tilde{\mu}\sim N(2.8,0.49)$
$\tilde{\tilde{r}}_{(12)13}\sim N(\mu,0.25)\ with\ \tilde{\mu}\sim N(2.7,0.49)$
$\tilde{\tilde{r}}_{(13)13}\sim N(\mu,0.04)\ with\ \tilde{\mu}\sim N(3.5,1.00)$