

## An approximation method for design applications related to sway in RC framed buildings

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### Abstract

*In this paper, an approximate method is proposed for determining sway of multistory RC buildings subjected to various types of lateral loads. The calculation of both fundamental period and stability index in RC building requires the sway term at each story level. Using approximate method design engineers can estimate sway terms at each story level. The developed analytical expressions are inserted into fundamental period and stability index equations to replace the sway terms, which yields modified equations for fundamental period and stability index without any sway terms. It is fairly easier to employ these equations developed by eliminating all sway terms. Results obtained from the equations are remarkably close to those generated by the related computer program. Consequently, design engineers can reliably use the simple equations to calculate stability index and fundamental period, which enables the determination of these parameters without referring to the complex sway terms. The capability and accuracy of the proposed equations are demonstrated by a numerical example in which computer program results are compared with the proposed methodology.*

**Keywords:** Analytical methods; Sway; Framed buildings; Fundamental period; Stability index.

### 1. Introduction

In earthquake prone areas, lateral loads, more or less, affect all structural members of the building and hence require lateral load and sway analyses except the floor slabs. This does not mean that the floor slabs are trivial with respect to the building's response towards lateral loads. As a whole, slabs are crucial for contributing to resistance of the building against lateral loads, as a result of forming the infinitely stiff diaphragms which distribute the lateral loads among the beam-column joints in proportion to their relative beam-column joint stiffness. As generally encountered in practice, slabs are assumed to be infinitely stiff in their own planes except those exhibiting unusual geometric properties.

During the design stage of reinforced concrete buildings, a designer must satisfy not only the strength requirements but also the serviceability requirements as well. To satisfy the serviceability requirements in tall reinforced concrete buildings, an accurate assessment of deflection under lateral and gravity loads is necessary. In recent years, high-rise and slender buildings have been constructed using high-strength steel and concrete.

Therefore, the serviceability limit state for lateral drift becomes a much more important design criteria and must be satisfied to prevent large second-order P-delta effects [1].

Drift is the main cause of structural damage in buildings subjected to earthquake ground motions. Drifts are also responsible for earthquake-induced damage on many types of nonstructural elements in buildings [2]. When the structural drift in RC building is not limited, stability problems occur due to excessive second order moments. Being aware of the importance of story drift, particularly during seismic events, building codes [3, 4] require the calculation of seismic drift and impose restrictions on maximum values.

Determination of sway and the related design requirements are essential in building design. The realization of the importance of sway in building design dates back to long years. Westergaard [5] showed that structural sway that occurs under pulse loading could be critical. Similar studies followed [6], but the researchers' interest mostly focused on moment resisting building frames modeled as shear beams. Smith *et al.* [7] presented an approximate method for estimating the sway of buildings. For the buildings which have uniform story heights, the results of Smith's method close results from computer analysis which use stiffness matrix. Hasan *et al.* [8] investigated the numerical modelling analysis and design of non-sway and sway method for multi-storey reinforced concrete frames. Dinh and Ichinose [9] used probabilistic techniques in prediction of the seismic story drifts in buildings. The subject of study followed by

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some other complementary researches such as Caterino et al. [10], which explored the possibility to use expeditious methods to evaluate lateral interstory drifts and stiffnesses during the preliminary design of a given RC frame structure. Xie and Wen [11], which developed a method based on the continuous Timoshenko beam model to provide an estimate of maximum interstory drift demands for earthquake ground motions. Zou and Chan [12] presented an effective numerical optimization technique for the seismic drift performance design of buildings under multiple earthquake loading conditions. Lu *et al.* [13] provided a simple alternative method for the prediction of story drift distribution and critical drift concentrations in a RC frame. Garcia and Miranda [14] presented the implementation of a probabilistic approach to estimate residual drift demands during the seismic performance of existing multi-story buildings. Lin and Miranda [15] introduced two approximate methods for the estimation of the maximum inelastic roof displacement of multi-story buildings.

Presently, the calculation of sway is generally carried out by computer program. As the design parameters change, the number of sway solutions of the 3-D building increase, hence the computer modeling effort becomes more tedious. The practicing engineer confronted with equivalent lateral static loads needs a simple analytical method which remains robust when encountering changes in geometry and types of lateral loads. A well developed analytical method should also be versatile and reflect all of the important parameters that presumably affect drift [16]. Ideally, engineers utilizing this particular analytical method should develop an insight into the behavior of buildings exposed to various lateral loads. This skill is of utmost importance as crucial or even catastrophic errors can arise if the design process does not encompass a structural feel.

This study proposes a quite accurate analytical method for reinforced concrete framed buildings in comparison to computer program results. Additionally, the method is versatile and robust. Consequently, the method can be practically applied for both preliminary

and final designs. Both the fundamental period and stability index equations include sway terms for each story. Design engineers can use the simple analytical expression to calculate the fundamental period and the stability index. Building codes provide empirical formulas for estimating the building's fundamental period. The formulas contain sway terms at each story level. Similarly, according to ACI 318 [3], to decide whether a building is subject to sway or not, Q index must be calculated. Calculation of Q index requires the determination drift. If the developed analytical expressions are substituted into sway terms in fundamental period and stability index equations, they can be eliminated from fundamental period and stability index equations. Use of these equations is quite simple as they are free of sway terms.

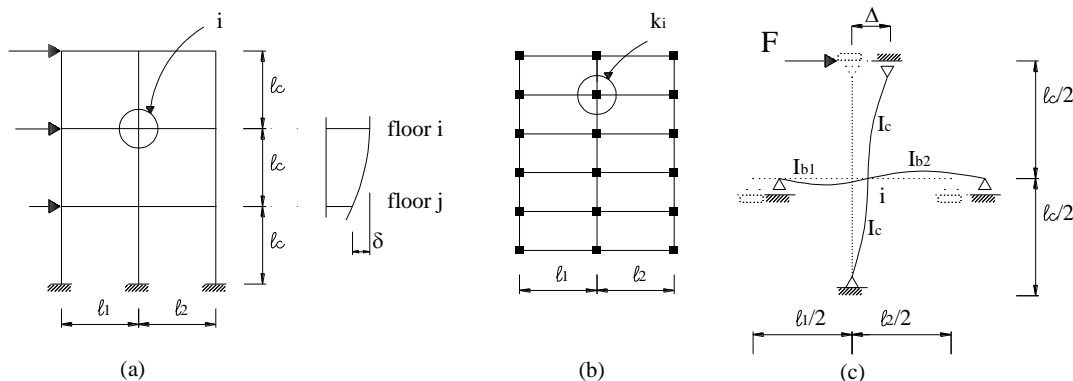
## 2. Proposed Method to Calculate Sway of Reinforced Concrete Framed Buildings

The basic assumptions made in development of the proposed method are listed as follows:

a) The material is linear and elastic; b) Floors are infinitely rigid in their own planes; c) The building is not subject to floor torsion; d) Rigidity is constant along the height of the building, and e) Torsion rigidity of beams is neglected.

The elevation and plan of such a typical moment resisting frame are displayed in Fig. 1 (a) and Fig. 1(b), respectively. The frame resists lateral loads in proportion to the beam-column joint stiffness that can be defined on the typical elastic line of joint (i), Fig. 1(c). The summation of all joint stiffness values in the story provides the total stiffness of a story of length  $\ell_c$ . The joint stiffness ( $k_i$ ) can be derived by using Fig. 1(c) by determining sway ( $\Delta$ ).

$$\Delta = \frac{F \ell_c^3}{12 E_c I_c} \left( 1 + \frac{12 \ell_2 I_c}{6 I_{b2} \ell_c} - \frac{12 I_c I_{b1} \ell_2^2}{6 I_{b2} \ell_c (I_{b1} \ell_2 + I_{b2} \ell_1)} \right) \quad (1)$$



**Fig. 1** Frame subject to lateral load and the drift between two consecutive floors

In the equation above, F is the concentrated load, and  $E_c$  is the modulus of elasticity of concrete.  $I_c$  and  $I_b$  are the moments of inertia, and  $\ell_c$  and  $\ell_i$  are the lengths of column

and beam, respectively. Denoting the expression within the parenthesis in Eqn.(1) as A, the following expression is obtained:

$$\Delta = \frac{F \ell_c^3}{12 E_c I_c} A \quad (2)$$

To determine the stiffness of the beam-column joint under consideration, Eqn. (2) is rearranged by equating  $\Delta=1$ , Eqn. (3). The term  $F$  in Eqn. (3) represents the stiffness of the beam-column joint.

$$F = \frac{12 E_c I_c}{\ell_c^3} \frac{1}{A} \quad (3)$$

The term  $A$  in Eqn. (3) can further be rearranged and simplified and then the stiffness of the beam-column joint shown in Fig. 1 is obtained by Eqn. (4).

$$F = \frac{12 E_c I_c}{\ell_c^3} \cdot \frac{1}{1 + \frac{2 I_c}{\ell_c \left( \frac{I_{b1}}{\ell_1} + \frac{I_{b2}}{\ell_2} \right)}} \quad (4)$$

The joint stiffness represents a height of  $\ell_c/2 + \ell_c/2 = \ell_c$  of the framed building as given in Eqn. (4). If  $F$  is divided by  $\ell_c$ , the framed building turns into a continuous shear beam having a shear rigidity of  $GA$  per unit height, as per Eqn. (5):

$$GA_i = \frac{F}{\ell_c} \quad (5)$$

Substituting Eqn. (4) into Eqn. (5) gives the sway rigidity of one joint, as per Eqn. (6), [Fig. 1(c)]:

$$GA_i = \frac{12 E_c I_c}{\ell_c^2} \cdot \frac{1}{1 + \frac{2 I_c}{\ell_c \left( \frac{I_{b1}}{\ell_1} + \frac{I_{b2}}{\ell_2} \right)}} \quad (6)$$

It can be observed from Eqn. (6) that the continuous shear rigidity,  $GA$ , of a framed building depends on the relative rigidities of columns and beams that come together at a joint [17]. The summation of all joint stiffnesses on the floor gives the sway rigidity of that story, as seen in Eqn. (7), [Fig. 1(b)], where  $n$  denotes the total number of joints in the floor plan:

$$GA = \sum_{i=1}^{i=n} GA_i \quad (7)$$

Total sway of any story (j) above the ground is obtained from the summation of all story drifts up to story (j). The moment resisting frame can be turned into a continuous shear beam by distributing the drift ( $\delta$ ) evenly along the story height and applying any type of lateral load continuously along the structural height which transforms the summation process into integration [17].

$$\delta = \sum_{i=1}^{i=n} \frac{1}{\frac{12 E_c I_c}{\ell_c^3} \cdot \frac{1}{1 + \frac{2 I_c}{\ell_c \left( \frac{I_{b1}}{\ell_1} + \frac{I_{b2}}{\ell_2} \right)}}} \quad (8)$$

In Eqn. (8),  $\delta$  is the relative sway between two consecutive floors,  $E_c$  is the modulus of elasticity, and  $n$  is the total number of joints in the floor plan.

$$\Delta_i = \delta V_{oi} \quad (9)$$

In Eqn.(9),  $\Delta_i$  is the relative sway, and  $V_{oi}$  is the total shear force at  $i^{\text{th}}$  floor. The total sway of any floor  $k$  as denoted by  $y$  is the summation of all relative sways from the ground floor to level of  $k^{\text{th}}$  floor, Eqn. (10).

$$y = \sum_{i=1}^{i=k} \Delta_i = \delta \sum_{i=1}^{i=k} V_{oi} \quad (10)$$

In the specific case where a continuous shear beam with continuous  $GA$  is subjected to any applied lateral force along the height of the building, Eqn. 10 can be turned into an integral operation and consequently becomes a differential equation [17].

$$y = \frac{\delta}{\ell_c} \int_0^x V_o(x) dx \quad (11)$$

By substituting Eqn. (8) in Eqn. (11), Eqn. (12) is obtained.

$$y = \frac{1}{\sum_{i=1}^{i=n} (GA)_i} \int_0^x V_o(x) dx \quad (12)$$

Similarly, by substituting Eqn. (7) in Eqn. (12), Eqn. (13) is obtained.

$$y = \frac{1}{GA} \int_0^x V_o(x) \quad (13)$$

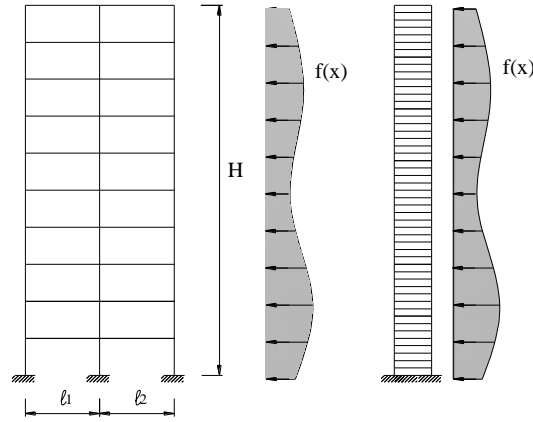
Double differentiation of Eqn. (13) yields the differential equation of a framed building modeled as a shear beam subject to the distributed lateral load defined by the function  $f(x)$ , Eqn. (14).

$$GA(y'') = -f(x) \quad (14)$$

The solution of Eqn. (14) is given in Eqn. (15) for sway at any  $x$  level where  $x$  is the height from the foundation level,  $M_o(0)$  is the moment at the base of the

building, and  $M_o(x)$  is the moment at any  $x$  level [18-21].

$$y = \frac{M_o(0) - M_o(x)}{GA} \quad (15)$$



**Fig. 2** Continuous shear beam model of a framed building subject to lateral load  $f(x)$

*Sway under Different Types of Loads: Triangularly Distributed, Uniformly Distributed and Concentrated at Any Level of  $x$*

The equivalent static triangular load of  $f(x)=px/H$  has a frequent use in earthquake analysis as exemplified in Fig. 3(a), and the emergent sway is a quantity the designer inevitably seeks to determine. The equation of sway, expressed in Eqn. (16), can be adopted:

$$y(x) = \frac{pH^2}{2GA} \left( k - \frac{k^3}{3} \right), \quad k = \frac{x}{H} \quad (16)$$

where  $p$  is the maximum intensity of distributed lateral load, and  $H$  is the total height of the building.

The sway at any level  $x$  above the base can be evaluated for uniformly distributed load of  $f(x)=p$  as shown in Fig. 3(b), and Eqn. (17) is obtained:

The lateral load may have any distribution of  $f(x)$ , Fig. 2.

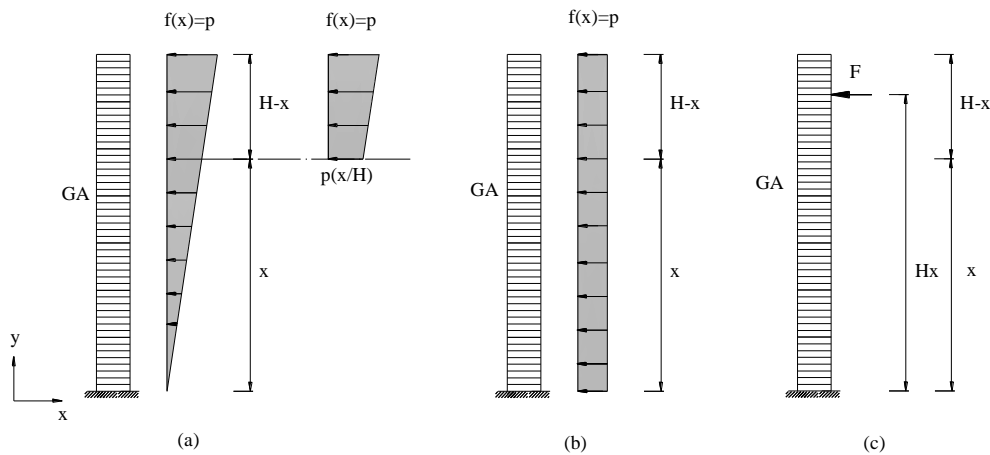
$$y(x) = \frac{pH^2}{2GA} (2k - k^2), \quad k = \frac{x}{H} \quad (17)$$

With the application of a concentrated load at any  $x$  level as shown in Fig. 3(c), the equation of the sway is obtained as expressed in Eqn.(18).

$$y(x) = \frac{1}{GA} F x \quad (18)$$

The sway at roof level can be evaluated for a concentrated load applied at roof level. This assessment is often necessary if a part of the total seismic force is applied at roof level to account for higher mode effects.  $\Delta$  is the maximum sway at roof level, Eqn. (19).

$$y = \frac{FH}{GA} = \Delta, \quad x = H \quad (19)$$



**Fig. 3** Framed building modeled as a continuous shear beam subject to (a) Triangularly distributed load (b) Uniformly distributed load (c) Concentrated load at any level of  $x$

### 3. Verification of the Proposed Method to Calculate Sway

The verification of the proposed method is conducted on the framed building whose plans are shown in Fig. 4 for different type of lateral loads. An alternative floor plan can also be possible as long as it is symmetrical in the direction of bending. Sway analyses are only conducted in the y direction. The shear rigidity  $GA$  can be calculated for the columns using the gross concrete dimensions and the typical value of  $E_c=28500$  MPa for concrete.

The number of floors varies among 5, 10, and 20 stories. Sway profiles obtained from analytical methods and computer programs are compared in Fig. 5. Obviously, the results of the two methods are in close agreement. Lack of symmetry in the floor plan leading to floor torsion and

rotation of the building about the vertical axis is a different issue that is beyond the scope of this study.

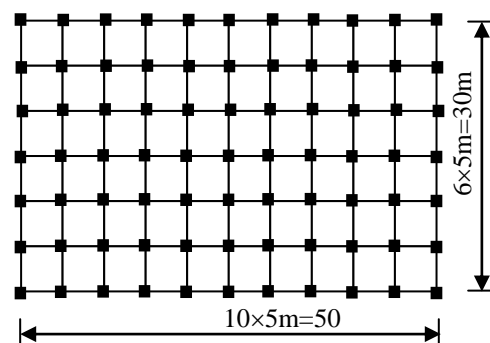


Fig. 4 Plan of RC framed building [21]

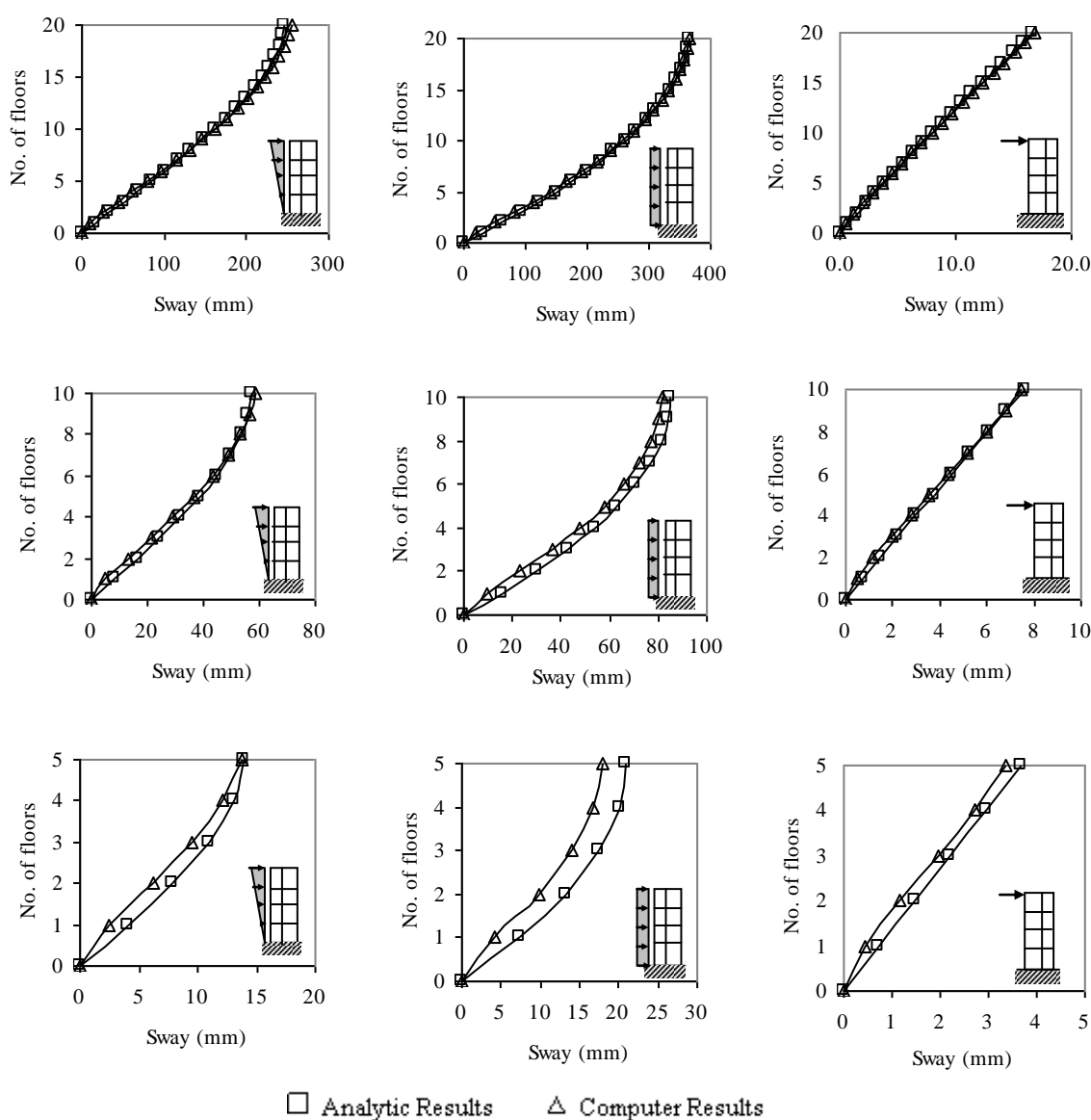


Fig. 5 Sway profile of framed buildings with different number of floors subject to varying lateral loads

For the symmetrical framed building, the sway profiles obtained by the proposed method and computer program almost perfectly agree with the exception of 5-story

building, which is quite understandable since the discrete sway stiffness at each floor is modeled as a continuous shear rigidity  $GA$  according to the proposed method.

#### 4. Design Applications for Related Sway Issues in Framed Buildings

A fault rupture releases a tremendous amount of seismic energy which radiates through the ground, originating at the seismic event's epicenter and reaching the building. The building absorbs a share of this energy as a function of its fundamental period. An appropriately designed building must dissipate the energy influx and in doing so must not collapse. The main design principle allows the building to dissipate seismic energy through sustaining controlled damages at locations that avoids an ensuing collapse.

##### 4.1. Checking total sway

The first line of defense against seismic shocks is non-structural elements which dissipate energy; but nevertheless remains insufficient. The main energy dissipation must take place in the building by flexural failure which necessitates the presence of plastic hinges at

the ends of beams and at the bases of shear walls.

The resultant sway does not get out of control to the extent of culminating in stability problems that originate from excessive values of second order moments. The building codes seek to avoid such an event by limiting the total emergent drift. The interstorey drift is limited by building codes as follows:

$$\Delta_i / h_i \leq 0.02 / R \quad [4, 22, 23] \quad (20)$$

$$\Delta_i / h_i \leq 0.025 / R \quad [24, 25] \quad (21)$$

$$\Delta_i / h_i \leq 0.004 \quad [26] \quad (22)$$

$$\Delta_i / h_i \leq 0.01 / R \quad [27] \quad (23)$$

where  $\Delta_i$  is the story drift;  $h_i$  is the story height, and  $R$  is the behavior factor reflecting the amount of seismic energy dissipation. The story drift ( $\Delta_i/h_i$ ) are calculated by using the sway equations developed for different types of loads in Section 2. The obtained results are compared with computer results in Fig. 6. Additionally, the drift limitations given in different codes can be showed in Fig. 6.

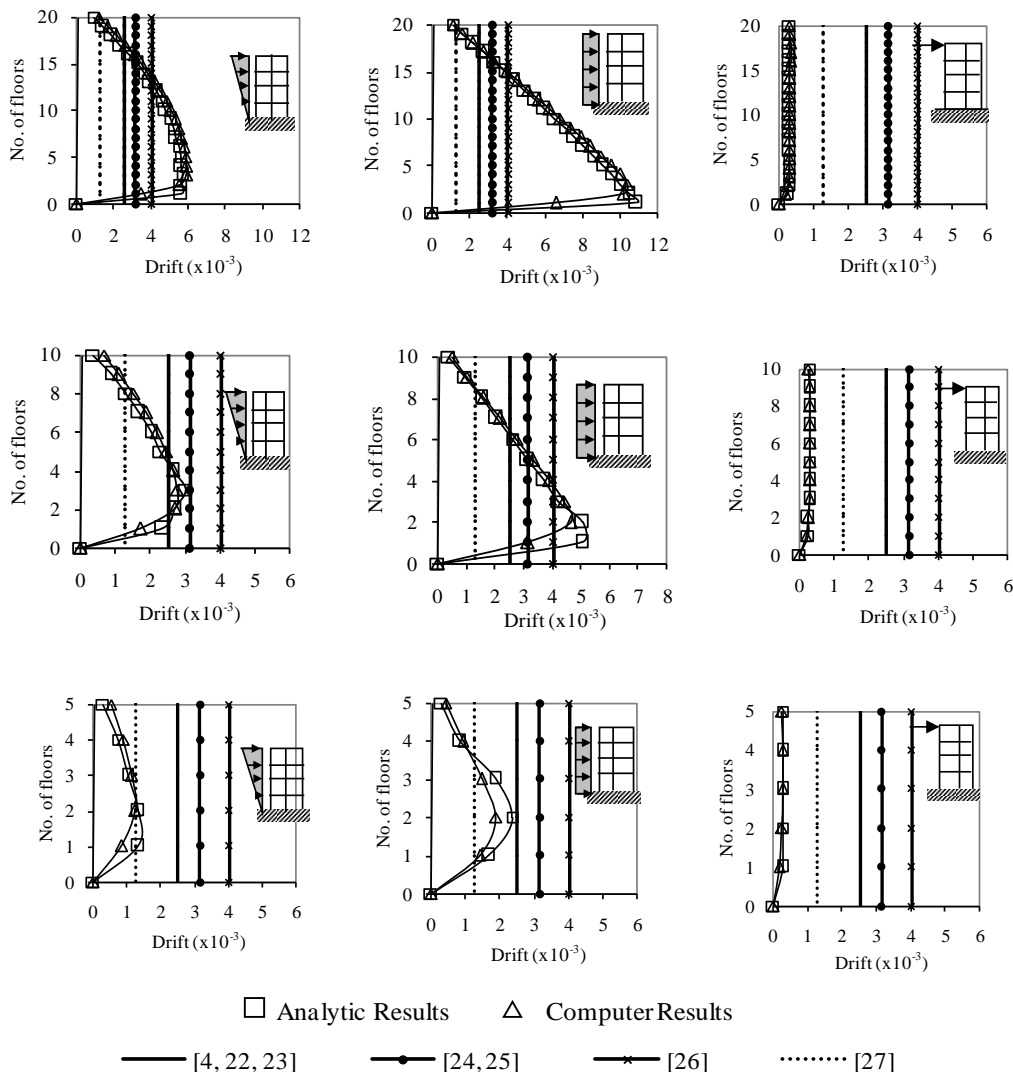


Fig. 6 Drift profile of framed buildings and the drift limitations given in codes

#### 4.2. Evaluation and assessment of the fundamental period

Building codes provide empirical formulas for estimating the fundamental period of the building. These formulas are developed on the basis of observed periods of real buildings during ground motions and the period is generally expressed as a function of the building's height and construction type (frame or shear wall), etc. [28].

Since the seismic design loads are a function of natural periods, which in turn are the functions of a building's structural mass and stiffness, any modification to the structural stiffness and mass during the design process will require a repetition of load calculations, implying the initiation of an iterative reanalysis and redesign process [12].

Engineers working with seismic design are compelled to calculate the fundamental period. For guidance purposes, building codes offer approximate formulae primarily obtained from field tests. However, the formulas are rather raw and calculating and modifying the elastic period to reflect the presence of partition walls and other non-structural elements is an alternative approach. Some codes suggested the use of fundamental period expression given below:

$$T = C_t H^{3/4} \quad [4, 29] \quad (24)$$

where  $H$  is height of the building;  $C_t$  is 0.049 as per UBC [4]; and given as 0.07 for RC framed buildings in

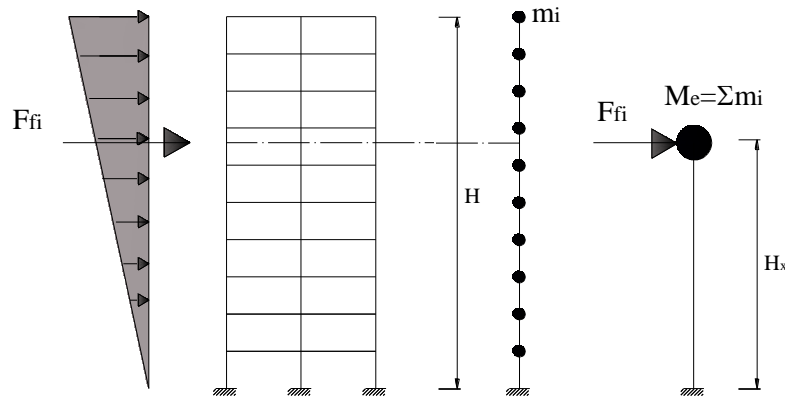
TEC [29].

Turkish Earthquake Code [22] and International Building Code [23] require the calculation of fundamental period by Rayleigh's equation:

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^N m_i \delta_i^2}{\sum_{i=1}^N F_i \delta_i}} \quad [22, 23] \quad (25)$$

where  $m_i$  is the mass at level  $i$ ,  $\delta_i$  is the lateral displacement at level  $i$ ,  $F_i$  is the equivalent lateral force at level  $i$ , and  $N$  is the number of stories. Apparently, Eqn. (25) contains sway terms at each story level. In this context, a simple, quick, and accurate method of calculating the fundamental period is of utmost importance.

The distribution of seismic force on the building can be considered as an inverse triangle as shown in Fig. 7. The total effective mass can be considered to exist at centroid of the distributed load,  $H_x$ . If the story masses are different and a concentrated load exists at roof level, the story forces are determined by using the product of  $m_i x_i$ . Coupled with the concentrated force at roof level, a new centroid,  $H_x$ , is calculated and the total mass is presumed to concentrate at the determined  $H_x$ .



**Fig. 7** Single-degree of freedom modeling for multi-degree of freedom buildings

At this stage, the multi-degrees of freedom framed and dual buildings are turned into single-degree of freedom systems (Fig. 7). The fundamental period is obtained by a simple calculation, examples of which will be presented in the following sections for illustrative buildings.

By dividing the numerator and the denominator of Eqn. (25) with  $\delta_i^2$ , Eqn. (26) is obtained as given below:

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^N m_i}{\sum_{i=1}^N \frac{F_i}{\delta_i}}} \quad (26)$$

The analytical expression developed for a concentrated load at any level of  $x$  in Section 2 can replace and eliminate the sway terms from Eqn. (26), yielding the fundamental periods for the multi-degree of freedom framed buildings, as per Eqn. (28):

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^N m_i}{\sum_{i=1}^N \frac{F_i}{\delta_i}}} = 2\pi \sqrt{\frac{W \cdot H_x}{GA \cdot g}}, \quad H_x = \frac{2 \cdot H}{3} \quad (27)$$

$$T = 2\pi \sqrt{\frac{2WH}{3GA}} \quad (28)$$

where  $g$  is the acceleration of gravity, and  $GA$ ,  $W$ , and  $H$  are shear rigidity, weight, and height of the building, respectively.

#### 4.3. Evaluation of second order column end moments

Building codes require all beam-column joints to be “strong column-weak beam” designs. Under pure gravity load, if hinging is present, it must occur in the beams rather than the columns. Obviously, the hinges occurring at ends of beams are specially designed for seismic energy dissipation. The columns are of vital importance; hence their design must be carefully evaluated.

In frame and member analysis, it is often appropriate to take account of the second order load effects on sway, moments, and stability caused by axial loads acting on the displacements of the frame and frame members [30].

The column failure ensues from two load actions: the axial load and the moment combination ( $N_i + \Delta N_i$ ,  $M_i + \Delta M_i$ ), where  $\Delta N_i$  is the increase in axial load due to the overturning effect, and  $\Delta M_i$  is the moment increase at ends of the column due to the second order effect. A direct second order analysis is preferred in accordance with the ACI 318 [3], but another approximation method called the Moment Magnification Method can be an acceptable alternative in case this is not practically applicable. The Moment Magnification Method is based on the magnification of first order moments by a factor of  $M_d = \beta M_i$ , where  $M_d$  denotes the design moment containing any second order effects, while  $M_i$  is the value of greater of the column end moments. Nevertheless, the calculation of  $\beta$  depends on whether the building is exposed to sway or not. All buildings exhibit sway characteristics to a certain degree and the limit is determined by the magnitude of change in their end moments. This limit is defined as 5% as per the ACI 318 [3]. Likewise, the calculation of this limit can be performed by evaluation of the  $Q$  factor in cases where a second order analysis is not applicable.

$$Q = \frac{\sum P_u \Delta_o}{V_u \ell_c} \leq 0.05 \quad (29)$$

where  $\Delta_o$  is the story drift;  $\ell_c$  is the story height, and  $P_u$  and  $V_u$  are total vertical load and the story shear, respectively. As Eqn. (29) shows, the  $Q$  factor contains the story drift term  $\Delta_o$  which can differ from story to story. The design engineer is then faced with the difficulty of calculating sway of a 3D building. Hence, a practical, versatile and relatively accurate hand-calculation method is of utmost importance.

The triangularly distributed lateral load is an appropriate example. Any other lateral load or load combinations can be used as well. In light of the sway

equations given in Section 2, by using the equation of  $\Delta_o$  for the triangularly distributed load in the  $Q$  expression and rearranging terms, the following equation is obtained:

$$\frac{\Delta_o}{\ell_c} = \frac{Q V_u}{\sum P_u} \quad (30)$$

By substituting  $V_u$  obtained for the triangularly distributed seismic lateral load in Eqn. (30), Eqn. (32) is obtained as shown below:

$$V_u = \frac{p(H^2 - x^2)}{2H} \quad (31)$$

$$\frac{\Delta_o}{\ell_c} = \frac{Q p(H^2 - x^2)}{2H \sum P_u} \quad (32)$$

Greater magnitudes of sway means greater magnification of column end moments especially in framed buildings compared to dual buildings. By adopting the first derivative of the sway equation (Eqn. 16) for the triangularly distributed load, Eqn. (33) is obtained as follows:

$$\frac{\Delta_o}{\ell_c} = y' = \frac{pH^2}{2GA} \left( \frac{1}{H} - \frac{x^2}{H^3} \right) \quad (33)$$

Equating Eqn. (32) and Eqn. (33) and arranging terms, Eqn. (34) is obtained:

$$Q = \frac{\sum P_u}{GA} \quad (34)$$

Apparently, the previous equation frees expression of the  $Q$  index from  $\Delta_o$ . Another observation is that the type of lateral load bears no significance. Thus, the design engineer does not need to model and perform a lateral load analysis as the type of lateral load used is trivial.

## 5. Verification of the Design Applications for Related Sway Issues

The framed buildings whose plans are displayed in Fig. 4 are examples of verifying the developed design equations. The obtained fundamental period and stability index results with proposed method are compared with SAP 2000 computer program results [31].

### 5.1. Verifying the developed equation to calculate the fundamental period

The developed equation to calculate the fundamental period is verified on frame type buildings. The results are presented in Table 1. As seen in the table below, fundamental periods for 5, 10, 15, and 20 story buildings have a maximum error of 10%.



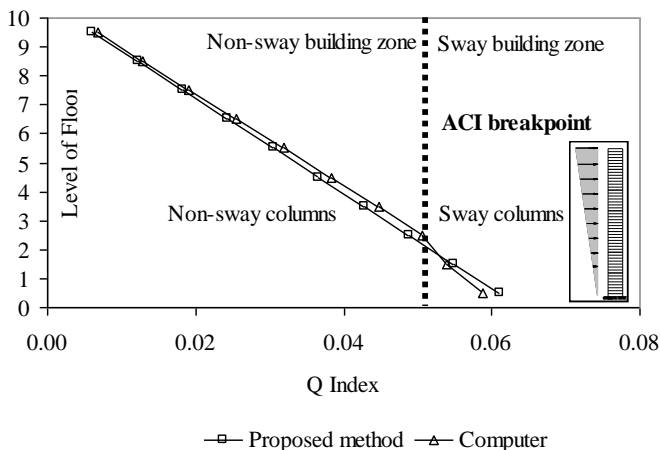
**Table 1** Fundamental period results obtained by computer program [31] and versus the proposed method

No. of story	H (m)	$H_x$ (m)	T (sn)		
			SAP 2000	Proposed method	Error (%)
20	60	40	2.43	2.48	2
15	45	30	1.79	1.86	4
10	30	20	1.17	1.24	5
5	15	10	0.57	0.62	8

## 5.2. Verifying the developed equation to calculate the stability index

The Q index calculated for the illustrative 10 story framed building appears in graphic form in Fig. 8. As Fig. 8 shows, the proposed method and computer program results are in almost perfect agreement.

Consequently, design engineers can use the simple analytical expression to calculate the Q index without relying on computer program to calculate sway and drift. Another observation is that the Q index decreases for upper stories for a building exhibiting uniform sway stiffness along its height. As the sway stiffness of the building decreases, the Q index will increase and intersect the ACI breakpoint.



**Fig. 8** Q index results obtained by computer program [31] versus the proposed method

## 6. conclusions

In this study, the approximate method has been presented to estimate storey sway or interstory drift in multi-storey framed building under lateral load. For a designer, approximate analyses may be useful to obtain estimates of building behavior during preliminary design or to verify the results of a more sophisticated computer analysis.

The proposed sway equations are relatively simple. Different types of lateral equivalent static forces can be handled as triangularly distributed load, uniformly distributed load, and a concentrated load at any level along the height of the building. The engineer can use these methods in the preliminary design stage as well as the final design stage to calculate the required parameters.

For the symmetrical framed building, the sway profiles obtained by the proposed method and computer program agree almost perfectly, with the exception 5-story building. This outcome is in line with the expectations, because in the proposed methodology the discrete sway stiffness at each floor is modeled as a continuous shear rigidity GA.

During the design stage of reinforced concrete buildings, a designer must satisfy not only the requirements pertaining to strength but also serviceability. Consequently, building codes introduce limits on acceptable values of drift, which can be calculated and controlled by the presented equations. Interstory drift values deriving from the proposed procedure are in good agreement with those obtained by computer program.

The fundamental period and stability index equations include sway terms for each story. Instead of sway terms, the developed analytical expressions are substituted into the fundamental period and stability index equations so that fundamental period and stability index equations without any sway terms could be obtained. Results of equations, free of sway terms of fundamental period and stability index, are remarkably close to the results obtained from the computer program. Consequently, design engineers can reliably use the simple equations to calculate stability index and fundamental period, which enables the determination of these parameters without referring to the complex sway terms.

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