



Modified Weighted Least Squares Method to Improve Active Distribution System State Estimation

M. Ajoudani*, A. Sheikholeslami^{*(C.A.)}, and A. Zakariazadeh**

Abstract: The development of communications and telecommunications infrastructure, followed by the extension of a new generation of smart distribution grids, has brought real-time control of distribution systems to electrical industry professionals' attention. Also, the increasing use of distributed generation (DG) resources and the need for participation in the system voltage control, which is possible only with central control of the distribution system, has increased the importance of the real-time operation of distribution systems. In real-time operation of a power system, what is important is that since the grid information is limited, the overall grid status such as the voltage phasor in the buses, current in branches, the values of loads, etc. are specified to the grid operators. This can occur with an active distribution system state estimation (ADSSE) method. The conventional method in the state estimation of an active distribution system is the weighted least squares (WLS) method. This paper presents a new method to modify the error modeling in the WLS method and improve the accuracy SVs estimations by including load variations (LVs) during measurement intervals, transmission time of data to the information collection center, and calculation time of the state variables (SVs), as well as by adjusting the variance in the smart meters (SM). The proposed method is tested on an IEEE 34-bus standard distribution system, and the results are compared with the conventional method. The simulation results reveal that the proposed approach is robust and reduces the estimation error, thereby improving ADSSE accuracy compared with the conventional methods.

Keywords: Active Distribution System, State Estimation, Weighted Least Squares, Smart Meter, State Variables.

1 Introduction

TODAY, state estimation (SE) is considered an essential part of energy management systems (EMS) in transmission control centers [1]. In fact, EMSs play an important role in monitoring and controlling transmission grids, and SE forms its framework [2]. In contrast, accurate monitoring of the grid in distribution systems has not been necessary due to their passive

operation, and they could easily be managed without the need for advanced control tools. As a result, advanced distribution grids have not been used in the last few years [3]. However, there have already been significant changes at the distribution system level that have affected their performance [4]. Most important of all, there has been an increase in DG units at the distribution grid level, which has invalidated the passive grid assumption [5]. In addition, bidirectional power flow has been made possible on the grid, and this requires more sophisticated and more active grid management [6]. Moreover, new actors, like electric vehicles, are entering distribution grids and they thus have significant effects on their performance. These vehicles, if not properly managed, can pose significant grid problems due to their high demand for charging [7]. Proper charging management of them can improve the performance of grids by contributing to voltage control and reducing grid losses.

Iranian Journal of Electrical and Electronic Engineering, 2020.
Paper first received 02 October 2019, revised 21 November 2019, and accepted 29 November 2019.

* The authors are with the Department of Electrical and Computer Engineering, Babol Noshirvani University of Technology, Babol, Iran. E-mails: ajoudani@bandargaziau.ac.ir and asheikh@nit.ac.ir.

** The author is with the Department of Electrical Engineering, University of Science and Technology of Mazandaran, Behshahr, Iran. E-mail: zakaria@mazust.ac.ir.
Corresponding Author: A. Sheikholeslami.

In addition, electric vehicles, which can be regarded as energy storage devices and demand-side management, are expected to support the performance of future distribution grids [8]. All these options are generally called distributed energy resources. They are an essential solution for achieving higher investment returns on renewable energy sources and improving the efficiency and reliability of distribution grids. To coordinate the operation of this tool at the distribution system level, intelligent distribution management systems equipped with advanced control functions should be developed [9]. In this context, the SE will play a decisive role. In fact, for the proper functioning of control tools, it is essential to have in-depth knowledge of the operating conditions [10]. Superficial knowledge can lead to wrong decision-making by distribution management systems due to technical problems and unexpected costs. Therefore, like transmission systems, SE is essential for processing the measured data, increasing their accuracy, and providing a reliable picture of operating conditions for the higher-level functions within distribution management systems [10]. The first step in SE is to collect input data from the distribution grid level. These data are the measurements and grid load information, which are collected in different ways. Measurements can be analog or digital. Digital measurements include switching mode, power switches mode, and tap changer in transformers. Analog measurements include active and reactive power, voltage phasor, current phasor, etc. in the distribution grid [11]. The measurements are divided into three categories: 1) real; 2) pseudo; and 3) virtual measurements. The real measurements are data measured in real-time by sensors. The pseudo measurements are predicted or assumed from the load curves with high error percentage probability factors. The virtual type is not measured but generally known as zero injection node data [12]. SE is typically formulated as a WLS problem [13]. The main methods of SE are divided into two classes: the first is based on statistical criteria such as maximum similarity criterion, minimum variance criterion, and weighted least squares criterion [14]. The second is based on the formulation of the estimated load state [15]. The WLS method has been used in most SE studies [16], but the short-term variations of the load are not considered in the interval of data transmission, which is usually considered by default 15 minutes [17].

References [18, 19] show that the use of SM improves the accuracy of SE but grid changes are not considered in the duration between two data transmissions and whenever the SE is required, the data collected at the beginning of the period are used, which impairs the precision of the method. It is shown in [20, 21] that smart meters, similar to transmission lines, improve the accuracy of the ADSSE, assuming that SM measurements are performed synchronously. However, if the behavior of all signals is assumed synchronous, it

results in a lower quality of the ADSSE since the time difference between the measurements is significant. References [22, 23] consider this delay in order to increase the accuracy of SM measurements. In [22], the authors consider two levels of error (two or 10%) in measuring smart meters, whereas in [23], the authors consider all measurements made by the meters with a 10% error. Moreover, in [24, 25], the patterns of short-term variations of load and the time elapsed between sampling the measured parameters and executing the SE are not taken into account. Alternative methods in the process of implementing SE in a distribution system involve modeling grid loads using neural grids. The problem with this approach is that the training of the neural grid needs a reliable analysis of power flow across the distribution grid for a full year [26]. Studies in this field have shown that the researchers have covered the lack of grid information between two sequential measurements, taking into account a constant percentage of error, so the SE has not been highly accurate.

For the purposes of this paper, all sources of information summarized below:

- 1) Remote terminal unit (RTU) measurements captured at HV-MV substations, collected by the SCADA system of the DMS at rates ranging from few seconds to about a minute (in general, much lower refreshing rates than those employed at transmission-level SCADAs [27, 28]).
- 2) Distributed generation is already a reality and will increasingly spread in many radial feeders worldwide. Depending on the specific regulation and rated power, the production of distributed generators (DGs) is required to be monitored at different rates, ranging from day-ahead hourly forecasting to real telemetry periodically submitted to the DMS [29].
- 3) Distribution utilities have customarily kept a more or less elaborated database of historic load patterns/profiles. This information originates in several sources; including load forecasting, load allocation techniques in combination with feeder head measurements, characteristic power factor values of aggregated loads and systematic metering campaigns performed at specific points. The feeder-level state estimators can benefit from these not very precise values of P and Q, which can be used as pseudo-measurements to extend the observable area [30].
- 4) The latest and eventually most important addition to the list of information sources at the feeder level comes from the automatic meter reading/advanced metering infrastructure (AMR/AMI) infrastructure (typically smart meter concentrators), provided the right communication bridge is built between AMI and DMS subsystems. Nowadays this information is collected once a day in many systems but,

depending on bandwidth availability, snapshot latencies of up to 15 min have been reported [20, 30].

This paper introduces a new method and considers grid LVs in the time interval between two consecutive measurements and their transmission time to the supervisory control and data acquisition (SCADA). It also modifies the equation of error in the traditional WLS method. Finally, the results of the proposed approach are compared with those of the traditional approach. The rest of the paper has been organized as follows. Section 2 describes the weighted least squares approach. Section 3 deals with the modeling of the measurements and assumptions. Section 4 presents the approach proposed for correcting the error equation in the traditional approach. In Section 5, a simulation is performed on a 34-bus IEEE active distribution system, and the results are compared with the results of the traditional approach.

2 Weighted Least Squares Method

A measured parameter from the active distribution grid can be considered as a random parameter with a Gaussian probability density function (GPDF). The mean value (MV) of the measured parameter is called the real value of this parameter, and the standard deviation (SD) of the random parameter indicates the measurement error. The GPDF for a random parameter is explained as [10, 31]:

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2} \quad (1)$$

where z represents the vector containing the measurements, μ is the MV, and σ is the SD [26]. In a grid, the joint GPDF, which indicates the probability function of K independent measurements, can be represented as the product of exclusive GPDFs (where it is assumed that any measure is independent of the other measurements) as [10, 31]:

$$f_k(z) = f(Z_1)f(Z_2)..f(Z_i)..f(Z_k) \quad (2)$$

where Z_i represents the i th measurement and $Z^T = [Z_1, Z_2, \dots, Z_k]$. The function $f_k(z)$ is called the likelihood function (LF) for Z . For a given collection of measured data and their related SD, when the MVs of the unknown closest to their actual values are selected, the LF will approach its maximum value. Therefore, the aim here is to maximize the LF by allocating MVs that are nearest to the measured values. The MV for a measured electric parameter is a function of the SVs that can be presented as

$$\mu_i = h_i(x_1, x_2, \dots, x_n) = h_i(X) \quad (3)$$

Given this assumption that the value of the mean noise for each measurement is zero, the described procedure

is formulated as

$$\begin{aligned} L &= \log f_k(z) \\ &= \sum_{i=1}^k \log f_k(z) \\ &= -\frac{1}{2} \sum_{i=1}^k \left(\frac{z_i - \mu_i}{\sigma_i}\right)^2 - \frac{k}{2} \log 2\pi - \sum_{i=1}^k \log \sigma_i \end{aligned} \quad (4)$$

in which without losing the generality, the logarithmic function is applied to simplify the optimization process. Since the last two components on the right side of (4) are constant, the process of maximizing $f_m(z)$ is equal to

$$\begin{aligned} &\text{maximize } \log f_k(z) \quad \text{or} \\ &\text{minimize } \sum_{i=1}^k \left(\frac{z_i - \mu_i}{\sigma_i}\right)^2 \end{aligned} \quad (5)$$

The above minimization scheme can be re-written in components of the residual r_i from measure i that is described below [31]

$$r_i = z_i - \mu_i \quad (6)$$

where the MV μ_i can be defined as the function $h_i(x)$, a nonlinear function that relates the i th measurement to the system state variable vector x . The square of any remaining r_i^2 is weighted using the coefficient $W_{ii} = \sigma_i^{-2}$, which inversely correlated with the error variance considered for that measurement. Therefore, the problem of minimizing (5) would be equivalent to minimizing the sum of the weighted coefficients of squares remaining or solving the optimization problem for the SVs vector x , which is stated as follows

$$\min \sum_{i=1}^k W_{ii} r_i^2 \quad (7)$$

$$z_i = h_i(x) + r_i \quad i = 1, 2, \dots, k \quad (8)$$

The above optimization problem is solved by an estimator called the WLS estimator for the state vector x .

3 Modeling the Measurements and Assumptions of the Problem

Consider the set of measurements obtained by the vector z [10]:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, \dots, x_m) \\ h_2(x_1, x_2, \dots, x_m) \\ \vdots \\ h_m(x_1, x_2, \dots, x_m) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} = h(x) + e \quad (9)$$

where the function $h_i(x)$ is the nonlinear function which relates the measurement of i to the system SV vector x , and e_i is the measurement error vector.

3.1 The Measurement Function $h(x^k)$

The measurements are of different types. The most common measurements are power flow between lines, the active and reactive power injected into the buses, the voltage magnitude in the buses, and the current magnitude in the lines. These measurements can be described from the view of SVs applying either the Cartesian coordinate system or the Polar coordinate system. When applying the polar coordinate system for a grid consisting of K buses, the SVs vector will contain $(2K-1)$ members, K voltage amplitude of the buses, and $(K-1)$ phase angles of the buses voltage, in which a bus is considered as the reference bus, and the phase angle for that is assigned an arbitrary value, such as zero. Where bus 1 is selected as the reference bus, so the SVs vector x will be as follows [10, 31]

$$x^T = [V_1, V_2, \dots, V_K, \theta_2, \theta_3, \dots, \theta_K]^T \tag{10}$$

The relationships for the types of measurement mentioned above are described below:

- Reactive and active power injection at bus i :

$$Q_i = V_i \left| \sum_{j=1}^K V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \right| \tag{11}$$

$$P_i = V_i \left| \sum_{j=1}^K V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \right| \tag{12}$$

- Reactive and active power flow from bus i to bus j :

$$Q_{ij} = -V_i^2 b_{ij} - V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \tag{13}$$

$$P_{ij} = V_i^2 g_{ij} - V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \tag{14}$$

- Current flow magnitude from bus i to bus j :

$$I_{ij} = \frac{\sqrt{P_{ij}^2 + Q_{ij}^2}}{V_i} \tag{15}$$

3.2 Assumptions of the Problem

Given the statistical characteristics of measurement errors, the following assumptions usually are taken into account in solving the problem [10, 31, 32]:

- $E(e_i) = 0 \quad i = 1, 2, \dots, m$
- Measurement errors are independent: $E(e_i, e_j) = 0$.

Hence, $cov(e) = E[e e^T] = R = diag\{\delta_1^2, \delta_2^2, \dots, \delta_m^2\}$.

The SD σ_i for any measurement i is computed to show the expected precision of each SM used. The estimator of the WLS method must minimize the following objective function.

$$J(X) = \sum_{i=1}^K \frac{(z_i - h_i(X))^2}{R_{ii}} = [Z - h(X)]^T R^{-1} [Z - h(X)] \tag{16}$$

At least, the first-order optimality conditions must be satisfied. These can be summarized as follows:

$$g(x) = \frac{\partial J(X)}{\partial X} = -H^T(X) R^{-1} [Z - h(X)] = 0 \tag{17}$$

In (17), $H(X) = \left[\frac{\partial h(X)}{\partial X} \right]$ is the Jacobin matrix of measurements. The structure of the Jacobian matrix is as follows:

$$H = \frac{\partial h(x)}{\partial x} = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial v} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial v} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial v} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial v} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial v} \\ 0 & \frac{\partial V_{mag}}{\partial v} \end{bmatrix} \tag{18}$$

The nonlinear function $g(x)$ extended around the state variable vector x^n into its Taylor series.

$$g(x) = g(x^n) + G(x^n)(x - x^n) + \dots = 0 \tag{19}$$

By ignoring the higher-order in the expressed Taylor expansion, an iterative solution is obtained known as the Gauss-Newton method, as shown below:

$$x^{n+1} = x^n - [G(x^n)]^{-1} . g(x^n) \tag{20}$$

where n is the iteration index and x^n is the solution vector at iteration n .

$$G(x^n) = \frac{\partial g(x^n)}{\partial X} = H^T(x^n) . R^{-1} . H(x^n) \tag{21}$$

$$g(x^n) = \frac{\partial g(X)}{\partial X} = -H^T(x^n) . R^{-1} . [Z - h(x^n)] \tag{22}$$

where $G(x)$ is called the gain matrix. The properties of the gain matrix are positive definite, symmetric, and sparse provided that the grid is entirely observable [10, 32]. The gain matrix is obtained using the Jacobian matrix of measurements, H and the covariance matrix of measurement error, R .

Given the assumption of measurements independent, the covariance matrix will be a diagonal matrix whose entries on the main diagonal will be filled with the variance of the measurements. The gain matrix is formed as follows:

$$G(x^n) = H^T R^{-1} H \tag{23}$$

The gain matrix $G(x)$ is generally not reversed (a matrix will generally be inverted when it is full rank, whereas here $G(x)$ is quite sparse). Matrix $G(x)$, to solve this problem, is decomposed into its triangular factors, and the below sparse linear sets of equations are solved applying the forward/back replacements (read [10] for more information) at any iterations n :

$$[G(X^n)] = \Delta x^{n+1} = H^T(x^n) \cdot R^{-1} \cdot [Z - h(x^n)] \tag{24}$$

where $\Delta x^{n+1} = x^{n+1} - x^n$.

3.3 Cholesky Analysis of G matrix

The matrix $G(x)$ can be written as the multiplying of a lower triangular sparse matrix in the transpose of that matrix. This process is known as the Cholesky decomposition of the matrix G . The decomposed form of G is as follows [10, 34, 35]:

$$G = LL^T \tag{25}$$

It must be mentioned that the Cholesky analysis may not exist for grids that are not observable. Triangular factors of Gain matrix $G(x)$ are not unique, and their sparsity highly depends on the way the analysis is carried out. Equations (1)-(25) are related to the fundamental relationships in the WLS method and the state estimation, which have been taken from [10], [31-35]. In most references focusing on a state estimation problem, these relationships have been expressed in brief or full details as required. In this paper, we try to develop a WLS method formulation aiming at improving the state estimation problem. Therefore, to clearly explain the changes made in the conventional method formulation as well as comparing and analyzing the results before and after the modifications, the fundamental relationships of the WLS method and state estimation were expressed.

4 Proposed Modified WLS State Estimation Algorithm

SE by the WLS method involves solving the iteration of the given normal equations by (23). For the state vector x^0 , an initial guess must be made. In the active distribution system, depending on the SVs considered to solve the state estimation problem, the initial guess will include either voltage or current phasors. In related references, the determination of the initial guess of SVs in the SE problem is carried out in two ways:

- Using network power flow to determine the initial guess of SVs;
- Using flat values as the initial guess for the SVs.

In this paper, tested both methods, ultimately determining the initial guess in the convergence of the SE problem was ineffective. Therefore, in this paper,

the initial guess is related to the voltage phasor, and all bus voltages are considered 1.0 per unit and in phase with each other. In the following, the method proposed in this paper is presented to modify the error equation and improve the results of SE in the WLS method.

4.1 Modified Error Equation in the WLS Method

According to the existing procedures, the sampling time of data varies depending on the smart meters. In this paper, it is assumed that the sampling time is 15 minutes. The t_{LU} variable represents the time elapsed since the last update. Given these, the t_{LU} variable can be anywhere between 0 to 900 seconds. As a result, the MV and SD employed in the ADSSE process for the proposed method can be calculated as follows:

$$\mu_{LV_i}(t_{LU}) = \frac{t_{LU}}{t_{LU_{max}}} \times (\mu_{LV_i}(t_{LU_{max}}) - \mu_{LV_i}(0)) + \mu_{LV_i}(0) \tag{26}$$

$$\sigma_{LV_i}^2(t_{LU}) = \frac{t_{LU}}{t_{LU_{max}}} \times (\sigma_{LV_i}^2(t_{LU_{max}}) - \sigma_{LV_i}^2(0)) + \sigma_{LV_i}^2(0) \tag{27}$$

$$\mu_{total_i} = \mu_{m_i} + \mu_{LV_i} = \mu_{LV_i} \tag{28}$$

where $\sigma_{LV_i}(0)$, $\sigma_{LV_i}(t_{LU_{max}})$, $\mu_{LV_i}(0)$, and $(\mu_{LV_i}(t_{LU_{max}}))$ are the calculated SD and mean of LV for the starting and ending points of the measurement interval, and $t_{LU_{max}}$ denotes the time interval between the two sequential samplings. Using (26) and (27), SD and MV can be calculated for each point within the sampling interval. Therefore, the ADSSE process can be implemented at each point within the sampling interval. In order to implement the proposed method, SD and mean of LV for any sampling time should be calculated based on the grid historical data. While ADSSE is running, the variance of any measurement is updated by assessing $(\sigma_{total_i}^2 = \sigma_{m_i}^2 + \sigma_{LV_i}^2)$ and results in an update of the covariance matrix R . As stated, in (4), it is assumed that the MV of error in the noise measurement is zero.

However, in this paper, to achieve the maximum accuracy, the MV of the total error is computed based on (26) and (28). As a result, the WLS method formulation in the proposed approach is modified as follows:

$$Z = h(x) + \epsilon + \Delta lv \tag{29}$$

where Δlv is the short-term LV vector that is included in the main equation of the regular WLS method to enhance accuracy and model the effect of LVs in short-term periods. Thus, the objective function in (4) should be modified to:

$$\max f(z) = \max \log(f(z))$$

$$= -\frac{1}{2} \sum_{i=1}^k \left(\frac{z_i - \mu_i - \mu_{LV_i}}{\sigma_i} \right)^2 - \frac{k}{2} \log 2\pi - \sum_{i=1}^k \log \sigma_i \quad (30)$$

where μ_{LV_i} is the mean of load variation computed by (26). So, the measured value in each measurement must be subtracted from the mean of the error, and (17) is corrected as follows:

$$g(x) = \frac{\partial J(x)}{\partial x}$$

$$= -H^T(x)R^{-1}[Z - \mu_i - \mu_{LV_i} - h(x)] = 0 \quad (31)$$

where $\mu = \mu_i + \mu_{LV_i}$ is a vector referring to the mean of all the LV errors. A flowchart of SE by the WLS method is shown for both the proposed and the conventional method in Fig. 1.

5 Simulation Results

In this section, the effectiveness of the proposed approach is validated by using an IEEE 34-bus standard active distribution grid. In this paper, there is a mix of

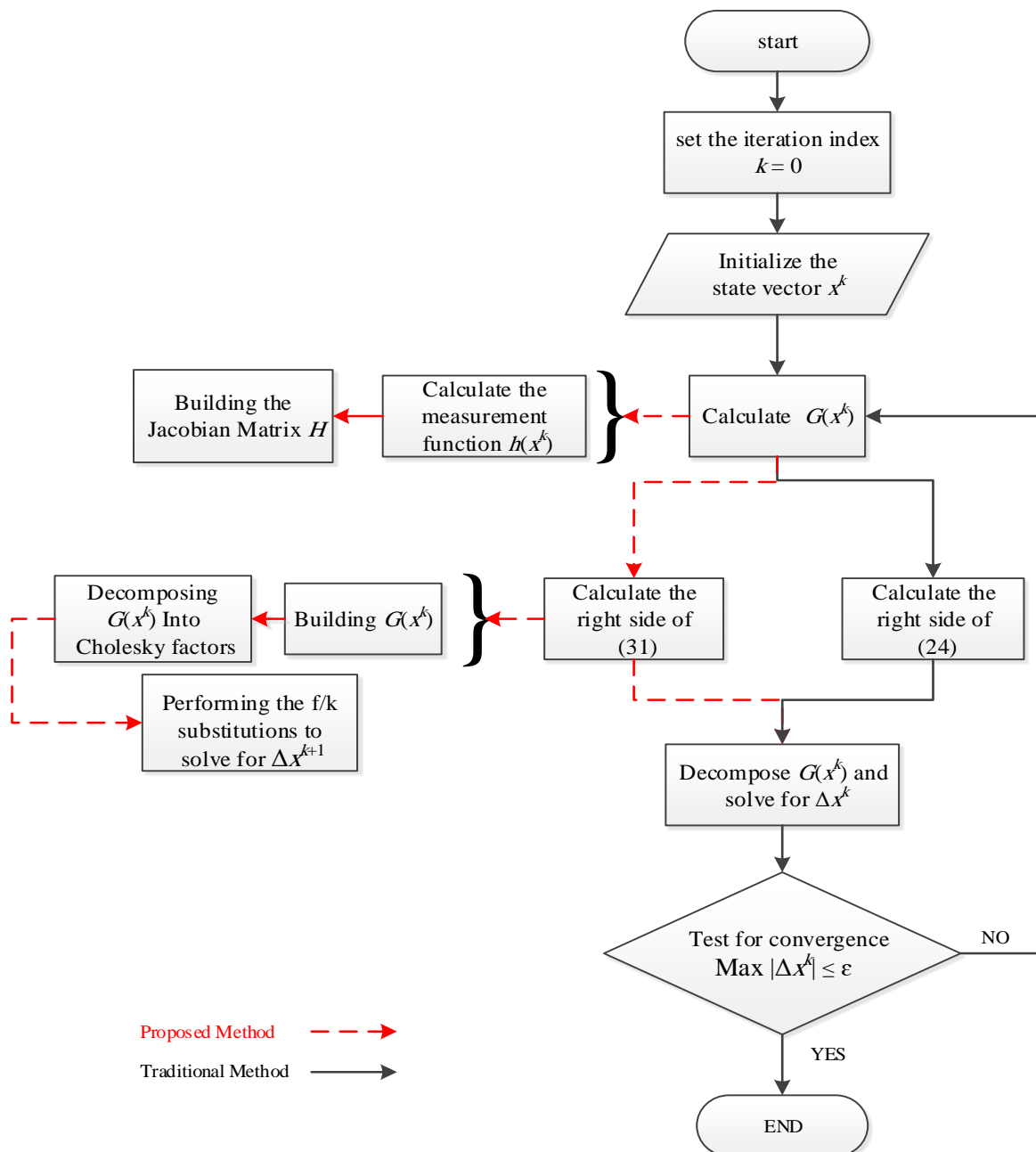


Fig. 1 Flowchart of the weighted least squares method with the traditional and proposed approach.

load types such as urban load, rural load, and industrial load. Also, there are two units of DG, which are diesel generator type. In this paper, we tested a standard IEEE 34-bus distribution system in that the number and type of measurement devices and their installation spots as well as the number of DG units, sizes and spots were predetermined and we made no changes in them. The forward/back power flow, which is suitable for distribution systems, is applied to the case study in the first step. Any time of day, the SE operation is performed on the active distribution system; the values obtained from the SE are compared with the values obtained from the power flow as real values of the grid SVs. Given the fact that the load of the grid is constantly changing, it is necessary to run the power flow with new values of the loads whenever the operator performs SE. There are few tests to verify that a collection of data has been distributed normally [36-38]. Regarding the statistical tests (the Anderson-Darling and Shapiro-Wilk tests) on grid load changes, it was found that LVs had a normal distribution. Therefore, according to different load patterns, LVs in the program for power flow were considered to have normal distribution as shown in Fig. 2. According to the load pattern, three-time ranges are considered. The time interval is based on low-load (between 00:00 AM and 07:00 AM), high-load (between 07:00 am and 07:00 pm), and mid-load (between 07:00 pm and 12:00 pm) calculations. As stated, the obtained values of the forward/back power flow calculation are used as the reference or actual values of the measured parameters in the grid. The number of SVs in the grid is 67, which includes 34 voltage magnitudes and 33 phase angles.

Measurements can be of a variety of types. In this paper used measurements are the line power flows, bus power injections and line current magnitudes. It is assumed that measuring devices transmit data to the control center every 15 minutes; meanwhile, there is no information about load changes in the grid. Two scenarios are presented below. Scenario I considers the traditional method while Scenario II presents the method proposed in this paper.

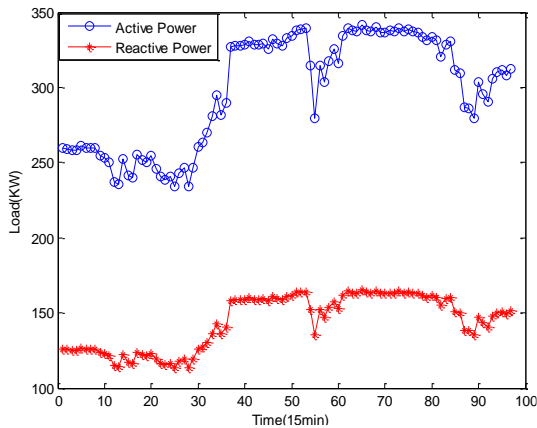


Fig. 2 The load variation pattern curve for sample bus 7 from the studied grid for 24h.

Scenario I: In this case, given that there is no information within the 15-minute interval of the grid, the values measured at the beginning of each interval are used to estimate SVs. The program is executed every one minute for 24 hours for 24-hour. Afterward, for each of the three periods of low-load, mid-load, and high-load, the sample time is considered for the display. For the low-load time interval, the moment (01:34:00), for the peak consumption time interval, the moment (10:47:00) and for the mid-load time interval, the moment (19:59:00) is displayed as the sample. The voltage profile obtained from the instantaneous power flow and the values of SVs estimated by the traditional method for the three sample moments are shown in Fig. 3.

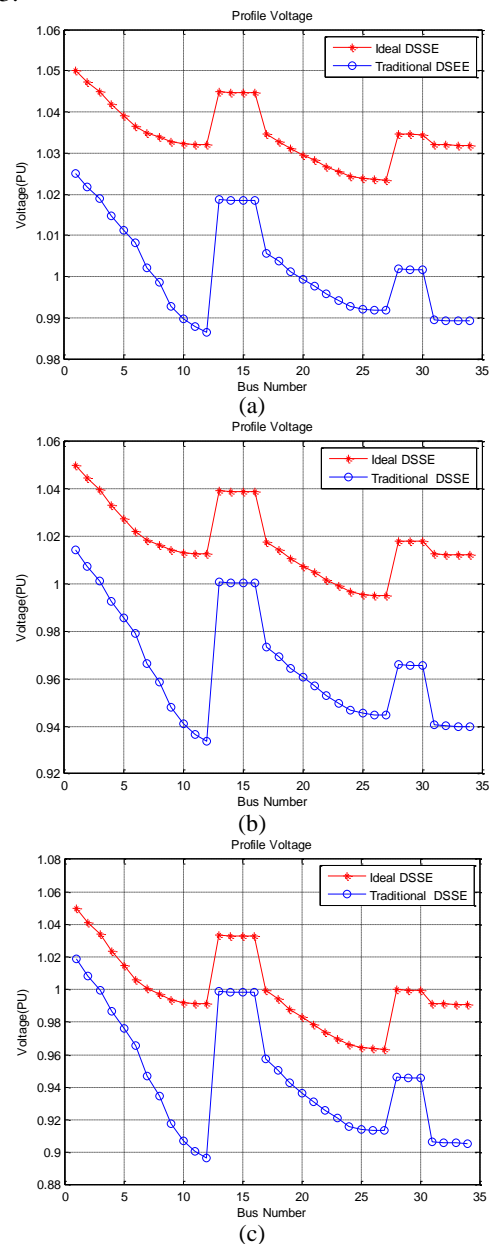


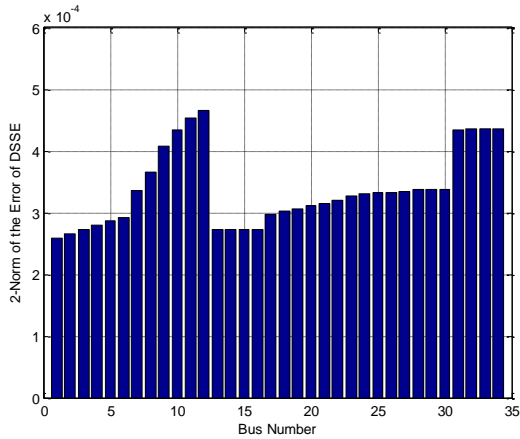
Fig. 3 Voltage profile curve in ideal ADSSE and estimated with the traditional method; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.

In this paper, the two-norm error of ADSSE that is defined below is used for all bus voltages.

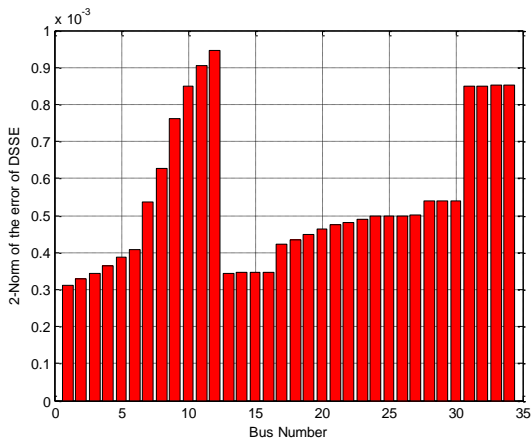
$$error = \sqrt{\sum_{i=1}^{nbus} (v_i^{est} - v_i^{act})^2} \quad (32)$$

where $nbus$ is the number of grid buses, v_i^{est} is the

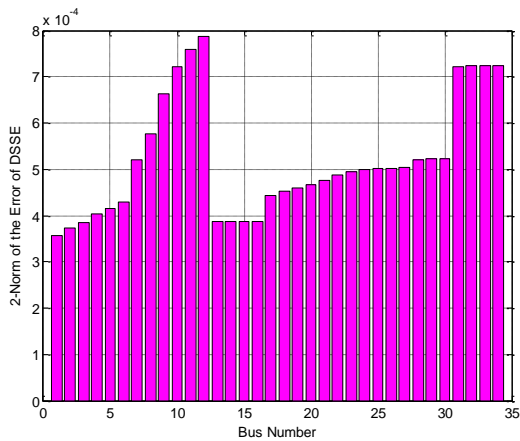
voltage magnitude estimated for the i -th bus, and v_i^{act} is the real value of voltage magnitude for the i -th bus. Equation (32) is also used for the phase angle. Figs. 4 and 5 depict the two-norm error for the actual and estimated values of SVs for three sample times.



(a)

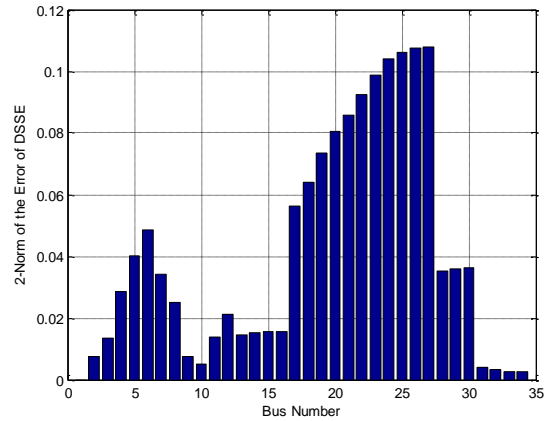


(b)

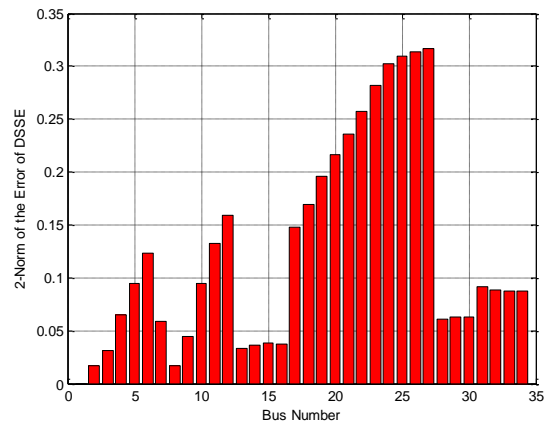


(c)

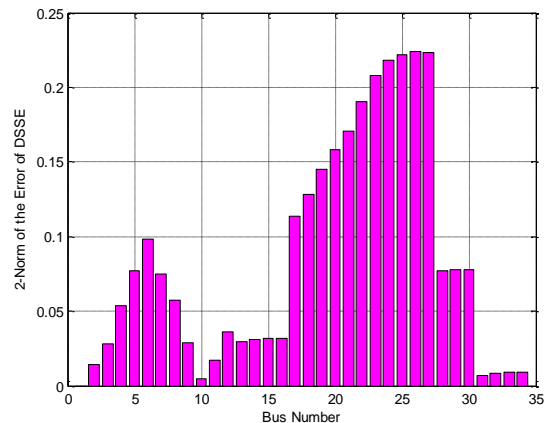
Fig. 4 The two-norm error for the actual and estimated values of buses voltage with the traditional method; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.



(a)



(b)



(c)

Fig. 5 The two-norm error for the actual and estimated values of phase angles with the traditional method; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.

Scenario II: In this scenario, the proposed method is considered. Variations of load at time intervals without measurement are considered by inserting (26)-(28) in the previous equations which are related to the WLS method and finally the error equation is corrected by (29). According to the first scenario, in this case, the program is running all day and within a time interval of

one minute. Sampling times in this scenario are similar to those in Scenario I. Fig. 6 shows the voltage profile obtained from the instantaneous power flow and the values of SVs estimated by Scenario II for the three sample moments. In addition, Figs. 7 and 8 present the two-norm error for the actual and estimated values of SVs in Scenario II for the three sample moments.

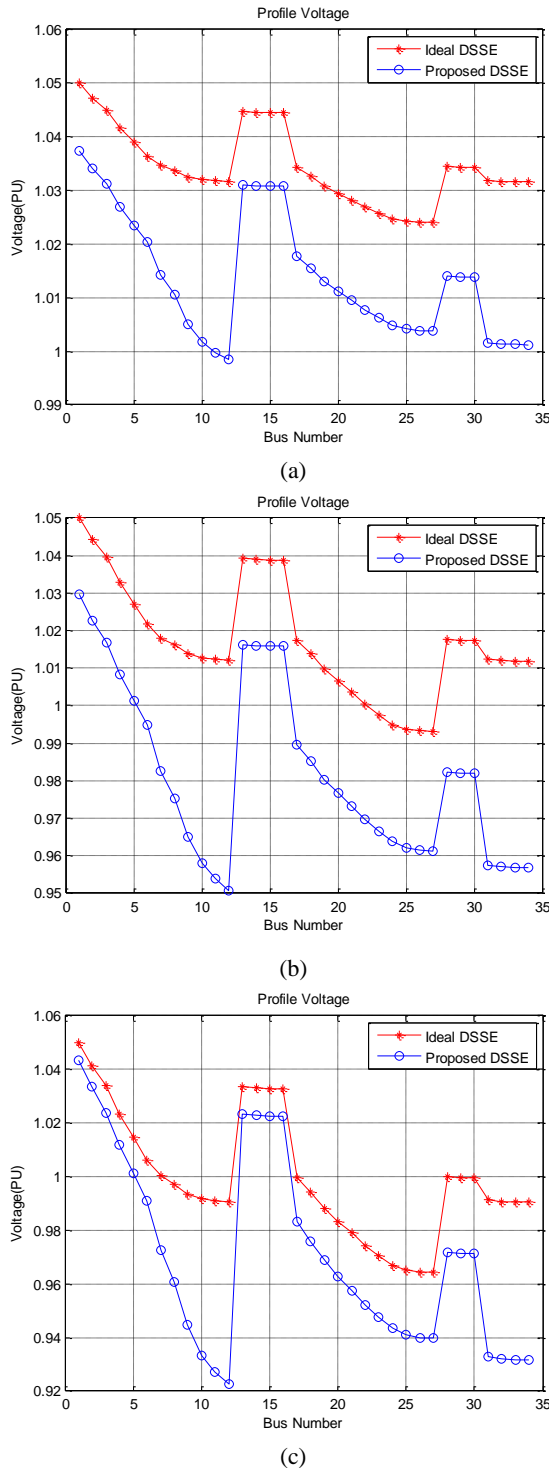


Fig. 6 Grid voltage profile in actual and estimated ADSSE with the proposed method; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.

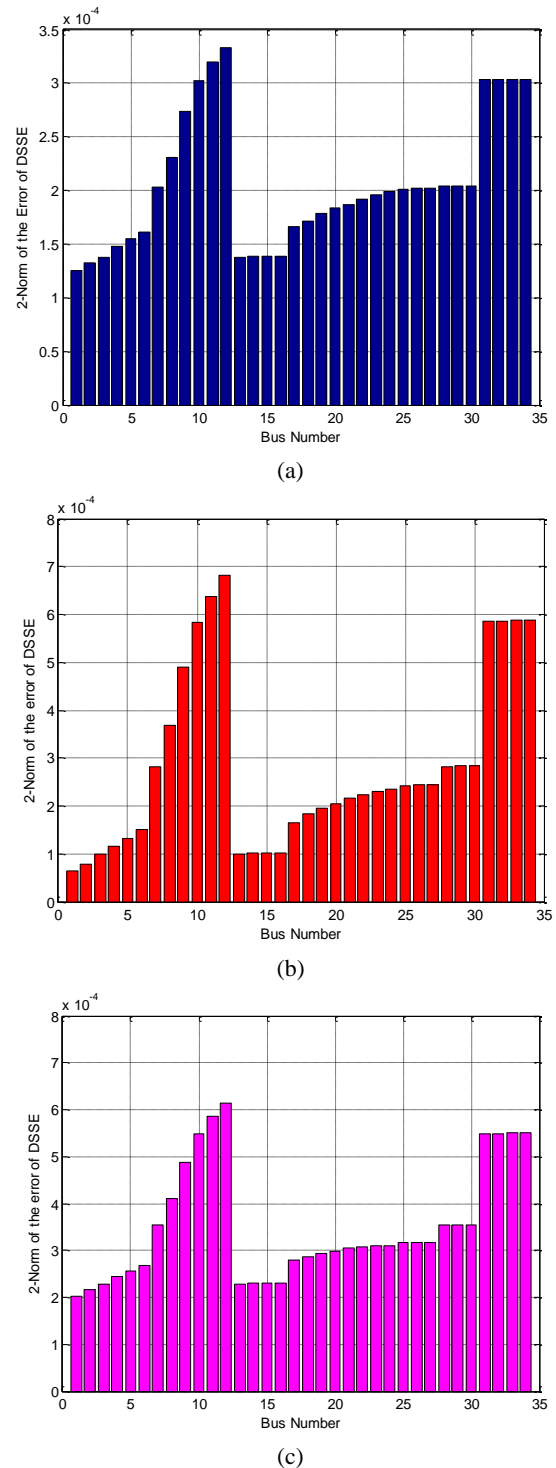
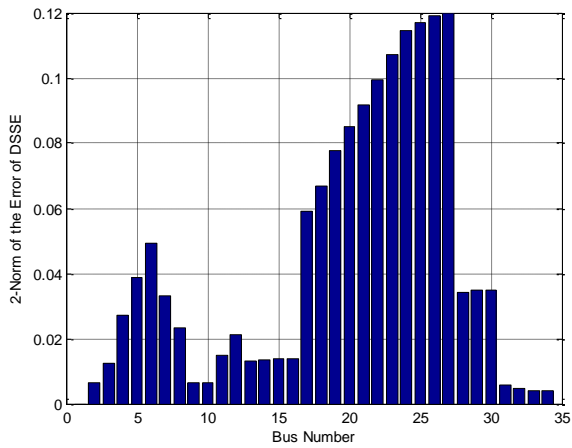


Fig. 7 The two-norm error for the actual and estimated values of buses voltage with the proposed method; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.

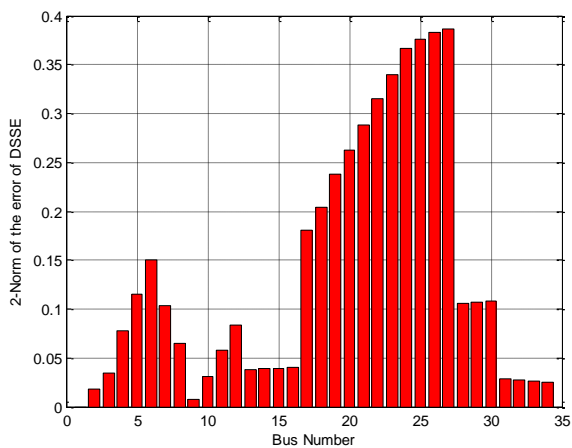
The results obtained from the two scenarios are compared in Fig. 9. As expected, by applying the proposed modification in the SE algorithm, the estimated values of SVs are closer to the actual value. Also, the proposed method has a lower error rate than

the traditional method.

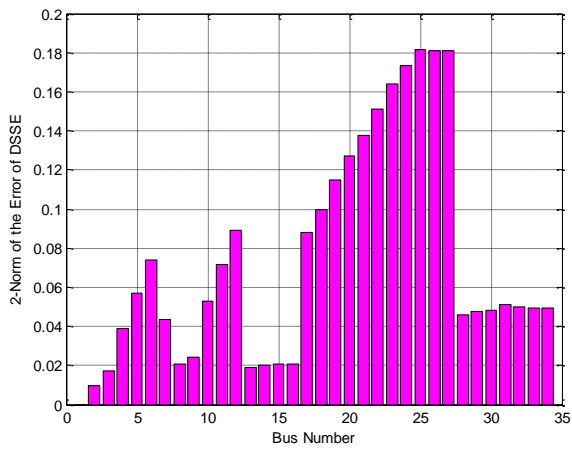
Figs. 10 and 11 compare the errors obtained from the estimated values and the real values for the voltage phasor (including amplitude and phase angle) in the test distribution system for two scenarios, respectively. As



(a)

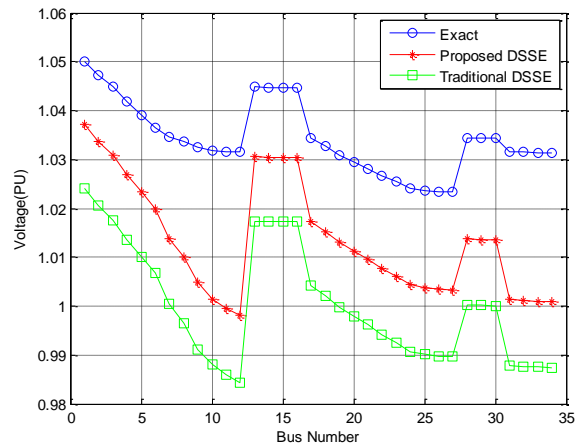


(b)

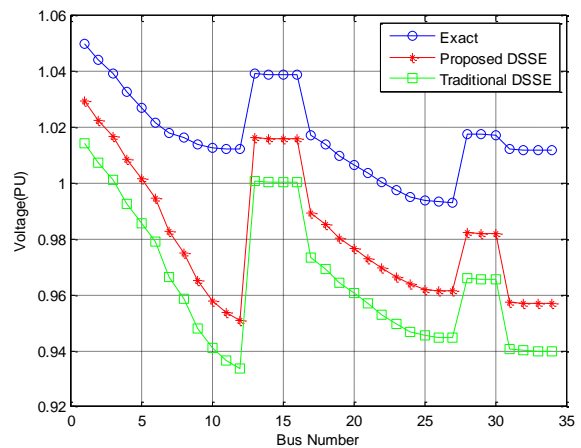


(c)

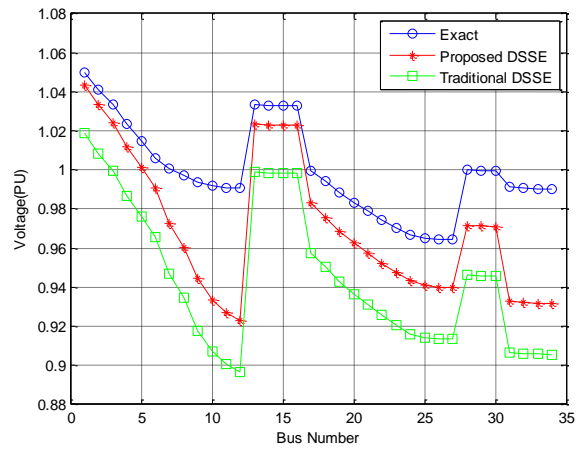
Fig. 8 The two-norm error for the actual and estimated values of phase angles with the proposed method; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.



(a)



(b)

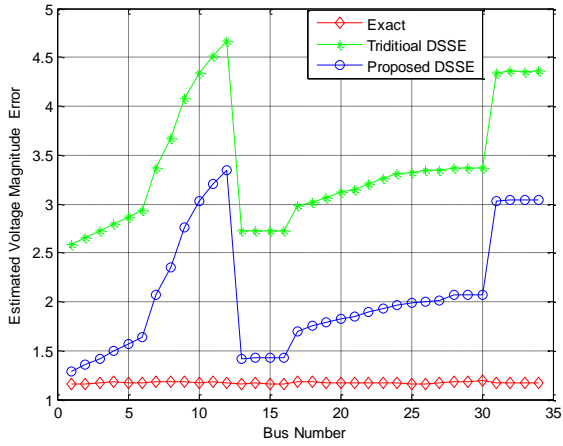


(c)

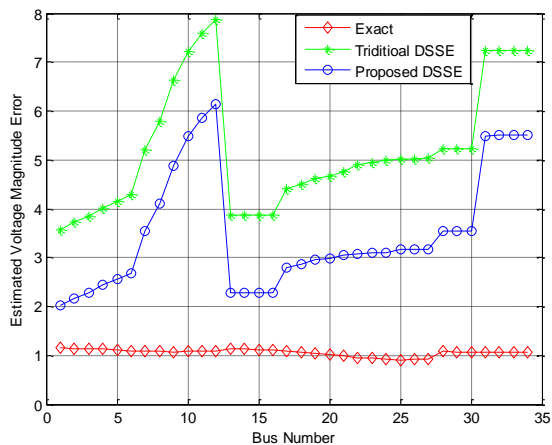
Fig. 9 Comparing the grid voltage profile in base state and estimated values in two scenarios; a) for the time 1: 34:00, b) for the time 10:47:00, and c) for the time 19:59:00.

expected, the best case for ADSSE is achieved under the exact case (ideal) in which it has been assumed that all SMs would always be updated in real-time (see Figs. 10 and 11, "Exact case"). Load changes throughout the day

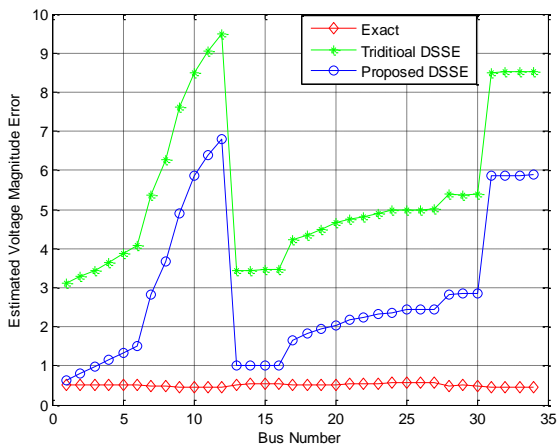
are also considered, so the ADSSE error rate also changes accordingly (see Fig. 2). When the LVs rate is high, the error rate in ADSSE also increases, i.e., in the morning when the load is increased and in the evening



(a)

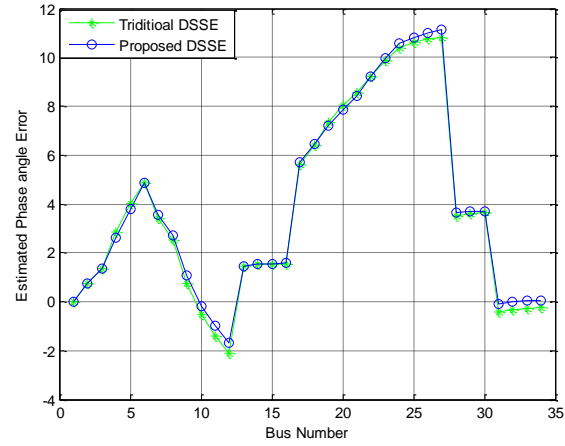


(b)

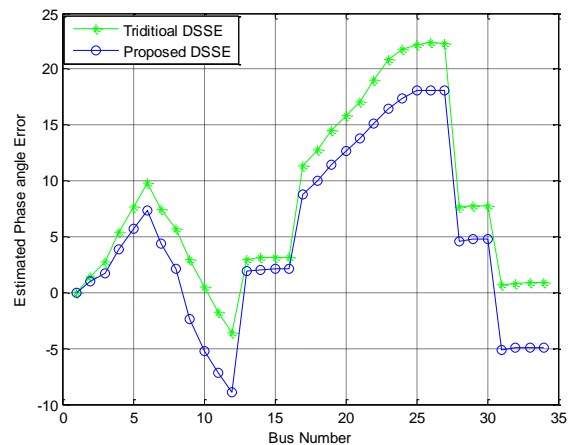


(c)

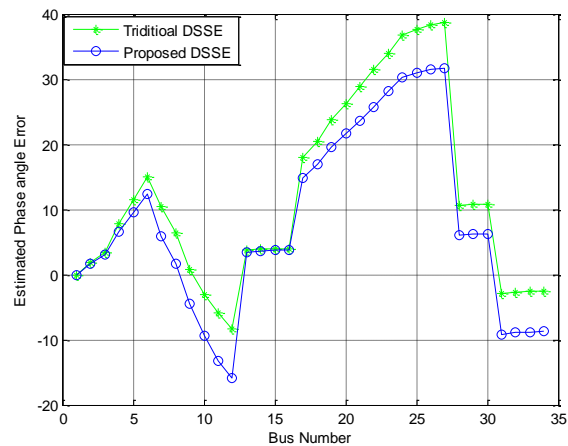
Fig. 10 Percentage error of the estimated values of the voltage magnitude for exact ADSSE, traditional ADSSE and proposed ADSSE; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.



(a)



(b)



(c)

Fig. 11 Percentage error of the estimated values of the phase angle for exact ADSSE, traditional ADSSE and proposed ADSSE; a) for the time 1:34:00, b) for the time 10:47:00, and c) for the time 19:59:00.

Table 1 Statistical performance of ADSSE error for a typical load curve in the IEEE 34-bus active distribution grid.

Sample Time	Best DSSE(least total error)			Mean DSSE(total error)			Worst DSSE(most total error)		
	1:34'	10:47'	19:59'	1:34'	10:47'	19:59'	1:34'	10:47'	19:59'
Exact ADSSE	1.1550	0.9033	0.4457	1.7335	1.1191	0.5143	1.1877	1.1550	0.5630
Conventional Method	2.5900	3.5600	3.1100	3.746	5.836	6.373	4.6700	7.8600	9.4800
Proposed Method	1.2900	2.0300	1.765	2.431	4.204	3.821	3.3400	6.1400	6.8100
Improvement of ADSSE accuracy by the proposed DSSE compared to the traditional ADSSE [%]	50.19	42.97	43.24	35.1	27.96	40.04	28.47	21.88	28.16

when the load is decreased (see Figs. 10(b) and 11(b)). However, when LVs are low, the error rate in ADSSE is small, and the results of the proposed ADSSE and the traditional ADSSE are approximately identical, which can be seen from Figs. 10(a) and 11(a). While the proposed ADSSE and traditional ADSSE have almost the identical error in their accuracy, the proposed approach treats the measurements in a different way by suitably adjusting their variances, thereby providing higher precision. In addition, the best mean and worst performances for both scenarios are summarized in Table 1. As can be observed in Figs. 10 and 11 and Table 1, the total error of ADSSE in Scenario I (traditional approach) varies from 2.59 to 9.48 whereas it varies from 1.29 to 6.81 in Scenario II.

For better understanding, Table I also presents the percentage of error reduction obtained from the proposed ADSSE approach as compared to the traditional ADSSE approach. As can be seen, the proposed method improves the precision of ADSSE by an average of 35.1% for the low-load time interval, 40.04% for the mid-load time interval, and 27.96% for the peak consumption time. The results indicate that the method presented in this paper has considerable potential for improving the quality of ADSSE in active distribution grids.

6 Conclusion

Measurement devices located in distribution grids measure electrical variables asynchronously. Therefore, there is an important challenge in using these measurements in SE algorithms because they are designed on the assumption that the measurements are performed synchronously. This paper proposed a solution to this problem. Initially, two well-known statistical tests were used to prove that the changes in the electrical loads are normally distributed in the short run. Based on this achievement, a new method was proposed in that MV and SD of measurement error in the data sent through smart meters are adjusted and thereby the problem of non-synchronicity in data transmission and expiration of measurements between two consecutive sampling interval due to lack of measurement is resolved. In order to show the effectiveness of the proposed method, it was compared to the traditional distribution system state estimation. As previously mentioned, in the traditional method, the

non-synchronicity in measurements is not taken into account and also, whenever a DSSE between the two consecutive intervals is performed, the measured data is used at the beginning of the interval. According to the simulation results, the quality of the proposed method is quite evident compared to the traditional method. Based on the comparisons whose results are summarized in Table 1, the accuracy of the state estimation has been improved by 50% in the best condition and by about 21% in the worst condition. The advantage of the proposed method is that it can be easily incorporated into WLS-based state estimators already used by electric companies without any major changes in their framework. Also, as any iteration uses the values obtained from the previous iteration, the convergence speed of the algorithm is also increased. Based on the results achieved in this paper and the benefits of the proposed method, it is believed that the proposed method can be an essential tool for the implementation of SE in active distribution systems.

References

- [1] C. N. Lu, J. H. Teng, and W. H. E. Liu, "Distribution system state estimation," *IEEE Transactions on Power Systems*, Vol. 10, No. 1, pp. 229–240, 1995.
- [2] F. Bouhafs, M. Mackay, and M. Merabti, "Links to the future: Communication requirements and challenges in the smart grid," *IEEE Power and Energy Magazine*, Vol. 10, No. 1, pp. 24–32, 2012.
- [3] S. Lefebvre, J. Prevost, and L. Lenoir, "Distribution state estimation: A necessary requirement for the smart grid," in *IEEE PES General Meeting Conference & Exposition*, National Harbor, MD, 2014.
- [4] Cisco Systems Inc., "Smart grid reference architecture, vol. 1, using information and communication services to support a smarter grid," *International Business Machines Corporation, Edison Company*, San Jose, CA, USA. [Online] Available: www.pointview.com/data/files/1/636/218_1.pdf, 2016.
- [5] G. T. Heydt, "The next generation of power distribution system," *IEEE Transactions on Smart Grid*, Vol. 1, No. 3, pp. 225–235, Dec. 2010.

- [6] A. Rankovic and A. T. Saric, "Modeling of photovoltaic and wind turbine based distributed generation in state estimation," in *Power Electronics and Motion Control Conference (EPE/PEMC)*, Novi Sad, 2012.
- [7] S. Shafie, M. F. Firuzabad, and M. Rastegar, "Investigating the impacts of plug in hybrid electric vehicles on power distribution systems," *IEEE Transactions on Smart Grid*, Vol. 4, No. 3, pp. 1351–1360, 2013.
- [8] R. C. Leou, "Optimal charging/discharging control of electric vehicles considering power system constraints and operating costs," *IEEE Transactions on Power Systems*, Vol. 31, No. 3, pp. 1854–1860, 2016.
- [9] C. Lu and J. Teng, "Distribution system state estimation," *IEEE Transactions Power System*, Vol. 10, pp. 229–240, Feb. 1995.
- [10] A. G. Expósito and A. Abur, *Power system state estimation: Theory and implementation*. New York, NY, USA: Marcel Dekker, 2004.
- [11] M. Baran and T. McDemott, "Distribution system state estimation using AMI data," in *IEEE Power System Conference and Exposition (PSCE)*, pp. 1–3, 2006.
- [12] N. Nusrat, M. R. Irving, and A. TaylorG, "An overlapping zone-based state estimation method for distribution system," *IEEE Transactions on Smart Grid*, Vol. 6, No. 4, pp. 2126–2133, 2015.
- [13] B. Hayes and M. Prodanovic, "State estimation techniques for electric power distribution systems," in *European Modelling Symposium (EMS)*, pp. 303–308, 2014.
- [14] X. Tai, Z. Lin, M. Fu, and Y. Sun, "A new distributed state estimation technique for power networks," in *IEEE American Control Conference*, pp. 3338–3343, Jun. 2013.
- [15] Z. Wu, J. Xu, and X. Yu, "Commentary on active distribution system state estimation technology," *Power System Automation*, pp. 1–9, 2017.
- [16] R. Sigh, B. Pal, and R. Jabr, "Choice of the estimator for distribution system state estimation," *IET Generation, Transmission & Distribution*, Vol. 3, No. 7, pp. 666–678, 2009.
- [17] S. M. Mahaei and M. R. Navayi, "Power system state estimation with weighted linear least square," *International Journal of Electrical and Computer Engineering (2088-8708)*, Vol. 4, No. 2, pp. 169–178, 2014.
- [18] K. Samarakoon, J. Wu, J. Ekanayake and N. Jenkins, "Use of delayed smart meter measurements for distribution state estimation," in *IEEE Power and Energy Society General Meeting*, San Diego, 2011.
- [19] E. B. Alzate, M. Bueno, X. Jian, and K. Strunz. "Distribution system state estimation to support coordinated voltage-control strategies by using smart meters", *IEEE Transactions on Power Systems*, 2019.
- [20] J. Ziyu, C. Jiaxiong, and L. Yuan, "State estimation in distribution system considering effects of AMI data," in *Proceedings of IEEE Southeastcon*, Jacksonville, FL, USA, 2013.
- [21] A. Al-Wakeel, J. Wu, and N. Jenkins, "State estimation of medium voltage distribution networks using smart meter measurements," *Applied Energy*, Vol. 184, pp. 207–218, 2016.
- [22] J. Wu, Y. He, and N. Jenkin, "A robust state estimator for medium voltage distribution networks," *IEEE Transactions on Power Systems*, Vol. 28, No. 2, pp. 1008–1016, May 2013.
- [23] D. D. Giustina, M. Pau, P. A. Pegoraro, F. Ponci, and S. Sulis, "Electrical distribution system state estimation: measurement issues and challenges," *IEEE Instrumentation & Measurement Magazine*, Vol. 17, No. 6, pp. 36–42, 2014.
- [24] D. Atanackovic and V. Dabic, "Deployment of real-time state estimator and load flow in BC Hydro DMS-challenges and opportunities," in *IEEE PES General Meeting*, Vancouver, BC, 2013.
- [25] L. Junqi et al., "Trade-offs in PMU deployment for state estimation in active distribution grids," *IEEE Transactions on Smart Grid*, Vol. 3, No. 2, pp. 915–924, Jun. 2012.
- [26] E. Manitsas, R. Singh, B. C. Pal, and G. Strbac, "Distribution system state estimation using an artificial neural network approach for pseudo measurement modeling," *IEEE Transactions on Power Systems*, Vol. 27, No. 4, 2012.
- [27] A. Gómez-Expósito, C. Gómez-Quiles and I. Džafi, "State estimation in two time scales for smart distribution system," *IEEE Transactions on Smart Grid*, Vol. 6, No. 1, Jan. 2015.
- [28] P. Kansal and A. Bose, "Bandwidth and latency requirements for smart transmission grid applications" *IEEE Transactions on Smart Grid*, Vol. 3, No. 3, pp. 1344–1351, Sep. 2012.
- [29] A. Bose, "Smart transmission grid applications and their supporting infrastructure" *IEEE Transactions on Smart Grid*, Vol. 1, No. 1, pp. 11–19, Jun. 2010.

- [30] A. Gómez-Expósito, A. Abur, A. de la Villa Jaén, and C. Gómez-Quiles, "A multilevel state estimation paradigm for smart grids," in *Proceedings of the IEEE*, Vol. 99, No. 6, pp. 952–976, Jun. 2011.
- [31] A. Alimardani, F. Therrien, and J. Jatskevich, "Distribution system state estimation based on nonsynchronized smart meters," *IEEE Transactions on Smart Grid*, Vol. 6, No. 6, pp. 2919–2928, 2015.
- [32] L. Holten, A. Gjelsvik, S. Aam, F. F. Wu, and W. H. E. Liu, "Comparison of different methods for state estimation," *IEEE Transactions on Power Systems*, Vol. 3, No. 4, pp. 1798–1806, Nov. 1988.
- [33] C. W. Hansen and A. S. Debs, "Power system state estimation using three-phase models," *IEEE Transactions on Power Systems*, Vol. 10, No. 2, pp. 818–824, May 1995.
- [34] F. L. Alvarado, W. F. Tinney, and M. K. Enns, "Sparsity in large-scale network computation," *Advances in Electric Power and Energy Conversion System Dynamics and Control*, Vol. 41, pp. 207–272, 1991.
- [35] A. Gomez and L. G. Franquelo, "Node ordering algorithms for sparse vector method improvement," *ZERE Transactions on Power Systems*, Vol. 3, No. 1, pp. 73–79, Feb. 1988.
- [36] G. E. Dallal and L. Wilkinson, "An analytic approximation to the distribution of Lilliefors's test statistic for normality," *The American Statistician*, Vol. 40, No. 4, pp. 294–296, 1986.
- [37] S. S. Shapiro and M. B. Wilk, "An analysis of variance test for normality (complete samples)," *Biometrika*, Vol. 52, Nos. 3–4, pp. 591–611, 1965.
- [38] N. M. Razali and Y. B. Wah, "Power comparisons of Shapiro–Wilk, Kolmogorov–Smirnov, Lilliefors, and Anderson–Darling tests," *Journal of Statistical Modeling and Analytics*, Vol. 2, No. 1, pp. 21–33, 2011.



M. Ajoudani received the B.Sc. and M.Sc. degree in Electrical Engineering from IAU, Iran, and Shahid Chamran University of Ahvaz, Iran, in 2006 and 2009, respectively. He is currently pursuing the Ph.D. degree in the Department of Power Electronic of the Babol Noshirvani University of Technology. Since 2010, he has been a faculty member of the Islamic Azad University of Golestan Province, Iran. His research interests include smart grid, distribution system Automation, and renewable energy.



A. Sheikholeslami was born in Dorood, Iran, in 1956. He received the B.Sc. degree in Electrical Engineering from Mazandaran University Babol, Iran, in 1979, and M.Sc. and Ph.D. from Strathclyde University, Glasgow, UK, 1986 and 1990, respectively. He is currently an Associate Professor of the Babol Noshirvani University of Technology. His research interests include power electronics, power quality, smart grids, and renewable energy.



A. Zakariazadeh received the B.Sc. (Hons.) degree in Electrical Engineering from the University of Mazandaran, Babol, in 2007 and the M.Sc. (Hons.) and the Ph.D. degrees in Electrical Engineering from Iran University of Science and Technology, Tehran in 2010 and 2014, respectively. Since 2016, he has been an Assistant Professor with the University of Science and Technology of Mazandaran, Behshahr, Iran. His research interests include smart grid, demand response, electric vehicles, and stochastic optimization.



© 2020 by the authors. Licensee IUST, Tehran, Iran. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0) license (<https://creativecommons.org/licenses/by-nc/4.0/>).