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Research Paper

Reliability Assessment of Distribution Transformers Considering Load Harmonics

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Abstract: The progressive application of non-linear loads in distribution systems (DS) increases current harmonics flow in DS's apparatuses, especially distribution transformers (DTs). Since DTs' operating temperature rises due to the harmonics flow, their loading should be reduced such that the hot spot temperature (HST) is preserved under its permissible value. This means that DTs' available capacity is influenced by load harmonic content. In this paper, a novel formulation for DTs' failure rate in the presence of harmonics is presented as a function of load harmonic contents. Using the suggested equivalent failure rate, DTs' available capacity in harmonic polluted DS is mathematically formulated. Additionally, the presence of the harmonic increases the HST, leading to DTs' aging acceleration. Therefore, the impact of harmonic components on DTs' aging is arithmetically modeled. To evaluate the efficacy of the suggested reliability model, it is applied to three distinct DTs having respectively industrial, commercial, and residential loads. The obtained results indicate that the available capacity of DTs with the same rated capacity would be different regarding to their load harmonic contents. On the other hand, it is comprehended from the achieved results that the aging acceleration factor (Faa) of the DTs increases owing to their load harmonic contents.

Keywords: Available Capacity, Distribution Transformer, Harmonics; Aging, Reliability Assessment.

1 Introduction

TRANSFORMERS are one of the most valuable equipment of the power systems, play a vital role in appropriate function of such systems.

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Therefore, the optimal utilization of such devices has been a great concern of both academic and experimental environments. Among the various studies concerning to the transformers' planning/operation, the reliability studies has a specific significance. Generally speaking, the reliability studies of devices, such as transformers, are structurally divided into two main categories namely chance-based events and wear out effects [1]. In the chance-based reliability studies, the main purpose is to determine the available capacity of the transformers, using their failure rates obtained from the historical data. In this way, some recent reliability-based researches have been taken this aspect of transformer reliability into their considerations. The reliability model of a high voltage transformer in oil natural air forced (ONAF) and oil natural air natural (ONAN) cooling modes are presented in [2],[3], respectively, where the transformer's available capacity is analyzed in terms of cooling system.

On the other hand, in the wear out-oriented reliability studies, the key goal is to determine the useful lifespan of the devices. It is evident that the accurate reliability modeling of the transformers with considering the entire effective parameters aids the utilities to determine an appropriate maintenance/replacement scheduling. In this way, a reliability-oriented maintenance strategy have been absorbed the attraction of the authors of [4]. In the studies of [4], a condition assessment model is suggested to evaluate the health status and consequently relationship between failure rate and health of the transformer.

The impact of transformer aging on its reliability is discussed in [5] where the results of the suggested Arrhenius-Weibull probability distribution function have been compared with the corresponding results achieved from Gaussian probability distribution. In [6], an analytical method is developed to evaluate the transformer's lifetime according to the ambient temperature. Based on the mathematical model presented in [6], the proper time of transformer overhaul /replacement can be determined. In [7], based on Arrhenius function, a statistical failure model is presented to evaluate the thermal degradation of the transformer's paper insulation. By applying the suggested model on two populations of transformers, strategies have been defined with the aim of maximizing the transformer utilization and also replacement deferral. At the same context a probabilistic framework to decide the optimal replacement strategy of aging power equipment, including power transformers, is introduced in [8]. Distribution transformers (DTs) have a great role in reliable feeding of loads in distribution systems (DSs). The lifespan of DTs considering diverse levels of distributed generation (DG) penetration is evaluated in [9-10]. The results of these studies demonstrate that the DG penetration causes decrease in DT's loading, hotspot temperature and consequently extends its lifespan. Besides, the more interrelation between DG generation, load demand and daily high ambient temperature, the more extension in DT's lifespan will be. At the same context, the impact of zero-energy buildings on the aging of DTs is discussed in [11]. In [12], a risk assessment model has been developed to evaluate the effectiveness of the reliability-centered maintenance (RCM) applied to DTs, as large population small-size assets of DNs. The suggested RCM evaluation model of [12] is proved to be efficient when there is inadequate historical data about the populous assets of DNs. In [13], using the related IEC and IEEE standards [14-15] concerning to transformer heat balance models, the loss of life of wind farms' transformers as a function of their loading and ambient temperature is calculated. The obtained results have been used to calculate the transformers' overloading percentage, duration time and consequently the transformers' sizing.

Power systems' equipment is designed to operate under the rated frequency. However, in the nowadays real-world power systems presence of non-linear loads cause the power system equipment, such as transformers, to tolerate other disturbing frequencies (known as harmonics) besides the fundamental one. In this case, the current magnitude due to the presence of the harmonics will be increased leading to power loss escalation and accordingly reduction in the useful lifetime of the equipment. Therefore, studying the destructive impact of harmonics on the operational indices of the power systems should be significantly taken into account. In [16], a mathematical model based on finite element method (FEM) is presented to calculate the hot spot and top oil temperature (TOT) of the power transformers under harmonic condition. Using the presented model of [16], the useful lifetimes of the power transformers as well as their loading capability under harmonic conditions are analyzed.

As the major proportion of the non-linear loads are located in distribution side of the power system, harmonic studies in DS is of major concern for both scientific researchers and distribution companies (DISCOs) in practice [17-20]. Within the harmonic-based studies of DSs, special attention is paid to the appropriate performance of DTs. The mathematical framework presented in [20] considers the influences of the current harmonics on extreme heat generation in a DT and so its insulation aging because of the created thermal stresses. The authors of [21] introduced a new method to solve the optimal DTs' sizing in a harmonic-polluted DS. In [22], the effect of appropriately designed passive filters on enhancing the loading capability of DTs in the presence of mixed linear and non-linear loads are investigated. Furthermore, in [23], a stochastically-formulated model is suggested to define the impact of high dispersion level of fully electric vehicles on the thermal ageing of DTs and so their useful lifespan. The influence of harmonic loads on additional losses of a 3 phase, dry-type, 440/380 V, 4.5 kVA laboratory transformer is experimentally investigated in [24] via connecting different types of load (linear active and nonlinear active-reactive load) to the transformer. The aim of the study performed in [24] was experimental confirmation of the mathematical model proposed in the IEEE Std. C57.110-1998. Moreover, in [25], the aging and so lifespan estimation of three various DTs are technically assessed based on the recorded real data. The attained results of [25] confirm the remarkable reduction in useful lifespan of some under-study DTs from 30 years to 17 years. In this study, the variation of harmonic magnitudes over the time is neglected. Since the temperature rise of transformers, as the main reason of their aging acceleration, depends on the transformer's thermal time constant, neglecting the variation of the harmonics' magnitude over the time would introduce errors in the study results.

As it can be seen, the above mentioned literatures consider the impact of harmonics either on the aging process of transformers while neglecting the harmonics' magnitude variation or on the chance-based events.

Since the overall reliability of DTs is directly affected by both chance-based events and aging process, in this paper, a modified failure rate of the DTs in the presence of harmonics is firstly formulated using continuous Markov processes. By doing so, the available capacity of DTs in harmonic-polluted DSs throughout its useful lifespan can be calculated. The resultant available capacity can help DISCO's planner to optimally select the DTs' rated capacity to ensure not to exceed their temperature rise limit in harmonic condition within their useful lifespan. In the following, the impact of harmonics on the aging process and accordingly on the overall reliability of DTs is mathematically discussed. In the second part of the study, the variation of harmonics magnitudes over time is considered, and using this, the variation of DT's available capacity over time is evaluated. To illustrate the performance of the suggested reliability-based model, it is developed in MATLAB programming environment and tested on three distinct DT types. The achieved results justify the effectiveness of the suggested reliability model compared to optimistic previous results (i.e. considering the DT's original failure rate). To the best of the authors' knowledge, the suggested model assigned for the overall reliability calculation of the harmonic-polluted DTs is specific to this paper and has not been observed in the previous research works in the area.

The remainder of the paper is organized as follows. In section 2 as a general review, the losses of a DT under harmonic conditions are mathematically described. Section 3 includes the details of the suggested Markov-based reliability model for assessing the available capacity of DTs in the presence of harmonics. Afterward, the mathematical formulation for studying the impact of harmonics on the DT's aging is fully elucidated in section 4. The simulation results pertaining to various types of the DT's loading is presented and discussed in section 5. Finally, section 6 concludes the paper.

2 DT Losses under Harmonic Conditions

The transformer losses are generally divided into two parts known as no-load and load losses, illustrated as follows:

$$P_T = P_{NL} + P_{LL} \tag{1}$$

Where, P_{NL}, P_{LL} and P_T are no-load loss, load loss and total loss of the transformer, respectively.

No load losses (P_{NL}) include core losses in the core materials (i.e. hysteresis and eddy current losses), dielectric losses in the insulation system and copper losses in the winding due to exciting current. This type of losses depends on the voltage waveform.

Load losses (P_{LL}) include transformer Ohmic losses resulting from the load current passing through the DC resistance of the coil, eddy current losses of the coil and other stray losses inside the transformer tank and other metallic parts except the coils. Thus, the mathematical representation of the P_{LL} is as below:

$$P_{LL} = P_{dc} + P_{TSL} \tag{2}$$

Where, P_{dc} is the DT's ohmic losses. Moreover, P_{TSL} denotes for the total stray losses calculated as follows:

$$P_{TSL} = P_{EC} + P_{OSL} \tag{3}$$

In Eq. (3), P_{EC} is the eddy current losses of the coils and P_{OSL} stands for the other stray losses. Based on the IEEE C57.110 standards [15], 33% of the total stray losses in oil-immersed DTs belong to eddy current losses of the coil and so the remaining part, i.e. 67% of the total stray losses, is related to other stray losses. In dry type DT, the portion of other stray losses in total stray losses is 33%. Also, the total stray losses are commonly 20%-30% of the DT's load losses [26].

Generally speaking, both DT’s voltage and current can be polluted by harmonics. The voltage THD is very low (less than 5%) in most DSs. Therefore, from voltage harmonic viewpoint, the no load losses of the DTs are commonly calculated based on merely fundamental component of the voltage (the amount declared by the manufacturers). However, it should be underlined that the current harmonics are so essential in DSs since they can increase the effective load current leading to increment of the DC losses.

2.1 Ohmic Losses of DTs

Under the harmonic conditions, the ohmic losses of a DT are obtained as follows:

$$\begin{aligned} P_{dc} &= R_{dc} \times I^2 \\ &= R_{dc} \times \sum_{h=1}^{h=h_{max}} I_h{}^2 = R_{dc} \times I_R^2 \times \sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R}\right]^2 \\ &= P_{dc-S} \times \sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R}\right]^2 \end{aligned} \tag{4}$$

Where, R_{dc} is the equivalent ohmic resistance of windings, I is the effective value of the load current, I_R is the effective value of rated current under pure sinusoidal conditions, h is harmonic order, I_h is the effective value of the h^{th} harmonic's current and $P_{dc\text{-}S}$ is the ohmic losses of the DT under its sinusoidal nominal load announced by the manufacturer.

2.2 Eddy Current Losses of DTs' Windings

When a conductor is exposed to a variable electromagnetic field, a definite voltage is induced inside it leading to formation of eddy current. The eddy current inside of the DT generates losses and so increases the conductor temperature. The eddy current losses of the coils are proportional to the square of both current harmonics' order and magnitude, as presented in Eq. (5) [15]:

$$P_{EC} = P_{EC-R} \times \sum_{h=1}^{h=h_{\max}} h^2 \left[\frac{I_h}{I_R}\right]^2 \tag{5}$$

Where, $P_{EC\text{-}R}$ is the rated eddy current losses of the coils.

The proportion of the eddy current losses to the ohmic losses under the harmonic conditions can be calculated as Eq. (6) where "*p.u*" stands for per unit value based on the $P_{dc\text{-}S}$.:

$$\begin{aligned} \frac{P_{EC}}{P_{dc}} &= \frac{P_{EC-R} \times \sum_{h=1}^{h=h_{\max}} h^2 \left[\frac{I_h}{I_R}\right]^2}{P_{dc-S} \times \sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R}\right]^2} \\ &= P_{EC-R}(pu) \times F_{HL} \end{aligned} \tag{6}$$

Where, F_{HL} is the harmonic losses factor related to the eddy current losses of the coil, computed as below:

$$F_{HL} = \frac{\sum_{h=1}^{h=h_{\max}} h^2 \left[\frac{I_h}{I_R}\right]^2}{\sum_{h=1}^{h=h_{\max}} \left[\frac{I_h}{I_R}\right]^2} = \frac{\sum_{h=1}^{h=h_{\max}} h^2 I_h^2}{\sum_{h=1}^{h=h_{\max}} I_h^2} \tag{7}$$

2.3. Other Stray Losses of DTs

In addition to the DT's coil, the eddy current losses are formed in other metal parts of a DT known as the other stray losses. The mentioned losses are generated in iron parts of a transformer such as yoke, tank walls and clamps subjected to several factors including voltage level of the DT and also materials of the tanks and clamps [16]. In the presence of harmonic currents, other stray losses are calculated as follows [15]:

$$P_{OSL} = P_{OSL-R} \times \sum_{h=1}^{h=h_{\max}} h^{0.8} \left[\frac{I_h}{I_R}\right]^2 \tag{8}$$

Where, $P_{OSL\text{-}R}$ is the rated value of the other stray losses proclaimed by the manufacturers. The ratio of other stray losses to the ohmic losses under the harmonic conditions is calculated as follows:

$$\begin{aligned} \frac{P_{OSL}}{P_{dc}} &= \frac{P_{OSL-R} \times \sum_{h=1}^{h=h_{\max}} h^{0.8} \left[\frac{I_h}{I_R}\right]^2}{P_{dc-S} \times \sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R}\right]^2} \\ &= P_{OSL-R}(pu) \times F_{HL-STR} \end{aligned} \tag{9}$$

Where, $F_{HL\text{-}STR}$ is the harmonic losses factor for other stray losses specified as follows [27]:

$$F_{HL-STR} = \frac{\sum_{h=1}^{h=h_{\max}} h^{0.8} \left[\frac{I_h}{I_R}\right]^2}{\sum_{h=1}^{h=h_{\max}} \left[\frac{I_h}{I_R}\right]^2} = \frac{\sum_{h=1}^{h=h_{\max}} h^{0.8} I_h^2}{\sum_{h=1}^{h=h_{\max}} I_h^2} \tag{10}$$

3 Modeling of Harmonic Impacts on DTs' Availability

As generally known, the current harmonics lead to additional heat in DTs. In this circumstance, if a DT is operated at its rated capacity, the additional heat generated by the harmonic losses raises its HST over the maximum permissible value resulting the insulation degradation. To avoid this issue, the maximum loading of DTs under harmonic conditions should be lower than their rated capacity.

3.1 Maximum Allowable Current of a DT under Harmonic Load Conditions

Regarding to Eqs. (2)-(10), the DT's load losses under harmonic conditions can be calculated using Eq. (11).

$$P_{LL-H} = P_{dc} + P_{EC} + P_{OSL} = P_{dc-S} \times \sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R}\right]^2 + P_{EC-R} \times \sum_{h=1}^{h=h_{max}} h^2 \left[\frac{I_h}{I_R}\right]^2 + P_{OSL-R} \times \sum_{h=1}^{h=h_{max}} h^{0.8} \left[\frac{I_h}{I_R}\right]^2 \qquad (11)$$

Where, P_{LL-H} is the load losses of the DT under harmonic conditions. Based on P_{dc-S}, Eq. (11) can be rewritten in per unit form as Eq. (12):

$$\begin{aligned} P_{LL-H}(pu) &= \sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R}\right]^2 \times \left[1 + F_{HL} \times P_{EC-R}(pu) + F_{HL-STR} \times P_{OSL-R}(pu)\right] \\ &= I^2(pu) \times \left[1 + F_{HL} \times P_{EC-R}(pu) + F_{HL-STR} \times P_{OSL-R}(pu)\right] \end{aligned} \qquad (12)$$

In this way, the maximum permissible load current (in p.u.) should be restricted to a value that the DT's per unit load losses is equal to unity [28]. Thus, it can be mentioned that:

$$I_{max}(p.u) = \frac{1}{\sqrt{1 + F_{HL} \times P_{EC-R}(pu) + F_{HL-STR} \times P_{OSL-R}(pu)}} \qquad (13)$$

As a result, based on Eq. (13), the maximum capacity of the DT under non-sinusoidal loads is formulated as follows:

$$S_{Har} = I_{max}(pu) \times S_{Rated} \qquad (14)$$

Where, S_{Rated} and S_{Har} are the nominal capacity and the maximum DT's allowable loading under non-sinusoidal conditions, respectively.

3.2 The DT's Markov Model under Harmonic Conditions

During the useful lifetime, the state space diagram for a single repairable component like a DT can be displayed with a two-state (i.e. *Up* and *Down* states) Markov model, as shown in Fig. 1. In Fig.1, λ and μ are failure rate (fault/year) and repair rate (repair/year) of the component, respectively.

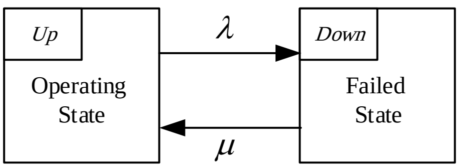


Fig.1 State space diagram of a repairable component.

In a transformer with ONAF cooling system, outage of one or more cooling fan causes capacity decrement. Therefore, in addition to *Up* and *Down* states, the state space model of such transformers includes other derated states with capacity between zero and the rated value. However, since the cooling system of DTs (up to 33kV) is ONAN, it is assumed that in the case of failure for such transformers, the entire capacity is missed. Therefore, from reliability aspect, the Markov model of DTs is accordingly modeled with two state of *Up* and *Down*, i.e. derated state is not considered. Thus, based on the state space model of Fig. 1, the capacity outage probability table (COPT) of the DT in non-harmonic conditions is demonstrated in Table 1.

Table 1 COPT of DT in non-harmonic conditions

Capacity	Individual probability
S_{Rated}	*A*
0	*U*

In Table 1, S_{Rated} is DT's rated capacity and, *A* and *U*, are the availability and unavailability of the DT in non-harmonic conditions, respectively that are calculated as follows:

$$A = \frac{\mu}{\lambda + \mu} = \frac{m}{T} \qquad (15)$$

$$U = \frac{\lambda}{\lambda + \mu} = \frac{r}{T} \qquad (16)$$

Where, *m* and *r* represent the mean time to failure (MTTF) and mean time to repair (MTTR), respectively. Also, *T* represents the cycle time of the system which is equal to the summation of *m* and *r*.

Under the harmonic conditions, the capacity values of Table 1 change to the values presented in Table 2.

Table 2 COPT of transformer in harmonic conditions

Capacity	Individual probability
S_{Har}	A
0	U

The mean time/state diagram and chronological transition between the *Up* and *Down* states of the DT, is presented in time graphs of Fig. 2 where the DT capacity is inscribed at the top of each time duration in all graphs. The graph (a) is related to the non-harmonic conditions where the DT's available capacity is S_{Rated} in duration m and zero in duration r. As previously described, the presence of harmonics decreases the average available capacity of the DT to a value lower than the S_{Rated}, i.e. S_{Har} in the period of m. Considering S_{Har} as the maximum available capacity in the period m, the *Up* and *Down* states of the DT and its associated mean time/state diagram can be displayed as graph (b) of Fig. 2. In this graph, it is seen that in mean time duration m, the S_{Har} (shown by red solid line) is less than its corresponding S_{Rated} (depicted by dashed black line). Based on the capacity rounding approach [29], it can be assumed that in a mean time/state diagram with fixed cycle time T, the harmonic-polluted DT is available with the rated capacity for duration m' shorter than m and is unavailable for duration r' longer than r. Regarding to the mentioned assumption, the graph (c) is represented in Fig. 2.

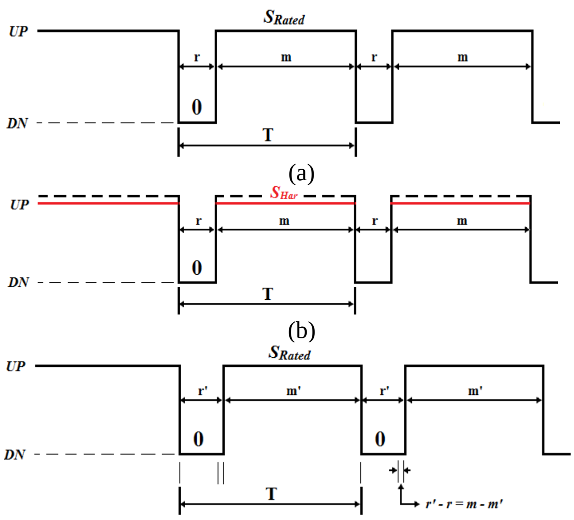


Fig. 2 The mean time/state diagram of a DT
(a) Non-harmonic conditions
(b) Harmonic conditions before the capacity rounding
(c) Harmonic conditions after the capacity rounding

Regarding to graph (c) of Fig. 2, it is possible to convert the capacity values of the Table 2 into the capacity values of the Table 1. The new COPT is presented in Table 3.

Table 3 COPT with rounded values in harmonic conditions

Capacity	Individual probability
S_{Rated}	$K\times A$
0	$U + (1\text{-}K)\times A$

In Table 3, the coefficient K is calculated as follows:

$$K = \frac{S_{Har}}{S_{Rated}} \xRightarrow{From\ (14)} K = I_{max\,(pu)} \tag{17}$$

In non-harmonic conditions, S_{Har} is equal to S_{Rated} leads to unity value for the K. However, this value is less than unity under harmonic conditions.

According to graph (c) of Fig. 2 and also Eq. (17), the relationship between m and m' is obtained as follows:

$$m' = K\times m \tag{18}$$

Given that the m is inverse of λ, the failure rate of the DT under harmonic conditions is obtained as follows:

$$\lambda' = \frac{\lambda}{K} = \lambda\times\sqrt{1+F_{HL}\times P_{EC-R(pu)} + F_{HL-STR}\times P_{OSL-R(pu)}} \tag{19}$$

According to Eq. (19), the equivalent failure rate of the DT under harmonic conditions is greater than the corresponding one under non-harmonic conditions (i.e. $\lambda' > \lambda$). Furthermore, the more harmonic content of the load, the more equivalent failure rate of the DT. Thus, in useful lifetime, the DT's reliability in harmonic conditions can be formulated based on (19) [1]:

$$R(t) = e^{-\lambda' t} \tag{20}$$

4 Mathematical Modeling of the Harmonic Impact on DT's Aging

Generally speaking, existing of moisture and oxygen in transformer's oil and thermal stress, lead to acceleration in its insulation aging. In this case, the generated heat escalates the HST of the DTs which is the main factor for evaluating the insulation aging [15].

The HST of the DT is statistically a function of its total losses including harmonic-oriented losses. Hence, harmonic loads affect the DT's lifetime and consequently its reliability in the aging stage.

4.1 The Impact of Temperature on the DT's Aging

The aging acceleration factor (F_{aa}) is a parameter that presents the relationship between the HST and the lifetime of the DT. The F_{aa} index shows how many hours of DT's lifetime would be spent for each hour operating time. This factor is mathematically calculated as follows [30]:

$$F_{aa} = e^{\left[\left(\frac{B}{\theta_{H-R}+273} - \frac{B}{\theta_H + 273} \right) \right]} \tag{21}$$

Where, B is the aging constant which is typically 15000℃; θ_H is the HST of the DT's winding and θ_{H-R} is the HST under the rated load. It is worthwhile to mention that for the permissible temperature rise of the coil over the ambient temperature of 55℃ and 65℃, the θ_{H-R} is 95℃ and 110℃, respectively [15].

In the oil-immersed DTs, the HST at any time can be computed as follows:

$$\theta_H(t) = \Delta\theta_{HS}(t) + \Delta\theta_{TO}(t) + \theta_A(t) \tag{22}$$

Where, $\Delta\theta_{HS}$ is the HST rise over the TOT for a specific load loss (i.e. P_{LL}), $\Delta\theta_{TO}$ is the TOT rise over ambient temperature and, θ_A is the ambient temperature.

Assuming a constant load losses of P_{LL}, the $\Delta\theta_{HS}$ and $\Delta\theta_{TO}$ can be calculated as follows [15]:

$$\Delta\theta_{HS} = \Delta\theta_{HS-R} \times \left(\frac{P_{LL}}{P_{LL-R}} \right)^{0.8} \tag{23}$$

$$\Delta\theta_{TO} = \Delta\theta_{TO-R} \times \left(\frac{P_{LL}+P_{NL}}{P_{LL-R}+P_{NL}} \right)^{0.8} \tag{24}$$

Where, $\Delta\theta_{HS-R}$ and $\Delta\theta_{TO-R}$ are the values of $\Delta\theta_{HS}$ and $\Delta\theta_{TO}$ in the case of P_{LL-R}, respectively.

At each time interval with fixed loading and accordingly fixed load losses of the DT, the variation trend of the $\Delta\theta_{HS}$ and $\Delta\theta_{TO}$ can be represented by an exponential function as below:

$$\Delta\theta_{HS}(t) = \left(\Delta\theta_{HS-U} - \Delta\theta_{HS-i} \right) \times \left(1 - e^{-\frac{t}{\tau_w}} \right) + \Delta\theta_{HS-i} \tag{25}$$

$$\Delta\theta_{TO}(t) = \left(\Delta\theta_{TO-U} - \Delta\theta_{TO-i} \right) \times \left(1 - e^{-\frac{t}{\tau_{TO}}} \right) + \Delta\theta_{TO-i} \tag{26}$$

Where, $\Delta\theta_{HS-i}$ and $\Delta\theta_{HS-U}$ are the initial and ultimate value of the $\Delta\theta_{HS}$, respectively, $\Delta\theta_{TO-i}$ and $\Delta\theta_{TO-U}$ are the initial and ultimate value of $\Delta\theta_{TO}$, respectively, τ_w and τ_{TO} are the thermal time constant of the DT's coil and oil, respectively, and t is the time variable of the interval.

4.2 The DT's Reliability in Aging Area

In most of the recent studies concerning to the DT's reliability, the analyses have been performed based on the DT's performance within its useful lifetime considering a constant failure rate [2-3]. However, in spite of determining the *MTTF* value in these studies, a DT may fail long time before reaching the *MTTF* in practice, due to the aging process. Throughout the aging period, the failure rate is not constant and has a raising behavior. Thus, it cannot be explicitly expected that the DT does not fail over this period and so, the effect of aging on the DT's reliability should be taken into account. The probability of the DT not to be failed due to wear out along a period of time t within the interval of T to $(T + t)$ can be expressed as follows [1]:

$$R_w(t) = \int_{T+t}^{\infty} f(t)dt \Big/ \int_{T}^{\infty} f(t)dt = \frac{R_w(T+t)}{R_w(T)} \tag{27}$$

Where, R is reliability function, subscript w implies the wear out and $f_{(t)}$ is the failure probability distribution function (PDF) of the DT.

When the chance-based failures are also incorporated into the evaluation of the DT's reliability, its overall reliability within the interval t (i.e. T to $T+t$), can be determined as follows:

$$R(t) = R_c(t) \times R_w(t) = e^{-\lambda' t} \frac{R_w(T+t)}{R_w(T)} \tag{28}$$

Where, subscript c stands for the chance-based events. Considering the lifetime beginning from T=0, the Eq. (28) is changed as below:

$$R(t) = e^{-\lambda' t} \frac{R_w(0+t)}{R_w(0)} \overset{R_w(0)=1}{=\!=\!=\!=} e^{-\lambda' t} R_w(t) \tag{29}$$

The PDF function in the wear out period can be approximated using a normal distribution around the mean lifetime of the DT, as depicted in Fig. 3 [1]. In Fig. 3, t denotes the age of the DT and M is the mean lifetime of the DT announced by the manufacturer.

If the effect of loading and harmonics on the wearing process is ignored, the average lifetime of the DT is the standard lifetime declared by the manufacturer. Therefore, to obtain the value of $R_w(t)$ in Eq. (29) aiming to use the table of the standard normal PDF, the following substitution should be performed:

$$Z_1 = \frac{t - M}{\sigma} \tag{30}$$

Where, σ is the standard deviation of DT's lifetime in the aging area. Since each hour operation at the HST of θ_H spends the DT's lifetime for F_{aa} hours, after t hours operation of the DT, the actual amount of the spent DT's lifetime would be the sum of the aging acceleration factors (F_{aa}) associated with each operation hour. As a result, Z_1 in Eq. (30) should be substituted with Z_2 suggested in Eq. (31):

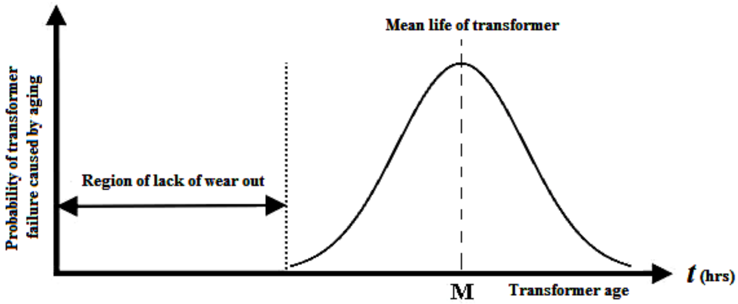


Fig. 3 The failure distribution function of the transformer considering wear out period.

$$Z_2 = \frac{F_{aacu} - M}{\sigma} \tag{31}$$

Where, F_{aacu} is the cumulative aging acceleration factor in hours.

5 Simulation Results and Discussion

To evaluate the impact of the harmonics on DT's availability, aging and consequently overall reliability, the recorded data associated with the loading of DTs, installed in one of the northern city of Iran named Sari, is utilized. Since the harmonic generation of the load is related to its type, three DTs with three distinct load types, i.e. residential, commercial and industrial, are considered for the purpose of simulation studies.

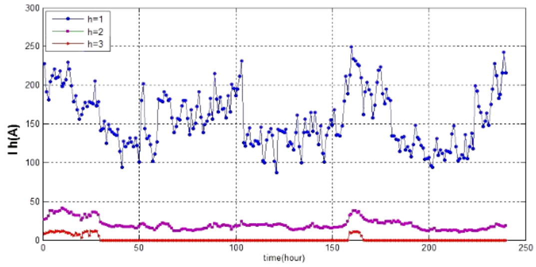
5.1 DTs and Their Load Harmonics Data

The selected DTs' belong to the North Sari electricity distribution affairs and their descriptive data have been considered according to Table 4.

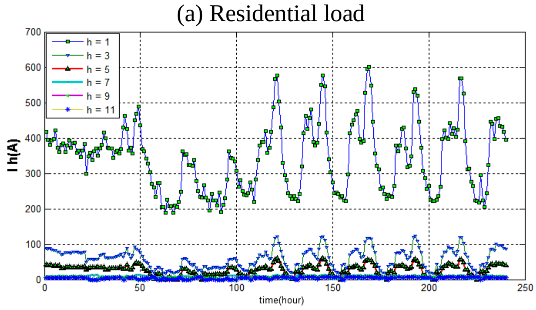
Table 4 The DTs' descriptive data [31]

Transformer Parameters	Transformer load type		
	Residental	Commercial	Industrial
S_{Rated} (KVA)	630	630	2000
Voltage ratio (kV)	20 / 0.4	20 / 0.4	20 / 0.4
P_{NL} (W)	1300	1300	3200
P_{LL-R} (W)	6500	6500	23000
Ohmic resistance of the primary winding (Ω/phase)	7.549	7.549	2.347
Ohmic resistance of the secondary winding (Ω/phase)	0.00262	0.00262	0.000764
The average ambient temperature (^{oC})	30	30	30
permissible temperature rise of the coil over the ambient temperature (^{oC})	65	65	65
λ (f/yr)	0.05	0.05	0.05
τ_{TO} (hrs)	3	3	3
τ_w (hrs)	2.167	2.167	1.583

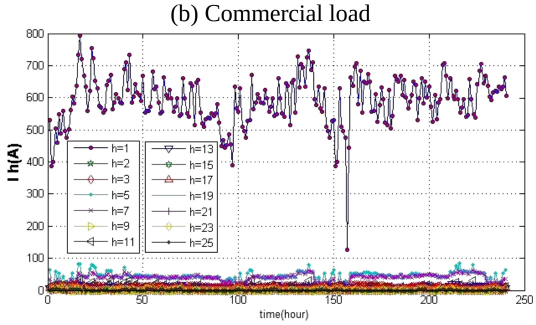
The mean lifetime *M* plays an important role in determining the wearing period. Considering the normal PDF, the wearing stage of DT starts almost at (M - 3σ) and its failure probability approximate to unity near the ($M + 3\sigma$). The values of M and σ are assumed to be 120,000 and 10,000 hours, respectively [31]. The numerical data pertaining to the loading of the three DTs have been registered by power quality analyzer *Unilizer 902*. In the mentioned analyzer up to the 25th harmonic of the current passing through the DTs are stored with the rate of one sample per 10 minutes over a period of one year. However, due to space limitation, only the hourly average recorded data associated with the first 10 days of the measurement are presented in Fig.4. As seen in Fig.4, the residential load has the current harmonic up to 3rd order, while in commercial and industrial loads, up to 11th and 25th order are recorded, respectively. On the other hand, the commercial load has not any even order harmonic while the second order is evidently seen in the residential and industrial loads.



(a) Residential load



(b) Commercial load

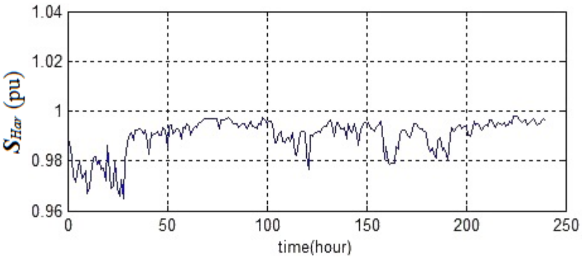


(c) Industrial load

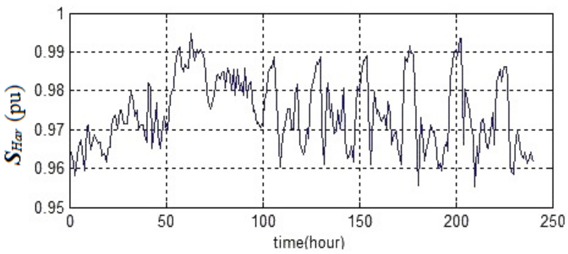
Fig. 4 Diagrams of the DT's harmonic amplitude associated three kinds of loads.

5.2 Harmonics impacts on the availability of three DTs

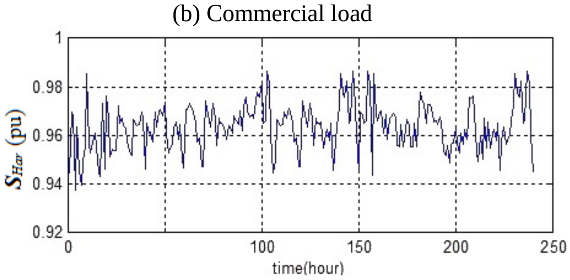
Based on Eqs. (13) to (19) and the graphical data presented in Fig. 4, the available capacity related to each of the three DTs in the presence of the harmonics (i.e. S_{Har}) are displayed in Fig. 5. As depicted in Fig.5, the presence of the harmonics reduces the available capacity such that the mean available capacities of the DTs with residential, commercial and industrial loads are obtained as 0.985, 0.975 and 0.960 p.u., respectively. Additionally, from another aspect, regarding to Eq. (19), these values are equivalent to the failure rates of 0.0508, 0.0513 and 0.0521, respectively and accordingly 1.6%, 2.6% and 4.2% increase in the initial failure rates, respectively. Evaluating the obtained results, it is comprehended that the associated harmonic contents of the load, in addition to its rated value, should be essentially taken into account by the DISCO's planners when selecting the DT's rated capacity.



(a) Residential load



(b) Commercial load



(c) Industrial load

Fig. 5 The available capacity of DT associated with the first 10 days of measurement in the presence of the harmonic (S_{Har}) for three kinds of loads.

5.3 Harmonics impacts on the aging process of three DTs

To investigate the effect of the load harmonics on the aging acceleration of DTs throughout their useful lifetime, a computational program has been developed within the MATLAB software. The step-by-step algorithm of the mentioned program is as follows:

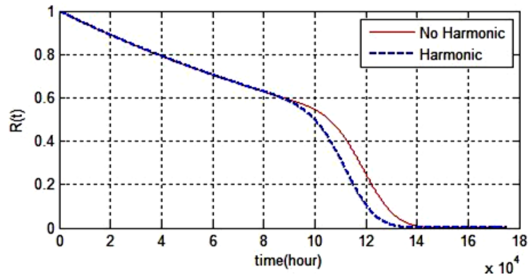
1- In each time period, based on the measurements and also using relations Eqs. (7)-(10), the F_{HL} and $F_{HL\text{-}STR}$ values are calculated for that time period.
2- The $P_{LL\text{-}H}$ value is calculated based on Eq. (12).
3- Using Eq. (24) and Eq. (26), the $\Delta\theta_{TO}$ value is computed.
4- Using Eq. (23) and Eq. (25), the $\Delta\theta_{HS}$ value is calculated.
5- Using the ambient temperature data and Eq. (22), the θ_H value is obtained.
6- The F_{aa} value is determined using Eq. (21).
7- Regarding to the value of $F_{aa\mathrm{cu}}$ presented in Eq. (31), $R_w(t)$ is calculated using standard normal distribution function.
8- Based on the value of λ' obtained by Eq. (19) and also $R_w(t)$, the overall reliability in that time interval is calculated using Eq. (29).
9- The steps 1 to 8 are repeated for the remaining time intervals till the end of evaluation period is reached.

For each time interval, the values associated to the end of the previous time interval are utilized as the initial values.

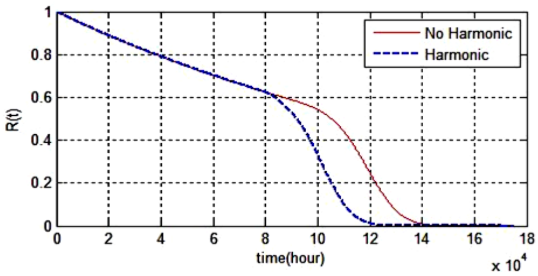
It is noted that for the sake of studying the impact of aging on the DT's overall reliability, an evaluation period longer than M should be taken into account. Therefore, far more data is needed rather than the data recorded merely for one year. For this reason, the DT's loading, including all harmonic levels are grown with the annual rate of 5% for each year of a 20-year evaluation period.

It should be remarked that if the loading of every DT exceeds its corresponding rated capacity (S_{Rated}), it is supposed that the excessive load will be transferred to another similar DT. In this case, the loading growth for that DT will be stopped at the beginning of the next year. Regarding to the above-mentioned explanation, the DTs' overall reliability with and without considering the harmonics effects is illustrated in Fig. 6.

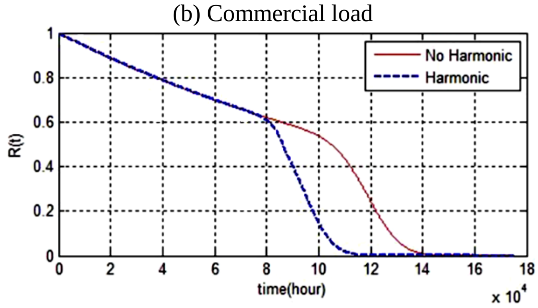
As seen in Fig.6, considering harmonics impact on the DT's overall reliability calculation, especially in the aging period, would lead to more realistic results compared to the case of no harmonics. For instance, after 120,000 and 140,000 hours operation in the case of no harmonics, the overall reliability of all three DTs decreases to 0.25206 and 0.01025, respectively. The details of the calculations are presented in Eq. (32).



(a) Residential load



(b) Commercial load



(c) Industrial load

Fig.6 The comparative diagram of the DTs' overall reliability with and without considering the harmonic.

$$
\begin{aligned}
&R(120,000) = \\
&\quad e^{-\left(0.05\times\frac{120,000}{8760}\right)} \times R_w(M) = 0.25206 \\
&R(140,000) = \\
&\quad e^{-\left(0.05\times\frac{140,000}{8760}\right)} \times R_w(M+2\sigma) = 0.01025
\end{aligned}
\tag{32}
$$

Also, when the impact of harmonics is considered, the overall reliability curve for three DTs is below the curve related to the case without harmonics consideration. Additionally, it is perceived that the DTs' reliability curves have various negative slops due to the different harmonics contents. In this way, the overall reliability curve has lower negative gradient for the residential DT because of the lower harmonics content while the corresponding curve for the industrial DT has the fastest drop. As another comparative result, the overall reliability values reduce to 0.25206 after (111,453), (103,776) and (95,738) hours for the residential, commercial and industrial DTs, respectively, while this time was 120,000 hours for all three DTs in the case of no harmonics.

6 Conclusion

In this paper, the impact of load harmonic contents on the DT's failure rate and overall reliability, including available capacity and aging acceleration factor, is mathematically modeled and extensively analyzed. Using the suggested model, the impact of load harmonics on the DT's overall reliability has been studied. To do so, one year recorded data pertaining to the harmonic contents of three distinct DTs feeding three different load types including residential, commercial and industrial have been used for the simulation study. The results of the studies show that the presence of the harmonics causes reduction in DT's available capacity which has been mathematically equalized with a new modified value for both DT's failure rate and lifetime. Moreover, the trend of harmonic contents illustrate that the more harmonic content, the more reduction in DT's available capacity and lifetime. Furthermore, incorporating the recorded harmonics data to the suggested reliability calculation model, the residential, commercial and industrial DTs' equivalent failure rates are increased by 1.6%, 2.6% and 4.2%, respectively. In addition, based on the DT's modified failure rate, the overall reliability of the residential, commercial and industrial DTs decreases to 0.25206 after (111,453), (103,776) and (95,738) hours of operation instead of 120,000 hours when no harmonics is considered. These results can help DISCOs to organize DTs' loading and to schedule their overhaul/replacement program more accurately.

Intellectual Property

The authors confirm that they have given due consideration to the protection of intellectual property associated with this work and that there are no impediments to publication, including the timing to publication, with respect to intellectual property.

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H. A. Shiraeh: Data Curation, Software and Simulation, Original Draft Preparation, **I. Ahmadi:** Idea & Conceptualization, Supervision, Analysis, Revise & Editing, Verification, **M. Mirzaie:** Revise & Editing, **M. Ahmadigorji:** Revise & Editing.

Declaration of Competing Interest

The authors hereby confirm that the submitted manuscript is an original work and has not been published so far, is not under consideration for publication by any other journal and will not be submitted to any other journal until the decision will be made by this journal. All authors have approved the manuscript and agree with its submission to "Iranian Journal of Electrical and Electronic Engineering".

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