Closed-Form Analytical Equations to Transient Analysis of Bang-Bang Phase-Locked Loops

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Abstract: In this paper an exact transient analysis of Bang-Bang PLLs (BBPLLs), as a nonlinear system, is presented. New equations are proposed for expression of transient behavior of the BBPLL with first order filter in response to phase step input. This approach gives new insights into the transient behavior of BBPLLs. Approximating transient response to reasonable specific waveform; the loop transient time characteristics such as locking time, peak time, rise time and over shoot are derived with acceptable accuracy. The validity of the resulted equations is verified through simulations using MATLAB SIMULINK. Simulation results show the high accuracy of the proposed method to model BBPLLs behavior.

Keywords: Bang-Bang PLL, Nonlinear Operation, Transient Analysis, CDR Circuits.

1 Introduction

The use of bang-bang phase locked loops has become increasingly common in a lot of communication systems, especially in clock and data recovery (CDR) systems [1]-[4]. Also these types of PLLs are widely used at the higher speed process. These systems use a bang-bang phase detector (BBPD) in the loop, which samples data as a part of the phase detection process, therefore there exists no systematic phase error between the data signals and the recovered clock signals. BBPD quantizes the phase error between input (data) and output (clock) with 1-bit resolution. For this reason, these types of the PLLs suffer hard nonlinearity. This behavior of the BBPLLs causes that the researchers make an effort to analyze it. BBPLL circuits must satisfy some specifications such as speed, stability, capture range and phase noise, posing challenges to system designers. Most researches focused on jitter analysis (see [5]-[10]) but there exists a few researches on the transient analysis. Design of the BBPLL in higher speeds is the most important reason to analyze the BBPLL in time domain. Transient behavior of a BBPLL in response to the phase and frequency step inputs is a main issue in time domain that this paper has focused on phase step response. Because of the nonlinear behavior of the BBPLL, these inputs cause responses with two different general forms depending on the input step size. Some simple equations have been presented to express the locking time of the BBPLL in [11] and [12]. These equations are limited only for particular conditions which is called cycle slipping. These researches are focused on the frequency step response. Also there exists no expression for other features of transient time; rise time, peak time, settling time and over shoots.

This paper presents a novel method to derive the closed form equations to clarify the transient behavior of the BBPLLs. In this method a parameterized waveform of the output excess phase is estimated and equations for unknown parameters are derived using differential equations governing on the BBPLL.

The rest of the paper is as follows: Section 2 introduces BBPLL model with first order filter and a brief review of the existing methods. Section 3 describes the proposed methodology to derive the equations for locking time, peak time, rise time and over shoot. A new technique based on an estimated model for output is presented to derive equations for transient response in this section. Simulation results will show validity of this method in section 4. Finally, section 5 gives conclusions.

2 BBPLL with First Order Filter

Consider two cases of the BBPLL; BBPLL with zero order filter and BBPLL with first order filter. References [2] and [5] have illustrated the operation of these two cases clearly. Due to the low speed and small capture range of the BBPLLs with zero order filter, their

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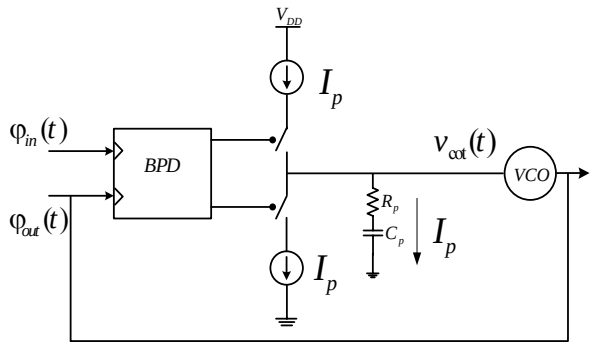

Fig. 1 BBPLL architecture with first order filter

use is very limit [2]. In contrary, the BBPLL with first order filter is noticeable. Figure 1 shows a BBPLL with first order filter. This structure adds an integral path to the loop filter of the system. This integral path provides bigger capture range and makes the loop faster. Because of these compelling advantages, the bang-bang loop with first order filter has become a common design choice for state-of-the CDR designs. For this reason, in this paper, we have focused on the BBPLL with first order loop filter used as a CDR.

As mentioned above, the purpose of this paper is to derive the all information of the transient time. At first, the previous works are briefly discussed. Only reference [2] has indicated phase step response of the BBPLL by phase trajectory with discrete time difference equations for BBPLL with first order filter as (1).

$$\frac{\theta(t)}{\theta_{bb}} = \Delta - (n - \frac{n^2}{\xi}) \tag{1}$$

In above equation $\theta(t)$ refers to excess phase difference, θ_{bb} indicates loop phase step that equals to $2\pi I_p R_p K_{vco} t_{update}$ and $n = t/t_{update}$ and $t_{update} = 1/f_c$. f_c is the central frequency of VCO. A stability factor, ξ is defined in (1) which equals to $2R_pC_p/t_{update}$ and a normalized transient phase step as $\Delta = \theta_{step}/\theta_{bb}$. In equation (1), the time required to reach steady state, given a step of Δ is always less than or equal to Δ time steps, independent of ξ [2]. The resulted phase trajectories show changes of oscillatory transient for the different values of ξ. These trajectories don't express the details of transient behavior of the BBPLL, because there are no equations to indicate features of transient time. Therefore it is critical to analysis the BBPLL in time domain and extract all important times.

Consider a BBPLL with first order filter shown in Figure 1. The BBPLL consist of a BBPD, charge pump with current $I_{p,}$ a loop filter with resistance R_p and capacitor C_P connected series, and voltage controlled oscillator (VCO) with gain K_{VCO}. The output voltage of bang-bang phase detector $v_{pd}(t)$ provides a binary form of voltage as $\text{sign}(\varphi_{in}(t) - \varphi_{out}(t))$ where it has indication of the phase difference between the input (data) and the output (clock). The parameters $\varphi_{in}(t)$ and $\varphi_{out}(t)$ refer to phase of input signal and the output signal of VCO, respectively. $v_{pd}(t)$ is $+1$ when the phase difference is positive and -1 when phase difference is negative. $v_{pd}(t)$ is shown as (2).

$$v_{pd}(t) = \text{sign}(\varphi_{in}(t) - \varphi_{out}(t)) \tag{2}$$

when $v_{pd}(t)$ is $+1$, I_p charges the loop filter and when is -1, discharges the loop filter. This model is the same BBPLL discussed in [4] and [5]. To introduce BBPLL, at the first we express the basic equations governing on the BBPLL. Note that the VCO used in Figure 1 is assumed linear. So we can write equation (3) for VCO.

$$\omega_{out}(t) = \omega_c + k_{vco} v_{cot}(t) \tag{3}$$

where $\omega_{out}(t)$, ω_c and $v_{cot}(t)$ refer to the output frequency, VCO central frequency and control voltage of VCO, respectively.

In the BBPLL based CDR, with the input data signal shown in Figure 2, in the steady state or locked condition the output phase (frequency) is twice of the input phase (frequency) [1]. So when the input phase (frequency) changes from its steady state condition, the PLL experiences a transient time and the output phase (frequency) reaches to a new steady state; twice of the input new phase (frequency). So in locked condition, the output phase (frequency) is twice of the input (data). So in locked condition at the VCO central frequency ω_c (nominal condition), we consider the input frequency equal to $\omega_c/2$ in equations governing on the BBPLL.

Figure 2 shows the input data and cock data with a phase error. It is seen from Figure 2 that the frequency of clock is twice of input data frequency. Using the relation between phase and frequency as $\varphi(t) = \int \omega(t)dt$ and introducing the phase step as $\Delta\varphi_{in}$, we can write the set of following equations for BBPLL. Input phase and output phase are defined as (4) and (5).

$$\varphi_{in}(t) = \int \omega_{in}(t)dt + \Delta\varphi_{in} u(t) = \frac{\omega_c}{2} t + \Delta\varphi_{in} u(t) \tag{4}$$

$$\varphi_{out}(t) = \int \omega_{out}(t)dt = \omega_c t + 2\pi k_{vco} \int v_{cot}(t)dt \tag{5}$$

Since at the nominal steady state, the phase error, $\varphi_{in}(t) - \varphi_{out}(t)$, is zero, we express the dynamical

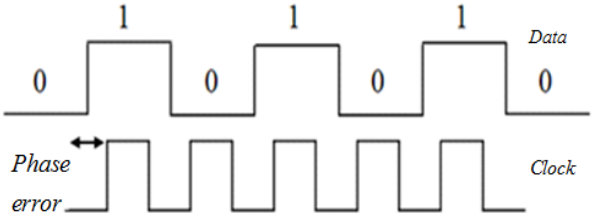

Fig. 2 Input (data) signal and output (clock) signal

equations of the BBPLL in term of the excess phases. So $\Delta\varphi_{in}u(t)$ is the input excess phase denoted by $\varphi_{in,ex}(t)$ and $K_{vco}\int v_{cot}(t)dt$ is the output excess phase, denoted by $\varphi_{out,ex}(t)$. So excess phase error is written as (6).

$$\begin{aligned}\varphi_e(t) &= \varphi_{in,ex}(t) - \varphi_{out,ex}(t) \\ &= \Delta\varphi_{in} - 2\pi k_{vco}\int v_{cot}(t)dt\end{aligned} \tag{6}$$

After locking, as mentioned above, the output phase is twice of the input, so according to (4) and (5), we have (7).

$$\omega_c t + 2\pi k_{vco}\int v_{cot}(t)dt = 2\left(\frac{\omega_c}{2}t + \Delta\varphi_{in}u(t)\right) \tag{7}$$

As we know, in the locked condition, the output frequency is twice of the input. Using this point along with equation (7), (6) should be rewritten as (8).

$$\varphi_e(t) = 2\Delta\varphi_{in} - 2\pi k_{vco}\int v_{cot}(t)dt \tag{8}$$

Or we can consider a factor of 1/2 for output excess phase ($K_{vco}\int v_{cot}(t)dt$) instead of factor of 2 for input excess phase ($\Delta\varphi_{in}u(t)$) in equation (8). Equation (8) guaranties that $\varphi_e(t)=0$ is equal to the locked condition. This also can be investigated and verified using simple simulations.

3 Proposed Method

3.1 General Procedure of the Proposed Method

The method proposed in this paper is based on the exact differential equations governing on the BBPLL. These equations are simply obtained using circuit analysis methods and the equations (1) to (8). At the beginning, we obtain $v_{cot}(t)$ by analyzing Figure 1 as (9).

$$v_{cot}(t) = R_p I_p \text{sign}(\varphi_e(t)) + \int \frac{I_p \text{sign}(\varphi_e(t))}{C_p}dt \tag{9}$$

In equation (9), the term $\text{sign}(\varphi_e(t))$ is the role of phase detector output on the filter charging current that is in the form of $\pm R_p I_p \pm (I_p/C_p)t$.

To derive the differential equation of $v_{cot}(t)$, we need to define $v_{pd}(t)$.

As mentioned before, the output voltage of BBPD ($v_{pd}(t)$) is as a sign function, this function imposes a hard nonlinearity on the loop. To overcome this problem, we introduce a proper function instead of sign. Because of discontinuity of sign at zero, we suggest replacing sign with another function which is continuous and also is very similar to sign function. So it is possible to apply integral and derivative operators on it. Among different alternatives, trigonometric functions such as $\tanh(\alpha\varphi_e(t))$ and $\arctan(\alpha\varphi_e(t))$ are selected, because these functions are more similar to sign function. See Figure 3, when α is increased $\tanh(\alpha\varphi_{ex}(t))$ converges to sign function. α is an input parameter. By changing α, we can make an acceptable approximation for sign function. We solve equation (9) by replacing sign function with $\tanh(\alpha\varphi_{ex}(t))$.

Substituting (8) into $v_{pd}(t)$ and $v_{pd}(t)$ into (9) and finally differentiating both sides of (9), results in differential equation of $v_{cot}(t)$ as follows:

$$\begin{aligned}&\frac{dv_{cot}(t)}{dt} = \\ &\alpha.I_p.R_p(1-\tanh^2(\alpha.(2\Delta\varphi_{in} - 2\pi K_{vco}\int v_{cot}(t).dt)))\mu_X \\ &\times(-2\pi K_{vco}.v_{cot}(t)) + \frac{I_p}{C_p}\tanh(\alpha.(2\Delta\varphi_{in} - 2\pi K_{vco}\int v_{cot}(t).dt))\end{aligned} \tag{10}$$

To solve (10), consider the set of following equations in state form:

$$\begin{aligned}x_1(t) &= 2\pi K_{vco}\int v_{cot}(t)dt \\ x_2(t) &= v_{cot}(t) \\ \frac{dx_1(t)}{dt} &= 2\pi K_{vco}x_2(t)\end{aligned} \tag{11}$$

Substituting the above equations into (10), result in a differential equation as

$$\begin{aligned}\frac{dx_2(t)}{dt} &= -\alpha.I_p.R_p(1-\tanh^2(\alpha.(2\Delta\varphi_{in} - x_1(t)))).\left(\frac{dx_1(t)}{dt}\right) \\ &+\frac{I_p}{C_p}\tanh(\alpha.(2\Delta\varphi_{in} - x_1(t)))\end{aligned} \tag{12}$$

$x_2(t)$ refers to the control voltage of oscillator and $x_1(t)$ indicates the output excess phase. To evaluate the effect of approximation of $\tanh(\alpha\varphi_{ex}(t))$ on the behavior of the system, we numerically solved the equation (12) in

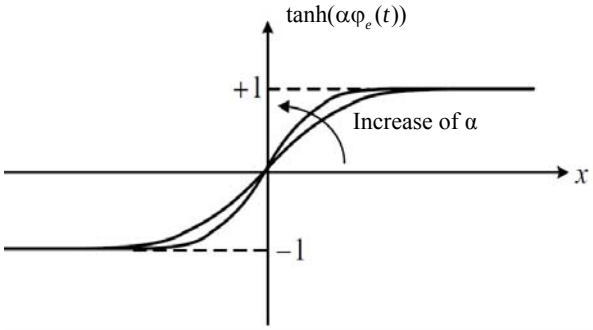

Fig. 3 $\tanh(\alpha\varphi_e(t))$ function compared with sign.

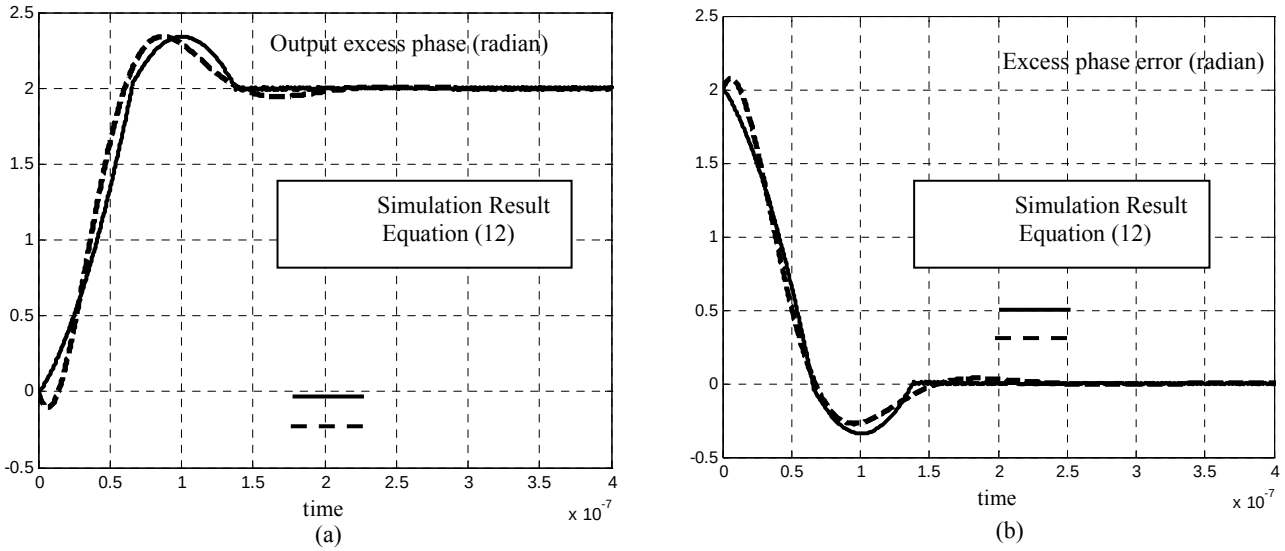

Fig. 4 Numerical Solution of the Eq. (12) for $\Delta\phi_{in}$= 1 rad. compared with simulation. (a) Output excess phase. (b) excess phase error.

MATLAB. Figure 4 (a) illustrates output excess phase and (b) shows the excess phase error ($2\Delta\varphi_{in} - x_1(t)$). The simulation results show the equation mentioned in (12) models the system behavior with acceptable accuracy. Since (12) is a nonlinear equation, the direct solution of the system is very complicated. So we introduce an approximate method.

Based on what reported in the previous works and many simulations performed (Fig. 4), we found out that the output excess phase transient has a well defined shape that can be simply approximated by an analytical function.

We solve equation (12) by fitting a reasonable waveform for $x_1(t)$. In a BBPLL, before applying a phase step ($\Delta\varphi_{in}$), the output excess phase ($x_1(t)$) is zero, according to what mentioned before, if a phase step enters, $x_1(t)$ reaches a constant value ($2\Delta\varphi_{in}$) after the oscillatory transient and after the locking point, the final value does not change, (see Fig.4). So we consider a damping oscillatory waveform as (13) to model the output excess phase. Equation (13) is the key simplifying assumption that leads to straightforward analysis along with accurate results. Equation (13) could be considered as an initial simple estimation for output excess phase response, it is necessary to improve the approximate solution using a simple approach. Assuming this, the analysis is devoted to find d, a, and b.

$$x_1(t) = d(1 - e^{-at}\sin(bt) - e^{-at}\cos(bt)) \qquad (13)$$

Regarding (12), it is necessary to find $dx_1(t)/dt$ and $dx_2(t)/dt$. Based on the equations indicated in (11) and (13), $dx_2(t)/dt$ and $dx_1(t)/dt$ can easily be obtained as (14) and (15), respectively shown at follows:

$$dx_2(t)/dt = \frac{d}{2\pi K_{vco}}(e^{-at}\sin(bt)(-2ab - a^2 + b^2) + e^{-at}\cos(bt)(-a^2 + b^2 + 2ab)) \qquad (14)$$

$$dx_1(t)/dt = de^{-at}\sin(bt)(b + a) + de^{-at}\cos(bt)(a - b) \qquad (15)$$

Using bit mathematical operations, (12)-(15) can be combined and equation (16) is resulted; the basic trigonometric equation of the proposed method.

$$\frac{d}{2\pi K_{vco}}(e^{-at}\sin(bt)(-2ab - a^2 + b^2) + e^{-at}\cos(bt)(-a^2 + b^2 + 2ab)) = -\alpha.I_p.R_p(1 - \tanh^2(\alpha.(2\Delta\varphi_{in} - d(1 - e^{-at}\cos(bt) - e^{-at}\sin(bt))))) \times de^{-at}\sin(bt)(b + a) + de^{-at}\cos(bt)(a - b) + \frac{I_p}{C_p}\tanh(\alpha.(2\Delta\varphi_{in} - d(1 - e^{-at}\cos(bt) - e^{-at}\sin(bt)))) \qquad (16)$$

Equation (16) holds for every time especially for the special times that we have:

$$t \to \infty \Rightarrow e^{-a.t}\cos(b.t) = 0 \; e^{-a.t}\sin(b.t) = 0 \qquad (17)$$

$$e^{-at}\sin(bt) = -e^{-at}\cos(bt) \qquad (18)$$

$$t = 0 \Rightarrow \quad e^{-a.t}\cos(b.t) = 1 \; e^{-a.t}\sin(b.t) = 0 \qquad (19)$$

The special times used above make it very simple to find the fitting parameters. Indeed these times leave simple equations to solve.

Substituting equation (17) into (16), we can find equation (20) for parameter d in term of input phase step.

$$\tanh(\alpha.(2\Delta\varphi_{in} - d)) = 0 \Rightarrow d = 2\Delta\varphi_{in} \qquad (20)$$

when the BBPLL is locked, the final value of output excess phase can be calculated by (20).

Substituting (18) into (16), a closed form equation is derived for parameter a in term of circuit parameters as

$$a = \frac{2\pi I_p R_p \alpha K_{vco}}{2} \tag{21}$$

It can be seen from (21) that the parameter a is directly proportional to I_p, R_p, K_{vco} and input parameter α.

To obtain b, we assume that the left side of (16) $(dx_2(t)/dt)$ is as (22).

$$\frac{dx_2(t)}{dt}_{new} = \frac{dx_2(t)}{dt}_{old} - d\frac{2ab + a^2}{2\pi K_{vco}} e^{-\beta t} \tag{22}$$

In added exponential term, β is so large. This assumption is used to provide a proper initial condition for the left side of (16) (equation (14)) at $t = 0$. Note that this assumption has no effect on other equations indicated in (20) and (21), it just leads to an appropriate response for parameter b. Considering assumption of (22) and substituting (20) in (16), we have

$$d(b^2 - 2a^2) = -2\pi\alpha . I_p . R_p K_{vco} \times (1 - \tanh^2(2\alpha\Delta\varphi_{in}$$

$$\times d(a - b) + 2\pi\frac{I_p K_{vco}}{C_p}\tanh(2\alpha\Delta\varphi_{in}) \tag{23}$$

Since $\tanh(2\alpha\Delta\varphi_{in}) \approx 1$ and knowing a and d, a closed-form equation for unknown parameter b can be derived as (24).

$$b = \sqrt{\frac{2\pi I_p K_{vco}}{dC_p} + 2a^2} \tag{24}$$

It can be seen from (24), b is a function of circuit parameters and input phase step.

To calculate d, a and b, we need to define the parameter α. As mentioned before, $\tanh(2\alpha\Delta\varphi_{in})$ is approximately equal to $+1$. We assume $\tanh(2\alpha\Delta\varphi_{in}) = 0.99999$ that results in (25) for parameter α.

$$\alpha = 3/\Delta\varphi_{in} \tag{25}$$

For simplicity, we assumed, α only depends on the input step. The expression (25) can help to find a proper α for each value of $\Delta\varphi_{in}$. The simulation results verify the accuracy of (25).

The closed form equations obtained in this section can represent the unknown parameters a, d and b just knowing the circuit parameters and the size of phase step input.

3.2 Transient Features of BBPLL

In previous section, we have approximated a mathematical model for output excess phase in response to phase step input. This model has been shown in (13). To describe the transient behavior of the BBPLL, we need to know the features of its transient time such as rise time, over shoot, peak time and settling time. Equation (13) is similar to the step response of a linear second order system (see Fig. 4(a)). According to [14], we can easily rewrite (13) as (26).

$$x_1(t) = d(1 - \sqrt{2}e^{-at}\sin(bt + \frac{\pi}{4})) \tag{26}$$

Using what mentioned in [14], the transient features of (26) can be easily obtained as following equations. Rise time can be written as (27).

$$t_r = \frac{\pi - \theta}{b} \Rightarrow t_r = \frac{3\pi}{4b} \tag{27}$$

where $\theta = \pi/4$. Equation (27) shows that the rise time is reversely proportional to parameter b. Also as illustrated in [14], the peak time can be defined as (28).

$$t_p = \frac{dx_1(t)}{dt} = 0 \Rightarrow t_p = \frac{\pi}{b} \tag{28}$$

The peak time presented in (28) is also reversely proportional to b. And finally the over shoot is found as

$$M_p = \frac{x_1(t_p) - x_1(\infty)}{x_1(\infty)} = e^{-\frac{a\pi}{b}} \tag{29}$$

From (29), it is seen that the over shoot is function of a and b. If we consider the tolerance, %5, hence the settling time can be derived as (30).

$$t_s = \frac{3}{a} \tag{30}$$

The settling time presented in (30) is reversely proportional to a. Resulted equations can manifest the transient behavior of the BBPLL clearly.

This method can describe the transient behavior of BBPLL for each input phase step and different values of design parameters. The proposed method in this paper can help BBPLL designers to design a BBPLL with higher speed and more stable in transient region.

4 Simulation Results

The proposed method to obtain the features of the BBPLL is evaluated by simulations for different values of input phase step $\Delta\varphi_{in}$, current I_p, resistance R_p, capacitor C_p, and K_{vco}.

In this paper a BBPLL based CDR with an Alexander PD is simulated in MATLAB simulink. A numerical method using MATLAB software is used to solve (12) for the case that $\Delta\varphi_{in}$ increases. Figure 5 (a) to (d) show the numerical and approximated solution of Eq. (12), and simulated results of output excess phase, when $\Delta\varphi_{in}$ increases. Figure 5(a) to (d) show results with different values of $\Delta\varphi_{in}$ = 0.8 rad, 1.0 rad, 1.2 rad

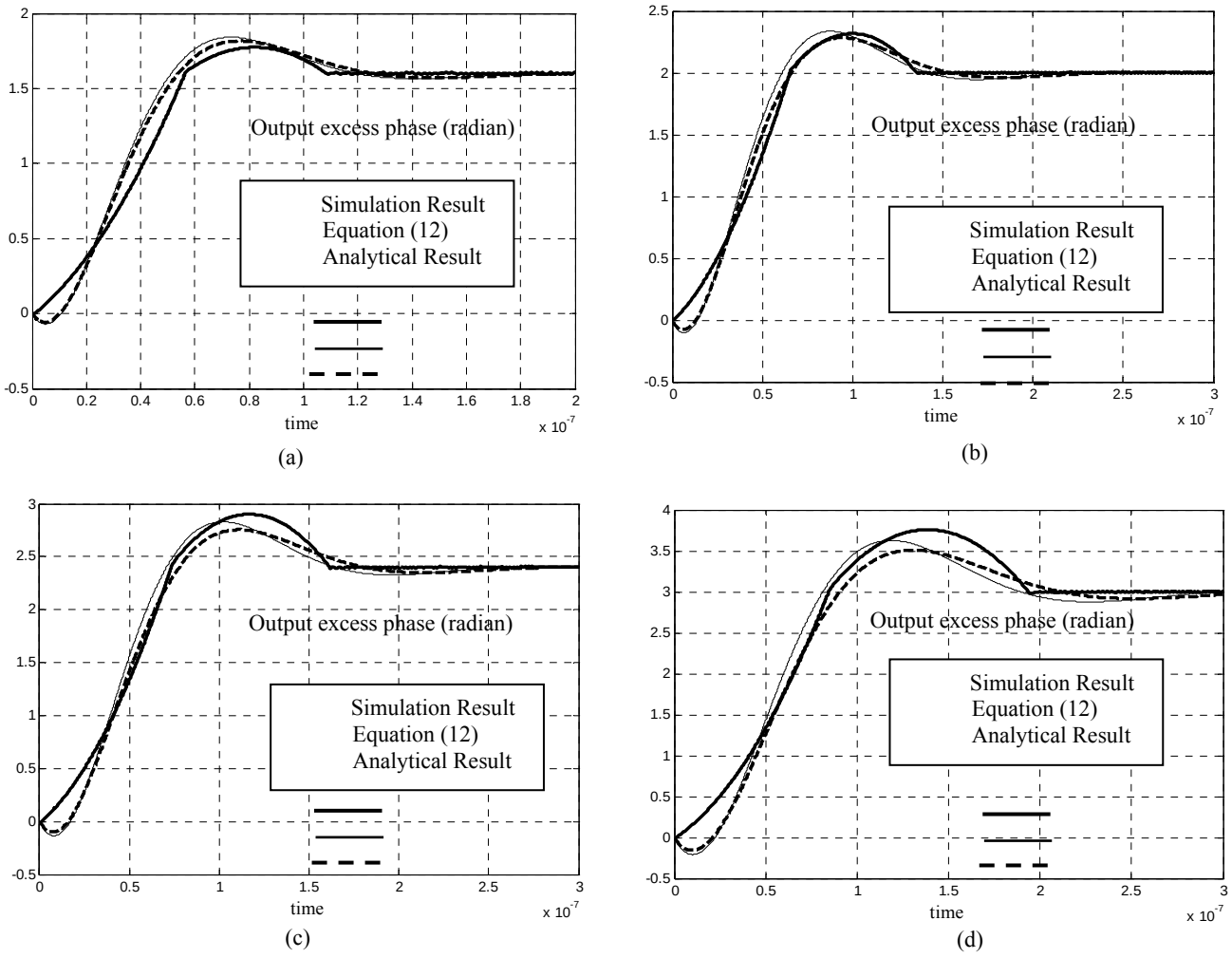

Fig. 5 Comparison between approximated and numerical results of Eq. (12) and simulation results for output excess phase when input phase step increases. (a) $\Delta\varphi_{in} = 0.8\text{rad}$, (b) $\Delta\varphi_{in} = 1\text{rad}$, (c) $\Delta\varphi_{in} = 1.2\text{rad}$, (d) $\Delta\varphi_{in} = 1.5\text{rad}$.

and 1.5 rad, respectively. First, the value of α is calculated for each phase step using (25). In second step the unknown parameters of the estimated output are calculated by (20), (21) and (24). Finally the transient features t_r, t_p, M_p, and t_s are calculated by (27)-(30). Expression (25) shows, each phase step results one α, this shows that the parameter a obtained in (21) depends on the input phase step; consequently, b is depended on the input phase step. Therefore all important times in transient region depend on the input phase step. Table 1 gives simulation parameters. A comparison between estimated and real values shows the accuracy of analytical equations. Estimated results and actual undergoes the variation of input phase step are given in Table 2. The numerical, estimated and simulation results presented in Table 2 and Figure 5 show, when $\Delta\varphi_{in}$ increases, the rise time, peak time, settling time and over shoot increase and this is exactly what we expect in the BBPLL.

Based on the presented plots and Table in this paper, the difference between precise and approximate solutions for transient features is negligibly small. Comparison between estimated and real values shows the accuracy of proposed equations.

Table 1 Simulation parameters.

Simulation Parameters	
Resistance R_p	300Ω
Capacitor C_p	100pf
Current I_p	40uA
VCO gain K_{VCO}	200MHz/v
VCO Frequency	2GHz

Table 2 Comparison between simulation, calculated and numerical results when input step increases.

Calculated Results				Numerical Result				Simulation Results				
t_s (μs)	t_p (μs)	t_r (μs)	M_p	t_s (μs)	t_p (μs)	t_r (μs)	M_p	t_s (μs)	t_p (μs)	t_r (μs)	M_p	$\Delta\varphi_{in}$ (rad)
0.1	0.072	0.054	0.13	0.15	0.069	0.049	0.14	0.1	0.08	0.056	0.1	0.8
0.132	0.088	0.066	0.136	0.137	0.082	0.06	0.142	0.136	0.094	0.065	0.15	1
0.16	0.1	0.078	0.142	0.18	0.097	0.07	0.149	0.162	0.12	0.073	0.2	1.2
0.199	0.126	0.095	0.15	0.22	0.01	0.085	0.18	0.196	0.13	0.085	0.23	1.5

5 Conclusions

This paper has presented a novel method to derive exact enough analytical equations for the transient response of BBPLL; rise time, settling time, peak time, and over shoot. The output excess phase is assumed a damping oscillatory waveform where the unknowns are parameters which result in rise time, peak time, settling time and over shoot. Nonlinear behavior of the BBPLL can be explained easily with the presented equations. This method can provide insight into BBPLL with different BBPDs which can help designers for better and much easier designs.

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