Analyzing Capacity Withholding in Oligopoly Electricity Markets Considering Forward Contracts and Demand Elasticity

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Abstract: In this paper capacity withholding in an oligopolistic electricity market that all Generation Companies (GenCos) bid in a Cournot model is analyzed and the capacity withheld index, the capacity distortion index and the price distortion index are obtained and formulated. Then a new index, Distortion-Withheld Index (DWI), is proposed in order to measure the potential ability of market for capacity withholding. In these indices the impact of demand elasticity on capacity withholding is considered and it is shown that demand elasticity plays an important role for capacity withholding and market power mitigation. Due to the significant role of forward contracts for market power mitigation and risk hedging in power markets, the impacts of these contracts on capacity withholding are considered. The effects of GenCos' strategic forward contracts on capacity withholding are also discussed. Moreover, the relationship between capacity withholding of GenCos and market price distortion is acquired. A two-settlement market including a forward market and a spot market is used to describe GenCos' strategic forward contracting and spot market competition.

Keywords: Market Power, Capacity Withholding, Forward Contract, Demand Elasticity.

1 Introduction

Along with the deregulation of power market, issues how to assess market power and how to mitigate it have been brought about. Market power to a seller is the ability to profitably maintain prices above competitive levels for a significant period of time. The two components of market power strategy are quantity withholding and financial withholding. Financial withholding is used when GenCos raise their bidding prices at a considerable level to obtain higher profits and quantity withholding (capacity withholding) is used when GenCos withhold capacity in order to push up the market clearing price [1]. However, as regulator approximately knows cost of a specific unit, financial withholding cannot be realized but capacity withholding can cause more expensive units to operate and raise the market clearing price [2]. According to the game theory, financial withholding is used in the Bertrand games and quantity withholding is used in the Cournot games.

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Market power can be measured using market power indices. The indices such as Herfindahl─Hirschman Index (HHI), Lerner Index (LI), Price-Cost-Mark-up index (PCMI), etc have been used in market power analysis.

The HHI is defined as the sum of the square of market shares of all suppliers and it is used to measure the market power exercising.

$$\mathrm{HHI} = \sum_{i=1}^{N} S_i^2 \tag{1}$$

where N is the number of GenCos and S_i is the percentage market share of GenCo i. Market power exists if the HHI is larger than 1000 in percentage basis. In [3] it has been shown that the HHI cannot reflect local market power. In order to solve this problem, System HHI and Group HHI are defined and applied in [4]. According to regulations set by Federal Energy Regulatory Council (FERC), a market participant can exercise market power only if he/she owns 20% or more of the total market share. In [5], this benchmark is used as a threshold for market power estimation.

The LI is the most common comparison index which assesses the market power comparing the levels of prices under imperfect and perfect competition. It is defined as follows:

$$\mathrm{LI}_i = \frac{\pi - \mathrm{MC_i}}{\pi} \tag{2}$$

where LI_i is the Lerner index for GenCo i, π and $\mathrm{MC_i}$ are market clearing price and marginal cost of GenCo i, respectively. If the LI of a GenCo is larger than zero it possesses the market power. PCMI is similar to LI.

$$\mathrm{PCMI}_i = \frac{\pi - \mathrm{MC_i}}{\mathrm{MC_i}} \tag{3}$$

Both the HHI and LI are usually used to measure system market power without considering forward contracts.

Other indices such as Residual Supply Index (RSI) and Local Market Power Index (LMPI) are also introduced. The RSI for GenCo j is defined as follows

$$\mathrm{RSI}_j = \frac{\sum_{i=1}^{N} \mathrm{AC_i} - \mathrm{AC_j}}{\mathrm{Y}} \tag{4}$$

where $\mathrm{AC_j}$ is the available capacity of GenCo j and Y is the total demand. This index is developed and applied in [6]. In [7], LPMI is calculated according to market concentration and demand-supply ratio.

All of the indices which were mentioned above can not detect the GenCos' capacity withholding. In [8] the Withholding Capacity Ratio (WCR) is studied.

$$\mathrm{WCR}_i = 1 - \frac{\mathrm{BP_i}}{\mathrm{AC_i}} \tag{5}$$

where $\mathrm{BP_i}$ is the bid power of GenCo i. The WCR reflects how much capacity is not bided in the market, ranges from 0 to 1. The WCR is an unreliable index for capacity withholding measuring because some parameters are ignored in it, such as demand elasticity and GenCos' generation costs. Demand elasticity can importantly affect the market performance contributing to mitigate the strategic behavior of the producers [9]. For example, the thermal units use banking approach in off-peak hours that electrical energy consumption is lower than the peak hours and demand elasticity is high to avoid paying cold start up costs in the peak hours. The banking approach requires that sufficient energy be input to boiler to just maintain operating temperature [10]. With this approach the available capacity increases and then the WCR increases toward 1. In fact, there is no capacity withholding and the growth of WCR is because of low electrical energy consumption in off-peak hours, high demand elasticity and the banking approach of thermal units.

However, issues such as the FERC recommendation to use a fixed threshold to determine the presence of market power and the lack of accounting for demand elasticity have been proved inadequate in actual operation [11]. Up to now, some of research works on the strategic bidding behaviors of GenCos considered the demand side as given demand curves [12-13]. Only a few have considered the impacts of demand elasticity on the strategic behavior in the electricity markets [9].

On the other hand, it is widely recognized that forward contracts play an important role as means for market power mitigation in power markets. Theoretical analyses of forward and spot market interactions have been presented in many references [14-15]. Allaz and Vila [16] argued that for a symmetric duopoly forward trading could make the spot market more competitive. Unfortunately, forward contracts are overlooked in the market power indices and many of the research works on capacity withholding have not considered the influences of forward contracts on capacity withholding in the power markets [17-20]. Moreover, there are some contracts, in which both the spot and forward sales of GenCos are taken as strategic variables. These contracts referred to as strategic forward contracts, which are considered in this paper.

Therefore, three problems in capacity withholding assessment are needed to be studied further.

First, the effects of demand elasticity on capacity withholding needs more study.

Second, the relation between price distortion and capacity withholding has not been modeled yet.

Third, the impacts of forward contracts on capacity withholding have not been assessed well yet.

In this paper with comparison the perfectly competitive market and the oligopoly market that all GenCos bid in the Cournot model, the capacities of GenCos that are shutdown and withheld from the market are computed and the impacts of demand elasticity and forward contracts on capacity withholding are assessed.

This paper is organized as follows. The Capacity withheld index ($\Delta y_i^{\mathrm{withheld}}$), the capacity distortion index ($\Delta y_i^{\mathrm{distort}}$) and the price distortion index ($\Delta\pi^{\mathrm{distort}}$) are defined in next section. In section 3, mathematical formulations are presented. In section 3-1 capacity withholding in oligopoly electricity markets is analyzed for spot markets and the related indices are acquired. In section 3-2, the impacts of forward contracts on capacity withholding are considered and in a two-settlement market including a spot market and a forward market are modeled and formulated. In section 3-3 the impacts of strategic contracting in the forward market on capacity withholding are analyzed. Numerical results are analyzed in section 4. In section 5 a larger test system is used and comparison between HHI and DWI is presented. The conclusions are summarized in section 6.

2 The Definition of Capacity Withheld Index, Capacity Distortion Index and Price Distortion Index

The parameters $\Delta y_i^{withheld}$, $\Delta y_i^{distort}$ and $\Delta\pi^{distort}$ are defined as follows:

$$\Delta y_i^{withheld} = y_i^p(\pi^e) - y_i^e \quad (6)$$

$$\Delta y_i^{distort} = y_i^p - y_i^e \quad (7)$$

$$\Delta\pi^{distort} = \pi^e - \pi^p \quad (8)$$

where y_i^p is the competitive output of GenCo i and π^p is the competitive market clearing price. After capacity withholding, the output of the GenCo i decreases to y_i^e and the market clearing price increases to π^e. $y_i^p(\pi^e)$ is the competitive output which would be produced at π^e by GenCo i. In a market with N GenCos, $\Delta Y^{withheld} = \sum_{i=1}^{N} y_i^{withheld}$ and $\Delta Y^{distort} = \sum_{i=1}^{N} \Delta y_i^{distort}$.

Figure 1 presents these indices. To simplify the illustration, it is assumed in Fig. 1 that there is only one GenCo in the market. Therefore the subscript *i* is omitted from y_i^p, y_i^e and $y_i^p(\pi^e)$. The capacity withheld index shows that how much the supply curve is shifted to left. It can be observed that the capacity distortion is considerably smaller than the capacity of the GenCo that is withheld, as shown in Fig. 1.

In order to solve three problems that were mentioned above, these indices are extended and a new integrated index, called DWI, is proposed in next section.

3 Mathematical Formulation

3.1 Spot Market

Suppose there are N GenCos in the electricity market. Each GenCo has a generator and is characterized by the following quadratic cost function

$$C_i = \frac{a_i}{2} y_i^2 + b_i y_i \quad (9)$$

where y_i is the quantity generated by GenCo i; a_i and b_i are the coefficients of the cost function of GenCo i. The marginal cost of GenCo i is defined as follows:

$$MC_i = \frac{dC_i}{dy_i} = a_i y_i + b_i \quad (10)$$

The aggregate demand function is

$$\pi = -\alpha Y + \beta, \qquad \alpha > 0 \quad (11)$$

where $Y = \sum_{i=1}^{N} y_i$ and there is negligible transmission loss. α and β are coefficients of the aggregate demand function. In a perfectly competitive market, all GenCos compete with each other, and each of them is a price taker. Each GenCo should increase its generation up to the point where its marginal cost is equal to the market price as shown in the following:

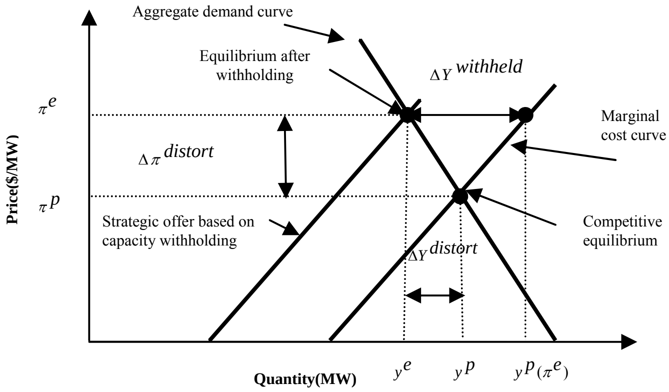


Fig. 1 Illustration of capacity withheld index, capacity distortion index and price distortion index.

$$\pi = a_i y_i + b_i \quad (12)$$

Imperfect competition can be modeled using either a Cournot model or a Bertrand model. We consider the Cournot model, in which GenCos decide how much they produce. GenCo i, like other GenCos, seeks to maximize its profit (Ω_i) and the optimization problem faced by each GenCo, expressed as follows:

$$\Omega_i = y_i \pi - C_i \quad (13)$$

The derivative of profit of GenCo i with respect to its decision variable (y_i) can be written as:

The derivative of profit of GenCo *i* with respect to its decision variable (y_i) can be written as:

$$\frac{\partial\Omega_i}{\partial y_i} = y_i \frac{\partial\pi}{\partial y_i} + \pi - MC_i \quad (14)$$

By setting (14) to zero it can be written as:

$$\pi + y_i \frac{\partial\pi}{\partial y_i} = MC_i \quad (15)$$

In [21], it has been shown that (15) can be written as:

$$\pi\left(1 - \frac{S_i}{|\varepsilon|}\right) = MC_i \quad (16)$$

where S_i is the market share of GenCo *i* ($S_i = y_i / Y$) and ε is the demand elasticity ($\varepsilon = \partial y / \partial\pi \times \pi / Y$).

3.1.1 Capacity Withholding Index

In order to assess capacity withholding, we use the comparison indices, in which market outcomes in actual markets are compared with perfect competition. For the specific features of the power markets, the actual power markets may be better described in terms of oligopoly. In this paper we use the oligopoly market that all GenCos bid in the Cournot model, as the actual power market.

According to (12), $y_i^p(\pi^e)$ can be written as:

$$y_i^p(\pi^e) = \frac{\pi^e - b_i}{a_i} \quad (17)$$

By using (16), the actual output of GenCo i can be obtained by:

$$y_i^e = \frac{\pi^e\left(1 - \frac{S_i}{|\varepsilon|}\right) - b_i}{a_i} \quad (18)$$

Substituting (17) and (18) into (6) yields:

$$\Delta y_i^{withheld} = \frac{\pi^e \frac{S_i}{|\varepsilon|}}{a_i} \quad (19)$$

By using the aggregate demand function in (11), S_i and ε can be written as:

$$S_i = \frac{y_i^e}{\frac{\beta}{\alpha} - \frac{\pi^e}{\alpha}} \quad (20)$$

$$\varepsilon = -\frac{1}{\alpha} \times \frac{\pi^e}{\frac{\beta}{\alpha} - \frac{\pi^e}{\alpha}} \quad (21)$$

Substituting (20) and (21) into (19) yields:

$$\Delta y_i^{withheld} = \frac{\alpha}{a_i} y_i^e \quad (22)$$

The fraction of $\Delta y_i^{withheld}$ and y_i^e can be written as:

$$\frac{\Delta y_i^{withheld}}{y_i^e} = \frac{\alpha}{a_i} \quad (23)$$

According to (23) in an oligopoly market with Cournot model, α / a_i of the generation power of each GenCo is withheld from the spot market. As explained above, demand elasticity has a significant role to reduce the capacity withholding of GenCos. When α is decreasing, the demand becomes more elastic ($\varepsilon \propto 1/\alpha$) and then the fraction in (23) becomes smaller (the capacity withheld of GenCo i is decreases). It can also be observed that the fraction is affected by the slope of GenCos' marginal cost functions. If a_i is larger than a_j (for any i and $j \in 1,\dots,N$), α / a_i is smaller than α / a_j. According to (23), $\Delta y_i^{whithheld} / y_i^e < \Delta y_j^{whithheld} / y_j^e$ can be obtained. It means that a GenCo with a large slope of marginal cost function will have less incentive to withhold the capacity.

Moreover, the term α / a_i can be used as a criterion to determine that which player is more strategic than other players and it can be an alternative for the FERC recommendation. In fact, a GenCo with large α / a_i will have more incentive to exercise market power.

3.1.2 Capacity Withholding Index

According to (12) y_i^p can be written as:

$$y_i^p = \frac{\pi^p - b_i}{a_i} \quad (24)$$

Substituting (24) and (18) into (7) yields:

$$\Delta y_i^{distort} = \frac{\pi^e \frac{S_i}{|\varepsilon|}}{a_i} - \frac{\pi^e - \pi^p}{a_i} \quad (25)$$

According to (8), the second term on the right-hand side of (25) is $\Delta\pi^{distort}$. Substituting (20) and (21) into (25) yields:

$$\Delta y_i^{distort} = \frac{\alpha}{a_i} y_i^e - \frac{\Delta\pi^{distort}}{a_i} \quad (26)$$

Equation (27) is obtained by using (11).

$$\Delta\pi^{distort} = \pi^e - \pi^p = -\alpha Y^e + \alpha Y^p = \alpha \Delta Y^{distort} \quad (27)$$

where $Y^p = \sum_{i=1}^{N} y_i^p$, $Y^e = \sum_{i=1}^{N} y_i^e$ and $\Delta Y^{distort} = Y^p - Y^e$. Ideally, with the increase of $\Delta Y^{distort}$, the expected result should be increase in $\Delta\pi^{distort}$. Moreover, $\Delta\pi^{distort}$ is influenced by the slope of the aggregate demand function. It means that in a market with a large slope of demand function, the GenCos will have more incentive to lift up the market price.

Substituting (27) in to (26) yields:

$$\Delta y_i^{distort} = \frac{\alpha}{a_i}(y_i^e - \Delta Y^{distort}) \quad (28)$$

The value of $\Delta y_i^{distort}$ may be positive or negative. For inexpensive GenCos (GenCos with large α / a_i, with respect to other GenCos in the market), $\Delta y_i^{distort}$ can be positive. Positive values of $\Delta y_i^{distort}$ implies that for these GenCos, $y_i^e > \Delta Y^{distort}$ or $y_i^e < y_i^p$ (see (7)). In fact, these GenCos can indirectly control the market clearing price and make the expensive GenCos (GenCos with small α / a_i) become the marginal GenCos by capacity withholding. It may cause the expensive GenCos generate more than their competitive generations (y_i^p). Then for the expensive GenCos $y_i^e > y_i^p$ and $\Delta y_i^{distort} < 0$ may be obtained.

The $\Delta Y^{distort}$ can be also defined as the sum of the capacity distortions of all GenCos:

$$\Delta Y^{distort} = \sum_{i=1}^{N} \frac{\alpha}{a_i} y_i^e - \sum_{i=1}^{N} \frac{\alpha}{a_i} \Delta Y^{distort} \quad (29)$$

Equation (29) can be written as:

$$\Delta Y^{distort} = \frac{\sum_{i=1}^{N} \frac{\alpha}{a_i} y_i^e}{1 + \sum_{i=1}^{N} \frac{\alpha}{a_i}} \quad (30)$$

A new index to assess the potential ability of market for capacity withholding is addressed here. This index named Distortion-Withheld Index (DWI). DWI is a new index to identify $\Delta Y^{distort}$ - $\Delta Y^{withheld}$ ratio. According to (30), DWI is expressed as:

$$DWI = \frac{\Delta Y^{distort}}{\Delta Y^{withheld}} = \frac{1}{1 + \sum_{i=1}^{N} \frac{\alpha}{a_i}} \quad (31)$$

where $\Delta Y^{withheld} = \sum_{i=1}^{N} \alpha / a_i \times y_i^e$. DWI ranges between 0 and 1, which means $\Delta Y^{distort}$ is smaller than $\Delta Y^{withheld}$. When DWI is lower, it is more likely that the market has more potential ability for capacity withholding. In contrast, the higher DWI, the market is closer to competition.

In order to assess the impact of elasticity of demand on DWI, we suppose that the slope of aggregate demand function is decreased from α to α'. When α is decreasing, the DWI becomes larger ($\Delta Y^{distort}$ - $\Delta Y^{withheld}$ ratio is becomes larger). Graphically, the more elastic demand curve (the demand curve with α') will be located in lower position with respect to the demand curve with α. The fact that consumers are more responsive to price changes has an effect on the optimal behavior of producers, which will find it better to decrease their capacity withholding. This is shown in Fig. 2 by the shift from strategic offer 1 to the strategic offer 2. As a result of the modifications in both demand and supply curves, the price distortion is decreased from $\Delta\pi^{distort}$ to $\Delta\pi'^{distort}$. The capacity distortion is decreased form $\Delta Y^{distort}$ to $\Delta Y'^{distort}$ and the capacity withheld is decreased from $\Delta Y^{withheld}$ to $\Delta Y'^{withheld}$. Therefore the increase in DWI is coherent with increase in competition and decrease in capacity withholding.

After clearing the day ahead electricity market by market operator, all necessary data, such as GenCos' bid data, for calculating the DWI index are available. This index can be used for monitoring of market competition level. Decrease in DWI is coherent with increase in market power. Market power can be prevented by competition or by monitoring and enforcement. Competition is preferable but does not automatically reach satisfactory levels. The two key determinates of the competitiveness of a power market are demand elasticity and the extent of forward contracting. From market operator's point of view for a market with low DWI forward contracting should be encouraged rather than inhibited. The most effective form of forward contracting is long-term forward contracting and market operator can use appropriate signals to encouraging market players to forward contracting. On other hand from market designer's point of view demand elasticity should be increased for a market with low levels of DWI.

3.2 Two-Settlement Market

In this paper we use the two-stage game model to formulate the two-settlement market consisting of a forward contract and a spot market.

In the first stage, GenCos enter forward market and compete with each other in the forward market by choosing the quantity of their forward contracts (y_i^c), which they are willing to sell at the forward market price (π_i^c). Suppose that the forward contracts are observable for all GenCos in spot market. In the second stage, the GenCos bid in the spot market by using the data observed in the forward market [17]. The Cournot model is employed to model the spot market and the GenCos are risk neutral. In this section, it is assumed that the forward contracts are not the strategic contracts.

The GenCo i's optimization problem can be determined by:

$$\max\left\{\Omega_i = \pi(y_i - y_i^c) + \pi_i^c y_i^c - C_i\right\} \quad (32)$$

$$\frac{\partial \Omega_i}{\partial y_i} = (y_i - y_i^c)\frac{\partial \pi}{\partial y_i} + \pi - MC_i \quad (33)$$

Notice that the generation of GenCo i in the spot market is $y_i - y_i^c$ and $y_i > y_i^c$. By setting the derivative of GenCo i's profit with respect to its decision variable (y_i) as zero, (33) can be written as:

$$\pi + (y_i - y_i^c)\frac{\partial \pi}{\partial y_i} = MC_i \quad (34)$$

Equation (34) is similar to (15). It can be written as:

$$\pi\left(1 - \frac{S_i'}{|\varepsilon|}\right) = MC_i \quad (35)$$

where S_i' is the spot market share of GenCo *i*.

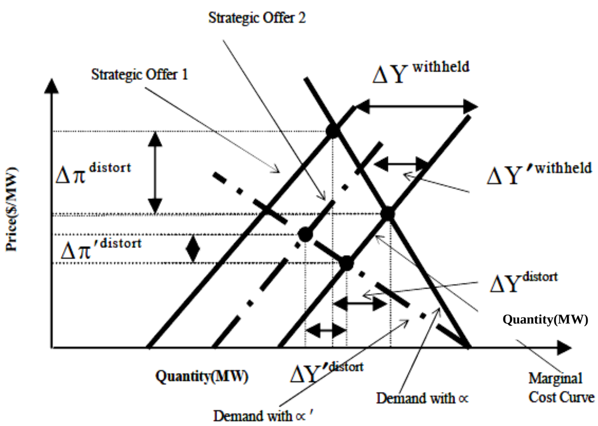


Fig. 2 Impact of the increased demand elasticity on capacity withholding of market.

3.2.1 Capacity Withheld Index

According to (35) y_i^e can be determined by:

$$y_i^e = \frac{\pi^e\left(1-\frac{S_i'}{|\varepsilon|}\right)-b_i}{a_i} \quad (36)$$

Substituting (17) and (36) into (6) yields:

$$\Delta y_i^{withheld} = \frac{\pi^e \frac{S_i'}{|\varepsilon|}}{a_i} \quad (37)$$

where

$$S_i' = \frac{y_i^e - y_i^c}{\frac{\beta}{\alpha}-\frac{\pi^e}{\alpha}} \quad (38)$$

Substituting (38) and (21) into (37) yields:

$$\Delta y_i^{withheld} = \frac{\alpha}{a_i}(y_i^e - y_i^c) \quad (39)$$

It can be observed that due to the role of forward contracts for market power mitigation $\Delta y_i^{withheld}$ is decreased. In other words a GenCo with a large y_i^c will have less incentive to withhold capacity in the spot market.

3.2.2 Capacity Distortion Index and Price Distortion Index

By Substituting (36) and (18) into (7), the capacity distortion index can be determined by:

$$\Delta y_i^{distort} = \frac{\alpha}{a_i}(y_i^e - y_i^c - \Delta Y^{distort}) \quad (40)$$

The capacity distortion index and DWI can be written as:

$$\Delta Y^{distort} = \frac{\sum_{i=1}^{N}\frac{\alpha}{a_i}(y_i^e - y_i^c)}{1+\sum_{i=1}^{N}\frac{\alpha}{a_i}}$$

$$DWI == \frac{\Delta Y^{distort}}{\Delta Y^{withheld}} = \frac{1}{1+\sum_{i=1}^{N}\frac{\alpha}{a_i}} \quad (41)$$

According to (41), due to the forward contract, both $\Delta Y^{withheld}$ and $\Delta Y^{distort}$ are decreased but the $\Delta Y^{distort}$-$\Delta Y^{withheld}$ ratio (DWI) is constant. Hence, the DWI reflects the market's potential ability for capacity withholding as it only depends on the slope of aggregate demand function and the slope of GenCos' marginal cost functions.

The price distortion index can be obtained by:

$$\Delta\pi^{distort} = \alpha\Delta Y^{distort} = \alpha\frac{\sum_{i=1}^{N}\frac{\alpha}{a_i}(y_i^e - y_i^c)}{1+\sum_{i=1}^{N}\frac{\alpha}{a_i}} \quad (42)$$

Equation (42) shows the impact of the forward contract on the spot market clearing price. With the increase of y_i^c the GenCos will have less incentive to raise the market clearing price in spot market.

3.3 Two-Settlement Market with Strategic Contracting in Forward Market

In this section it is assumed that in the forward market, each GenCo aims to maximize its profit by choosing its forward sales (y_i^c), taking into account the impacts of these forward sales on the spot market decisions. These forward contracts are settled financially in the spot market. In the forward market we assumed that all GenCos are risk neutral and there are enough arbitrageurs who are risk neutral. The forward price will be an unbiased estimator of the spot market price. The GenCos' competition in the market is assumed to be in a Cournot manner. Each GenCo solves the following problem to choose its forward market output so that to maximize its profit.

$$\max\left\{\Omega_i = \pi(y_i - y_i^c) + \pi_i^c y_i^c - C_i\right\}$$

$$\text{s.t.} \quad (43)$$

$$\pi_i^c = \pi$$

In the first stage the decision variable is y_i^c and in the second stage the decision variable is y_i. This problem has been solved in [22]. The relationship between y_i^c and y_i can be determined by:

$$y_i^c = \frac{A_i}{1+A_i}y_i \quad (44)$$

where

$$A_i = \sum_{\substack{j=1\\ j\neq i}}^{N}\left(\frac{\alpha}{\alpha + a_j}\right) \quad (45)$$

3.3.1 Capacity Withheld Index

According to (44) y_i^c can be written as:

$$y_i^c = \frac{A_i}{1+A_i}y_i^e \quad (46)$$

Substituting (46) into (39) yields:

$$\Delta y_i^{withheld} = \frac{\alpha}{a_i}y_i^e \times \frac{1}{1+A_i} \quad (47)$$

According to (45), $0 < 1/(1+A_i) < 1$ can be obtained. In (47), the coefficient $1/(1+A_i)$ is an attenuation factor that decreases the capacity withholding of the GenCos. Moreover, the term $1/(1+A_i)$ helps to moderate the increase in the capacity withholding of GenCos due to the higher demand curve associated to the decreased demand elasticity. With the decrease of the demand elasticity, the expected result should be the increase in the capacity withholding in the market. According to (45) with the increase of α, A_i

becomes larger and the coefficient $1/(1+A_i)$ becomes smaller. It means that this attenuation factor can moderate the impact of demand elasticity reduction on the capacity withholding of GenCos. Furthermore, the attenuation factor is influenced by the slope of GenCos' marginal cost functions. The attenuation factor of a GenCo with a large slope of marginal cost is larger than the attenuation factor of a GenCo with a smaller one.

3.3.2 Capacity Distortion Index and Price Distortion Index

By substituting (46) into (40), the capacity distortion index can be obtained by:

$$\Delta y_i^{\text{distort}} = \frac{\alpha}{a_i}\left(\frac{1}{1+A_i} y_i^e - \Delta Y^{\text{distort}}\right) \tag{48}$$

where

$$\Delta Y^{\text{distort}} = \frac{\sum_{i=1}^{N} \frac{\alpha}{a_i}\left(\frac{1}{1+A_i} y_i^e\right)}{1+\sum_{i=1}^{N} \frac{\alpha}{a_i}}$$

$$\text{DWI} == \frac{\Delta Y^{\text{distort}}}{\Delta Y^{\text{withheld}}} = \frac{1}{1+\sum_{i=1}^{N} \frac{\alpha}{a_i}} \tag{49}$$

The price distortion index can be determined by:

$$\Delta \pi^{\text{distort}} = \alpha \Delta Y^{\text{distort}} = \alpha \frac{\sum_{i=1}^{N} \frac{\alpha}{a_i}\left(\frac{1}{1+A_i} y_i^e\right)}{1+\sum_{i=1}^{N} \frac{\alpha}{a_i}} \tag{50}$$

It is obvious that due to the attenuation factor $1/(1+A_i)$, $\Delta y_i^{\text{distort}}$, $\Delta Y^{\text{distort}}$ and $\Delta \pi^{\text{distort}}$ are decreased considerably.

4 Numerical Results

In this section, a market with three GenCos in [22] is used to validate the formulas and theoretical analyses in section 3 and the effectiveness of the capacity withheld index, capacity distortion index, price distortion index and DWI, in detection and measurement the capacity withholding of GenCos in the market. There are three case studies for this market. In Case A, there is the Cournot type competition in the spot market without the forward contracts. In Case B, there is the Cournot type competition in the spot market with the forward contracts. The Cournot type competition in the spot market with the strategic forward contracts is considered in Case C.

The cost coefficients of the three GenCos market are listed in Table 1. In this market the aggregate demand parameters in (11) are $\alpha = 2$ \$/(MWh.GWh) and $\beta = 90$\$/MWh. In [22], a co-evolutionary genetic algorithm (CGA) is employed to determine the market equilibrium in different simulation cases without and with forward contracts.

The capacity withheld index, capacity distortion index, price distortion index and DWI of Case A, Case B and Case C are listed in Tables 2 and 3.

In all cases it can be observed that a GenCo with a larger α/a_i will have more incentive to withhold capacity. When comparing Cases B and C with Case A, it is clear that the market price is lower and generation output increases when GenCos enter the forward market. The market price in Case B is higher than Case C and generation output in Case C is larger than Case B. The effect of the strategic forward contracts on capacity withholding and price distortion is studied by the attenuation factor $1/(1+A_i)$. By comparing Case B with Case C it can be observed that $\Delta\pi^{\text{distort}}$ is lower and $\Delta Y^{\text{withheld}}$ decreases when GenCos use strategic forward contracts. Notice that the DWI in all cases is constant.

The impacts of the demand function's slope on capacity withholding also studied. Cases C_1 and C_2 are the same as Case C expect for $\alpha = 4$ and $\alpha = 1$ respectively. The simulation results and capacity withholding indices of Cases C_1 and C_2 are listed in Tables 4 and 5 respectively. In Case C_1 with the increase of α the capacity withheld index and capacity distortion index become larger and the price distortion is higher than Case C. It can be observed that the capacity distortion of GenCo 3 is negative. It means that this GenCo prefers to generate more than its competitive output. In Case C_1 the DWI is smaller than Case C_2 and the GenCos become less competitive.

To investigate the impacts of cost parameters on the capacity withholding, Cases D_1 and D_2 are performed. Cases D_1 and D_2 are the same as Case C except for $a_1 = 2$ and $b_1 = 8$, respectively. The simulation results and capacity withholding indices of Case D_1 and D_2 are shown in Tables 4 and 5 respectively. In Case D_1, $\alpha/a_1 = \alpha/a_3$ and $A_1 = A_3$. The ability of GenCo 1 for capacity withholding is the same as GenCo 3. But $y_1 < y_3$ and $\Delta y_1^{\text{withheld}} < \Delta y_3^{\text{withheld}}$. In this Case DWI is increased with respect to Case C and $\Delta Y^{\text{distort}}$, $\Delta Y^{\text{withheld}}$ and $\Delta\pi^{\text{distort}}$ are decreased. By comparing Cases $D_1$and C, it can be observed that the GenCos become more competitive in Case D_1. The DWI in Cases D_2 and C is equal. It means that the GenCos' withholding ability in Case D_2 is the same as Case C. The GenCo 1 in Case D_2 becomes cheaper than Case C and the output of market in Case D_2 is slightly higher than Case C. Then $\Delta Y^{\text{distort}}$, $\Delta Y^{\text{withheld}}$ and $\Delta\pi^{\text{distort}}$ in Case D_2 are slightly higher than Case C.

5 Comparison between HHI and DWI

We apply the obtained indices to a five GenCos test system to get some insights on the outcomes of larger system and to show the applicability to real systems. In this market $\alpha = 10$£/(MWh.GWh) and $\beta = 350$£/MWh. The cost parameters of five GenCos which are based on the cost data for the five strategic firms in England and Wales subsequent to 1999 [23], are shown in Table 6. The simulation results and the capacity withholding indices for Cournot type competition in five GenCos test system (Case E) are listed in Tables 7 and 8.

According to Tables 2 and 7 the values of DWI for Cases A and E are 0.18 and 0.05, respectively.It means that market power and capacity withholding in the three GenCos system is lower than the five GenCos system. Moreover, the values of $\Delta\pi^{distort}$ for Cases A and E are 13.96$/MWh and 53.5£/MWh. It means that the effect of capacity withholding on market price in Case A is lower than Case E.

In order to compare the HHI and DWI, we calculated the HHI of Cases A and E. The values of HHI for Cases A and E are 3346 and 2027 respectively. It means that from HHI point of view the market power of Case E is lower than Case A. But according to Tables 2 and 8 it is clear that the market power in Case A is lower than Case E.

From above analysis, HHI can not give any information about capacity withholding. It is the fact that the market participants can withhold capacity even though the HHI is not very high but in low demand elasticity. Contrast to HHI, DWI considers capacity withholding. DWI is simple and adaptive in electricity market to detect capacity withholding, taking demand elasticity into consideration.

Notice that $\Delta y_i^{withheld}$, $\Delta y_i^{distort}$, $\Delta\pi^{distort}$ and DWI are comparison indices. In these indices market outcomes in actual markets are compared with perfect competition. Therefore we need to the perfect competition simulation. But all of the formulations of the capacity withholding indices mentioned in this paper, just depend on the values of y_i^e, a_i and α. It means that there is no need to the perfect competition simulation when we use these indices for capacity withholding analyzing. But in other comparison indices such as LI and PCMI, perfect competition simulation is necessary.

Table 1 Cost coefficients of the three GenCos.

GenCo No	1	2	3
Cost Paramete a_i \$/(MWh.GWh)	1	1.5	2
Cost Parameter b_i \$/MWh	12	10	8

Table 2 Simulation results and DWI for Cases A, B and C.

Case	GenCo No	y_i (GWh)	y_i^c (GWh)	α / a_i	A_i	DWI
A	1	9.11	-	2	-	0.18
	2	8.38	-	1.33	-	
	3	7.83	-	1	-	
B	1	10.70	5	2	-	0.18
	2	9.18	4	1.33	-	
	3	8.03	3	1	-	
C	1	10.59	5.48	1.32	1.07	0.18
	2	9.42	5.07	1.85	1.16	
	3	8.58	4.75	2.38	1.24	

Table 3 Capacity withholding indices for Cases A, B and C.

Case	GenCo No	$\Delta y_i^{withheld}$ (GWh)	$\Delta y_i^{distort}$ (GWh)	$\Delta Y^{distort}$ (GWh)	$\Delta\pi^{distort}$ (\$/MWh)
A	1	18.22	4.26	6.98	13.96
	2	11.17	1.86		
	3	7.83	0.85		
B	1	11.4	2.66	4.37	8.74
	2	6.90	1.08		
	3	5.03	0.66		
C	1	10.23	2.78	3.72	7.44
	2	5.81	0.84		
	3	3.83	0.11		

Table 4 Simulation results and DWI for Cases C_1, C_2, D_1 and D_2.

Case	GenCo No	y_i (GWh)	y_i^c (GWh)	α / a_i	A_i	DWI
C_1	1	12.31	7.17	4	1.39	0.10
	2	11.17	6.64	2.67	1.46	
	3	10.29	6.22	2	1.53	
C_2	1	7.66	3.24	1	0.73	0.31
	2	6.89	3.13	0.67	0.83	
	3	6.37	3.02	0.5	0.91	
D_1	1	7.89	4.08	1	1.07	0.23
	2	10.16	5.08	1.33	1	
	3	9.24	4.78	1	1.07	
D_2	1	12.04	6.23	2	1.07	0.18
	2	8.94	4.82	1.33	1.16	
	3	8.18	4.53	1	1.24	

Table 5 Capacity withholding indices for Cases C_1, C_2, D_1 and D_2.

Case	GenCo No	$\Delta y_i^{withheld}$ (GWh)	$\Delta y_i^{distort}$ (GWh)	$\Delta Y^{distort}$ (GWh)	$\Delta\pi^{distort}$ (\$/MWh)
C_1	1	20.57	3.67	4.22	16.90
	2	12.08	0.83		
	3	8.14	-0.29		
C_2	1	4.42	1.71	2.71	2.72
	2	2.51	0.71		
	3	1.67	0.32		
D_1	1	3.81	0.34	3.47	6.94
	2	6.77	2.15		
	3	4.46	0.99		
D_2	1	11.62	3.84	3.89	7.78
	2	5.52	0.31		
	3	3.65	-0.24		

Table 6 Cost coefficients of the five GenCos.

GenCo No	1	2	3	4	5
Cost Parameter a_i £/(MWh.GWh)	2.68	4.61	1.78	1.93	4.61
Cost Parameter b_i £/MWh	12	12	8	8	12

Table 7 Simulation results and DWI for Cases E.

Case	GenCo No	y_i (GWh)	y_i^c (GWh)	α / a_i	A_i	DWI
E	1	5.39	-	3.72	-	0.05
	2	4.68	-	2.16	-	
	3	6.14	-	5.59	-	
	4	6.07	-	5.18	-	
	5	4.68	-	2.16	-	

Table 8 Capacity withholding indices for Case E.

Case	GenCo No	$\Delta y_i^{withheld}$ (GWh)	$\Delta y_i^{distort}$ (GWh)	$\Delta Y^{distort}$ (GWh)	$\Delta\pi^{distort}$ (£/MWh)
E	1	20.05	0.14	5.35	53.5
	2	10.13	-1.45		
	3	34.31	4.41		
	4	31.45	3.73		
	5	10.13	-1.45		

6 Conclusion

In this paper, we resort to a set of comparison indices that allow to measure and analyze capacity withholding comparing the oligopoly outcomes with the ideal benchmarks represented by perfect competition. The DWI can provide a quantitative way for market designer or regulator to monitor the overall potential ability of market for capacity withholding. Comparison between HHI and DWI shows that HHI can not interpret the capacity withholding in electricity markets. It is found that the GenCos' capacity withholding depends significantly on the slope of their marginal cost function and the slope of system demand function. The increment in the demand elasticity provides the expected positive results on capacity withholding in terms of the increased DWI and the reduced capacity withheld, capacity distortion and price distortion in the market. More importantly, it has been found that the strategic forward contracting is helpful to mitigate the GenCos' capacity withholding and improve the competition in electricity markets.

References

[1] Stoft S., *Power System Economics: Designing Markets for Electricity*, New York: Wiley-IEEE; 2002.

[2] Liu J. D., Lie T. T. and Lo K. L., "An empirical method of dynamic oligopoly behavior analysis in electricity markets", *IEEE Trans. Power Syst.*, Vol. 21, No. 2, pp. 499-506, 2006.

[3] Gan D. and Bourcier D. V., "Locational market power screening and congestion management: experience and suggestions", *IEEE Trans. Power Syst.*, Vol. 17, No. 1, pp. 180-185, 2002.

[4] Yang J. and Jordan G., "System dynamic index for market power mitigation in the restructured electricity industry", *Power engineering society summer meeting, IEEE 2000*, Vol. 4, pp. 2217-2222, 2000.

[5] Kanagala A., Sahni M., Sharma S., Gou B. and Yu J.," A probabilistic approach of HHI to determine possibility of market power acquisition", *Power systems conference and exposition, IEEE PES 2004*, Vol. 3, pp. 1-6, 2004.

[6] Rahimi A. F. and Sheffrin A. Y., "Effective market monitoring in deregulated electricity markets", *IEEE Trans. Power Syst.*, Vol. 18, No. 2, pp. 486-493, 2003.

[7] Li C., Xia Q., Kang C. and Jiang J., "Novel approach to assess local market power considering transmission constraints", *Elsevier. Electrical Power and Energy Systems*, Vol. 30, No. 1, pp. 39-45, 2008.

[8] Dun-nan L., Guang-yu H., Rui-qing L. and Xue-qing C., "Analytical method of suppliers' behaviors in electricity market", *Power Systems Conference and Exposition, IEEE PES 2004*, Vol. 2, pp. 881-885, 2004.

[9] Bompard E., Ma Y. C., Napoli R. and Abrate G., "The demand elasticity impacts on the strategic bidding behavior of the electricity producers", *IEEE Trans. Power Syst.*, Vol. 22, No. 1, pp. 188-197, 2007.

[10] Wood A. J. and Wollenberg B. F., *Power Generation, Operation, and Control*, Second ed. New York: Wiley; 1996.

[11] Sahni M., Corley H. W. and Gou B., "Application of the HHI in a probabilistic framework for a parametric approach to determine possible market power", *Power Systems Conference and Exposition, IEEE PES 2009*, pp. 1-8, 2009.

[12] Bompard E., Wane L. and Napoli R., "Network constraint impacts on the competitive electricity markets under supply side strategic bidding", *IEEE Trans. Power Syst.*, Vol. 21, No. 1, pp. 160-170, 2006.

[13] Badlick R., "Electricity market equilibrium models: the effect of parameterization", *IEEE Trans. Power Syst.*, Vol. 17, No. 4, pp. 1170-1176, 2002.

[14] Kamara A., "Issues in future markets: a survey", *Futures Markets*, Vol. 2, No. 3, pp. 261-294, 1982.

[15] Newbery D. M., ”Competition, contracts and entry in the electricity spot market”, *Rand J. Econ*, Vol. 29, pp. 726-749, 1998.

[16] Allaz B. and Villa J. L., “Cournot competition, forward markets and efficiency*”, J. Econ. Theory*, Vol. 59, pp. 1-16, 1993.

[17] Xiaohong G., Yu-chi H. and Peypyne D. L.,” Gaming and price spike in electric power markets”, *IEEE Trans. Power Syst.*, Vol. 16, No. 3, pp. 402-408, 2001.

[18] Tellidou A. C. and Bakirtzis A. G., “Agent-based analysis of capacity withholding and tacit collusion in electricity markets”, *IEEE Trans. Power Syst.*, Vol. 22, No. 4, pp. 1735-1742, 2007.

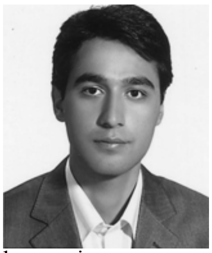
[19] Badri A., Jadid S. and Parsa-Moghaddam M., “Impacts of participants' market power and transmission constraints on GenCos' nash equilibrium point”, *Iranian Journal of Electrical & Electronic Engineering (IJEEE),* Vol. 3, No. 1 & 2, pp. 1-9, 2007.

[20] Wang X., Li Y. and Zhang S.,” Oligopolistic equilibrium analysis for electricity markets: a nonlinear complementarity approach”, *IEEE Trans. Power Syst.*, Vol. 19, No. 3, pp. 1348-1355, 2004.

[21] Kirschen D. and Strbac G*., Fundamentals of Power System Economics*, Wiley, 2004.

[22] Zhang S. X., Chung C. Y., Wong K. P., Chen H.,” Analyzing two-settlement electricity market equilibrium by coevolutionary computation approach”, *IEEE Trans. Power Syst.*, Vol. 24, No. 3, pp. 1155-1164, 2009.

[23] Chen H., Wong K. P., Chung C. Y. and Nguyen D. H. M., ”A co-evolutionary approach to analyzing supply function equilibrium model”, *IEEE Trans. Power Syst.*, Vol. 21, No. 3, pp. 1019-1028, 2006.



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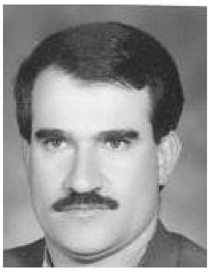

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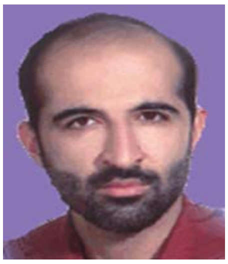

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