A Probabilistic Approach to Transmission Expansion Planning in Deregulated Power Systems under Uncertainties

H. Abdi, M. Parsa Moghaddam, and M. H. Javidi

Abstract: Restructuring of power system has faced this industry with numerous uncertainties. As a result, transmission expansion planning (TEP) like many other problems has become a very challenging problem in such systems. Due to these changes, various approaches have been proposed for TEP in the new environment. In this paper a new algorithm for TEP is presented. The method is based on probabilistic locational marginal price (LMP) considering electrical loss, transmission tariffs, and transmission congestion costs. It also considers the load curtailment cost in LMP calculations. Furthermore, to emphasize on competence of competition ability of the system, the final plan(s) is (are) selected based on minimization of average of total congestion cost for transmission system.

Keywords: Restructuring, Transmission expansion planning, Locational marginal price, Load curtailment cost.

1 Introduction

Restructuring of electric power industry has aimed to establish a competitive environment for electricity market. Key goal of restructuring is to reap benefits of competitive marketplaces. To achieve this, vertically integrated utilities (VIUs) have been disintegrated into separate components including generation, transmission and distribution. While in the new situation, generators as well as suppliers can compete with each other, the transmission system has preserved its monopolistic characteristics in most power systems.

Providing fair and non-discriminatory access to the system for all consumers and facilitating competition are the main objectives for TEP in the new environment. Cost and risk minimization for investors and increasing the flexibility of the system are some of important criteria that should be considered in TEP for deregulated systems.

From the view point of transmission planner, there are major differences between planning in regulated and deregulated power systems. Some of them are:

- While in integrated systems, TEP is considered only as a part of general expansion planning for the whole system; it is normally an isolated expansion planning for transmission system in deregulated environments [1].
- Deregulated power systems as compared with regulated ones, are faced with a lot of uncertainties. Therefore, TEP in such systems should be robust against those uncertainties [2, 3].
- In deregulated systems, transmission service pricing has more impact on TEP [4].

As the TEP problem is stated as a large-scale, non-linear and non-convex optimization problem, heuristic or meta-heuristic optimization algorithms can result in better solutions as compared with those obtained through classical techniques [5]. Some methods proposed so far for TEP in deregulated systems include multi-objective planning [6], fuzzy algorithm [7], cooperative game theory [8], multi-agent coalition formation [9], non-linear mixed integer programming [10], genetic algorithm [5], and LMP [3, 11-13]. However, as the pricing algorithms for purchasing and selling electric energy in deregulated power systems are essentially based on nodal pricing or LMP, methods based on LMP are of more importance. LMP is the price of supplying an additional MW of load at each bus in the system, considering generator and load bidding prices, the transmission system components experiencing congestion, losses and the electrical characteristics of the system.

In all proposed models for TEP which are based on LMP, major simplifications such as: not simultaneous consideration of loss and congestion costs, ignoring transmission tariffs, not modelling of complete uncertainties, and ignoring reactive power have been applied.

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This paper proposes a new algorithm for TEP in deregulated power systems based on probabilistic LMP. The novelty of this method as compared with our recently proposed algorithm [14, 15] is that the new method not only considers loss, congestion, transmission tariffs and uncertainties, but also it introduces cost of load curtailment in LMP computations. Furthermore, to emphasize on competence of competition ability of the system, the selection criteria for the final plan(s) is based on minimization of the average of total congestion cost utilizing AC optimal power flow (OPF) for transmission system which has never been referenced so far.

The proposed approach favours the following characteristics: i) introducing a new formulation for computing the probability density functions (pdfs) of LMPs; ii) modelling transmission tariffs for computation of LMPs; iii) using AC OPF considering the effect of reactive power on transmission plan; iv) introducing a new criteria for selecting the candidate lines as well as the final optimum plan(s) of TEP which emphasizes on competence of competition ability of transmission system as the key requirement for fair and non-discriminatory competition in power system.

Application of the proposed approach on an 8-bus test system confirms the advantages and the credibility of the proposed algorithm.

2 Model Overview

In a perfect competitive market the price is determined by interaction of all producers and consumers. In this environment each customer produces or consumes only a small portion of the market production. Therefore, producers or consumers can not affect the price alone and there is no discrimination among producers or consumers, i.e. all producers and consumers sell and buy at the same price. Moreover, in a competitive market there is no restriction for consumers to buy from any producer. To have a competitive electric market, the above conditions must be satisfied. In other word, to have a competitive electric market all power producers and consumers sell and buy electric energy at the same price and there should not be any restriction in power transfer. This means LMPs must be made equal at all buses and transmission lines will not be congested. Equalizing LMPs provides a non-discriminatory market and alleviating congestion eliminates power transmission constraints. Therefore, to facilitate a fair competition, a good approach for TEP is to expand the network in a way which flats the LMP profile as much as possible.

In the proposed method, first, LMPs in all network buses are calculated in their probabilistic forms as pdfs. To do this, OPF based on probabilistic load flow (PLF) has been applied. PLF is a load flow which utilizes the pdfs of the input variables instead of their deterministic values. As a result, the pdfs of output variables are calculated instead of their accurate values.

After calculation of probabilistic LMPs in all network buses, they are divided into source and sink sets based on their LMP mean values. Those buses at which, mean of LMPs are smaller than the total mean value of LMPs (LMP_{ave}) are grouped in the source set and the rest are considered as the set of sink buses.

A high mean of LMP at a sink bus indicates no access to cheap generation and a low mean of LMP indicates access to excess cheap generation and no access to enough loads. Hence, if a new transmission line is constructed between a sink and a source bus, the excess cheap generation at a source bus will be dispatched and electric energy will flow from this bus to a sink bus due to price potential difference.

The new transmission line has two effects; first, it alleviates the transmission constraints between these two buses and results in dispatching cheap generation that could not be dispatched because of the transmission constraints and second, it may decrease LMPs of some sink buses or increase LMPs of some source buses. Consequently, it decreases transmission constraints and price discrimination among customers. Therefore, if a line is added between a source and a sink bus, competition will be promoted among customers.

To reduce the number of lines nominated for expansion planning; only few buses among each of the above mentioned sets are selected to be connected through new lines. The criteria for choosing the nominated buses are as follows:

- Buses in the source set: $\mathrm{LMP}_{\mathrm{ave}} - \mathrm{LMP}_{\mathrm{k}} > \alpha\sigma_{\mathrm{k}}$ (1)

- Buses in the sink set: $\mathrm{LMP}_{\mathrm{k}} - \mathrm{LMP}_{\mathrm{ave}} > \alpha\sigma_{\mathrm{k}}$ (2)

To specify the flatness of a price profile, some indices are defined [3]. In a network with n buses, the pdf of LMPs have been computed for a given pdf for each input. Considering MLMP as a $1 \times n$ vector such that its *k*th element is the mean of LMP at bus *k*, and VLMP as a $1 \times n$ vector such that its *k*th element is the variance of LMP at bus *k*, the following parameters can be defined for determining the flatness of price profile:

- Mean of MLMP or LMP_{ave}: The less mean of MLMP indicates that cheaper generations are dispatched. This means a better condition for competition.
- Variance of MLMP: The smaller variance of MLMP indicates the flatter price profile and consequently better competition.
- Variance of VLMP: The smaller variance of VLMP indicates the more similar volatility of LMP at different buses and consequently the more similar risk in purchasing power from different buses.

Furthermore, the average of total congestion cost (TCC_{ave}) of network is considered as a very important criterion for selecting the optimal plans. TCC_{ave} shows how intensive transmission constraints are and consequently indicates how competitive electric market is.

3 Problem Formulation

To formulate the TEP problem in a deregulated system, some essential features should be simulated and considered in their mathematical forms. The most important parameters are as follows:

3.1 Uncertainties

The major uncertainties which should be modelled as inputs are generation and load bid prices which are introduced through normal pdfs. A normal pdf for a random variable *x* is expressed as follows:

$$f(x;\mu;\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3}$$

The generation bid prices with minimum and maximum powers which can be delivered to the network are announced to the independent system operator (ISO) by energy producers, whereas the load bid prices and their limitations are announced to ISO by customers.

3.2 Probabilistic LMP

As explained before, probabilistic LMPs are estimated using AC OPF. To do this, normally, the objective function of generation costs is minimized subject to equality and inequality constraints of power system. However, simultaneous inclusion of cost of load curtailment and transmission have not been addressed in the objective function so far or even if addressed they have just been estimated as a rough percentage of other costs. To estimate LMPs more accurately, we have included the cost of load curtailment and transmission in the objective function.

The proposed optimization model which is presented in Fig. 1 is as follows:

$$\text{Min } J_k=\sum_{i\in G}C_{gi}(P_{gi})+\sum_{i\in N}C_{di}(P_{di})+\sum_{\ell\in L}T_\ell P_{flow\,\ell} \tag{4}$$

s.t.

$$\sum_{i\in G}P_{gi}-\sum_{\ell\in L}P_{loss\,\ell}-\sum_{k\in N}P_{dk}=0 \tag{5}$$

$$P_{gi}-P_{di}-P_i(v,\delta)=0 \qquad i\in N \tag{6}$$

$$Q_{gi}-Q_{di}-Q_i(v,\delta)=0 \qquad i\in C \tag{7}$$

$$P_{gi}^m\le P_{gi}\le P_{gi}^M \tag{8}$$

$$Q_{gi}^m\le Q_{gi}\le Q_{gi}^M \tag{9}$$

$$\left|P_{flow\ell}\right|\le P_{flow\ell}^M \tag{10}$$

$$v_k^m\le v_k\le v_k^M \tag{11}$$

$$\delta_k^m\le\delta_k\le\delta_k^M \tag{12}$$

The objective function (4) represents the cost of operation. The first term in this equation represent the generation cost, and the second term states the load curtailment cost. Finally, the last term represents the transmission cost.

It should be mentioned that Eqs. (5) to (12) must be satisfied when the active load of bus *k* is increased (P_{dk}=1 MW), and all of the variables are changed, consequently. Furthermore, in the proposed algorithm, the well known loss coefficient method, developed by Kron and adopted by Kirchmayer has been applied for loss calculations [15].

Also, for adjusting power flows in overloaded lines, linear programming (LP) based on utilizing generation shift factors is used [16].

It should be noted that if LMP of one bus exceeds the bidding price of the corresponding customer, the load of that bus is curtailed until its LMP is reduced to a specified value. In fact each load can be modelled with a fix load which is never curtailed and an imaginary generator bidding load curtailment instead of generation. This imaginary generator such as other generators may be dispatched prior to dispatching more expensive generators. Then the price of curtailment of this load will be equal to the price of dispatched power of the imaginary generator.

3.3 Transmission Tariffs

Capital investment for new lines is the most important parameter for TEP. Here, we have used transmission tariffs for investment modelling. Transmission tariff is calculated according to the Levelized Transmitted Energy Cost (LTEC) as follows [18, 19]:

$$AC_\ell=(1+g)^{c_\ell}(PVC_\ell\,FCR_\ell+PVL_\ell\,FCRL_\ell+PVO_\ell CRF_\ell) \tag{13}$$

$$TE_\ell=8760\,P_{flow\,\ell}^M \tag{14}$$

$$LTEC_\ell=\frac{AC_\ell}{TE_\ell} \tag{15}$$

$$T_\ell=b*LTEC_\ell \tag{16}$$

As it can be observed, first the annual cost of line ℓ is calculated using Eq. (13) based on economic parameters. Then, $LTEC_\ell$ is calculated dividing the annual cost by total energy transmitted for line ℓ. Finally, the transmission tariff for line ℓ is obtained using Eq. (16).

3.4 Transmission Congestion Cost

Congestion cost of a line is defined as the opportunity cost of transmitting power through it. Generally congestion cost of line ℓ or the opportunity cost of its transmitting power is equal to:

$$CC_\ell=(LMP_r-LMP_q)P_{flow\,\ell} \tag{17}$$

After calculation of CC_ℓ for all transmission lines, total congestion cost of the network for each scenario is obtained as follows:

$$TCC = \sum_{\ell \in L} CC_{\ell} \qquad (18)$$

Also, average of total congestion cost for network, is equal to:

$$TCC_{ave} = \frac{1}{N_S} \sum_{i \in N_S} TCC_i \qquad (19)$$

4 Transmission Planning Procedure

The major steps of the proposed algorithm for TEP can be described as follow (Fig. 2):

A. Introducing α and pdfs of inputs, including generation and load bid prices during the peak load. This includes accessible power of generators as well as the customers' demands.

B. Simulating normal random variables and selecting the magnitudes for inputs using the method described in Ref. [20].

C. Calculation of TCC_{ave} for the base case based on calculation of LMPs for all network buses using optimization technique described in Fig. 1.

D. Calculation of pdfs of LMPs repeating steps B and C.

E. Specifying the sets of sink and source buses based on their LMP values and according to the introduced criteria.

F. Specifying the candidate lines for expansion planning.

G. Calculation of transmission tariffs for candidate lines.

H. Calculation of MLMP, VLMP for all buses and TCC_{ave} for all scenarios repeating steps B to D.

I. Choosing the best plan according to the TEP criteria.

5 Case Study

The presented approach has been applied to a typical 8-bus power system introduced in Ref. [3]. The network information is presented in Appendix B. However, line parameters have been modified to become more realistic.

Using 500 random generated samples from the pdfs of generation and load bid prices, MLMP and VLMP vectors, LMP_{ave}, *and* TCC_{ave} for the base case network are obtained as follow:

MLMP= [20.5163 23.7583 20.2190 21.9234 18.8210 19.8628 22.8886 24.9414] ($/MWh)

VLMP= [3.3552 2.4554 2.6808 2.5669 2.2432 2.2504 4.2106 3.6988] ($/MWh)

LMP_{ave}=21.6164 ($/MWh), TCC_{ave}=3,555.8 ($/h)

Sink and source buses and consequently candidate lines are specified comparing MLMP values at all buses of the network with LMP_{ave} value.

Set of source buses: {1, 3, 5, and 6}. Set of sink buses: {2, 4, 7, and 8}.

Transmission line candidates for expansion: {1-2,1-4, 1-7,1-8,3-2,3-4,3-7,3-8,5-2,5-4,5-7,5-8,6-2,6-4,6-7,6-8}.

Finally, based on the proposed algorithm, the optimal candidate line(s) for expansion is (are) specified as: {1-2, 3-8, 5-2 and 6-8} (Table 1).

It should be noted that besides decreasing the LMP_{ave} in the network, the total congestion cost must be minimized. This is due to the fact that less congestion in the network will result in less discriminative situation for competition in the network.

To investigate the validity of the proposed algorithm, all possible situations are considered in another approach without applying any screening procedure (Table 1). The results confirm that none of the optimal candidates are among those filtered candidates.

The major conclusions of the proposed method are as follow:

- As much as α is smaller, the candidate set of expansion buses will be bigger (Table 2).
- Comparing the results with those of Ref. [3], it can be observed that considering transmission tariffs and line resistance will result in more monotonous values for LMPs at different buses.
- By choosing an appropriate value for α and numbers of samples for input data, the calculation time is reduced efficiently, while it does not affect the solutions seriously.
- Partial changes in some parameters such as profit factor and power factor may lead to essential changes of the results.
- The optimum plan for expansion does not necessarily result in construction of the line between the buses with min-max LMPs. According to the calculations, while buses 8 and 5 have the maximum and the minimum LMPs, respectively, the optimum plan among all of the scenarios is line 5-2, instead of line 5-8. However, it should be mentioned that increasing α results in replacing the candidate line 5-2 with line 5-8 in the optimal plan.
- Adding the candidate lines which are limited to buses with min-max LMPs, results in decreasing TCC_{ave}.
- Line 2-3 is a critical line going to be congested in some scenarios. Therefore, as it is observed from the results, at least one side of the majority of proposed alternatives are ending with buses 2 or 3. This means that new candidates for TEP must be constructed between buses whose linking lines are going to be congested.
- The results confirm that the criterion for selection the optimum plan can be changed from minimization of LMP_{ave} [14, 15] to minimization of TCC_{ave}, which is a proper parameter for measuring price discrimination and customer constraints. In fact, it can be considered as a suitable criterion for measuring the degree of competitiveness in an electricity market.

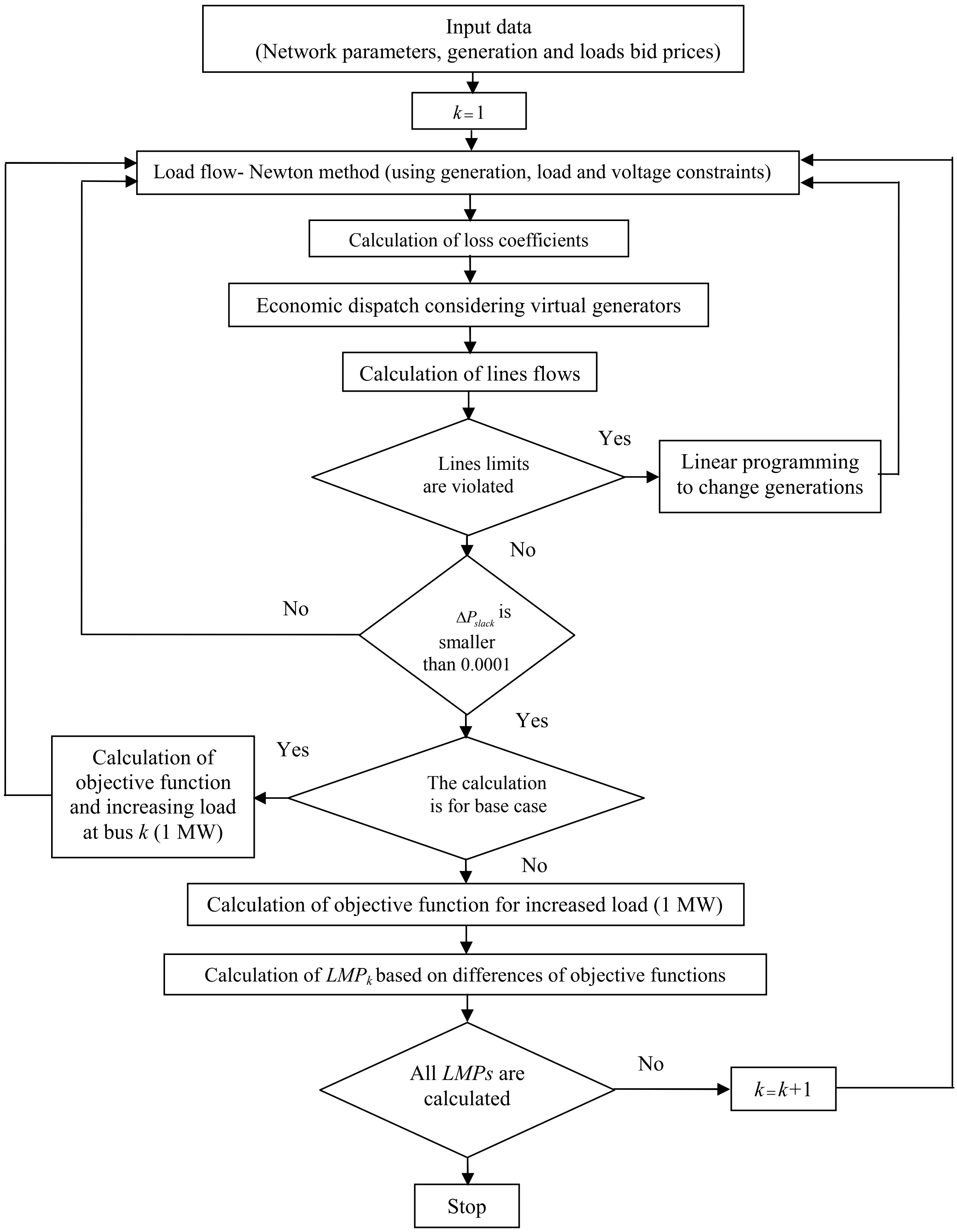

Fig. 1 General structure of the proposed OPF

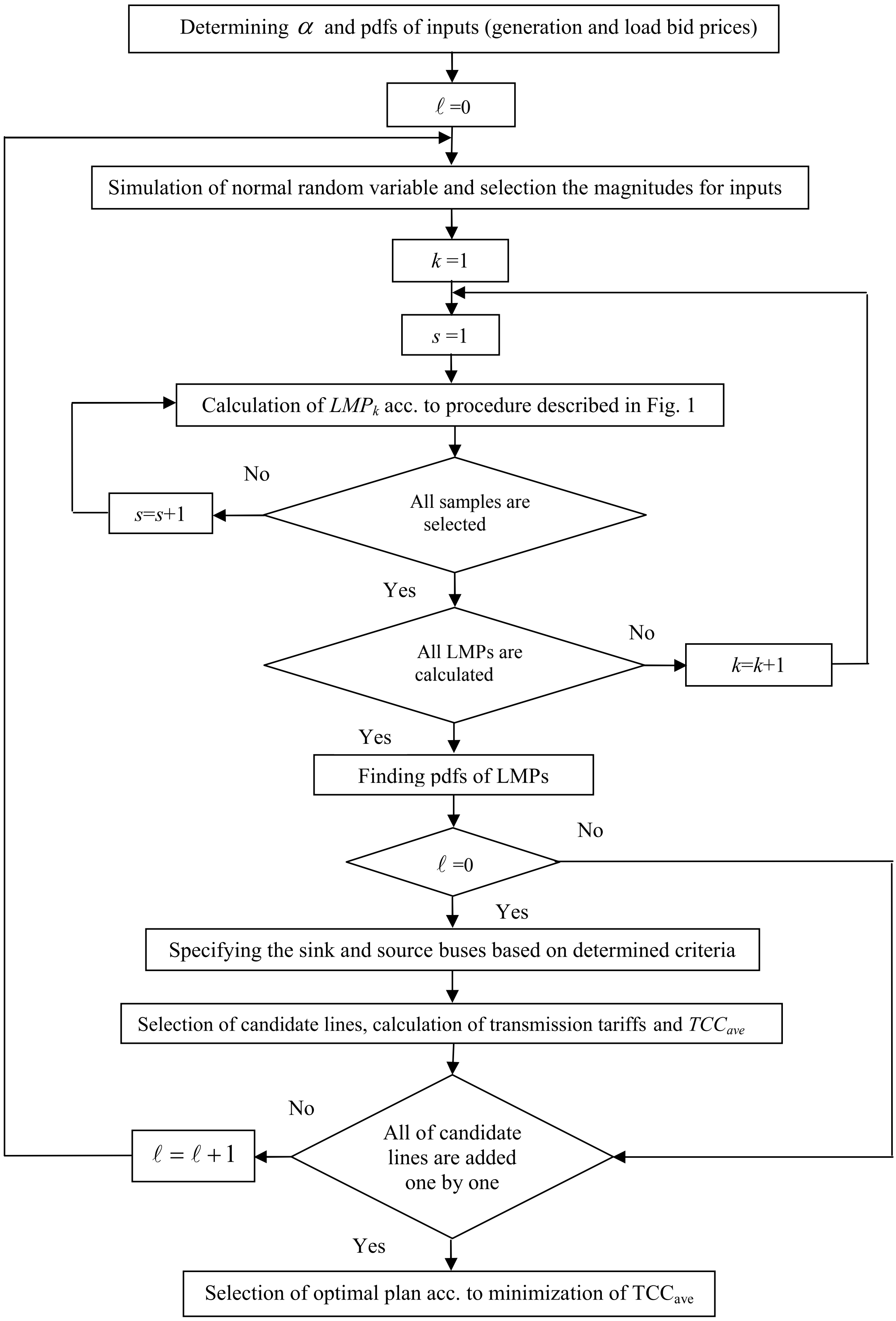

Fig. 2 Flowchart of the proposed algorithm for TEP

Table 1 The results of adding various candidate lines

New Line	Mean of MLMP ($/MWh)	Variance of MLMP ($/MWh)	Variance of VLMP ($/MWh)	Mean of TCC_{ave} ($/h)	Rank of TCC_{ave}	Lines close to congestion
1-2	20.7720	3.5861	1.9329	3,295	4	-
1-4	21.0790	4.1404	1.7668	3,472	10	-
1-7	21.0721	4.9000	0.8022	3,705	21	2-3
1-8	20.9569	3.9125	0.8126	3,450	9	2-3
3-2	21.2430	3.9984	2.6764	3,486	13	-
3-4	20.8789	4.0618	1.6759	3,548	16	-
3-7	21.5874	5.8734	4.9325	4,362	25	-
3-8	20.9872	3.7427	1.4942	3,317	5	-
5-2	20.2980	2.7281	1.7475	2,838	1	-
5-4	20.6840	3.2649	2.0552	3,380	7	5-4,2-3
5-7	21.4525	4.2895	29.9749	6,939	28	2-3
5-8	20.2596	2.4034	0.6007	3,256	3	2-3
6-2	20.6315	5.7135	2.6762	4,636	27	-
6-4	21.2087	4.4703	2.9807	3,586	19	-
6-7	20.9322	5.6949	2.2316	4,599	26	-
6-8	20.4205	2.9050	0.3355	3,160	2	-
Other feasible candidates						
1-3	21.3663	5.2794	1.9181	3,731	22	-
1-5	20.8841	3.6420	1.7087	3,369	6	-
1-6	20.8285	3.7264	2.3567	3,835	24	-
2-4	21.0697	3.6787	2.1841	3,491	14	-
2-7	21.3857	5.3662	0.7480	3,507	15	-
2-8	21.2215	5.0769	1.0957	3,479	12	2-3
3-5	21.2130	4.0986	1.7599	3,448	8	2-3
3-6	21.7463	5.1697	1.4272	3,577	18	-
4-7	20.9856	3.6661	3.0848	3,551	17	-
4-8	21.0072	4.2387	1.0291	3,598	20	-
5-6	21.3463	4.3781	0.9277	3,477	11	-
7-8	21.1706	4.5531	2.7951	3,760	23	-

Table 2 Different values of α and optimal options

α	Sink buses	Source buses	Candidate lines	Optimal options for each sink bus	Rank of optimal lines based on TCC_{ave}	Run time (Sec.)
All feasible candidates	-	-	28	1-2, 3-8, 5-2, 6-8	4, 5, 1, 2	143,150
$0 < \alpha < 0.012$	1, 3, 5, 6	2, 4, 7, 8	16	1-2, 3-8, 5-2, 6-8	4, 5, 1, 2	119,725
$0.012 < \alpha < 0.3021$	1, 3, 5, 6	2, 7, 8	12	1-2, 3-8, 5-2, 6-8	4, 5, 1, 2	73,834
$0.3021 < \alpha < 0.3279$	1, 3, 5, 6	2, 8	8	1-2, 3-8, 5-2, 6-8	4, 5, 1, 2	24,079
$0.3279 < \alpha < 0.5213$	3, 5, 6	2, 8	6	3-8, 5-2, 6-8	5, 1, 2	22,603
$0.5213 < \alpha < 0.7712$	5, 6	2, 8	4	5-2, 6-8	1, 2	20,524
$0.7712 < \alpha < 0.8723$	5	2, 8	2	5-2	1	10,433
$0.8723 < \alpha < 0.8989$	5	8	1	5-8	3	7,583

6 Conclusion

TEP in restructured power systems relies seriously on non-deterministic parameters. In such systems, there are many uncertainties about load and generation in the network. Therefore, probabilistic and heuristic methods instead of classical approaches may be applied to get better solutions.

In this paper a new algorithm based on probabilistic LMP for TEP in restructured power systems has been proposed. The novelty of the method as compared with our recently proposed algorithm is that the new method not only considers loss, congestion, transmission tariffs, and uncertainties, but also it introduces cost of load curtailment in LMP computations. Furthermore, to emphasize on competence of competition ability of the system, the selection criterion for the final plan(s) is (are) based on minimization of the average of total congestion cost utilizing AC OPF for transmission system.

Application of the proposed approach on an 8-bus test system confirms the advantages and the credibility of the proposed algorithm.

To promote the proposed algorithm following further works are under the study:

- The effect of considering different types of pdfs rather than normal function for modelling uncertainties.
- Modelling the changes of various parameters such as the profit and power factors which can influence the marginal transmission costs.

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Appendices

A. Nomenclature

J_k cost objective function, \$/h

LMP_k LMP at kth bus, \$/MWh

LMP_{ave} total mean value of LMPs, \$/MWh

α variable coefficient

σ_k standard deviation of pdf of LMP_k, \$/MWh

f normal probability density function, with mean value of μ and variance of σ^2,respectively

C_{gi} ,C_{di} bid prices for generator and load ith, \$/MWh

P_{gi}, Q_{gi} active and reactive powers generation of ith generator in the base case, MW, MVAr

P_{dk}, Q_{dk} active and reactive loads at bus k , MW,MVAr

ℓ line number starting from bus q and terminating to bus r

T_ℓ transmission tariff of line ℓ, \$/MWh

$P_{flow\,\ell}$ power flow of line ℓ, MW

$P_{loss\,\ell}$ loss of line ℓ at base case, MW

v_k, δ_k voltage magnitude and angle of bus k, kV ,degree

M, m indices for upper and lower limits

G, L, N, C sets of generators, lines, network buses and PQ buses, respectively

AC_ℓ annual cost of line ℓ, \$

g general inflation

c_ℓ duration for construction of line ℓ, year

PVC_ℓ, PVL_l, PVO_l present values of construction, land, and operation for line ℓ, \$

FCR_ℓ fixed charge rate of line ℓ

$FCRL_\ell$ fixed charge rate of land for line ℓ

CRF_ℓ capital return factor for line ℓ

TE_ℓ total energy transmitted by line ℓ, MWh

$LTEC_\ell$ levelized transmitted energy cost for line ℓ, \$/MWh

b profit factor, equal to 1.2

CC_ℓ congestion cost of line ℓ, \$/h

TCC_{ave} average of total congestion cost for network, \$/h

N_S number of samples.

B. Network information

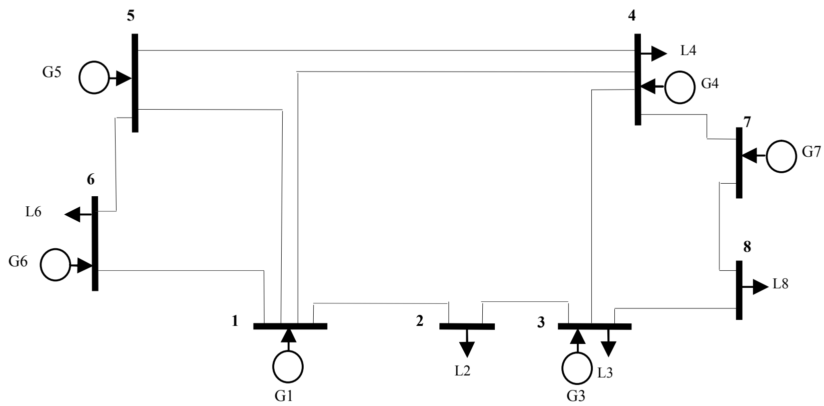

Fig. 3 Case study network

Table 3 Loads data ($\cos\phi$ =0.95)

Load No.	Bus No.	Load (MW) (μ,σ^2)	Bid (\$/MWh)
1	2	(300 , 10)	30
2	3	(300 , 12)	32
3	4	(300 , 15)	35
4	6	(300 , 5)	28
5	8	(250 , 9)	35

Table 4 Generators data

Gen. No.	Bus No.	P_{max}(MW)	Q_{max} (MVAr)	Bid (\$/MWh)
1	1	100	50	(15 , 1.8)
2	3	520	300	(30 , 1.5)
3	4	250	150	(30 , 2.0)
4	5	600	400	(10 , 3.0)
5	6	400	200	(20 , 2.1)
6	7	200	150	(20 , 1.5)

Table 5 Economic parameters

Description	Qty. for 1 sample line 230 kV
Construction cost of line	50,000 \$/Km
Price of land	25,000 \$/Km in width of right of way
Operation cost of line	1,000 \$/Km per year
Inflation rate	0.15
Duration for line construction	1 Year

Table 6 Lines data (R, X in p.u. on 100 MVA base)

Line	R	X	Limit (MW)	Tariff ($/MWh)
1-2	0.01675	0.06750	400	1.236
1-4	0.01122	0.09520	190	1.769
1-5	0.01340	0.05400	390	1.014
2-3	0.02364	0.09864	130	1.825
3-4	0.01770	0.12000	230	1.289
4-5	0.01340	0.11260	330	1.198
5-6	0.00680	0.05134	350	0.960
6-1	0.02400	0.15280	250	1.582
7-4	0.03480	0.22156	250	2.293
7-8	0.00800	0.06040	340	1.163
8-3	0.03240	0.20628	240	2.224

Table 7 Candidate lines data

Line	R	X	Limit(MW)	Length(Km)	Tariff($/MWh)
1-3	0.03729	0.25179	140	330	4.943
1-7	0.03600	0.22840	185	400	4.275
1-8	0.03015	0.25200	190	450	4.683
2-4	0.01273	0.10697	200	190	1.878
2-5	0.02345	0.09450	390	250	1.267
2-6	0.02100	0.19985	185	350	3.740
2-7	0.02111	0.08505	400	315	1.557
2-8	0.03540	0.22710	140	300	4.237
3-5	0.01822	0.15232	190	270	2.830
3-6	0.03240	0.21636	180	360	3.954
3-7	0.02950	0.18925	140	250	3.351
4-6	0.02700	0.17460	180	300	3.259
4-8	0.02310	0.19600	190	350	3.642
5-7	0.03600	0.22840	185	400	4.275
5-8	0.02700	0.25695	320	450	2.780
6-7	0.09850	0.40950	225	500	4.394
6-8	0.03600	0.23920	180	200	2.197