On the Achievable Rate-Regions for the Gaussian Two-Way Diamond Channel

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Abstract: In this paper, we study rate region of a Gaussian two-way diamond channel which operates in half- duplex mode. In the channel that we consider in this paper, Two Transceiver (TR) nodes exchange their messages with the cooperation of two relay nodes. We consider a special case of the Gaussian two-way diamond channels which is called Compute-and-Forward Multiple Access Channel (CF-MAC). In the CF-MAC, the TR nodes transmit their messages to the relay nodes which are followed by a simultaneous communication from the relay nodes to the TRs. Adopting rate splitting method in the terminal encoders and then using Compute-and-Forward (CF) relaying and decoding the sum of messages at the relay nodes, an achievable rate region for this channel is obtained. To this end, we use a superposition coding based on lattice codes. Using numerical results, we show that our proposed scheme outperforms the other similar methods and achieves a tighter gap to the outer bound.

Keywords: Diamond Channel, Lattice Codes, Superposition, Two Way Relay.

1 Introduction

Lattice structures are able to achieve the same rate which are achievable by independent identically distributed (i.i.d.) Gaussian random codes for some AWGN networks such as point to point channels [1], Multiple Access Channels (MAC) [2], Broadcast Channels (BC) [3] and relay networks [2]. Furthermore, lattice codes may also be used in achieving the capacity of Gaussian channels with interference or state known at the transmitter [4]. Noticeably, in some scenarios, it can be shown that lattice codes have a better performance than random codes. In relay networks, due to linearity of the lattice structures, by using lattice codes it's possible to achieve higher rate regions than i.i.d. random codes [2]. One of such relay networks that takes advantage of this linearity is the Gaussian Two-Way Relay Channel where two MSs communicate with each other through a Relay Station (RS) [5].

The Gaussian Two-Way Relay Channel consists of two phases: MAC phase and BC phase. In the MAC phase, instead of decoding the codewords separately, relays can use nested lattice codes to decode the linear combination of them. Afterward, in BC phase the sum of codewords can be sent to obtain the desired message in each MS since they can decode the received data using their own messages as side information [5].

Based on the fact that lattice codes have the best performance in order to achieve the sum of messages, the best rate region for the Gaussian two-way relay channel is established in [6]. In [7], it is shown that bursty amplify-and-forward can achieve the capacity region of the Gaussian N-relay diamond channel within a constant gap which is independent of channel gain. In [8], a diamond network with conferencing links between the relay nodes is considered and it is shown that a scheme based on the amplify-and-forward achieves rates which are closer to capacity region. In [9], the capacity regions of two-way diamond channels is studied. It is shown that for a linear deterministic model [10], the capacity of the diamond channel in each direction can be simultaneously achieved for all values of channel parameters. The Gaussian two-way diamond channel has been studied in [11] and using lattice codes some achievable rate-regions for different protocols such as CF-MAC and CF-BC are obtained. Based on rate-splitting and decoding the sum of messages in the relay nodes, a rate-region for the CF-MAC protocol is also obtained in [11].

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In this paper, we study a special case of the diamond networks which is called the Gaussian two-way diamond channel. In the channel we consider in this paper, two TR nodes with the help of two relay nodes aim to exchange their messages. We consider the Gaussian two-way diamond channel which operates in three phases and is called Compute-and-Forward Multiple Access Channel (CF-MAC), (as shown in Fig. 1). In this paper a new rate-region for this protocol is obtained by using superposition coding for nested lattice codes, although superposition coding has been used in this channel previously but it was based on random codes. In this type of coding, we split the message of each node into two parts and each transceiver sends one of them in different phases.

The remainder of the paper is organized as follows. System model is presented in Section 2. In Section 3, we first review the preliminaries of lattice codes and then analyze superposition coding based on nested lattices. In Section 4 we present our proposed scheme. By using numerical examples, achievable rate-regions of different cooperative protocols are compared in Section 5.

2 System Model

We consider the Gaussian two-way diamond channel in which two users (nodes A and B) exchange their messages with the help of multiple relay terminals (nodes R_1 and R_2) which operate in half-duplex mode as shown in Fig. 1. By half-duplex communication, we mean that each node can be in transmit or receive mode. Each TR node only communicates through relay nodes, that is, there is no direct link between TRs.

The m-th time slot is denoted by t_m and the symmetric CF-MAC protocol is modeled as:

Phase1: $$\mathbf{Y}_{r_1} = h_{a_1}\mathbf{X}_a^{(1)} + h_{b_1}\mathbf{X}_b^{(1)} + \mathbf{Z}_{r_1}^{(1)} \quad (1)$$

Phase2: $$\mathbf{Y}_{r_2} = h_{a_2}\mathbf{X}_a^{(2)} + h_{b_2}\mathbf{X}_b^{(2)} + \mathbf{Z}_{r_2}^{(2)} \quad (2)$$

Phase3: $$\mathbf{Y}_a = h_{a_1}\mathbf{X}_{r_1}^{(3)} + h_{a_2}\mathbf{X}_{r_2}^{(3)} + \mathbf{Z}_a^{(3)} \quad (3)$$

$$\mathbf{Y}_b = h_{b_1}\mathbf{X}_{r_1}^{(3)} + h_{b_2}\mathbf{X}_{r_2}^{(3)} + \mathbf{Z}_b^{(3)} \quad (4)$$

where $\mathbf{X}_j^{(m)}$ is the transmitted signal from node j in phase m and is the received signal in node j $(j \in \{a,b,r_1,r_2\})$. Also, $\mathbf{Z}_j^{(m)}$ is a zero-mean Gaussian noise with unit variance, i.e., $\mathbf{Z}_j \sim N(0,1)$ in phase m and node j. All noise variables are independent of each other and also independent of the channel inputs. Besides, $h_{in}, i \in \{a,b\}, n \in \{1,2\}$ are the channel gains. Average power constraint applies on the transmitted signals at node j and in the phase m as:

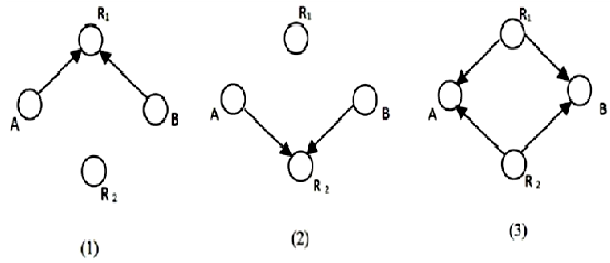

Fig. 1 The Gaussian Two-way Diamond Channel Model.

$$\frac{1}{n}E\,\|\mathbf{X}_j^{(m)}\|^2 \le P \quad (5)$$

A code $(2^{nR_a}, 2^{nR_b}, n)$ for the Gaussian diamond channel consists of two sets of integers $W_i = \{1,2,...,2^{nR_i}\}$ for $i \in \{a,b\}$, four encoding functions f_j for $j \in \{a,b,r_1,r_2\}$ and two decoding functions g_a and g_b.

Two sets of encoding function at the users A and B are defined as:

$$\mathbf{X}_i^{(m)} = f_i(\mathbf{W}_{im}) \quad (6)$$

Where the message of each user node i, can be splitted into two parts, $\mathbf{W}_{i_1}$ and $\mathbf{W}_{i_2}$.

Two sets of encoding relay functions at relay nodes R_1 and R_2 are represented by:

$$\mathbf{X}_j^{(m)} = f_j(\mathbf{Y}_j), \forall\, \mathrm{j} \ne \mathrm{i} \quad (7)$$

Decoding functions at user nodes are given by:

$$\hat{\mathbf{W}}_a = g_b(\mathbf{W}_b, \mathbf{Y}_b) \quad (8)$$

$$\hat{\mathbf{W}}_b = g_a(\mathbf{W}_a, \mathbf{Y}_a) \quad (9)$$

The average probability of error for this system is defined as:

$$\mathrm{P}_e^n = \sum_{i \in \{a,b\}} \Pr\{\hat{\mathbf{W}}_i \ne \mathbf{W}_i\} \quad (10)$$

Note that the condition $\mathrm{P}_e^n \to 0$ implies that individual average error probabilities also go to zero. We assume that the messages $\mathbf{W}_i$ are chosen independently and uniformly from the message sets $\mathbf{W}_i$. A rate pair (R_a, R_b) is said to be achievable for the Gaussian diamond channel if there exists a sequence of codes $(2^{nR_a}, 2^{nR_b}, n)$ such that $\mathrm{P}_e^n \to 0$ as $\mathrm{n} \to \infty$. The corresponding capacity region is the convex closure of all achievable rate pairs.

3 A Review of Nested Lattice Codes

3.1 Definitions

First, we give some definitions briefly [1, 2]. An n-dimensional *lattice* Λ^n is a set of points in the space which are closed on the subtraction and addition operations.

By considering **G** as generator matrix of lattice Λ^n, it can be constructed by:

$$\Lambda^n = \{\lambda = \mathbf{G}.z : z \in \mathbb{Z}^n\} \quad (11)$$

where $\mathbb{Z}$ is the set of integer numbers. Nearest neighbor *quantizer* Q_Λ maps each point in the space to the nearest lattice point.

$$Q_{\Lambda^{(n)}}(x) = \arg\min_{\lambda\in\Lambda} \| x - \lambda \|. \quad (12)$$

Fundamental voronoi region of lattice Λ^n is all points in the space that quantize to zero point of lattice Λ^n. Zero point belongs to all lattices and fundamental voronoi region is given by:

$$\upsilon_0(\Lambda^n) = \{\mathrm{x} \in \mathbb{R}^n : Q_{\Lambda^{(n)}}(x) = 0\} \quad (13)$$

Second moment of the lattice Λ^n is defined as:

$$\sigma^2(\Lambda^{(n)}) = \frac{1}{n}\frac{\int_{\upsilon(\Lambda)} \| x \|^2 dx}{\int_{\upsilon(\Lambda)} dx} \quad (14)$$

and the *normalized second moment* of lattice Λ^n can be presented as:

$$G(\Lambda^n) = \frac{\sigma^2(\Lambda^{(n)})}{[\int_{\upsilon(\Lambda)} dx]^{\frac{2}{n}}} = \frac{\sigma^2(\Lambda^{(n)})}{V^{\frac{2}{n}}} \quad (15)$$

where V is the volume of the voronoi region of lattice Λ^n. Lattice Λ^n is *good for quantization* or *Rogers-good* if:

$$\lim_{n\to\infty} G(\Lambda^{(n)}) = \frac{1}{2\pi e} \quad (16)$$

Suppose that $Z \sim N(0, \sigma_z^2 \mathbf{I}_n)$, then the lattice Λ^n is *good for AWGN coding* or *Poltyrev-good* if:

$$\mu(\Lambda,\varepsilon) = \frac{(Vol(\upsilon))^{\frac{2}{n}}}{2\pi e \sigma_z^2} > 1 \quad (17)$$

A *Nested lattice* consists of a coarse lattice and a fine lattice. A coarse lattice Λ^n is said to be *nested* in fine lattice $\Lambda_1^{(n)}$ if $\Lambda^{(n)} \subseteq \Lambda_1^{(n)}$. υ shows the fundamental Voronoi region of lattice Λ^n and a *nested lattice code* can be defined as

$$C = \{\Lambda_1 \cap \upsilon\} \quad (18)$$

The *rate* of a nested lattice code is given by

$$R = \frac{1}{n}\log|\mathrm{C}| = \frac{1}{n}\log\frac{Vol(\upsilon)}{Vol(\upsilon_1)} \quad (19)$$

In [12], Erez, Litsyn and Zamir show that there exists a sequence of lattices that are simultane- ously good for packing, covering, source coding (Rogers-good) and channel coding (Poltyrev-good). Before presenting our scheme, we review the concept of superposition coding based on lattice codes.

3.2 Superposition Coding for Lattice Codes

Consider the following nested lattices:

$$\Lambda_{sa}^{(\mathrm{n})} \subseteq \Lambda_{sb}^{(\mathrm{n})} \subseteq \Lambda_{m}^{(\mathrm{n})} \subseteq \Lambda_{c}^{(\mathrm{n})} \quad (20)$$

The fine lattice $\Lambda_c^{(\mathrm{n})}$ provides the codewords, while the coarse lattices $\Lambda_{sa}^{(\mathrm{n})}$ and $\Lambda_{sb}^{(\mathrm{n})}$ satisfy the power constraint. Based on this chain lattice, we define the following codebook:

$$C_i^{(n)} = \{\Lambda_c^{(n)} \cap \upsilon_{si}^{(n)}\}, \quad (21)$$

where their rates are given by

$$R_i = \frac{1}{n}\log|\mathrm{C}_i^{(n)}| = \frac{1}{n}\log\frac{Vol(\upsilon_{si}^{(n)})}{Vol(\upsilon_c^{(n)})}. \quad (22)$$

The meso-lattice [13] $\Lambda_m^{(\mathrm{n})}$ partitions the set of codewords for node i into two parts. To clear this, we define two additional codebooks as follows:

$$C_1^{(n)} = \{\Lambda_c^{(n)} \cap \upsilon_m^{(n)}\}, \quad (23)$$

$$C_{2i}^{(n)} = \{\Lambda_m^{(n)} \cap \upsilon_{si}^{(n)}\}, \quad (24)$$

where the associated coding rates are

$$R_1 = \frac{1}{n}\log(\frac{Vol(\upsilon_m^{(n)})}{Vol(\upsilon_c^{(n)})}), \quad (25)$$

$$R_{2i} = R_i - R_1 = \frac{1}{n}\log(\frac{Vol(\upsilon_m^{(n)})}{Vol(\upsilon_c^{(n)})}). \quad (26)$$

Now we can decompose each lattice codeword $\mathbf{V}_i \in C_i^{(n)}$ by $\Lambda_m^{(\mathrm{n})}$ into two points, $\mathbf{V}_{1i}$ and $\mathbf{V}_{2i}$ such that

$$\mathbf{V}_i = [\mathbf{V}_{1i} + \mathbf{V}_{2i}] \bmod \Lambda_{si}^{(\mathrm{n})}; \quad (27)$$

and

$$\mathbf{V}_{1i} = \mathbf{V}_i \bmod \Lambda_m^{(\mathrm{n})} \in C_1^{(n)}, \quad (28)$$

$$\mathbf{V}_{2i} = [\mathbf{V}_i - \mathbf{V}_{1i}] \bmod \Lambda_{si}^{(\mathrm{n})} \in C_{2,i}^{(n)}. \quad (29)$$

4 The Proposed Scheme

In our proposed scheme, we use superposition coding scheme based on lattice codes. That means node i using meso-lattice $\Lambda_m^{(\mathrm{n})}$ partitions its lattice codeword, $\mathbf{V}_i$, into two part: $\mathbf{V}_{1i}$ and $\mathbf{V}_{2i}$. In phase 1 and 2, each node sends one of those parts. Then the relays by using CF idea finds a linear combination of lattice codewords. Finally, in phase 3, the relay nodes send those linear combinations to TR nodes. Each TR node using its own message as side information, recovers the desired message. In the following, we explain encoding and decoding at nodes in more details.

By calculating the optimum time slot durations, t_1, t_2 and t_3, we can determine the codeword length in each phase as $n_1 = t_1 / T_s$, $n_2 = t_2 / T_s$ and $n_3 = t_3 / T_s$, where T_s is the sampling interval. In the following, without loss of generality, we assume that $h_a \geq h_b$. In order to apply the rate splitting, we choose a chain of lattices as Eq. (20), such that $\Lambda_{sa}^{(n)}$, $\Lambda_{sb}^{(n)}$ and $\Lambda_{m}^{(n)}$ are Rogers-good and Poltyrev-good while $\Lambda_{c}^{(n)}$ is Poltyrev-good. The generation of these lattices is fully explained in [12].

4.1 Phase 1

4.1.1 Encoding in Phase 1

In order to node i sends its message, first splits it such as, $W_i = W_{1i} + W_{2i}$, $i \in \{a,b\}$. Then, using a one to one mapping, it maps message W_{1i} to lattice point $\mathbf{V}_{1i}$ and message W_{2i} to lattice point $\mathbf{V}_{2i}$. We suppose that $h_{1i} = h_{2i} = h_i$. Now, in this phase, node i communicates the following signal over the channel:

$$\mathbf{X}_i^{(1)} = \frac{1}{h_i}[\mathbf{V}_{1i} + \mathbf{D}_{1i}] \bmod \Lambda_{si}^{(n_1)}, \tag{30}$$

where $\mathbf{D}_{1i}$ is a dither that is uniformly distributed over the Voronoi region of $\Lambda_{si}^{(n_1)}$, i.e., $\mathbf{D}_{1i} \sim Unif\ (\upsilon_{si})$. According to the channel power constraints, we choose the second moment of lattice $\Lambda_{si}^{(n_1)}$ as the following:

$$\sigma^2(\Lambda_{si}^{(n_1)}) = \mathrm{h}_i^2 P. \tag{31}$$

4.1.2 Decoding in Phase 1

Now, from [5], we know that if

$$\mathrm{R}_{1i} \leq \mathrm{R}^*_{1i}, \tag{32}$$

where

$$\mathrm{R}^*_{1i} \triangleq [\frac{1}{2}\log(\frac{h_i^2}{h_a^2 + h_b^2} + \mathrm{h}_i^2\mathrm{P})]^+, \tag{33}$$

and $[\mathrm{x}]^+ = \max\{0, \mathrm{x}\}$, then we can estimate the following linear combination correctly.

$$\mathbf{T}_1 = [\mathbf{V}_{1a} + \mathbf{V}_{1b} - \mathrm{Q}_{\Lambda_{sb}}(\mathbf{V}_{1b} + \mathbf{D}_{1b})] \bmod \Lambda_{sa}^{(n_1)}. \tag{34}$$

4.2 Phase 2

Encoding and decoding in this phase is exactly similar to phase 1. In this phase nodes A and B tries to send lattice points $\mathbf{V}_{2a}$ and $\mathbf{V}_{2b}$ to the relay node R_2 and in the relay node, we decode a linear combination of them. To end this, node i, $i \in \{a,b\}$, sends the following signal over the channel:

$$X_i^{(2)} = \frac{1}{h_i}[\mathbf{V}_{2i} + \mathbf{D}_{2i}] \bmod \Lambda_{si}^{(n_2)}, \tag{35}$$

where $\mathbf{D}_{2i} \sim Unif\ (\upsilon_{si})$. Now we can decode the following linear combination of lattice points $\mathbf{V}_{2a}$ and $\mathbf{V}_{2b}$ correctly.

$$\mathbf{T}_2 = [\mathbf{V}_{2a} + \mathbf{V}_{2b} - \mathrm{Q}_{\Lambda_{sb}}(\mathbf{V}_{2b} + \mathbf{D}_{2b})] \bmod \Lambda_{sa}^{(n_2)}, \tag{36}$$

If

$$\mathrm{R}_{2i} \leq \mathrm{R}^*_{2i}, \tag{37}$$

where $\mathrm{R}_{2i} = \mathrm{R}^*_{1i}$.

4.3 Phase 3

4.3.1 Encoding in the Relay Nodes

In this phase, the relay nodes $\mathbf{R}_1$ and $\mathbf{R}_2$ send $\mathbf{T}_1$ and $\mathbf{T}_2$ to nodes A and B. To do this, the relay nodes communicate the following signals over the channel:

$$\mathbf{X}_{r_1}^{(3)} = \frac{1}{h_a}[\mathbf{T}_1 + \mathbf{D}_{r1}] \bmod \Lambda_{sa}^{(n_3)}, \tag{38}$$

$$\mathbf{X}_{r_2}^{(3)} = \frac{1}{h_a}[\mathbf{T}_2 + \mathbf{D}_{r2}] \bmod \Lambda_{sa}^{(n_3)}, \tag{39}$$

where $\mathbf{D}_{r_1} \sim Unif\ (\upsilon_{s_1})$ and $\mathbf{D}_{r_2} \sim Unif\ (\upsilon_{s_2})$ Note that based on the Crypto lemma, the power constraints in the relay nodes are satisfied.

4.3.2 Decoding in the Node A

In the node A, based on the received signal,

$$\mathbf{Y}_a = h_a\mathbf{X}_{r_1}^{(3)} + h_a\mathbf{X}_{r_2}^{(3)} + \mathbf{Z}_a^{(3)}, \tag{40}$$

We perform the following operations in order to estimate message W_2:

$$\begin{aligned}
\mathbf{Y}_{da} &= [\alpha\mathbf{Y}_a - \mathbf{D}_{r_1} - \mathbf{D}_{r_2}] \bmod \Lambda_{sa}^{(n_3)} \\
&= [\alpha h_a\mathbf{X}_{r_1}^{(3)} + \alpha h_a\mathbf{X}_{r_2}^{(3)} + \alpha\mathbf{Z}_a^{(3)} \\
&\quad -\mathbf{D}_{r_1} - \mathbf{D}_{r_2}] \bmod \Lambda_{sa}^{(n_3)} \\
&= [\mathbf{T}_1 + \mathbf{T}_2 + \alpha h_a\mathbf{X}_{r_1}^{(3)} + \alpha h_a\mathbf{X}_{r_2}^{(3)} - \\
&\quad (\mathbf{T}_1 + \mathbf{D}_{r_1}) - (\mathbf{T}_2 + \mathbf{D}_{r_2}) + \alpha\mathbf{Z}_a^{(3)}] \bmod \Lambda_{sa}^{(n_3)} \\
&= [\mathbf{T}_1 + \mathbf{T}_2 + (\alpha - 1)h_a X_{r_2}^{(3)} \\
&\qquad + \alpha\mathbf{Z}_a^{(3)}] \bmod \Lambda_{sa}^{(n_3)} \\
&= [\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{Z}_{eff}] \bmod \Lambda_{sa}^{(n_3)},
\end{aligned} \tag{41}$$

where

$$\begin{aligned}
\mathbf{Z}_{eff} &= [(\alpha - 1)h_a X_{r_2}^{(3)} + (\alpha - 1)h_a X_{r_2}^{(3)} \\
&\qquad + \alpha\mathbf{Z}_a^{(3)}] \bmod \Lambda_{sa}^{(n_3)},
\end{aligned} \tag{42}$$

and Eq. (41) follows from the distributive law for the modulo operation and the modulo definition. Now, since $\mathbf{V}_a$ is available in the node 1, and thus $\mathbf{V}_{1a}$ and $\mathbf{V}_{2b}$, we can cancel the effect of them from $\mathbf{Y}_{da}$.

$$\begin{aligned}
\mathbf{Y}'_{da} &= [\mathbf{Y}_{da} - \mathbf{V}_a] \bmod \Lambda_{sb}^{(n_3)} \\
&= [[\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{Z}_{eff}] \bmod \Lambda_{sa}^{(n_3)} - \mathbf{V}_a] \bmod \Lambda_{sb}^{(n_3)} \\
&= [[\mathbf{V}_{1a} + \mathbf{V}_{1b} - Q_{\Lambda_{sb}}(\mathbf{V}_{1b} + \mathbf{D}_{1b})] \bmod \Lambda_{sa}^{(n_1)} \\
&\quad + \mathbf{V}_{2a} + \mathbf{V}_{2b} - Q_{\Lambda_{sb}}(\mathbf{V}_{2b} + \mathbf{D}_{2b}) + \mathbf{Z}_{eff} - \\
&\quad \mathbf{V}_{1a} - \mathbf{V}_{2a}] \bmod \Lambda_{sb}^{(n_3)}
\end{aligned} \tag{43}$$

$$= [\mathbf{V}_{1b} + \mathbf{V}_{2b} + \mathbf{Z}_{eff}] \bmod \Lambda_{sb}^{(n_3)} \tag{44}$$

$$= [\mathbf{V}_b + \mathbf{Z}_{eff}] \bmod \Lambda_{sb}^{(n_3)} \tag{45}$$

where Eqs. (43) and (44) are based on the fact that $\Lambda_{sa}^{(n_3)} \subseteq \Lambda_{sb}^{(n_3)}$ and the distributive law for the modulo operation and Eq. (45) follows from the definition

$\mathbf{V}_b = [\mathbf{V}_{1b} + \mathbf{V}_{2b}] \bmod \Lambda_{sb}^{(n_3)}$. Now we use the minimum Euclidean distance lattice decoding [1], [14] to estimate $\mathbf{V}_b$, correctly. Thus we get

$$\hat{\mathbf{V}}_b = Q_{\Lambda_c}(\mathbf{Y}'_{da}) \bmod \Lambda_{sb}^{(n_3)}$$
$$= Q_{\Lambda_c}(\mathbf{V}_b + \mathbf{Z}_{eff}) \bmod \Lambda_{sb}^{(n_3)}, \quad (46)$$

From Eq. (46), we can see that the estimation is incorrect if

$$\mathbf{Z}_{eff} \notin \upsilon_c. \quad (47)$$

Eq. (47) shows that the estimation of $\mathbf{V}_b$ is incorrect if the effective noise $\mathbf{Z}_{eff}$ leaves the Voronoi region surrounding the true codeword, i.e, $P_e = \Pr(\mathbf{Z}_{eff} \notin \upsilon_c)$. It can be shown that [1], [14], the error probability vanishes as $n_3 \to \infty$ if

$$\mu = \frac{(\mathrm{Vol}(\upsilon_c^{(3)}))^{\frac{2}{n_3}}}{2\pi e Var(\mathbf{Z}^*_{eff})} > 1, \quad (48)$$

where $\mathbf{Z}^*_{eff} \sim N(0, \mathrm{Var}(\mathbf{Z}_{eff}))$, $\mathrm{Var}(\mathbf{Z}_{eff}) = 2(\alpha-1)^2 h_a^2 P + \alpha^2$. Since $\Lambda_c^{(n)}$ is Poltyrev-good, the condition in Eq. (49) is satisfied. To minimize the variance of effective noise we choose $\alpha = 2h_a^2 P/(2h_a^2 P+1)$ and we get $Var(\mathbf{Z}_{eff}) = 2h_a^2 P/(2h_a^2 P+1)$. Now, from Eq. (19) for R_{ib}, we have:

$$R_{ib} = \frac{1}{n_3}\log(\frac{Vol(\upsilon_{sb}^{(n_3)})}{Vol(\upsilon_c^{(n_3)})})$$
$$= \frac{t_3}{2}\log(\frac{\sigma^2(\Lambda_{sb}^{(n_3)})}{G(\Lambda_{sb}^{(n_3)})(\mathrm{Vol}(\upsilon_c^{(n_3)}))^{\frac{2}{n_3}}}) \quad (49)$$
$$\le \frac{t_3}{2}\log(\frac{\sigma^2(\Lambda_{sb}^{(n_3)})}{G(\Lambda_{sb}^{(n_3)})2\pi e Var(\mathbf{Z}^*_{eff})}) \quad (50)$$
$$\le \frac{t_3}{2}\log(\frac{\sigma^2(\Lambda_{sb}^{(n_3)})}{Var(\mathbf{Z}^*_{eff})}) \quad (51)$$
$$= \frac{t_3}{2}\log(\frac{h_b^2 P}{Var(\mathbf{Z}^*_{eff})})$$
$$= \frac{t_3}{2}\log(\frac{h_b^2}{2h_a^2} + h_b^2 P),$$

where Eq. (50) follows from Eqs. (49) and (51) is based on Rogers goodness of $\Lambda_{sb}^{(n_3)}$ and the fact that $G(\Lambda_{sb}^{(n_3)}) \ge \frac{1}{2\pi e}$. Thus, in order to find V_b in node A, we must have

$$R_{1b} \le \frac{t_3}{2}\log(\frac{h_b^2}{2h_a^2} + h_b^2 P), \quad (52)$$

$$R_{2b} \le \frac{t_3}{2}\log(\frac{h_b^2}{2h_a^2} + h_b^2 P), \quad (53)$$

4.3.3 Decoding in the Node B

Using a similar decoding with decoding in node A, we can find lattice point $\mathbf{V}_a$ (and thus message W_a) if

$$R_{1a} \le \frac{t_3}{2}\log(\frac{h_a^2}{2h_b^2} + h_a^2 P), \quad (54)$$

$$R_{2a} \le \frac{t_3}{2}\log(\frac{h_a^2}{2h_b^2} + h_a^2 P). \quad (55)$$

Now, from Eqs. (32), (37), (52), (53), (54) and (55) and applying Fourier-Motzkin elimination, we can get the following rate-region for the Gaussian two-way diamond channel:

$$R_a \le \min((t_1R^*_{1a} + t_2R^*_{2a}), (t_2R^*_{2a} + \frac{t_3}{2}\log(\frac{h_a^2}{2h_b^2} + h_a^2 P)), (t_1R^*_{1a} + \frac{t_3}{2}\log(\frac{h_a^2}{2h_b^2} + h_a^2 P)), (\frac{t_3}{2}\log(\frac{h_a^2}{2h_b^2} + h_a^2 P) + \frac{t_3}{2}\log(\frac{h_a^2}{2h_b^2} + h_a^2 P))), \quad (56)$$

$$R_b \le \min((t_1R^*_{1b} + t_2R^*_{2b}), (t_2R^*_{2b} + \frac{t_3}{2}\log(\frac{h_b^2}{2h_a^2} + h_b^2 P)), (t_1R^*_{1b} + \frac{t_3}{2}\log(\frac{h_b^2}{2h_a^2} + h_b^2 P)), (\frac{t_3}{2}\log(\frac{h_b^2}{2h_a^2} + h_b^2 P) + \frac{t_3}{2}\log(\frac{h_b^2}{2h_a^2} + h_b^2 P))), \quad (57)$$

It should be noted that we assume $\Lambda_{sa}^{(n_3)} \subseteq \Lambda_{sb}^{(n_3)} \subseteq \Lambda_m^{(n_3)} \subseteq \Lambda_c^{(n_3)}$ and $\sigma^2(\Lambda_{sb}) \le \sigma^2(\Lambda_{sa})$ for this lattice structure. In other case that the channel between node B and Relays are better than the channel between node A and Relays, lattice chain differs as $\Lambda_{sb}^{(n_3)} \subseteq \Lambda_{sa}^{(n_3)} \subseteq \Lambda_m^{(n_3)} \subseteq \Lambda_c^{(n_3)}$. In this case, R_a and R_b constraints have the same equations as our first scenario.

5 Numerical Result

In this section, we evaluate the performance of our proposed method by numerical simulations. We compare the achievable rate-region by our proposed scheme based on lattice superposition coding with that of [11] based on random superposition coding in Figs. 2 and 3. As we can see the achievable rate-region using our proposed scheme is better that [11]. This is due to the fact that using superposition coding with lattice codes, we have significantly reduced the constraints over rates and this yields to a better rate region. For two cases of channel parameters our achievable rate region becomes closer to the outer bound than [11].

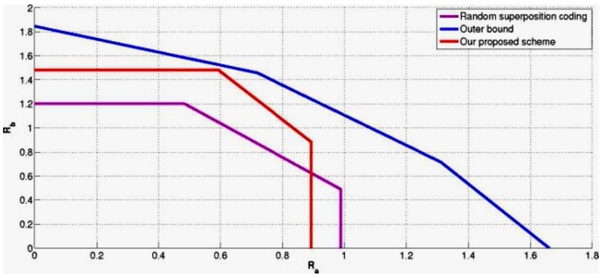

Fig. 2 Comparison between the rate regions of our proposed scheme with that of the proposed scheme in [11] and the outer bound (Channel parameters are: $h_a^2P=15$ and $h_b^2P=8$).

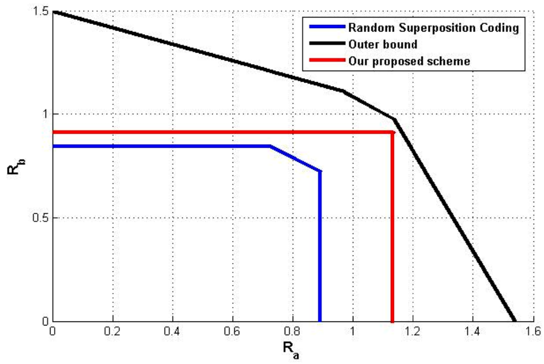

Fig. 3 Comparison between the rate regions of our proposed scheme with that of the proposed scheme in [11] and the outer bound (Channel parameters are: $h_a^2P=8$ and $h_b^2P=10$).

6 Conclusion

In this paper, we studied the Gaussian two-way diamond channel in half-duplex mode. Specially, we considered the Gaussian two-way diamond channel which operates in the CF-MAC protocol. Using lattice codes, we obtain a new rate-region for this protocol and as we saw this rate region is better than the obtained rate region.

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