

Multilevel PWM Waveform Decomposition and Phase-Shifted Carrier Technique

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Abstract: Multilevel PWM waveforms can be decomposed into several multilevel PWM components. Phase-shifted carrier (PSC) is an efficient decomposition technique. In this paper, we have first demonstrated the equality of PSC and alternative phase opposition disposition techniques. Second, we have modified PSC to accommodate other disposition techniques. Third, we have investigated the effects of using asymmetrical carriers on the spectrum of the resulting PWM waveform. Fourth, we have proposed a logical algorithm for decomposing all types of multilevel PWM waveforms.

Keywords: Carrier-Based PWM, Phase-Shifted Carrier, Multilevel inverters.

1 Introduction

Multilevel PWM inversion is an effective and practical solution for increasing power and reducing harmonics of AC waveforms. A multilevel inverter has four main advantages over the conventional bipolar inverter: First, the voltage stress on each switch is decreased, compared to existing topologies where switches are connected in series. Therefore, the rated voltage, and consequently, the total power of the inverter could be safely increased. Second, the rate of change of voltage (dv/dt) is decreased due to the lower voltage swing of each switching cycle. Third, harmonic distortion is reduced due to more output levels. Fourth, lower acoustic noise and EMI is obtained: [1].

Based on these advantages, various circuit topologies and modulation strategies have been reported for better utilization of multilevel inverters. Multilevel topologies are classified into three categories: Diode Clamped inverters, Flying Capacitor inverters, and Cascaded inverters (also called chain inverters). The topologies have an equal number of main switches. A 5-level inverter with different topologies is shown in Fig. 1. Diode Clamped inverter has the least number of capacitors among these three topologies but requires additional clamping diodes. Flying Capacitor inverter requires the most number of capacitors. Cascaded inverter has simple structure but it needs various separated DC sources. The main drawback of Diode

Clamped inverter is the unbalanced DC link capacitor. This limits the application of Diode Clamped inverter to applications with five or more levels. This problem may be mitigated using combinational topologies. For example, combining a 3-level Diode Clamped with H-bridge topology results in a 5-level mixed structure called neutral point clamped (NPC), mitigating the unbalanced DC link problem of the Diode Clamped configuration and also eliminating the need for separate DC sources: [2] to [5].

Various switching strategies have been developed to generate switching angles, to fit diverse topologies and applications. Carrier based strategies are of major interest due to their simplicity and flexibility. The most popular carrier-based techniques are:

1. Alternative phase opposition (APO) disposition
2. Phase opposition (PO) disposition
3. Phase (PH) disposition.

The main difference between these switching strategies is the shape of the undesired harmonics spectrum. While other phase displacements could also be used for contiguous triangular carriers, we focus on these three types because of their efficiency and popularity: [6] and [7].

In Section 2 of this paper, we first present PSC as an equivalent for APO disposition technique using a 5-level example. Further, we modify the PSC technique to accommodate with PO and PH disposition techniques. In Section 3, we will generalize the PSC technique for the 5-level example to N-level. In Section 4, we will explain the effects of using asymmetrical carriers on the spectrum of the multilevel PWM waveform. We then propose a logical decomposition technique for efficient

Iranian Journal of Electrical & Electronic Engineering, 2008.
Paper first received 4th July 2007 and in revised form 17th November 2008.

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decomposition of multilevel PWM waveforms. Using PSC technique, we will explain how to generate the control signals for switches for bipolar and unipolar configurations in Section 5.

2 Phase-Shifted Carrier Technique

2.1 General Definitions

To simplify our discussion about different dispositions and PSC technique, we have used a special notation for carrier and modulating signals. We have also defined some useful parameters:

1-Modulating signal:

$$y_m(A_m, \omega_m, \phi_m) = A_m \sin(\omega_m t + \phi_m)$$

2-Carrier signal:

$$y_c(A_c, \omega_c, \phi_c, w) = A_c \cdot \left\{ \begin{array}{ll} \frac{\omega_c t + \phi_c}{2\pi w} - \frac{1}{2} & 0 \leq \omega_c t + \phi_c < 2\pi w \\ -\frac{\omega_c t + \phi_c}{2\pi(1-w)} + \frac{1}{1-w} - \frac{1}{2} & 2\pi w \leq \omega_c t + \phi_c < 2\pi \end{array} \right.$$

As shown in Fig. 2 y_c is a periodic function with the period ($T_c = 2\pi/\omega_c$).

w is a parameter between 0 and 1 which represents the triangle width. For $w = 0.5$, y_c is a symmetrical triangular waveform and for $w = 0$ (1), y_c is a negative (positive) ramp sawtooth waveform.

3-Modulation index:

$$M = \frac{2A_m}{(N-1)A_c}$$

4-Modulation frequency ratio (Pulse number):

$$P = \frac{\omega_c}{\omega_m}$$

5-The number of strictly positive levels:

$$N' = \left\lceil \frac{N}{2} \right\rceil$$

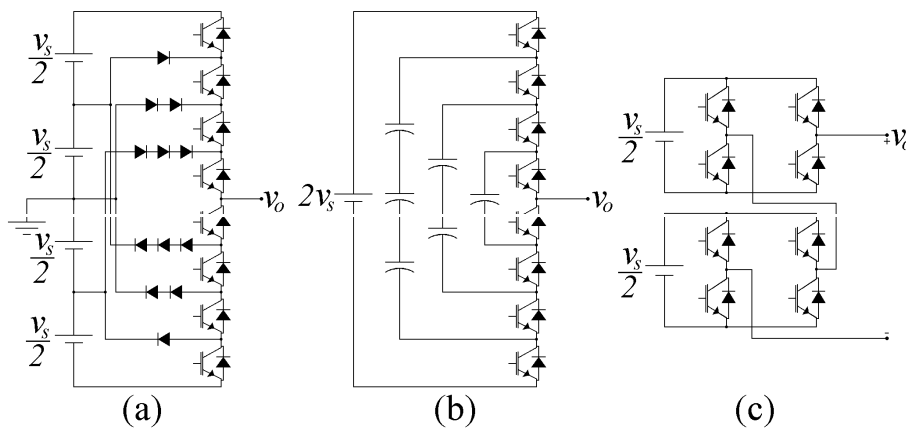


Fig. 1 Different circuit topologies for a 5-level inverter (a) Diode Clamped, (b) Flying Capacitor, (c) Cascaded.

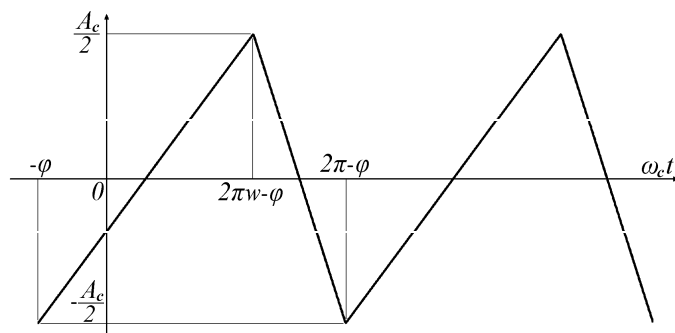


Fig. 2 Triangular carrier.

2.2 APO, PO, and PH Dispositions at a Glance

Carrier based N-level PWM operations consist of N-1 different carriers. The carriers have the same frequency ω_c , the same peak-to-peak amplitude A_c , and are disposed so that the bands they occupy are contiguous. They are defined as:

$$C_n = y_c(A_c, \omega_c, \phi_n, w) + (n - N/2)A_c, \quad n = 1, \dots, N-1$$

The zero reference is placed in the middle of the carrier set. And we choose the carrier C_1 as the reference carrier.

For APO disposition, all carriers are alternatively in phase opposition. The carriers are defined as:

$$C_n = (-1)^{n-1} y_c(A_c, \omega_c, \phi_c, 0.5) + (n - N/2)A_c \quad (1)$$

Fig. 3(a) shows the carriers and the modulating signal for a 5-level PWM using APO disposition technique with $M = 0.75$ and $P = 80$.

For PO disposition all the carriers above the zero reference are in phase among them but in opposition with those below. They are defined as:

$$C_n = (-1)^{\lfloor \frac{2n}{N} \rfloor} y_c(A_c, \omega_c, \phi_c, 0.5) + (n - N/2)A_c, \quad N \text{ odd}$$

Note that the PO disposition is undefined for even numbers of N. The Modulating signal and the carriers for the 5-level example using PO disposition is shown in Fig. 3(b).

For PH disposition, all carriers are in phase. They are defined as:

$$C_n = y_c(A_c, \omega_c, \phi_c, 0.5) + (n - N/2)A_c$$

Fig. 3(c) shows the arrangement of carriers in the 5-level example for PH disposition.

The harmonic content of the PWM waveforms using APO, PO, and PH dispositions are shown in Fig. 3(d,e,f) respectively. It is clear that the main difference is about the first set of undesired harmonics (those aggregated near P). For APO and PO dispositions, no harmonic exists at P due to odd symmetry of their PWM waveforms. For the PH case, however, the waveform is asymmetric and the harmonic at P is relatively high. Thus, APO and PO dispositions are more convenient to be used in single-phase inverters. For three-phase inverters, however, the triplen harmonics of voltage (current) will be eliminated due to Y (Δ) connection of the load. Choosing P as a multiple of three, we can

eliminate the harmonic at P. In this case, PH disposition is more convenient due to the very little values of other harmonics.

2.3 PSC as an Equivalent for APO

We start our discussion considering the 5-level PWM example and then generalize it to N levels. For the APO case, the four carriers are alternatively in phase opposition. Using Eq. (1), the carriers are defined as:

$$C_1 = y_c(A_c, \omega_c, 0, 0.5) - 1.5A_c$$

$$C_2 = y_c(A_c, \omega_c, \pi, 0.5) - 0.5A_c$$

$$C_3 = y_c(A_c, \omega_c, 0, 0.5) + 0.5A_c$$

$$C_4 = y_c(A_c, \omega_c, \pi, 0.5) + 1.5A_c$$

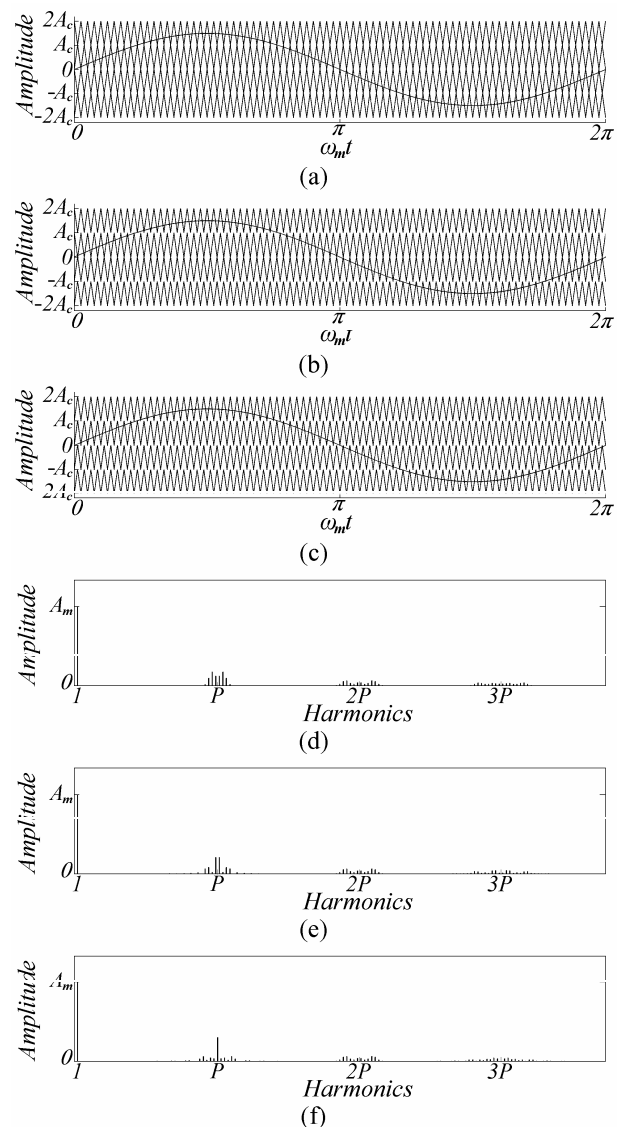


Fig. 3 A 5-level PWM operation with $M = 0.75$ and $P = 80$. Modulating signal and carriers for (a) APO, (b) PO, (c) PH. Waveform's spectrum for (d) APO, (e) PO, (f) PH dispositions.

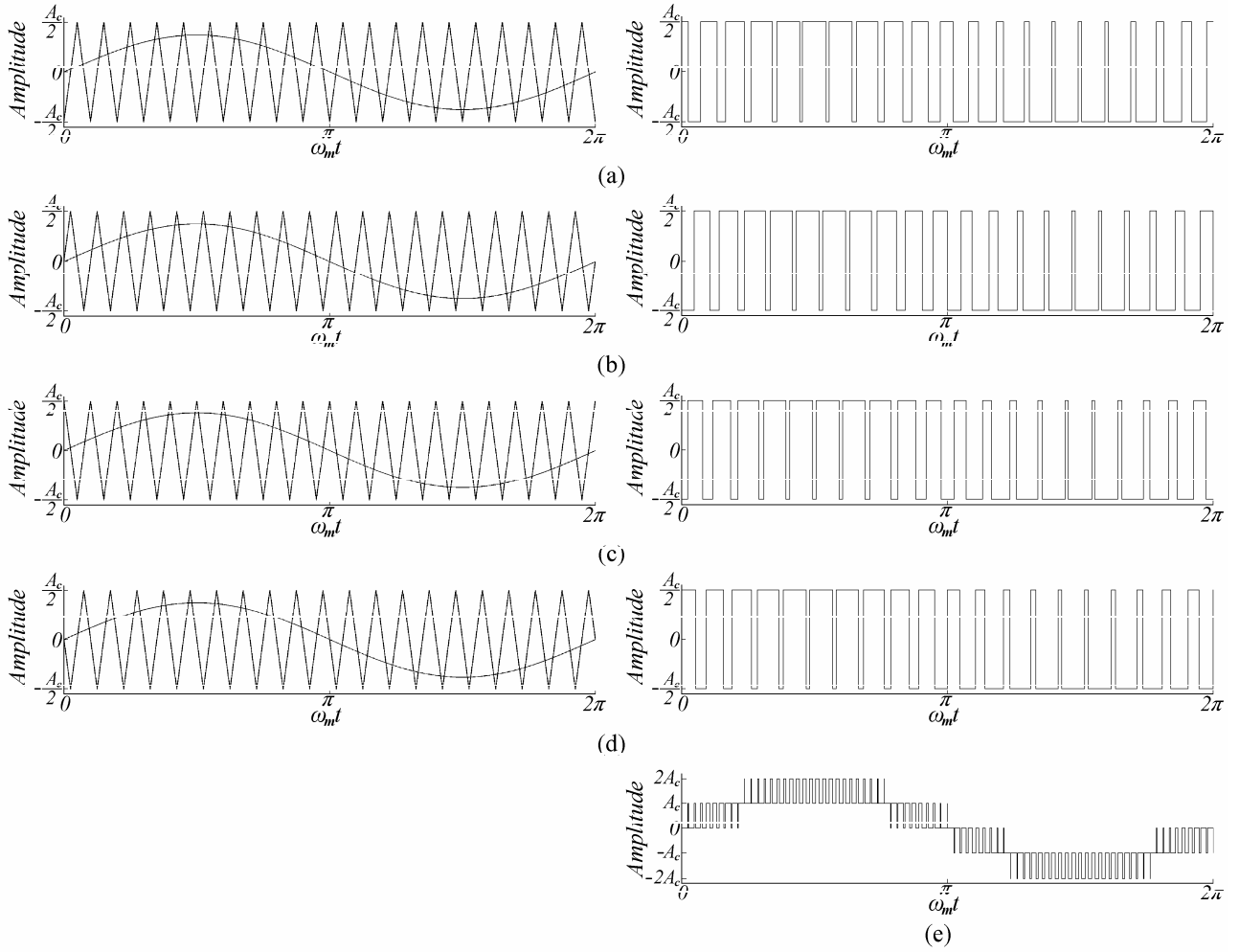


Fig. 4 Carrier and PWM waveforms for (a) C_1 , (b) C_2 , (c) C_3 , (d) C_4 and (e) resulting PWM waveform generated by adding up the last parts.

And the amplitude of the modulating signal is:

$$A_m = 2MA_c$$

The carriers and the spectrum of the resulting waveform are shown in Fig. 3(a,d). Since the switching frequency is equal to ω_c , the undesired harmonics are aggregated near the multiples of P . Note that no even harmonic exists due to odd symmetry of the PWM waveform.

Instead of the technique cited for the APO disposition, we can use PSC technique to generate the same switching angles. In this technique the 5-level operation splits into four different bipolar PWM operations. The frequency of each carrier is four times smaller than the frequency of carriers for APO case, and the carriers are phase shifted by $\pi/2$. They are defined as:

$$\begin{aligned} C_1' &= y_c(A_c, \omega_c', 0, 0.5) \\ C_2' &= y_c(A_c, \omega_c', \pi/2, 0.5) \end{aligned}$$

$$\begin{aligned} C_3' &= y_c(A_c, \omega_c', \pi, 0.5) \\ C_4' &= y_c(A_c, \omega_c', 3\pi/2, 0.5) \end{aligned}$$

Fig. 4(a,b,c,d) show the four bipolar operations using these carriers and the resulting bipolar PWM waveforms. The amplitude of the sinusoidal wave is:

$$A_m' = MA_c/2$$

As depicted in Fig. 4(e), summing up the four resulting bipolar PWM waveforms gives the same 5-level PWM wave as we had in the APO disposition technique. Note that the frequency of the resulting 5-level PWM is four times greater than that of each bipolar PWM waveform:

$$\omega_c = 4\omega_c', \quad P = 4P'$$

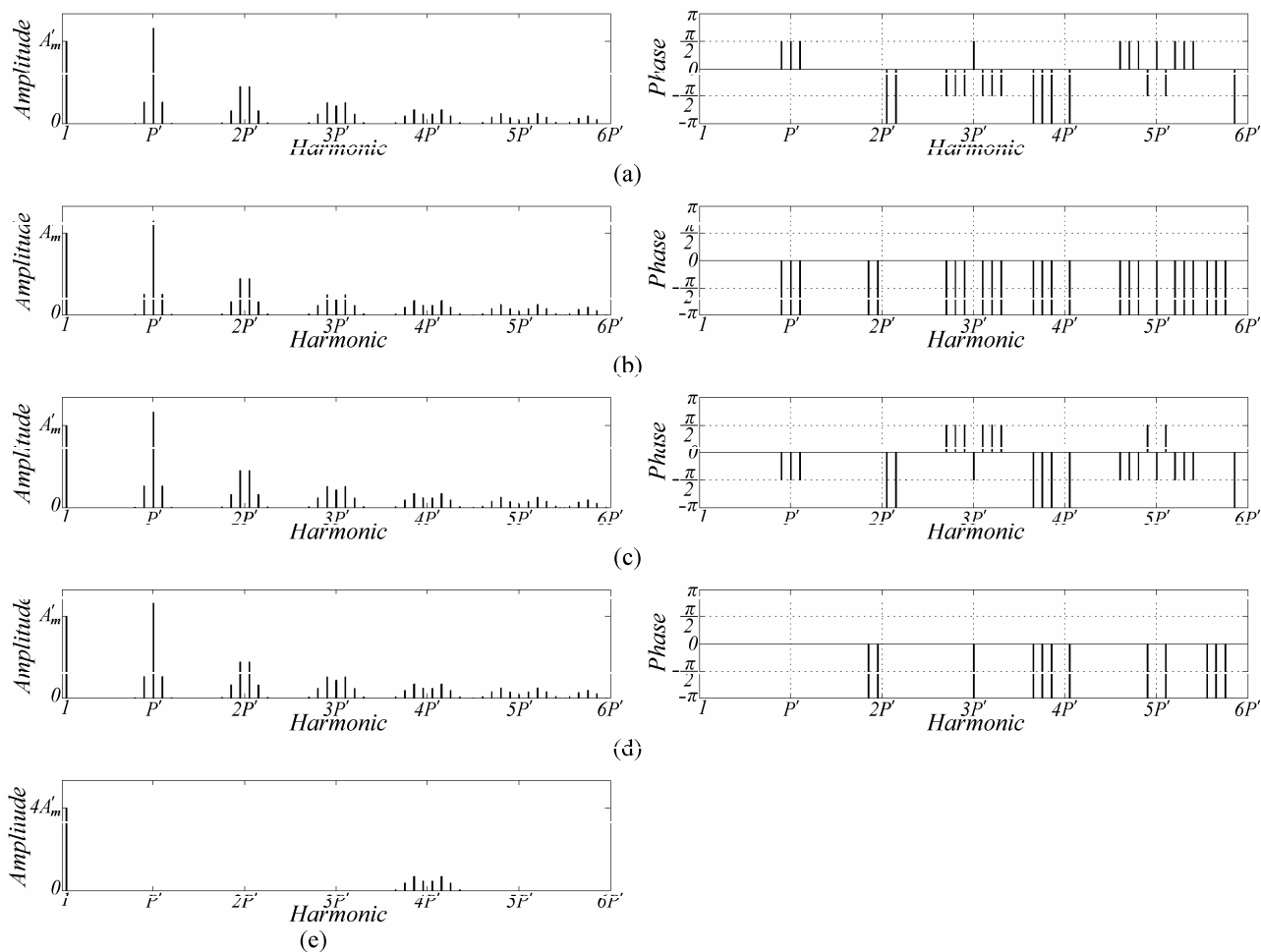


Fig. 5 Amplitude and phase in terms of harmonics for the PWM waveforms in Fig. 4 respectively.

Fig. 5(a,b,c,d) show the amplitude and phase of bipolar PWM waveforms generated by the carriers C_1 , C_2 , C_3 , and C_4 in terms of harmonics, respectively. Note that for better illustration, the phase angle of the harmonics with the amplitude more than $0.01A_c$ are shown and the others are set to zero. As depicted, for any amount of phase shifting the amplitude diagrams are the same and the difference is in phase angles. The undesired harmonics aggregated near the P' , $3P'$, $5P'$, ..., $(2m-1)P'$ harmonics are in phase opposition between C_1 and C_3 , and between C_2 and C_4 . Furthermore, the aggregated harmonics near the $2P'$, $6P'$, ..., $(2m-1)2P'$ harmonics are in phase opposition between $C_1(C_3)$ and $C_2(C_4)$. Therefore, these harmonics are eliminated when we sum up the four bipolar PWM waveforms and only the harmonics aggregated near $4mP'$ (mP) harmonics remain. The amplitude of the resulting 5-level PWM wave in terms of harmonics is shown in Fig. 5(e).

The spectrum of an N-level APO PWM or its PSC equivalent consists of the harmonics aggregated near $m(N-1)P'$ in the spectrum of bipolar sub-operations for

PSC. The other sets of harmonics in the spectrum of bipolar sub-operations are eliminated due to the opposition of their phase in different sub-operations.

2.4 Demonstration of Equality

To demonstrate the equality of the resulting waveforms, we can use mathematical relations. However, summing up four expressions to obtain the results that we already have for APO disposition is not quite interesting. Instead, we use a method originally developed by Bennett to obtain the mathematical relation which defines the PWM waveforms in the form of Fourier series. Using this approach gives a deeper understanding about different aspects of the concept and enables us to develop the equivalent PSC techniques for PO and PH dispositions: [8] and [9].

This method changes the problem of intersecting two periodic functions by the equivalent problem of intersecting a non-periodic function with a two orientation periodic function. It states that by referring to a bipolar modulation in which only one carrier signal is involved, to build the mathematical model of the

modulation process we can exploit the fact that the carrier is made of straight segments (it is a triangular wave). Each straight segment has an intersection point with the sinusoidal wave, but these straight segments are alternatively positive and negative ramps. Thus, we can consider the intersection of a negative ramp segment and the sinusoidal wave as the intersection of extending the last positive segment and a sinusoid biased by A_c and in phase opposition with the sinusoid of last segment. Putting it together, we can modify the problem by defining a surface ($z = F(x,y)$), which consists of different sinusoids alternatively in phase opposition toward y axis. The sinusoids subdivide the xy plane into different zones. The amount of z component in each zone is alternatively $+A_c/2$ and $-A_c/2$. Intersecting this surface with the plane ($y = Px$), gives us the PWM waveform. Fig. 6(a) shows the surface, the plane, and their intersection. To obtain the expression for the PWM waveform, the double Fourier series of the surface is calculated. The intersection is mathematically obtained introducing the relation:

$$y = Px$$

To find the time expression of the PWM waveform which is the intersection line between the surface and the plane we must introduce the following relations:

$$x = \omega_m t, \quad y = \omega_c t$$

In the case of over-modulation ($M > 1$) the sinusoids generated by the process described above partially overlap, delimiting new zones that must not be taken into account, the value of F inside these zones being the same as outside.

To demonstrate the equality of APO and PSC techniques, we just need to show that the resulting surfaces for these two techniques are equal. The surface presented, is periodic in both x and y . As depicted in Fig. 6(b) which shows the top view of the surface, ϕ identifies the position of the Reference point (R). Thus, according to periodicity of the surface and by assuming R as the zero point, considering one period of the surface gives us all we need to know about it:

$$0 \leq x < 2\pi, \quad -\pi \leq y < \pi, \quad R = (0,0)$$

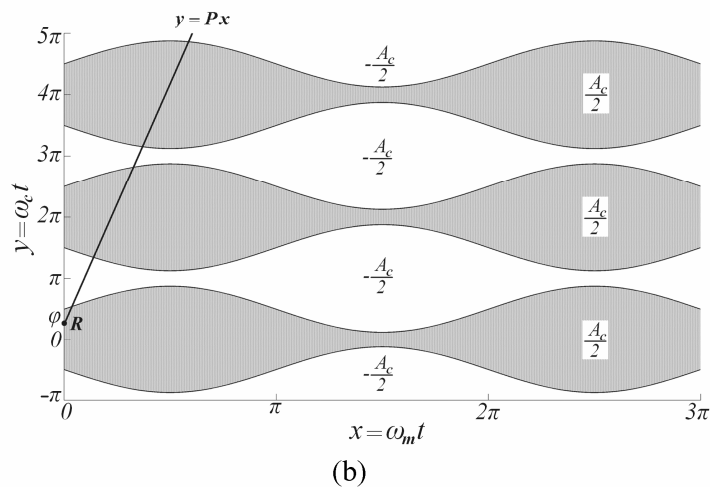
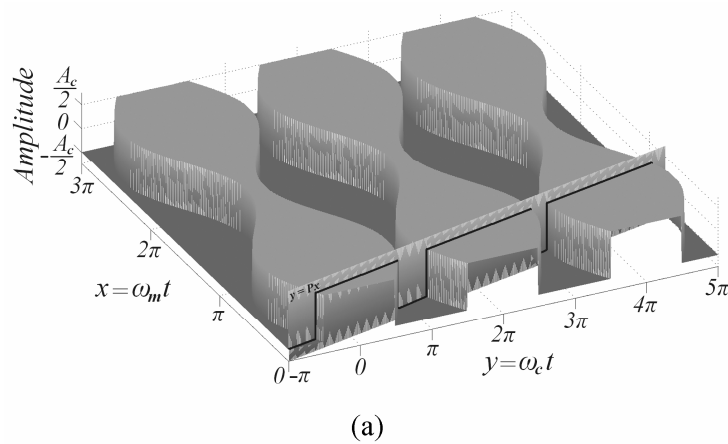


Fig. 6 Three-dimensional model for a bipolar modulation (a) the surface $z = F(x,y)$, the plane $y = Px$, and their intersection line. (b) top view of the surface and the plane with the carrier phase shifted by ϕ .

Fig. 7(a,b,c,d) show the top view of equivalent surfaces for C_1 , C_2 , C_3 , and C_4 , respectively. Note that here we subdivide the process into four bipolar processes for each carrier. Each bipolar process gives a surface which can assume only two values. These values are 0 and A_c for carriers above the zero reference, 0 and $-A_c$ for the carriers below. Superimposing these surfaces gives the resulting surface for the APO disposition technique. The resulting surface and its top view are shown in Fig. 7(e,f).

The top view of the equivalent surfaces for C'_1 , C'_2 , C'_3 , and C'_4 is shown in Fig. 8(a,b,c,d) respectively. Note that in this case the only difference between surfaces is about their reference point. Thus, each surface is shifted by $\pi/2$ toward y' axis. Adding up these surfaces gives the resulting surface for the PSC technique which is shown in Fig. 8(e,f). It is clear that the surface for APO disposition and the one for PSC technique are the same. The only difference that we see in the figures is that four periods of the surface for PSC is shown, comparing to one period that is shown for APO. It is because the y' axis in PSC is four times greater in scale than the y axis in APO disposition (Remember that $\omega_c = 4\omega'_c$).

As we illustrated, for our 5-level example, the surfaces and consequently the resulting PWM waveforms for APO disposition and PSC techniques are equal.

2.5 PSC Equivalents for PO and PH

In this section, we modify the presented PSC technique to accommodate with PO and PH dispositions. Considering our 5-level example, the surface F for PO disposition is obtained using the same process as APO. The surface and its top view are shown in Fig. 9(b,c).

As depicted in Fig. 9(a), we can subdivide the resulting surface for APO disposition into six different parts toward x axis. The parts are chosen based on the boundaries in which the sinusoidal wave travels from the region of one carrier into another. In our example the sinusoidal wave passes from the region of C_3 to that of C_4 at:

$$y_c = A_c \quad 0.75 \sin(\theta) = 0.5 \quad \theta = 41.81^\circ$$

So, the six parts are defined as:

$$\begin{aligned} B_1 : & 0 < x < \theta & B_4 : & \pi < x < \pi + \theta \\ B_2 : & \theta < x < \pi - \theta & B_5 : & \pi + \theta < x < 2\pi - \theta \\ B_3 : & \pi - \theta < x < \pi & B_6 : & 2\pi - \theta < x < 2\pi \end{aligned}$$

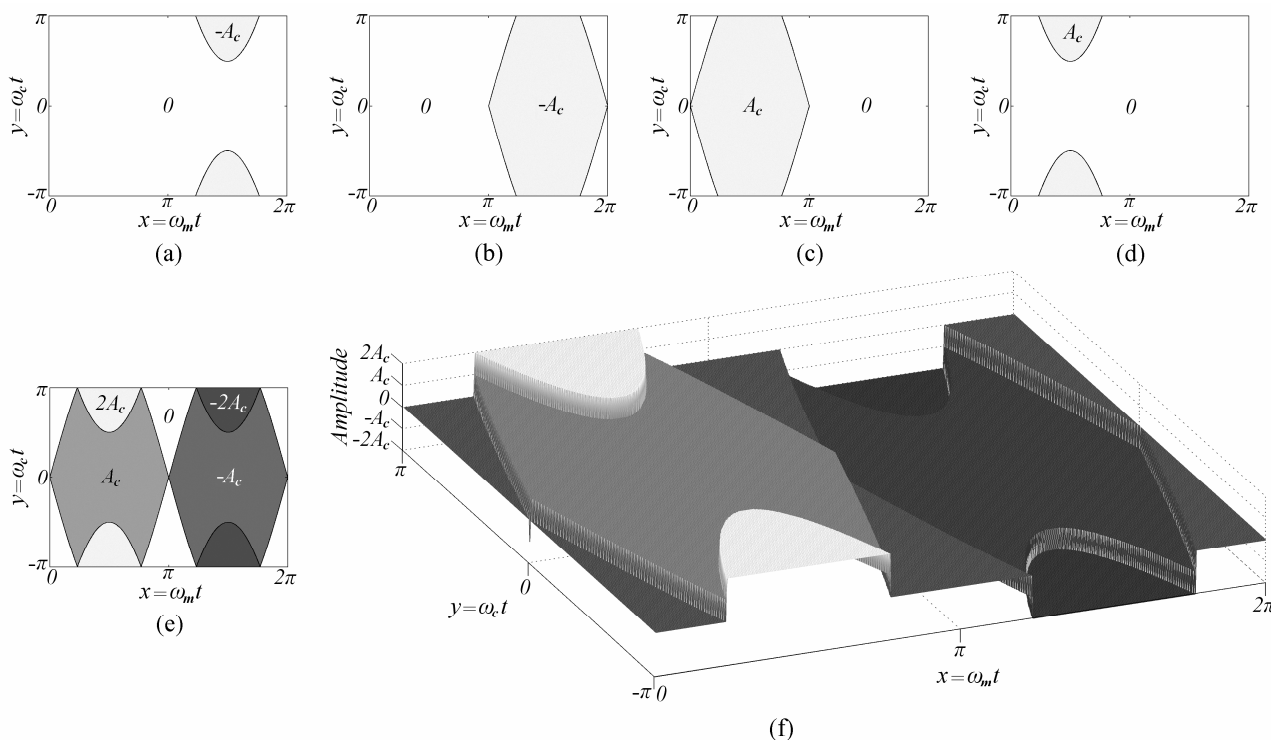


Fig. 7 The surface F corresponding to bipolar operations using (a) C_1 , (b) C_2 , (c) C_3 , (d) C_4 , (e), (f) corresponding surface and its top view for APO disposition technique.

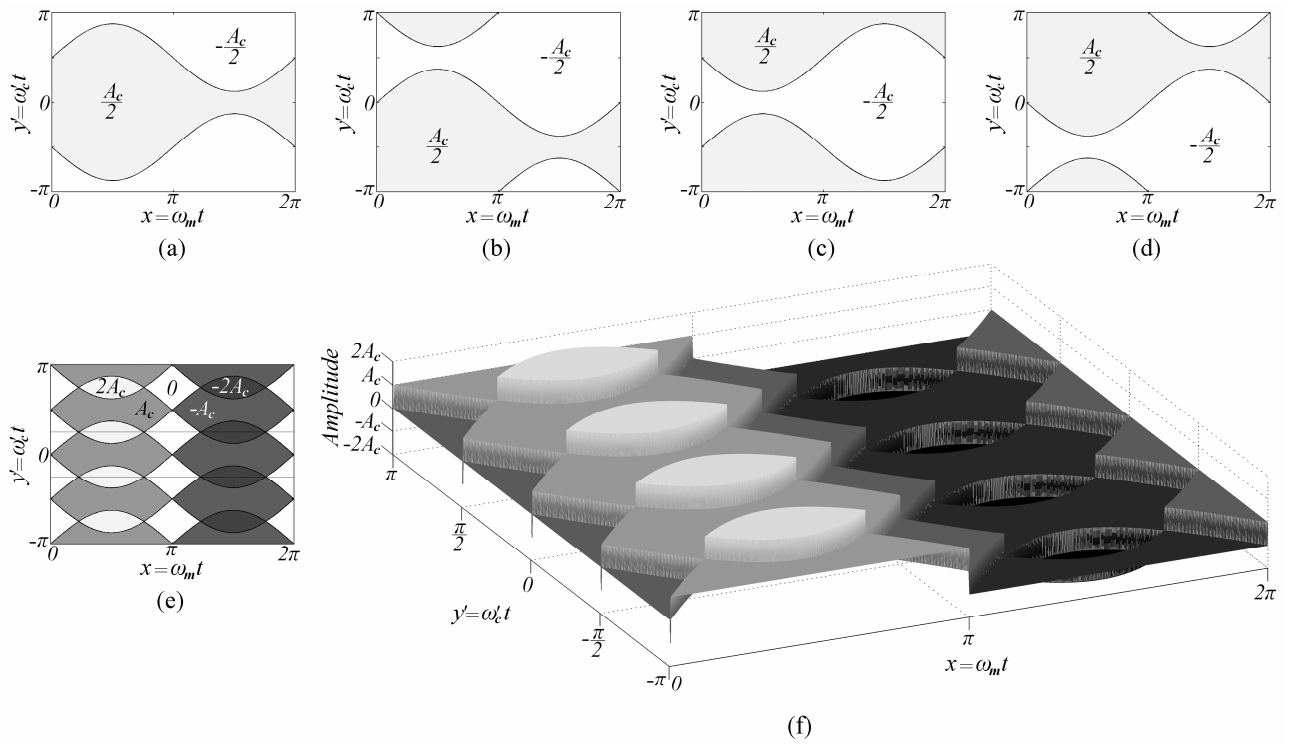


Fig. 8 The surface F corresponding to bipolar sub-operations using (a) C_1 , (b) C_2 , (c) C_3 , (d) C_4 , (e), (f) corresponding surface and its top view for PSC technique.

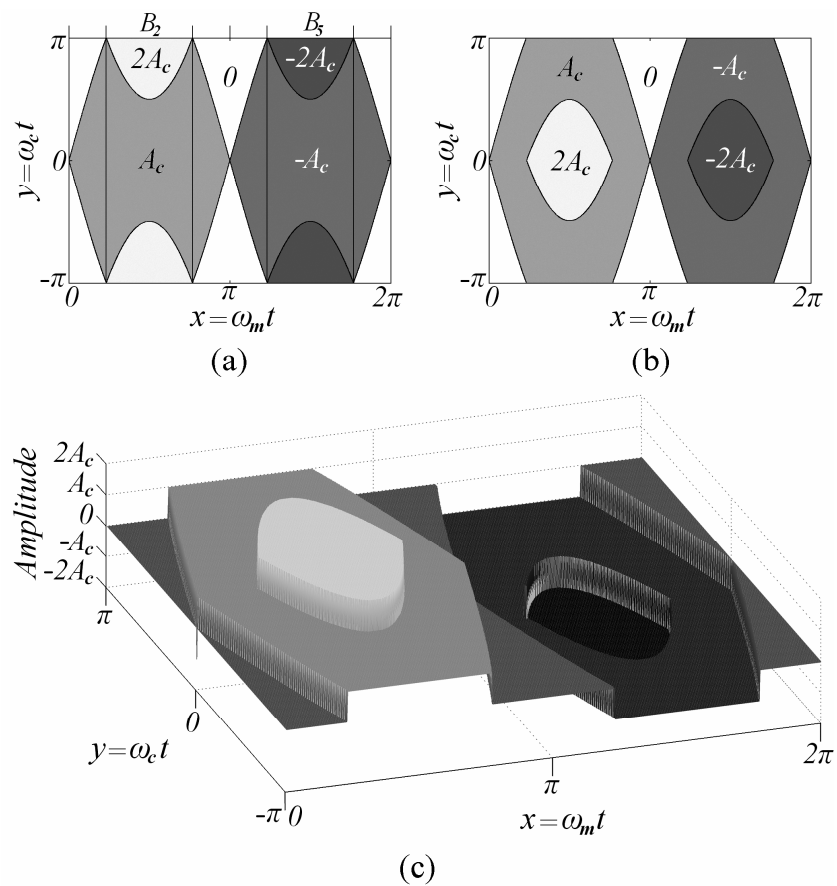


Fig. 9 PO disposition (a) parts that must be shifted, (b), (c) the corresponding surface and its top view.

Comparing the surfaces for APO and PO dispositions, it is evident that if we shift the parts B_2 and B_5 by π toward y axis in the APO surface, the PO surface will be obtained. To generate the carriers needed for the PSC equivalent, we should impose the shifting operation to the corresponding parts of carriers. In other words, we should shift the parts B_2 and B_5 in each carrier by $\pi/4$. Note that the y' axis is four times greater in scale than the y axis. That is why the amount of phase shifting has been divided by 4. For example, the PO equivalent for C'_3 is shown in Fig. 10(a) and is defined as:

$$C'_{3(PO)} = \begin{cases} y_c(A_c, \omega'_c t, \pi, 0.5) & \omega_m t : B_1, B_3, B_4, B_6 \\ y_c(A_c, \omega'_c t, \frac{5\pi}{4}, 0.5) & \omega_m t : B_2, B_5 \end{cases}$$

For the PH disposition, all we need to do is to shift the parts B_2 , B_4 , and B_6 by π toward y axis. Like the PO case, the amount of phase shifting toward y' axis is $\pi/4$. Parts that must be shifted, the surface and its top view are shown in Fig. 11. The PH equivalent for C'_3 is shown in Fig. 10(b) which is:

$$C'_{3(PH)} = \begin{cases} y_c(A_c, \omega'_c t, \pi, 0.5) & \omega_m t : B_1, B_3, B_5 \\ y_c(A_c, \omega'_c t, \frac{5\pi}{4}, 0.5) & \omega_m t : B_2, B_4, B_6 \end{cases}$$

3 Generalization to N Levels

The procedure we have introduced could be generalized to other conditions like N-level inversion or even over-modulation. The surface F corresponding to N-level PWM using APO disposition technique, has three public shapes. Fig. 12(a,b) show the surfaces for N odd (N' even and odd), respectively. And the surface for N even is shown in Fig. 12(c). The surfaces consist of

different zone separators which are a part of a sine wave. In fact the part of modulating signal which overlaps the region of each carrier configures a set of zone-separators in the corresponding surface which are placed due to the phase angle of that carrier and separate the zones that have two different z values corresponding to the boundary levels of that carrier. For N even the zone-separator starts from $\pm\pi/2$ where as in the case of N odd, it starts from the zero point. It is because unlike the odd case for N even the modulating signal starts from the middle point of $C_{N'}$ carrier.

The abscissas of the points in which the sinusoidal wave intersects the predefined levels between 0 and $\pi/2$ are denoted by $x_1, x_2, \dots, x_{N'}$. The number of strict positive levels required by the modulation process, the influence of the parameters N' and M on the shape of the figures is considered by defining N'' which is defined as:

$$N'' = \begin{cases} \left[\frac{MN' + 1}{2} \right] & N \text{ odd} \\ \left[\frac{MN' + 1 - M}{2} \right] & N \text{ even} \end{cases}$$

According to the symmetry of sinusoidal wave and uniform distribution of levels, the other angles are defined as:

$$\begin{aligned} 0 \leq x < \pi/2 & \quad x_1, \dots, x_{N'} \\ \pi/2 \leq x < \pi & \quad \pi - x_{N'}, \dots, \pi - x_2 \\ \pi \leq x < 3\pi/2 & \quad \pi + x_1, \dots, \pi + x_{N'} \\ 3\pi/2 \leq x < 2\pi & \quad 2\pi - x_{N'}, \dots, 2\pi - x_2 \end{aligned}$$

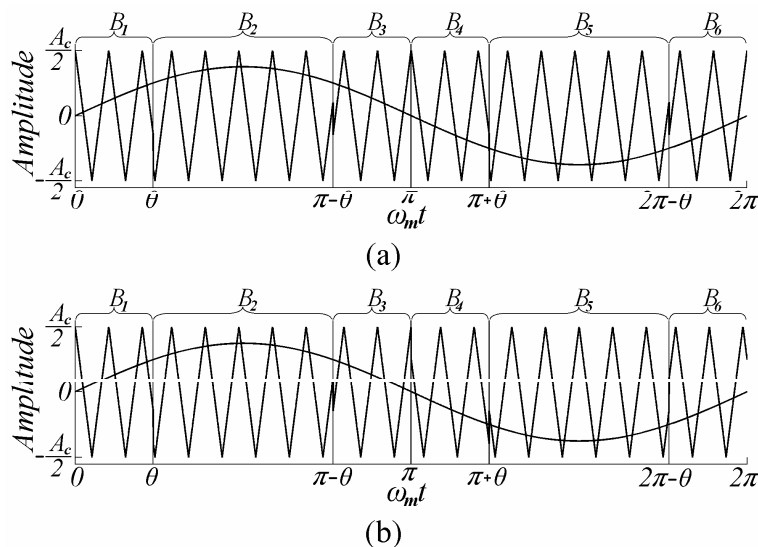


Fig. 10 (a) The modified carrier for PO disposition ($C'_{3(PO)}$), (b) The modified carrier for PH disposition ($C'_{3(PH)}$).

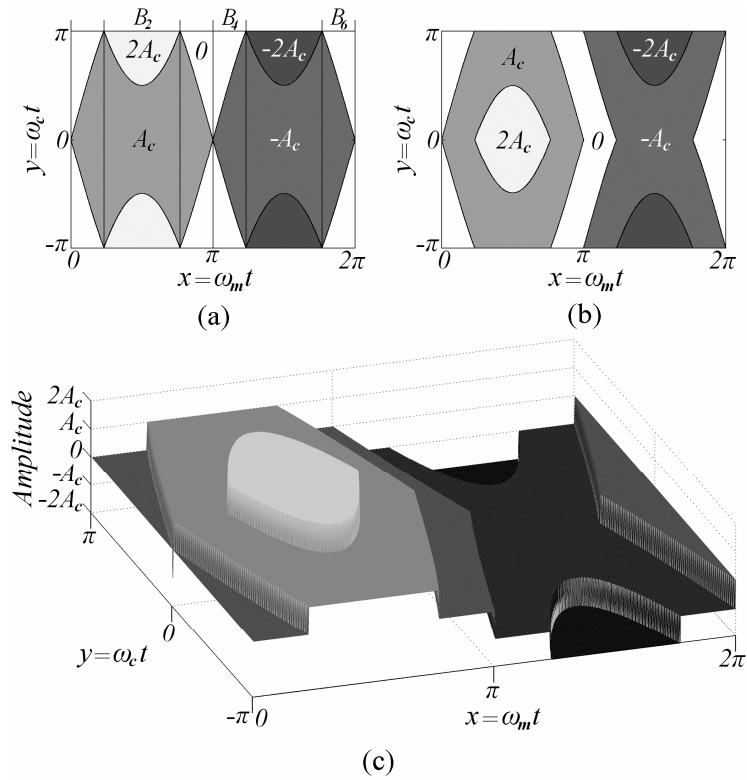


Fig. 11 PH disposition (a) parts that must be shifted, (b), (c) the surface F and its top view.

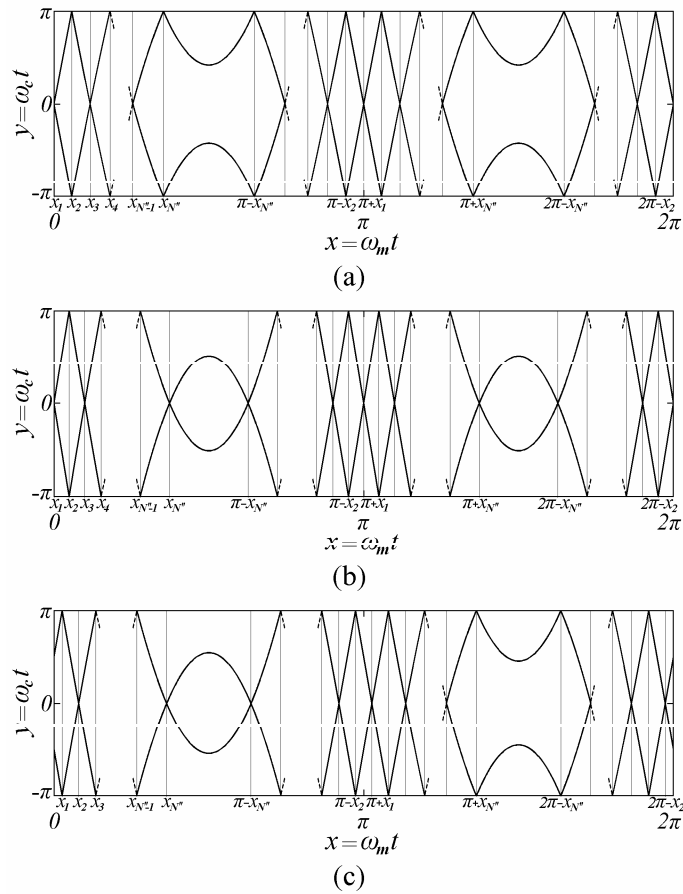


Fig. 12 General surfaces for APO disposition (a) N odd (N' even), (b) N odd (N' odd), (c) N even.

Based on this, we can subdivide the surface into parts between two consecutive angles which are called bases. There are four families of bases in APO disposition. Although to keep compatibility of this definition with those works which have done in this field we should add two more angles to our list, here we avoid doing this and we choose the name part instead of base to avoid any kind of misunderstanding. The parts are defined as:

$$\begin{aligned} B_1 &= [x_1, x_2), \dots, B_{2N-4} = [2\pi - x_2, 2\pi) \quad N \text{ odd} \\ B_1 &= [0, x_1), \dots, B_{2N-3} = [2\pi - x_2, 2\pi) \quad N \text{ even} \end{aligned}$$

The equality of the APO and PSC techniques is generally, because of the same configuration (same shape and positioning) of these parts when considering one period of APO and $1/(N-1)$ period of PSC. In fact, the phase shifted bipolar surfaces in the PSC technique partially cooperate to form the same surface as the one for APO.

In general, an N-level PWM using APO disposition technique consists of N-1 different carriers with the frequency ω_c which are defined by Eq. (1). The equivalent PSC technique consists of N-1 different carriers with the frequency $\omega_c/(N-1)$ which are defined as:

$$C'_n = y_c \left(A_c, \frac{\omega_c}{N-1}, \frac{(n-1)2\pi}{N-1}, w \right), \quad n = 1, \dots, N-1$$

In a more general case an N-level APO disposition PWM operation could be decomposed into m different $N^{(m)}$ -level APO, PO, or PH PWM operations in a way that:

$$m(N^{(m)} - 1) = N - 1, \quad m \in \mathbb{N}$$

The reference carrier for each $N^{(m)}$ -level operation is:

$$C_n^{(m)} = y_c \left(A_c, \frac{\omega_c}{m}, \frac{(n-1)2\pi}{m}, w \right) + \left(1 - \frac{N^{(m)}}{2} \right) \quad (2)$$

$$n = 1, \dots, m$$

The arrangement of the other carriers in each $N^{(m)}$ -level operation depends on the type of operation chosen (APO, PO, or PH) and must set according to their reference carrier. Although decomposing an APO operation into several PO or PH operations seems strange, but it was predictable. As depicted in Fig. 3 the first set of undesired harmonics differs among three dispositions, however the other sets are equal. Thus, for

example if we add two $N^{(2)}$ -level PH operations which are in phase opposition according to Eq. (2), the first set of undesired harmonics will be eliminated and the second set remains which is the same for APO, PO and PH dispositions.

The advantage of this generalization show itself specially, in the case of cascaded inverters, where we need to split an N-level operation into several 3-level operations to control the switches in each unipolar cell. Then, each 3-level operation uses one carrier but opposite phase sinusoids to generate the switching angles needed for each bipolar leg.

For PO and PH dispositions, the strategy is different. Fig. 13(a) shows the surface for PO disposition (definitely for N odd) in its general form. Comparing to the general surfaces illustrated for APO disposition, to accommodate the surface of PSC with PO we should shift the parts $B_2, B_4, \dots, B_{N-3}, B_N, B_{N+2}, \dots, B_{2N-3}$ by π toward y axis. In other words we must shift the even parts between 0 and π , and the odd parts between π and 2π by $\pi/(N-1)$ toward y' axis. Obviously, in the case of decomposing into m different $N^{(m)}$ -level operations, the partly phase shifting operation must be applied to all the carriers but the amount of phase shifting is π/m . Thus, the reference carriers for each $N^{(m)}$ -level operation are defined as:

$$C_{n(\text{PO})}^{(m)} = \left(1 - \frac{N^{(m)}}{2} \right) + \left\{ \begin{array}{l} y_c \left(A_c, \frac{\omega_c}{m}, \frac{(2n-1)\pi}{m}, w \right) \quad \omega_m t : \left. \begin{array}{l} B_{\text{even}} [0, \pi) \\ B_{\text{odd}} [\pi, 2\pi) \end{array} \right\} \\ y_c \left(A_c, \frac{\omega_c}{m}, \frac{(n-1)2\pi}{m}, w \right) \quad \omega_m t : \text{otherwise} \end{array} \right\}$$

For PH disposition, all even parts must be shifted by π/m . The general surfaces for PH disposition with N odd and even are shown in Fig. 13(b,c). The reference carriers are:

$$C_{n(\text{PH})}^{(m)} = \left(1 - \frac{N^{(m)}}{2} \right) + \left\{ \begin{array}{l} y_c \left(A_c, \frac{\omega_c}{m}, \frac{(2n-1)\pi}{m}, w \right) \quad \omega_m t : B_{\text{even}} \\ y_c \left(A_c, \frac{\omega_c}{m}, \frac{(n-1)2\pi}{m}, w \right) \quad \omega_m t : B_{\text{odd}} \end{array} \right\}$$

Here we developed the PSC technique to accommodate with APO, PO, and PH dispositions. Modifying the PSC technique for other possible phase displacements is out of scope of this paper but the surface represented here is a good means for handling those situations. Moreover, correct sign choice of the dynamic phase shift at each interval results in a more uniform distribution of switching angles. This effect is investigated in [10].

4 Special Features

4.1 Asymmetrical Carriers

Yet, our discussion was restricted to symmetrical triangular carriers. For the asymmetrical cases the situation is quite different. The effect of triangle width (w) on harmonic content of a bipolar PWM waveform is shown in Fig. 14. For $0.5 < w < 1$ the spectrum only differs in the phase angles. Obviously, the harmonic pollution in the spectrum of a PWM waveform using sawtooth carrier ($w = 0$ or 1) is more than that of a symmetrical triangular carrier ($w = 0.5$). In other words, the more w approaches from 0.5 to 0 or 1 the more harmonic distortion we will have at the inverter output. Thus using symmetrical triangular carrier is more convenient: [11].

Moreover, the more w approaches to 0 or 1 the more unwanted harmonics are likely to concentrate on the multiples of P . On the other hand, the first set of undesired harmonics for an N -level PO or PH dispositions are similar to $(N-1)$ th set of undesired harmonics for a bipolar modulation using $w = 0.25(0.75)$ or $w = 0(1)$ respectively. Therefore, we can use a regular PSC using sawtooth carriers instead of PH and a regular PSC using $w = 0.25(0.75)$ carriers instead of PO disposition. Although, from the harmonic pollution point of view this replacement is not efficient, but in some cases it is useful due to complexity of the carriers needed in the PSC equivalents of PO and PH techniques.

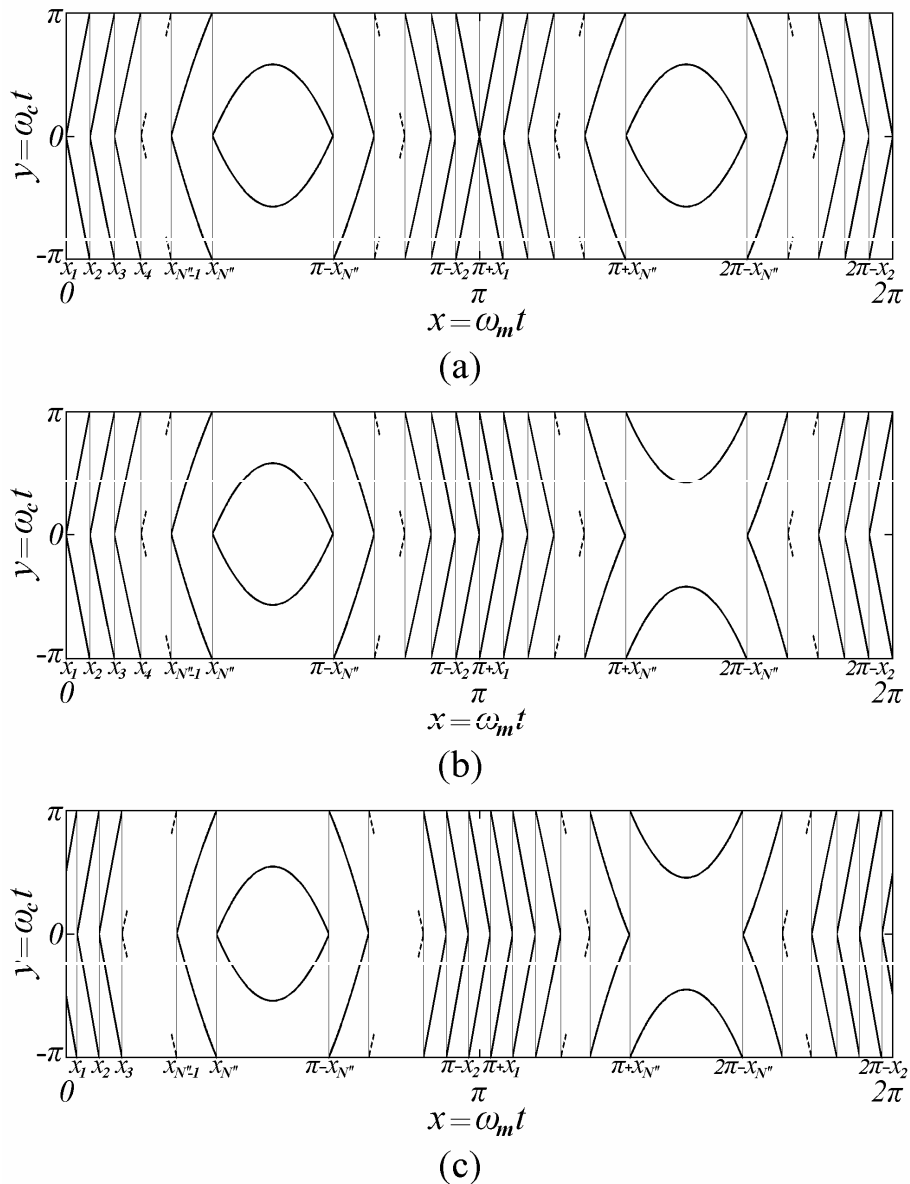


Fig. 13 General surfaces (a) PO (N odd), (b) PH (N odd), (c) PH (N even).

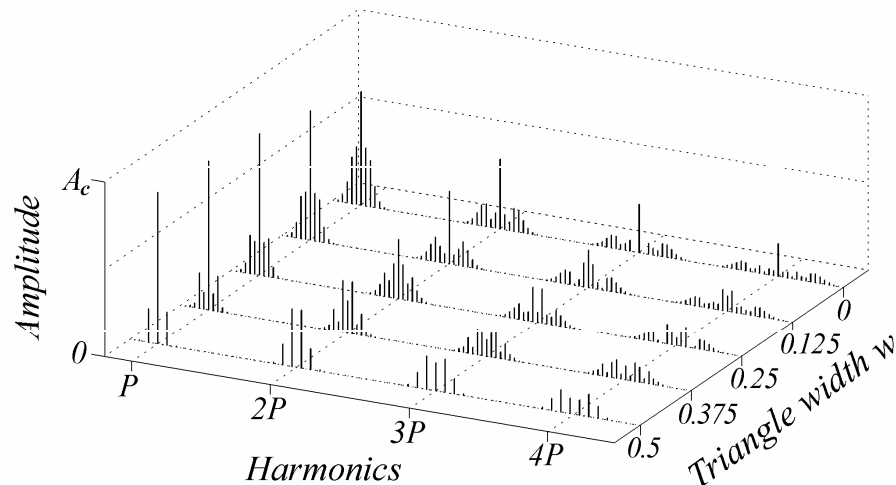


Fig. 14 The effect of triangle width (w) on the spectrum of corresponding bipolar PWM waveform.

4.2 Logical Decomposition

In the case of PSC equivalent for PO and PH dispositions, the major concern is about the implementation. Comparing to a regular PO or PH disposition, implementing a PSC equivalent needs twice more carriers. Moreover, the angles in which we should switch between carriers depend on the modulation index (M) and vary due to different amplitudes needed.

A somewhat easier way is to generate the resulting PWM waveform using conventional PO or PH methods and then decompose it to bipolar PWM sub-waveforms using a logical strategy. The strategy is based on preparing the most possible amount of time between two consecutive edges in the bipolar PWM sub-waveforms. To do this a stack is defined which saves the privilege level of each carrier to show which one is the best to be complemented. The stack consists of $N-1$ cells. Each cell can save a number from 1 to $N-1$ which refers to a specified sub-waveform. The privilege level is defined according to the distance of a cell from the end of stack. Thus, the cell at the start of stack has the most privilege where as the one at the end has the least. The algorithm is:

1. Choose an initial state for each bipolar sub-waveform in a way that their total sum is equal to the starting value of the main PWM waveform.
2. Load the stack with arbitrary initial sequence (e.g., the sequence 1, ..., $N-1$)
3. Keep the states of the carriers until an edge on the main wave is detected.
4. For rising (falling) edge, look up the stack from start to the end to find the most privileged cell which its corresponding carrier is in 0 (1) state.
5. Complement the state of corresponding carrier and transfer the cell to the end of stack (a circular shift using the stack from the selected cell to the end).
6. Go to step 3.

The results for a 5-level decomposition is shown in Fig. 15. The resulting bipolar PWM waveforms are the same as those in PSC technique.

Implementing such an algorithm needs a sequential logical circuit. At least two sequences are needed. One for read the stack and one for modify it. On the other hand, the pulses are generated by an analog process thus they might have very small amount of pulse widths. In this case, we should use a tiny pulse arrester or modifier circuit to prevent abnormal operation of our sequential circuit.

5 Applying to Parallel and Series Cells

The decomposition techniques that we have developed have two main advantages. First, a multilevel PWM waveform is decomposed into several sub-waveforms with lower frequency. Second, the main harmonics of sub-waveforms are equal. Thus, they are well suited to be used with parallel or series connected cells: [12].

The cells are usually bipolar (Fig. 16(a)) or unipolar (Fig. 16(b)). Applying the decomposition method to bipolar cells is quite simple. We should decompose the N -level PWM waveform into $N-1$ bipolar sub-waveforms and apply them to each bipolar leg (Q_1 must be fed by the bipolar waveform and Q_2 must be fed by its negation).

For the unipolar cells the situation is slightly different. The unipolar cells are naturally 3-level inverters. Thus we should decompose our N -level PWM waveform into $(N-1)/2$ 3-level sub-waveforms. Then each cell must generate one of these 3-level PWM sub-waveforms. Again, the 3-level waveform should be decomposed into two bipolar operations. The positive leg (Q_1 and Q_2') must be fed straightly and the negative leg (Q_3 and Q_4) must be fed inversely. In Brief, we have two strategies for unipolar cells:

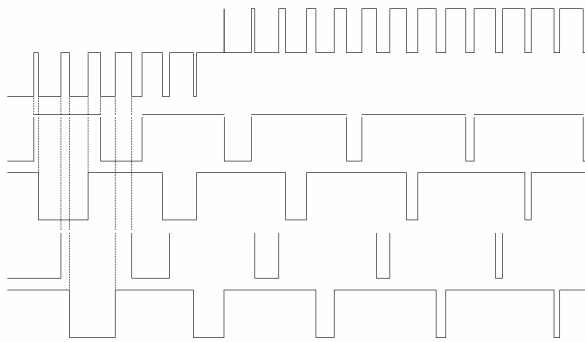


Fig. 15 Logical Decomposition of a 5-level PWM waveform into bipolar sub-waveforms.

1. Decompose the N-level operation into N-1 bipolar sub-operations. Use the PWM waveforms generated by carriers with phase $0 \leq \varphi_c < \pi$ to control the positive legs. For negative legs, use the negation of the PWM waveforms generated by the carriers with phase $\pi \leq \varphi_c < 2\pi$.
2. Decompose the N-level operation into N-1 bipolar sub-operations. Use the PWM waveforms generated by carriers with phase $0 \leq \varphi_c < \pi$ to control the positive legs. For negative legs, use the same carriers but the negation of modulating signal (opposite phase sine wave) to generate the PWM waveforms.

It is clear that the second strategy needs twice less number of carriers, while the results of using these strategies are the same.

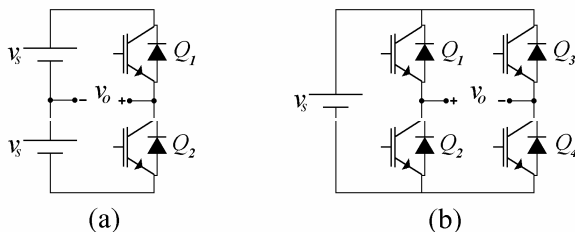


Fig. 16 (a) Bipolar cell, (b) Unipolar (H-bridge) cell.

5 Conclusion

In this paper we have presented PSC technique as an equivalent for APO disposition and have modified it to accommodate PO and PH disposition techniques. While the PSC technique and APO disposition generate the same waveform, PSC is more convenient to be used with cascaded and parallel multilevel topologies. We have developed the equivalent PSC techniques for PO and PH dispositions based on partly shifted carriers. These techniques are also useful for cascaded and parallel three-phase topologies. We have shown that using asymmetrical carriers in PSC technique can also help us to simply approach to the properties of PO and PH waveforms. We have proposed logical decomposition based on preparing the most possible

amount of time between two consecutive angles in each sub-waveform. It is useful for decomposing all types of multilevel PWM waveforms.

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