A Dissipative Integral Sliding Mode Control Redesign Method for Uncertain Nonlinear Switched Systems

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Abstract: This paper develops a new method of integral sliding mode control redesign for a class of perturbed nonlinear dissipative switched systems by modifying the dissipativity-based control law that was designed for the unperturbed systems. The nominal model is considered affine with matched and unmatched perturbations. The redesigned control law includes an integral sliding-based control signal such that the system always operates on the sliding mode and the dissipativity of the perturbed switched system is maintained from the initial time of the system operation for the norm bounded perturbations. The proposed techniques eliminate the restrictive design conditions on the derivative of storage functions offered in a recent work. In addition, the global dissipativity of the perturbed system is always maintained if the original unperturbed system is globally dissipative. Depending on the type of stability of the unperturbed system, the designed control law for the perturbed system guarantees robust exponential or asymptotic stability of the closed-loop system. The theoretical results are applied to nonlinear switched systems, and the convergence of the state vectors to the origin is verified by simulation in presence of nonlinear perturbations.

Keywords: Control Redesign, Dissipativity, Integral Sliding Mode Control, Nonlinear Switched Systems, Perturbation.

1 Introduction

A dynamical switched system is a system that switches among several subsystems based on a switching law. Many real-word systems such as automotive engine control systems [1], robot control systems [2], and haptic interfaces [3] demonstrate switching behaviors. As such, the switched systems have drawn considerable attention in the control and system analysis [4-6]. Employing a proper switching strategy in the control of linear or nonlinear systems often leads to better robustness and transient performance [7, 8]. Different methods of the stability analysis and stabilization have been purposed for the switched systems [9-11]. The Lyapunov base stability theory still plays a dominating role in the stability analysis and control design, [12, 13]. The common Lyapunov function method may be employed for different subsystems which guarantees the stability under an arbitrary switching law [10, 14]. However, since the common Lyapunov function method may make the design very conservative, the multiple Lyapunov function is proposed in [15], and became a standard analysis tool for most switched system designs

[16]. This method was further developed into dwell-time method [17, 18] and average dwell-time method [10, 19-21] which are only applicable for the switching systems with designable switching law.

Dissipativity and passivity introduced by Willems [22], are powerful tools for the stabilization analysis. Storage functions are usually related with the system's energy and can be candidates for the Lyapunov functions [23]. Additionally, dissipativity and passivity-base analysis can be helpful for proving the stability of the switched systems [24-26]. In [27], the classic passivity and the stability results were extended to cover the switched systems. The multiple storage functions and multiple supply rates for the switched systems were presented in [28, 29]. In [30], the feedback passivation conditions of the switched cascade nonlinear systems have been investigated. The average dwell-time base exponential stability of the switched nonlinear systems with passive and nonpassive subsystems is studied in [31].

Considering the fact that dissipativity and passivity are powerful tools for the analysis and the control design of the switched nonlinear systems, it would be interesting to be able to employ these tools for the perturbed switched systems, and to find how a dissipative switched nonlinear system can preserve its dissipativity property and consequently its stability in

presence of these perturbations. To the best of our knowledge, the dissipativity-based redesign method for deterministic switched nonlinear systems with uncertainties and perturbations are only considered in [32-34]; however, the presented method in [32] was developed only for matched perturbations, and the works in [33, 34] require vanishing unmatched perturbations and have restrictive inequality conditions. These restrictive conditions generally reduce the global dissipativity-based results of the original system into local results for the perturbed system. Other available robust methods for the switched nonlinear systems which are not based on passivity and dissipativity, require the switching law to be designable or restrict it [11, 35, 36].

On the other hand, sliding mode control is an effective robust control method for unmatched perturbations [37, 38] and its response, typically, includes two phases: reaching phase and sliding phase. In the reaching phase, system is sensitive to uncertainties and perturbations. To eliminate the reaching phase, an integral sliding mode is proposed in [39], and widely applied to various systems [40-42]. In [35, 43], integral sliding mode control is developed only for cascade uncertain switched systems with at least one linear part. Moreover, these designs are based on average dwell-time approach and restrict the switching law. Additionally, in [43], Lipschitz condition is needed for the nonlinear part.

In this paper, the storage function redesign method combined with an integral sliding mode control is offered for the first time by including a variable structure control law to guarantee the preservation of the dissipativity of the closed-loop nonlinear switched system in the presence of norm bounded perturbations. The dissipativity-base controller redesign offered in [34] for nonlinear switched systems with matched and unmatched vanishing perturbations forces restrictive conditions in the form of nonlinear inequalities. Usually these conditions results the local robust stability of perturbed switched system. Elimination of these restrictions while keeping the capability of the redesign approach in the presence of matched perturbations, motivated us to suggest a combined scheme consisting of this method and the integral sliding mode control for the nonlinear switched systems using multiple storage functions. The major advantage of this method is that it robustifies the available nominal controllers, thereby preserving the primary control objectives, while reducing the complexity of the robust controller design of the nonlinear switched systems. With this method, the switching law is not necessary to be designable, the restrictive conditions on the derivatives of the storage functions given in [32] are eliminated. The proposed method can only be applied to the perturbed switched systems whose control laws for their original unperturbed systems were designed based on dissipativity. Moreover, the original system must be affine, which is a usual assumption in passivity based stability analysis literature. To verify the theoretical results, the perturbed switched systems are considered in two examples, and the stability conditions are derived. Then, the simulation results are provided to show the effectiveness of the proposed method.

The rest of the paper is organized as follows: In Section 2, the model of the switched system is introduced, and the notion of dissipativity for the affine nonlinear switched systems is offered. Then, the sufficient conditions for the dissipativity of the affine nonlinear switched systems are derived. In Section 3, an integral sliding surface is presented which preserves the dissipativity property of the closed-loop system on the sliding surface. Then, a variable structure control is introduced which always places the switched system on the sliding mode. In Section 4, the theoretical results are illustrated by an example. Finally, the conclusion of the paper is presented in section 5.

2 The Preliminaries and the Problem Statement

Consider the affine nonlinear perturbed switched system of the form:

$$\dot{x} = f_i(x) + \Delta f_i(x,t)$$
$$+ g_i(x)\big((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t)\big) \qquad (1)$$
$$y = h_i(x,t)$$

where $i(t) \in M = \{1,2,\ldots,m\}$ is the switching signal that may depend on time, states or both, m is the number of subsystems, $x \in R^n$, $u_i \in R^p$ and $y \in R^p$ are the state, input and output vectors of the i-th subsystem, respectively, I is an identity matrix with proper dimension, and $f_i(x)$, $g_i(x)$ and $h_i(x,t)$ are given continuous functions. The bounded terms:

$$\|\Delta f_i(x,t)\| \le \rho_{f,i}(x,t), \ i = 1,2,\ldots,m \qquad (2)$$

are unmatched perturbations, and:

$$\|\Delta g_i(x,t)\| \le \rho_{g,i}(x,t) < 1, \qquad (3)$$

and:

$$\|\Delta_i(x,u_i,t)\| \le \rho_{u,i}(x,t), \qquad (4)$$

are matched perturbations with known nonnegative functions $\rho_{f,i}(x,t)$, $\rho_{g,i}(x,t)$ and $\rho_{u,i}(x,t)$.

The switching sequence between the subsystems is defined as $\Sigma = \{(i_0,t_0),(i_1,t_1),\ldots,(i_j,t_j),\ldots\}$, $i \in M$ and $j \in N$ where N is the set of nonnegative integer numbers of switches, t_0 is the initial time, t_j is the time that system switches to i_j-th subsystem and hold on it until $t \in [t_j,t_{j+1})$, assuming $i_j \ne i_{j+1}$ and $t_j < t_{j+1}$. More specifically, index i_j specifies that the i-th subsystem has become active at the j-th switching event.

The following assumption is used in the stability analyses that are based on multiple storage functions.

Assumption 1: The states of the switched system do not jump in the switching instances, and thus, the trajectory $x(t)$ is always continuous.

In following definition, multiple storage functions and multiple supply rates are introduced to characterize the dissipativity property for the affine nonlinear switched systems (1).

Definition 1 [34]: The nominal form of system (1) under the switching sequence Σ is said to be dissipative if for any subsystem i, $(i = 1,2,\ldots,m)$, there exist positive definite continuous storage functions $S_i(x)$, locally integrable supply rate functions $w_i(u_i,h_i)$, and locally integrable cross supply rate functions $w_{ij}(x,u_i,h_i,t)$, $(j = 1,2,\ldots,m, i \neq j)$ such that when the i-th subsystem is activated for the k-th time $(\forall s,t, \quad t_k \leq s \leq t < t_{k+1})$, the following conditions hold:

i)

$$S_{i_k}(x(t),t) - S_{i_k}(x(s),s) \leq \int_s^t w_{i_k}(u_{i_k},h_{i_k})dt,$$
$$k = 0,1,2,\ldots, \quad t_k \leq s \leq t < t_{k+1} \tag{5}$$

ii)

$$S_j(x(t),t) - S_j(x(s),s) \leq \int_s^t w_{i_k j}(x(\tau),u_{i_k}(\tau),h_{i_k}(\tau),\tau)d\tau$$
$$k = 0,1,2,\ldots, \quad t_k \leq s \leq t < t_{k+1} \tag{6}$$

iii) For all $i \neq j$ $(i = i_k$ and $i,j = 1,2,\ldots,m)$ and all $t \geq t_0$, there exist inputs $u_i(t) = \alpha(x(t),t)$ and $\varphi_{ij}(t) \in L_1^+[0,\infty)$, which may depend on the switching sequence Σ such that:

$$f_i(0) + g_i(0)\alpha(0,t) = 0, \tag{7}$$

$$w_i(u_i(t),h_i(x(t))) \leq 0, \tag{8}$$

and;

iv) For all $i \neq j$ there exist $\phi_{ij}(t) \in L_1^+[0,\infty)$, such that:

$$w_{ij}(x(t),u_i(t),h_i(t),t) \leq \phi_{ij}(t) \tag{9}$$

The common states between all subsystems causes the flow of energy from the active in to inactive subsystems, and is characterized by w_{ij} which denotes supply rate of energy from the i-th to the j-th subsystem as presented in condition (6). Eq. (7) guarantees the origin as the equilibrium point of the system. Inequalities (8) and (9) imply the existence of a control law u_i that decreases the energy of the i-th active subsystem and also the boundedness of the stored energy in the j-th inactive subsystem supplied by the i-th active subsystem.

Remark 1: In definition 1, for radially unbounded storage functions, the nominal control signal $u_i(t) = \alpha(x(t),t)$ globally stabilizes the nominal switched system. These conditions are sufficient for stability in the sense of Lyapunov and there exists other sufficient conditions, such as the form of supply rate

functions and the constraints on the switching law, that guarantee the other type of stability, e.g. the sufficient conditions for global asymptotic stability of unperturbed nonlinear switched systems are presented in [29]. These additional conditions are assumed to be considered in the primary control design and based on the desired primary control objectives.

Remark 2: If one can set supply and cross supply rate function to zero, i.e., $w_{ij} = 0$, $i, j = 1,2,\ldots,m$, the storage functions can be taken as Lyapunov functions. On the other hand, for a Lyapunov based stability analysis, the multiple Lyapunov functions can be considered as the storage functions and the system is dissipative with zero supply and cross supply rate functions.

Assumption 2: The unperturbed form of nonlinear switched system (1) is dissipative and globally stable under the given switching law Σ with controllers $u_i(t) = \alpha(x(t),t)$, i.e., there exist a set of radially unbounded storage functions that satisfy the dissipativity conditions of definition 1 under this switching law.

Assumption 3: Extending the dissipativity condition (7) for the closed loop perturbed system yields:

$$f_i(0) + \Delta f_i(0,t)$$
$$+ g_i(0)\big((I + \Delta g_i(0,t))u_i(0,t) + \Delta_i(0,u_i,t)\big) = 0 \tag{10}$$

The above condition can be satisfied in different ways, e.g., $f_i(0) = 0$, $\Delta f_i(0,t) = 0$ and $g_i(0) = 0$. Therefore, to satisfy (10), the perturbations do not need to be vanishing.

Remark 3: By considering $S_i(x) \in C^1$, $i = 1,2,\ldots,m$, the dissipativity conditions (5) and (6) can be rewritten as:

$$\dot{S}_i = \frac{\partial S_i}{\partial t} + \frac{\partial S_i}{\partial x}(f_i(x) + g_i(x)\alpha_i(x,t)) \leq w_i(u_i,h_i), \tag{11}$$

$$\dot{S}_j = \frac{\partial S_j}{\partial t} + \frac{\partial S_j}{\partial x}(f_i(x) + g_i(x)\alpha_i(x,t))$$
$$\leq w_{ij}(x,u_i,h_i,t) \tag{12}$$

where the t is belong to the interval of time that the i-th subsystem is active. These inequalities are used to derive the dissipativity conditions of the perturbed switched system.

Notice that any perturbation and uncertainty may cause the dissipativity conditions (5) and (6) and also the stability of the perturbed system no longer hold. The problem is to redesign the control law and to provide the sufficient conditions that preserve the dissipativity of the perturbed switched system as well as the primary control objectives. Therefore, the purpose is to robustify the controller using a sliding mode approach to compensate the side effects of perturbations (2), (3) and (4), so as to preserve the dissipativity and stability of the switched system (1) in the presence of norm bounded perturbation.

3 Main Results
3.1 The Dissipativity Condition and Integral Sliding Surface

Sufficient conditions for preserving the dissipativity of the perturbed switched system (1) is provided in the following Lemma.

Lemma 1: Considering Assumption 2 and the dissipativity conditions of the nominal system given in (5) to (9), if the following conditions:

$$\frac{\partial S_j}{\partial x}\left(\Delta f_i(x,t) + g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) - \alpha_i(x,t)\right.\right.$$
$$\left.\left. + \Delta_i(x,u_i,t)\right)\right) \le \psi_{ij}(t), \ i,j = 1,2,\ldots,m \quad (13)$$

hold for any $\psi_{ij}(t) \in L_1^+[0,\infty)$, where $\psi_{ii}(t) = 0$, then, the perturbed switched system (1) is dissipative with the same storage, supply, and cross supply rate functions as the nominal system.

Proof: Rewriting (5) and (6) for the perturbed switched system (1) yields:

$$\dot{S}_i = \frac{\partial S_i}{\partial t} + \frac{\partial S_i}{\partial x}\left(f_i(x) + \Delta f_i(x,t)\right.$$
$$+ g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t)\right))$$
$$\le w_i(u_i,h_i) + \frac{\partial S_i}{\partial x}\left(\Delta f_i(x,t)\right.\quad (14)$$
$$+ g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) - \alpha_i(x,t) + \Delta_i(x,u_i,t)\right))$$

$$\dot{S}_j = \frac{\partial S_j}{\partial t} + \frac{\partial S_j}{\partial x}\left(f_i(x) + \Delta f_i(x,t)\right.$$
$$+ g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t)\right))$$
$$\le w_{ij}(x,u_i,h_i,t) + \frac{\partial S_j}{\partial x}\left(\Delta f_i(x,t)\right.\quad (15)$$
$$+ g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) - \alpha_i(x,t) + \Delta_i(x,u_i,t)\right)).$$

The perturbations $\Delta f_i, \Delta g_i$ and Δ_i are characterized in (2), (3) and (4), respectively. Therefore one can easily see that if:

$$\frac{\partial S_j}{\partial x}\left(\Delta f_i(x,t) + g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) - \alpha_i(x,t)\right.\right.$$
$$\left.\left. + \Delta_i(x,u_i,t)\right)\right) \le \psi_{ij}(t), \ i,j = 1,2,\ldots,m \quad (16)$$

for any $\psi_{ij}(t) \in L_1^+[0,\infty)$ with $\psi_{ii}(t) = 0$, the inequalities (11) and (12) are satisfied; hence, the perturbed switched system (1) is dissipative.

Remark 4: Conditions (13) are the sufficient conditions to preserve the dissipativity of the perturbed switched system (1) which can be achieved by proper control signals $u_i(x,t)$, $i = 1,2,\ldots,m$. Therefore, any control objectives based on the storage, supply and cross supply rate functions for the nominal switched system, hold for the perturbed switched system (1) as well.

In following, a nonlinear integral sliding surface is used to achieve the sufficient conditions (13) and, consequently, the dissipativity of the perturbed switched system (1).

As in [40], the nonlinear integral sliding surface is chosen as:

$$S_s = H \times \left(x(t) - x(t_0) - \int_{t_0}^{t}(f_i + g_i\alpha_i)d\tau \right), i = 1,2,\ldots,m \quad (17)$$

where Hg_i is invertible and $H \in R^{p \times n}$ is known as the sliding surface matrix which is a constant matrix.

Remark 5: The sliding surface (17) depends on the initial time t_0 such that $S_s(t_0) = 0$, which eliminates the reaching phase in the sliding control approach. For this reason, the system is more robust against uncertainties and disturbances than the other sliding-base control systems with reaching phase [40].

The following theorem show that when the system is in the sliding surface, the dissipativity of the perturbed nonlinear switched system (1) is guaranteed.

Theorem 1: Suppose that the Assumption 2 holds. Then, the nonlinear perturbed switched system (1) is dissipative and globally stable on the integral sliding surface $S_s = 0$ in (17) with the same switching law, storage, supply and cross supply rate functions as ones of nominal switched system.

Proof: When the uncertain nonlinear switched system (1) operates in the sliding surface, we have:

$$S_s = 0 \quad (18)$$

Differentiating the sliding function (18) with respect to the time, one can obtain:

$$\dot{S}_s = H\left(\dot{x}(t) - f_i - g_i\alpha_i\right), \ i = 1,2,\ldots,m \quad (19)$$

Substituting (1) in to the (19) yields:

$$\dot{S}_s = H(f_i(x,t) + \Delta f_i(x,t) - f_i(x,t) - g_i(x,t)\alpha_i(x,t)$$
$$+ g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t)\right))$$
$$= H(\Delta f_i(x,t)\quad (20)$$
$$+ g_i(x)\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t) - \alpha_i(x,t)\right)).$$

Since the system is in sliding mode, then $\dot{S}_s = 0$. Solving the equation $\dot{S}_s = 0$ yields:

$$H(\Delta f_i(x,t) + g_i(x) \times$$
$$\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t) - \alpha_i(x,t)\right)) = 0. \quad (21)$$

According to the condition (3), $(I + \Delta g_i)$ is invertible. Additionally, Hg_i is nonsingular and H is full rank. The above equality yields a valid equivalent control as:

$$u_{ieq}(x,t) = (I + \Delta g_i(x,t))^{-1}(\alpha_i(x,t) - \Delta_i(x,u_i,t))$$
$$- (Hg_i(x)(I + \Delta g_i(x,t)))^{-1}H\Delta f_i(x,t). \quad (22)$$

Since H is full rank, Equation (21) results in:

$$\Delta f_i(x,t) + g_i(x) \times$$
$$\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t) - \alpha_i(x,t)\right) = 0. \quad (23)$$

Consequently, one can see that the inequality (13) is satisfied when the system (1) operates on the integral sliding surface (17). Thus, according to Lemma 1, the perturbed switched system (1) is dissipative with the same storage, supply and cross supply rate functions as nominal system. Notice that, satisfying (7) and (23) results the conditions (10) which implies that the origin is the equilibrium point of the perturbed switched system (1). Thus, any dissipative-based results about the equilibrium point of the unperturbed nonlinear switched system is still valid for the system in the presence of the perturbations.

Remark 6: Equation (23) holds for the perturbed switched system (1) on the sliding surface (19). Therefore, the dissipativity condition (7) for the perturbed switched system (1) is satisfied, and thus, the origin remains as the equilibrium point of the perturbed switched system on the sliding surface. Since these results are valid even for nonvanishing norm bounded perturbations, this control redesign method is robust for different types of perturbations more than the ones are considered in [32].

3.2 Finding the Robustifying Control Signal for the Norm Bounded Perturbations

By Theorem 1 in the previous subsection, we offered a nonlinear integral sliding surface such that the sliding mode is stable, and thus the dissipativity and stability properties are preserved. In this section, we design the sliding mode control law such that the trajectory of the perturbed switched system (1) remains on the sliding surface from the initial time. The following theorem guarantees the above objective.

Theorem 2: Consider the integral sliding surface (17) for the perturbed switched system (1) under the switching law Σ, and suppose that the functions $\rho_{f,i}(t,x)$, $\rho_{g,i}(t,x)$ and $\rho_{u,i}(t,x)$ in (2), (3), and (4) are the norm upper bound functions. Then, using a variable structure controller of the form:

$$u_i(x,t) = \alpha_i(x,t) - \frac{1}{1-\rho_{g,i}}\left(\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i}\right.$$
$$\left. + \rho_{u,i} + \|\alpha_i(x,t)\|\rho_{g,i} + \lambda_i\right)sign\left((Hg_i(x))^T S_s\right) \quad (24)$$

guarantees that the perturbed system operates on the sliding mode from the initial time and remains in it for all $t > t_0$, where λ_i's are arbitrary non-negative constant scalars which adjust the convergence rate to the sliding surface.

Proof: Choose the Lyapunov function candidate as follow:

$$V = \frac{1}{2} S_s^T S_s. \quad (25)$$

When the i-th subsystem is active, the time derivative of V along the trajectory of the switched system (1) is:

$$\dot{V} = S_s^T \dot{S}_s$$
$$= S_s^T H\left(f_i(x) + \Delta f_i(x,t) + g_i(x) \times \right.$$
$$\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t)\right)$$
$$\left. - f_i(x) - g_i(x)\alpha_i(x,t)\right) \quad (26)$$
$$= S_s^T\left(H\Delta f_i(x,t) + Hg_i(x) \times \right.$$
$$\left((I + \Delta g_i(x,t))u_i(x,t) + \Delta_i(x,u_i,t) - \alpha_i(x,t)\right).$$

Substituting the control signal (24) in to the above equation yields:

$$\dot{V} = S_s^T H\Delta f_i(x,t) + S_s^T Hg_i(x) \times$$
$$\left\{-\frac{1}{1-\rho_{g,i}}\left(\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i} + \rho_{u,i} + \|\alpha_i(x,t)\|\rho_{g,i}\right.\right.$$
$$\left. + \lambda_i\right)sign\left((Hg_i(x))^T S_s\right) + \Delta g_i\left(\alpha_i(x,t) - \frac{1}{1-\rho_{g,i}} \times\right.$$
$$\left(\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i} + \rho_{u,i} + \|\alpha_i(x,t)\|\rho_{g,i} + \lambda_i\right) \times$$
$$\left.\left. sign\left((Hg_i(x))^T S_s\right)\right) + \Delta_i(x,u_i,t)\right\}. \quad (27)$$

Using the matrix norm inequality in above equation and some manipulations results in:

$$\dot{V} \le S_s^T H\Delta f_i(x,t) + \left\|(Hg_i(x))^T S_s\right\| \times$$
$$\left\{-\left(\frac{1}{1-\rho_{g,i}}\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i} + \frac{\rho_{u,i}}{1-\rho_{g,i}}\right.\right.$$
$$\left.+ \frac{\rho_{g,i}}{1-\rho_{g,i}}\|\alpha_i(x,t)\| + \frac{\lambda_i}{1-\rho_{g,i}}\right) + \rho_{g,i}\|\alpha_i(x,t)\|$$
$$+ \frac{\rho_{g,i}}{1-\rho_{g,i}}\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i} + \frac{\rho_{g,i}}{1-\rho_{g,i}}\rho_{u,i}$$
$$\left.+ \frac{\rho_{g,i}^2}{1-\rho_{g,i}}\|\alpha_i(x,t)\| + \frac{\rho_{g,i}}{1-\rho_{g,i}}\lambda_i + \rho_{u,i}\right\}$$
$$\le S_s^T H\Delta f_i(x,t) + \left\|(Hg_i(x))^T S_s\right\| \times$$
$$\left\{-\left(\frac{1}{1-\rho_{g,i}}\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i} + \frac{\rho_{u,i}}{1-\rho_{g,i}}\right.\right.$$
$$\left.+ \frac{\rho_{g,i}}{1-\rho_{g,i}}\|\alpha_i(x,t)\| + \frac{\lambda_i}{1-\rho_{g,i}}\right)$$
$$+ \rho_{g,i}\|\alpha_i(x,t)\| + \frac{\rho_{g,i}}{1-\rho_{g,i}}\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i}$$
$$+ \frac{\rho_{g,i}}{1-\rho_{g,i}}\rho_{u,i} + \frac{\rho_{g,i}^2}{1-\rho_{g,i}}\|\alpha_i(x,t)\| + \frac{\rho_{g,i}}{1-\rho_{g,i}}\lambda_i + \rho_{u,i}\right\}$$
$$\le S_s^T H\Delta f_i(x,t) - \lambda_i\left\|(Hg_i(x))^T S_s\right\|$$
$$- \left\|S_s^T Hg_i(x)\right\|\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i} \quad (28)$$
$$\le S_s^T Hg_i(x)(Hg_i(x))^{-1}H\Delta f_i(x,t)$$
$$- \left\|S_s^T Hg_i(x)\right\|\left\|(Hg_i(x))^{-1}H\right\|\rho_{f,i} - \lambda_i\left\|(Hg_i(x))^T S_s\right\|$$
$$\le -\lambda_i\left\|(H_i g_i(x))^T S_s\right\|.$$

Since $Hg_i(x)$ is not singular, then, $S_s(x) \neq 0$ results in $\dot{V} < 0$. Therefore, since $S_s(x(t_0)) = 0$, the controller (24) preserves the sliding mode for all $t > t_0$. As shown in the last line of Inequality (28), λ_i's determine the decade rates of the Lyapunov function for each subsystem and can be adjusted by designer. The assumption $\rho_{g,i} < 1$ prevents zero division in (24). This completes the proof.

4 Simulation Results

To illustrate the results, we study the example is based on the one presented in [30]. The nonlinear switched system is comprised of two subsystems with:

$$f_1(x) = \begin{pmatrix} \dfrac{2}{\sqrt{x_1^2+1}} - x_1 + x_1 x_2 \\ 2x_1^2 \end{pmatrix}, \; g_1(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$f_2(x) = \begin{pmatrix} -3x_1 \sin^2 x_1 + x_1^3 \sin^2 x_1 + x_2 \cos x_1 \\ x_1 x_2 \end{pmatrix}, \quad (29)$$

$$g_2(x) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

The state feedback control laws designed in [30] for the subsystems are:

$$\alpha_1 = -x_1^4 - 2x_2^2 - x_2$$
$$\alpha_2 = x_1 \cos x_1 + x_1 x_2 + x_2, \quad (30)$$

and the switching strategy is:

$$\sigma(t) = \begin{cases} 1, & \text{if } x_1^4/4 \geq x_1^2/2 \\ 2, & \text{otherwise} \end{cases}, \quad (31)$$

which guarantee that the switching system is passive and asymptotically stable. The behavior of the unperturbed closed-loop system is verified in Fig. 1 in which the switching occurs at $t = 0.15$ sec. The initial state condition for all simulations of this example is considered as $x_0 = [3, -1]$. The storage functions used in [30] are considered as $S_1(x) = 0.5(0.5x_1^4 + x_2^2)$ and $S_2(x) = 0.5(x_1^2 + x_2^2)$.

Now, consider the uncertainties:

$$\Delta f_1 = \begin{pmatrix} 0.2x_1^3 \\ 0.2x_2^3 \end{pmatrix}, \; \Delta g_1 = \begin{pmatrix} 0 \\ 0.01x_1^3 \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 \\ 0.2x_2^3 \end{pmatrix}$$

$$\Delta f_2 = \begin{pmatrix} 0.2x_1^3 \\ 0.2x_2^3 \end{pmatrix}, \; \Delta g_2 = \begin{pmatrix} 0 \\ 0.01x_1^3 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 \\ -0.2x_2^3 \end{pmatrix} \quad (32)$$

for the switched system (1), the stability of switched system in the presence of nominal control signals (30) is lost as depicted in Fig. 2. Based on the results of Theorem 2, using $\lambda_i = 0$ for $i = 1,2$ and $H = [0 \; 1]$, the robustifying terms are obtained as:
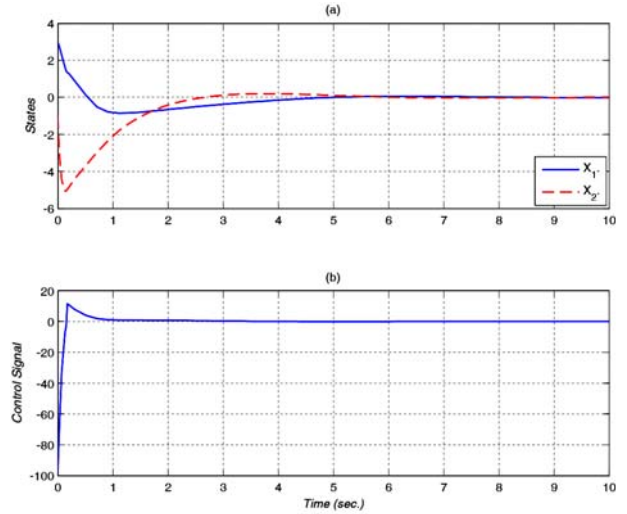

Fig. 1 (a) State trajectories and (b) control signal of the unperturbed and passive system.
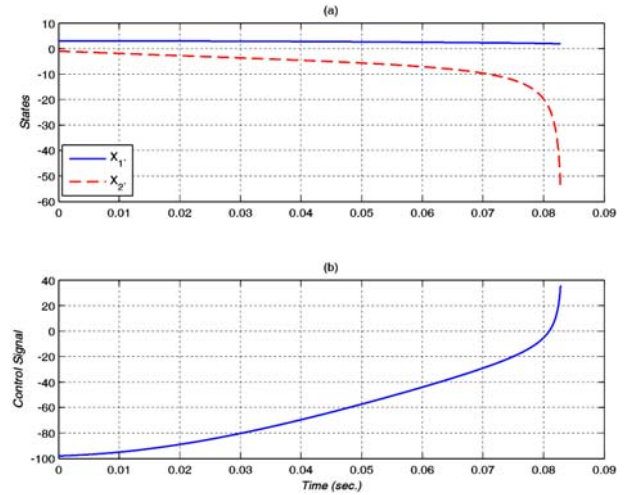

Fig. 2 The unstable closed-loop perturbed switched system: (a) state trajectories and (b) the control signal.

$$u_1(x,t) = -x_1^4 - 2x_2^2 - x_2 - \frac{1}{1-0.01|x_1|^3}\left(0.2\sqrt{x_1^6+x_2^6}\right.$$
$$\left. + 0.2|x_2|^3 + 0.01|x_1|^3|x_1^4 + 2x_2^2 + x_2|\right)sign(S_s), \quad (33)$$

$$u_2(x,t) = x_1 \cos x_1 + x_1 x_2 + x_2 + \frac{1}{1-0.01|x_1|^3}\left(0.2\sqrt{x_1^6+x_2^6}\right.$$
$$\left. + 0.2|x_2|^3 + 0.01|x_1|^3|x_1 \cos x_1 + x_1 x_2 + x_2|\right)sign(S_s), \quad (34)$$

which stabilize the perturbed switched system. The simulation results are depicted in Fig. 3 which show the stability of the perturbed system. Please note that the rate of convergence of the states to the sliding surface can be arbitrarily improved by increasing λ_i, $i = 1,2$ at the cost of increasing the amplitude of the control signal.
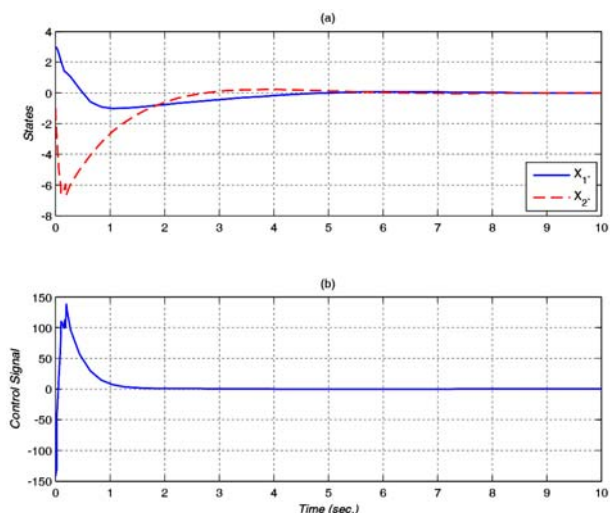
Fig. 3 The closed-loop switched system with perturbations: (a) state trajectories and (b) the robustifying sliding mode control signals (33) and (34).

5 Conclusions

In this paper, a redesigned robust integral sliding mode control is developed for a class of nonlinear dissipative switched systems with affine subsystems. The derived variable structure controllers guarantee the stability and convergence of the perturbed switched systems to the origin for norm bounded perturbations. The matched and unmatched perturbations are considered for the switched system, then the stable integral sliding surface is considered and the variable structure control is derived in Theorem 2. The redesigned variable structure controller preserves the stability of the closed-loop perturbed nonlinear switched system much similar to the unperturbed closed-loop switched system. Notice that, the existence of the dissipativity-based nominal control is required for the proposed redesign method. The simulation examples show the effectiveness of the proposed sliding method to dissipative nonlinear switched system in the presence of the matched and unmatched perturbations.

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