Pole Shape Optimization of Permanent Magnet Synchronous Motors Using the Reduced Basis Technique

A. Jabbari*, M. Shakeri* and S. A. Nabavi Niaki**

Abstract: In the present work, an integrated method of pole shape design optimization for reduction of torque pulsation components in permanent magnet synchronous motors is developed. A progressive design process is presented to find feasible optimal shapes. This method is applied on the pole shape optimization of two prototype permanent magnet synchronous motors, i.e., 4-poles/6-slots and 4-poles-12slots.

Keywords: PM Synchronous Motor, Torque Pulsation, Pole Shape, Optimization, Integrated Approach.

1 Introduction

Permanent magnet synchronous motors have been interested in high performance applications because of their high efficiency, power factor, and torque density. However, the noise and vibration caused by cogging torque and electromagnetic torque ripple seriously affects the motor performance. Pulsating torque is greatly affected by the configuration of the rotor and stator, therefore, pulsating torque minimization can be performed using pole shape optimization to obtain better magnetic field distribution and also to reduce torque ripple.

An extensive variety of techniques for minimizing cogging torque is documented in the literature for synchronous permanent magnet motors. These techniques can be categorized in two groups, i.e. size optimization and shape optimization techniques.

Many methods, such as pole skewing [1-4], pole segmentation [5], pole-arc optimization [6, 7, 8-10], pole displacing [2-4, 8-10] are classified in size optimization group. Although these methods are very successful in reducing the pulsating torque, their potential from the viewpoints of reducing the pulsating torque amounts and mechanical/magnetic constraints is limited.

Therefore shaping [2-4, 8-18] has been proposed to reduce the pulsating torque.

An innovative algorithm applicable for pole shape optimization problems is presented in this research and it is conducted to pole shape optimization of a 3-phase radial flux permanent magnet motor with 6 poles-18 slots. An efficient shape optimization methodology is developed by coupling a design variable linking technique with design of experiments method. In the RBT, some initial pole shapes are combined by assigning weight factors. By changing the weight factors, different resultant shapes can be generated. Therefore, the number of design variables is reduced to the number of weight factors and the optimization goal is to find the best possible combination of these weights to minimize cost function.

2 Method of Pole Shape Optimization

The RBT is suggested for pole shape design optimization in this research. Therefore, RBT should be adopted for pole shape optimization of electric motors.

2.1 RBT

The original concept in RBT is to built basis vectors, V^1, V^2, V^3,..., V^n with the boundary information of basis shapes and to add them linearly with the weighing coefficients, c_1, c_2, c_3,..., c_n, to each basis vector respectively.

$$V = \sum_{i=1}^{n} c_i V^i / C \tag{1}$$

where $0 \le c_i \le 1$ and C is scale factor and is defined as:

$$C = \sum_{i=1}^{n} c_i \tag{2}$$

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The basis vectors are constructed with boundary points of initial shapes.

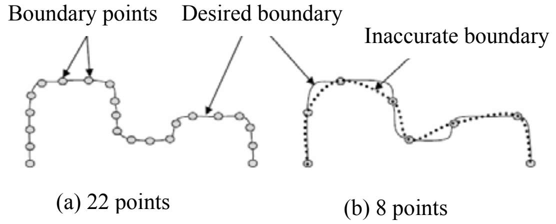
2.2 Vector Construction

The geometrical features of the basis shapes are defined in polar system by the r and θ co-ordinates of their boundary points. These co-ordinates define the basis vectors. All of the basis shapes have to be defined in the same fashion, and therefore, all the resulting basis vectors will have the same dimension. This will help to add them linearly with weights to each vector. The resulting vector will have a different shape than any of the basis shapes if at least two of the weights are non-zero. If the optimum magnet shape is any of the basis shapes, then the corresponding weight will be one and the others will be simply zero. The important factor that must be heeded is that the number of shape parameters should be as plentiful as possible. This means that the locations at which the co-ordinates are extracted should be as close to each other as possible in order to facilitate splinting and to get the detailed profile. Fig. 1(a) shows an edge defined by a set of points. There are 22 points that define the edge and the co-ordinates of these points form the basis vector. If a less number of points are used, for example eight points, then the resultant edge would look much different than the original one (Fig. 1(b)). To avoid this type of error, it is always safe to define the basis shapes with a large number of boundary points. Since this will not increase the number of design variables, there is no extra computational cost incurred by increasing the dimension of the basis vectors. This type of basis vector definition is useful for generating various possible shapes for optimization, but scaling of the resultant shapes is essential to maintain area restriction.

3 Pole Shape Optimization of the Prototype Motors

In [12] the influence of the shape of the magnets on the cogging torque and back-emf waveforms in the surface-mounted magnet motor having arc magnets is investigated. The magnet edges are assumed to be radial and the original thickness of the arc magnet is 3 mm.

The outer surface of each magnet is shaped sinusoidal and the edges of the magnet are varied from 3mm to 0mm. however, no optimal shape has been obtained in this work.



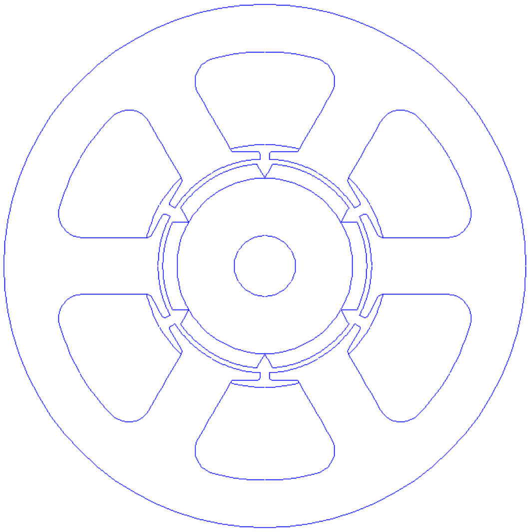
(a) 22 points

(b) 8 points

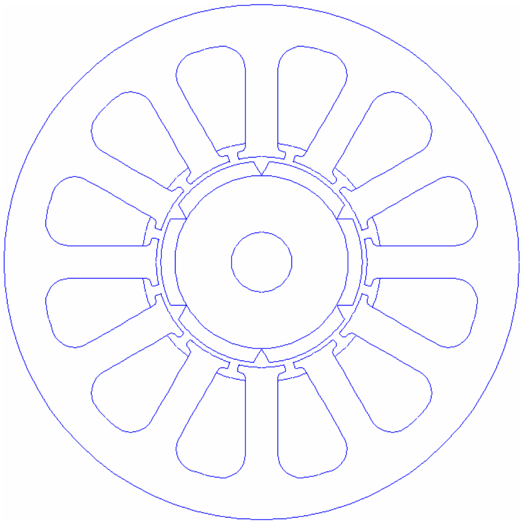
Fig. 1 Basis vectors construction

The pole shape optimization of two prototype motors studied in [12] is considered in this section. The first motor has 4 poles-6 slots and the second motor has 4 poles-12 slots. The finite element package is used to calculate the cogging torque in order to conduct DOE. Cross-sections and specifications of the investigated motors are shown in Fig. 2 and Table 1 respectively.

It is enough to consider one pole piece because of geometrical nature of the investigated motors. The next step of optimization procedure is to select appropriate basis shapes. Figs. 3 and 4 show three basis shapes assumed as three different B-Spline curves. B-Spline curves are smooth, empirical and can be manufactured by computer numerical control machines for motor I and motor II respectively. For all cases,



(a) 4 poles-6 slots motor



(b) 4 poles-12 slots motor

Fig. 2 Cross-sections of the studied motors, (a): the first motor, (b): the second motor

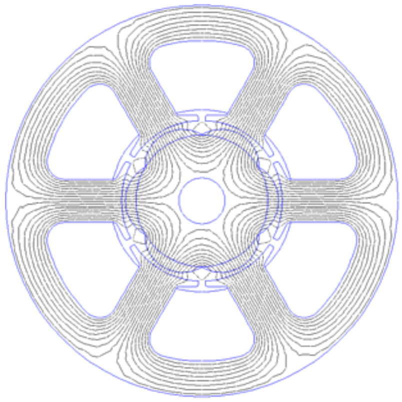
Table 1 Specification of the investigated motors

Parameter	**Motor I**	**Motor II**
Rotor Outer Diameter	43 mm	
Rotor Inner Diameter	13 mm	
Number of poles	4	
Pole Arc	59.9°	
Pole Thickness	3 mm	
Pole Arc R	10.6~21.5 mm	
Magnet material	NEOMAX-35EH	
Stator Outer Diameter	110 mm	
Stator Inner Diameter	45 mm	
Number of Slots	6	12
Stator Tooth Width	12 mm	7 mm
Stator Yoke Width	10 mm	8 mm
Slot Open	2 mm	
Tip Thickness	1.5 mm	
Slot Skew	0°	
Stator Length	50 mm	
Lamination material	M19-0.5mm	

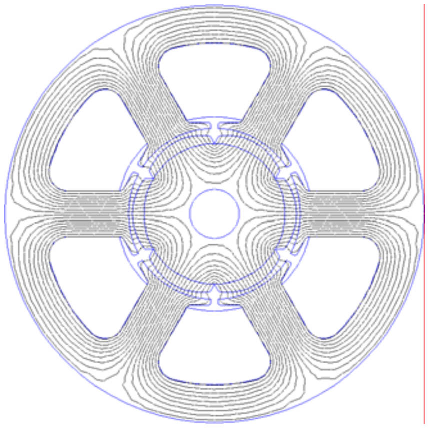
the fixed point is on R = 21.5 mm and co-ordinates of second point for Basis 1, Basis 2, and Basis 3 are 21.5 mm, 15 mm and 10.6 mm respectively. Each basis shapes is defined by one radial boundary point and basis vectors of each basis shape, R^1, R^2 and R^3, is constructed.

2-D FEA simulations of the basis shapes are performed to find the cogging torque for preliminary analysis as shown in Figs. 5-8. As seen from Figs. 7 and 8, for the first motor, the peak cogging torques of the Basis shapes are 0.550982, 0.27302, and 0.316556 (N.m), respectively. For the second motor, the peak cogging torques of the Basis shapes are 0.424898, 0.0935488, and 0.119773 (N.m), respectively. From this preliminary analysis, it can be said that the Basis 2 is more successful than the other two shapes in reducing the cogging torque. Therefore, the contribution of Basis 2 must be more than the other basis shapes, which must be recognized by the optimizer.

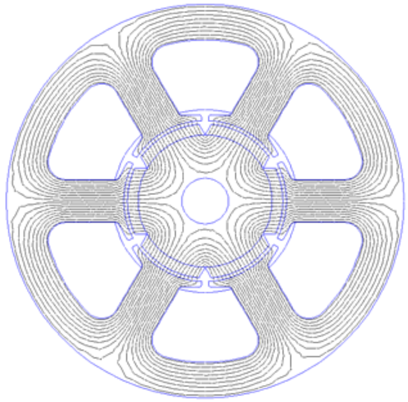
The RBT method is applied to basis vectors and the number of design variables is decreased to three, i.e. the weights for each basis vector. It is possible to obtain various resultant pole shapes by changing these weights, for the optimizer to find the best combination of these weights. Sixteen designs of experiment points are generated to conduct simulation. Geometrical scaling is applied to all pole shapes to maintain them in a limited area. Simulations are conducted at these designs of experiment points to find the cogging torque and to build the Taguchi models for optimization. Optimization is performed in QualiTek-4 to minimize the cogging torque.



(a) R=10.6 mm

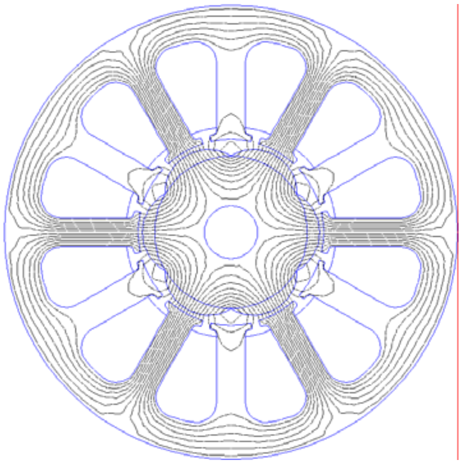


(b) R=15 mm

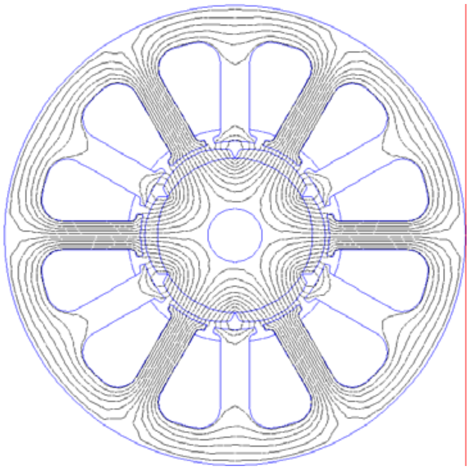


(c) R=21.5 mm

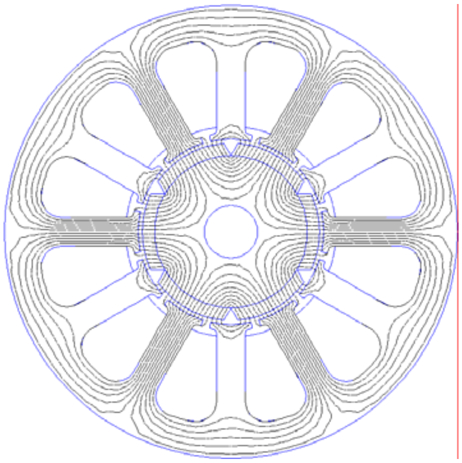
Fig. 3 Motor I; the selected Basis shapes and their contour maps



(a) R=10.6 mm

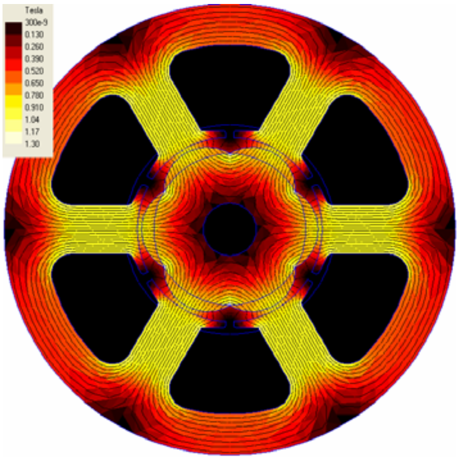


(b) R=15 mm

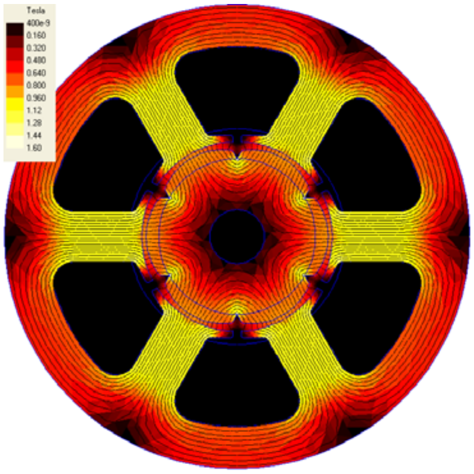


(c) R=21.5 mm

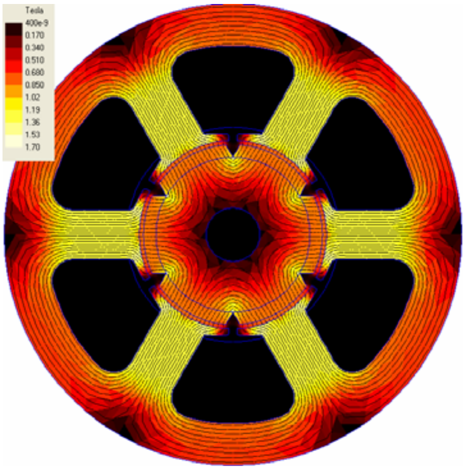
Fig. 4 Motor II, The selected Basis shapes and their contour maps.



(a) R=10.6 mm

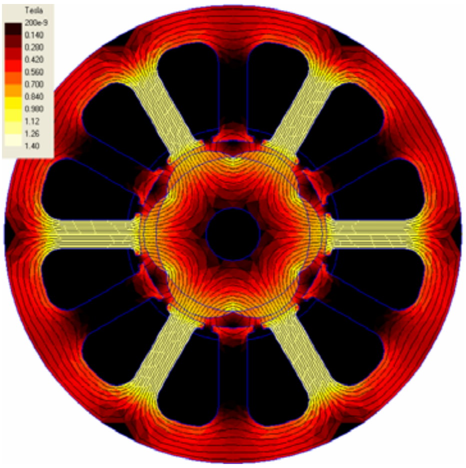


(b) R=15 mm

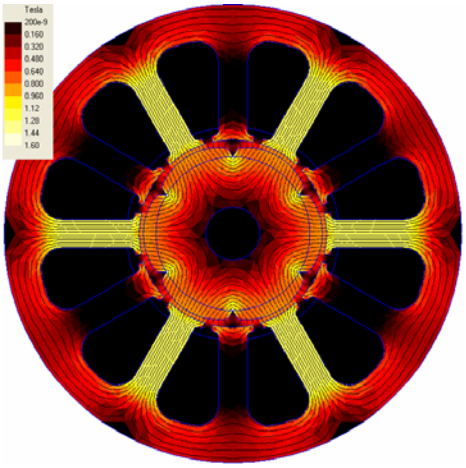


(c) R=21.5 mm

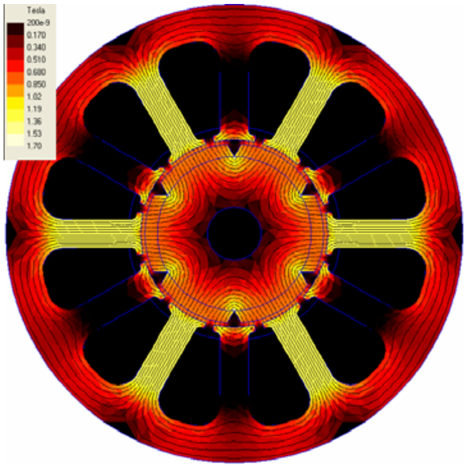
Fig. 5 Motor I; the selected Basis shapes and their contour maps.



(a) R=10.6 mm

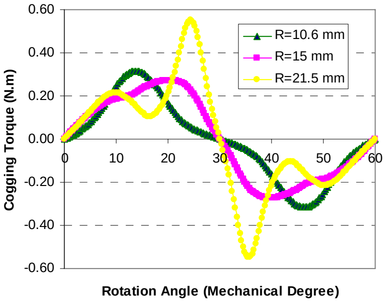


(b) R=15 mm

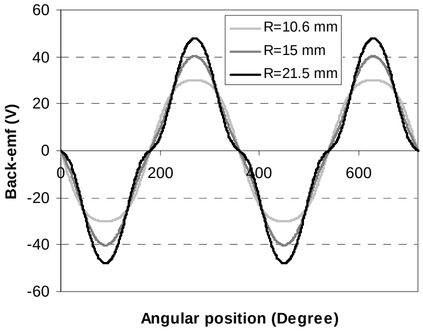


(c) R=21.5 mm

Fig. 6 Motor II; The selected Basis shapes and their contour maps.

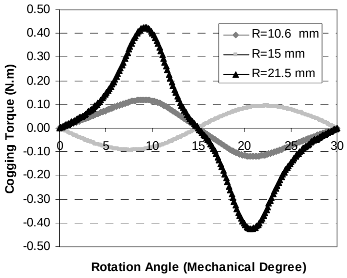


(a) Cogging torque waveform

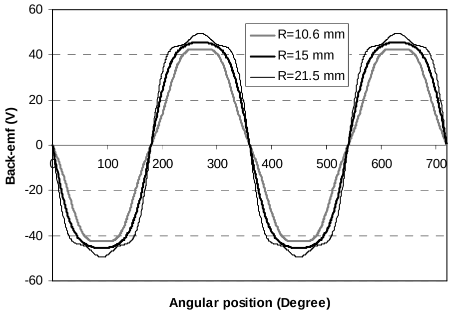


(b) Back-emf waveform

Fig. 7 A comparison of (a) cogging torques, (b) back-emf, for Basis shapes of type I motor.



(a) Cogging torque waveform



(b) Back-emf waveform

Fig. 8 A comparison of (a); cogging torques, (b): back-emf, for Basis shapes of type II motor

4 Design of Experiment Method

4.1 ANOVA Procedure

The Taguchi method [20] is used to investigate the effects of multiple variables simultaneously. Three weighting factors are studied to obtain the best combination. Table 2 shows the factors and their levels. Based on known variation of cogging torque with respect to different factors, each factor is considered to have four levels. The experiments are carried out using FEA and the cogging torque value obtained from each experiment is listed in Table 3.

Table 2 Weighting factors and their selected values

Factor	Level 1	Level 2	Level 3	Level 4
(R=10.6mm) c_1	0.3	0.5	0.7	0.9
(R=15mm) c_2	0.5	0.7	0.8	0.99
(R=21.5mm) c_3	0.01	0.1	0.3	0.5

Table 3 The values of cogging torque in sixteen experiments

Trial No.	Cogging Torque (kg.cm)	
	Motor I	Motor II
1	0.327650	0.294375
2	0.246571	0.123489
3	0.056873	0.304587
4	0.348765	0.092465
5	0.086750	0.234567
6	0.067534	0.356754
7	0.346597	0.218760
8	0.347650	0.450924
9	0.405465	0.326787
10	0.16875	0.120945
11	0.072346	0.237609
12	0.106754	0.458786
13	0.12834	0.457608
14	0.113493	0.179865
15	0.126586	0.078845
16	0.0506324	0.139866

4.2 Results Analysis

Considering cogging torque as target, the results are investigated. The main effects table, which presents the mean value of cogging torque for each factor at all levels, is shown in Table 4. The ANOVA table for cogging torque is also presented in Table 5. Predicted optimum combination of factors in this step is listed in Table 6.

4.3 Predicted Result Evaluation

Running the experiments at optimum combination of factors for cogging torque result (0.05 N.m for motor I and 0.03 N.m for motor II), these values are within the permissible limit and the predicted result is confirmed.

Confirmation means that for 90% confidence level, there is no need to repeat the procedure of designs of experiment with counting for interactions between factors. Fig. 9 shows the optimum resultant pole shape achieved by the reduced basis technique.

Table 4 The mean value of cogging torque for each factor at all levels for motor I / Motor II

Factor	Level 1	Level 2	Level 3	Level 4
c_1	0.203 /0.244	0.314 /0.211	0.285 /0.187	0.213 /0.104
c_2	0.327 /0.236	0.194 /0.148	0.209 /0.149	0.284 /0.212
c_3	0.256 /0.128	0.223 /0.141	0.314 /0.23	0.221 /0.247

Table 5 ANOVA table for motor I / Motor II

Factor	F	S	V	F
c_1	3	0.035/0.042	0.011/0.014	0.479/0.66
c_2	3	0.047/0.023	0.015/0.007	0.643/0.367
c_3	3	0.022/0.044	0.007/0.014	0.306/0.677
Other/error	6	0.148/0.129	0.024/021	
Total	15	0.254/0.24		

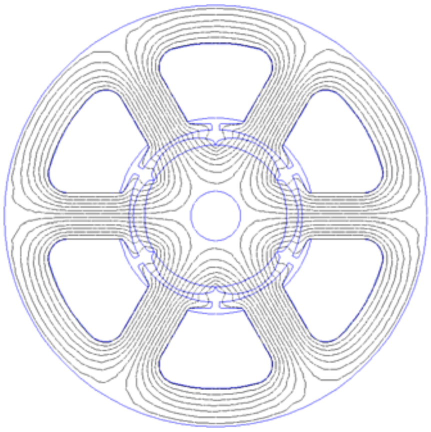
Table 6 Optimum conditions of cogging torque for motor I / Motor II

Factor	Value	Level	Contribution
c_1	0.3/0.9	1/4	-0.051/-0.083
c_2	0.9/0.7	3/2	-0.06/-0.039
c_3	0.01/0.01	1/1	-0.033/-0.058
Total contribution of factors			-0.144/-0.181
Grand average of performance			0.254/0.186
Expected result at optimum			0.11/0.006

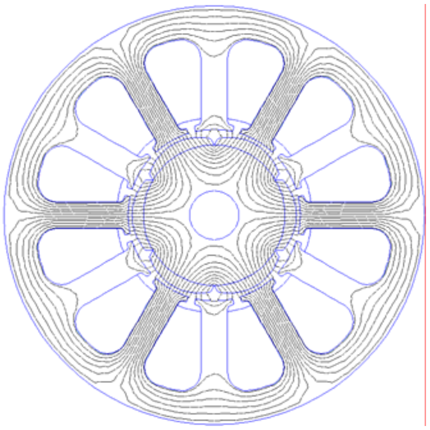
Therefore, the peak value of the cogging torque for optimum design has been reduced significantly by this method to 0.05 N.m for motor I and 0.03 N.m for motor II. A comparison of magnetic flux distribution of optimized designs is also shown in Fig. 10.

5 Conclusion

A pole shape optimization method for permanent pole motors is introduced in this paper using a robust method. The concept of a progressive design process is presented, which aids the designer in the selection of practical basis shapes that will give cogging torque reduction, but this will also increase computational cost. Increasing the number of basis shapes also enables the designer to obtain a better pole shape, but the computation time also increases to build a surrogate model. The developed algorithm has been applied on two prototype permanent magnet synchronous motors. The optimum pole shapes have been achieved by the implemented algorithms, starting from three different splines as basis shapes.

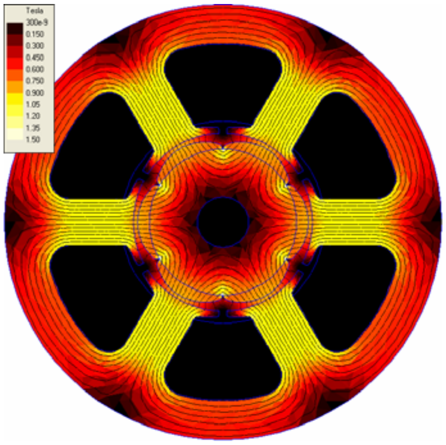


(a) R=12.34 mm

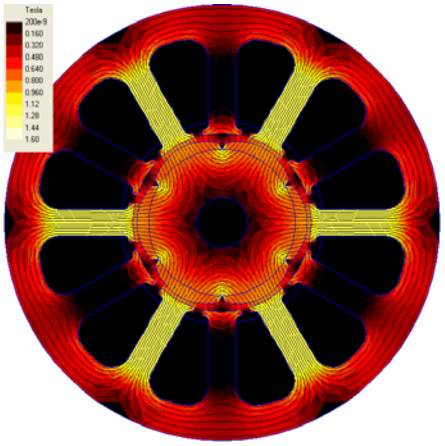


(b) R=13.68 mm

Fig. 9 Optimum resultant shape contour map



(a) Motor I: Optimum shape



(b) Motor II: Optimum shape

Fig. 10 Magnetic flux density distribution

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