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RESEARCH PAPER

Open Vehicle Routing Problem with Robust Optimization Approach

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ABSTRACT

One of the challenging issues in today's competitive world for servicing companies is uncertainty in some factors or parameters that they often derive from fluctuations of market price and other reasons. With regard to this subject, it would be essential to provide robust solutions in uncertain situations. This paper addresses an open vehicle routing problem with demand uncertainty and cost of vehicle uncertainty. Bertsimas and Sim's method has been applied to deal with uncertainty in this paper. In addition, a deterministic model of open vehicle routing problem is developed to present a robust counterpart model. The deterministic and the robust model is solved by GAMS software. Then, the mean and standard deviations of obtained solutions were compared in different uncertainty levels in numerous numerical examples to investigate the performance of the developed robust model and deterministic model. The computational results show that the robust model has a better performance than the solutions obtained by the deterministic model.

KEYWORDS: *Open vehicle routing problem*; *Uncertainty; Bertsimas*; *Robust optimization.*

1. Introduction

Open Vehicle Routing Problem (OVRP) is a well-known vehicle routing problem with many applications in the real world. This problem has been developed to decrease the cost of good transportation and servicing customers regarding increasing their satisfaction. In another classification, OVRP is an optimization problem. Optimization problems can be divided into two general categories. They are optimization in deterministic and non-deterministic spaces. Optimization aim is to find the best acceptable solution according to the available constraints and problem information. The values of all parameters in deterministic optimization problems are assumed to be known. However, problems often require deciding in the presence of uncertainty. In many real-life situations, there are parameters that their values do not reflect specified certainty. Because it is not possible to estimate the exact values of parameters in reality, in these kinds of problems, considering uncertainty in the decision-making process may lead to more stable solutions. That is why the uncertainty has recently attracted the attention of researchers. Moreover, it is necessary to embed the uncertainty in the model with a proper approach.

The presented approaches to deal with the uncertainty include stochastic programming, fuzzy programming, and robust programming. The first approach, i.e., stochastic programming, was introduced in the 1950s by Dantzig et al. [1]. Applying this approach is for probabilistic uncertainty. It means that scenarios with different occurrence probabilities are presented. By increasing the number of scenarios, the problem's dimension will increase, and leading to a computational challenge.

Based on the type of data uncertainty set, robust optimization problems are divided into three types: interval set uncertainty, set of elliptical uncertainties, and uncertainty set in the scenario.

The robust optimization considers an uncertainty bounded set instead of knowing the probability distribution. Because of OVRP importance in uncertain space in the real world, OVRP and its literature in the presence of uncertainty were investigated.

The first person who addressed the real-life application on OVRP was Bodin [2]. Then Sariklis and Powell [3] developed OVRP, and

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they used a heuristic method for solving this problem. It has been used for companies that they have not enough vehicles to service to all customers, or they have no vehicles at all. In this problem, the vehicles are not obligated to return to the distribution depot after servicing the last customer on its route [4, 5]. The main difference between OVRP and vehicle routing problem (VRP) is the type of route; OVRP and VRP are consists of Hamiltonian paths and Hamiltonian cycles, respectively [6]. The aim of OVRP is to find the minimum number of vehicles that are required to service all of the customers' demands. In this paper, the goal of OVRP is to decrease the cost of traveling and the cost of using vehicles. For solving OVRP, many methods have been used; for example, Fleszar et al. [6] applied variable neighborhood search (VNS) to solve it, that VNS is a heuristic method.

Generally, the presented methods to solve OVRP are divided into three categories; contains exact, heuristic, and meta-heuristic. Fu et al. [7] used a heuristic method, which prohibited searching for solving OVRP with capacity constraint and maximum path length. Liu et al. [8] used a meta-heuristic method of hybrid genetic algorithm for solving OVRP. Tarantilis et al. [9] selected list based threshold accepting algorithm (LBTA) to solve it. Letchford et al. [10] applied an exact method branch-and-cut. Li et al. [11] selected a record-to-record traveling heuristic algorithm and simulated an annealing meta-heuristic algorithm. Pisinger and Ropke [12] used an adaptive large neighborhood search. Repoussis et al. [13] considered a hybrid evolution strategy as a method to solve OVRP. Salari et al. [14] improved the integer linear programming heuristic method. MirHassani and Abolghasemi [15] applied a particle swarm optimization algorithm. López-Sánchez et al. [16] solved by a competitive algorithm with several starting points. Marinakis and Marinaki [17] used a bumble bees mating optimization algorithm for OVRP. Zachariadis and Kiranoudis [18] handled meta-heuristic methods based on local search. Cao et al. [19] considered the OVRP with uncertain demands and solved the robust optimization model by an improved differential evolution algorithm. Eduardo et al. [20] investigated a multi-depot OVRP and proposed new mixed-integer programming. Cao and Lai [5] considered an OVRP with fuzzy demands, and a hybrid intelligent algorithm solves its model. Liang et al. [21] considered a goal-robust-optimization model and proposed a heuristic algorithm and a particle swarm optimization based on genetic algorithms to solve OVRP with demand uncertainty [22] . Solved a robust counterpart model of open capacitated vehicle routing problem using LINGO and branch and bound solver to obtain the optimal routes. Yangkun et al. [23] solved an OVRP with soft time windows by Tabu Search algorithm. Yangkun, et al. [24] developed a capacitated OVRP with split deliveries by order considered and proposed an adaptive tabu search algorithm to solve it.

Some papers considered a robust vehicle routing problem. Sungur et al. [25] introduced the robust capacitated vehicle routing problem. They presented a branch-and-cut-based to solve the problem under uncertain customers' demands and uncertain travel times. Lee et al. [26] considered a dynamic programming algorithm to solve vehicle routing problems with uncertain demand, uncertain deadlines and uncertain travel time. Agra et al. [27] investigate the vehicle routing problem with time windows, and travel time uncertainty in which both time windows and travel time are modeled as interval data. Then, they adopted different robust optimization tools to handle the uncertainty.

In this paper, a comprehensive review has been done in OVRP and articles that considered stochastic OVRP. The related literature review presented in Table 1 and Figure 1 implies that the robust OVRP area is limited. Although OVRP has a special position in the real world because of its application, most research deterministic models have been addressed. Investigations indicate that none of the studies have addressed OVRP with customer demand uncertainty and vehicle uncertainty costs. Most of the research solved deterministic OVRP. However, in the real world, parameters are often uncertain, and it is needed to describe these non-deterministic parameters in the model.

According to the best of the authors' knowledge, there is no paper in the related literature to address OVRP with both demand uncertainty and cost of vehicles uncertainty under a robust approach. The robust counterpart model is presented, in which customers' demands and cost of vehicles belong to specific bounded uncertain sets with expected value and nominal value.

Among the previous research works, most of them considered customer demand and cost of vehicles as deterministic parameters, while these parameters are stochastic in the real-life. The effect of these uncertain parameters on the considered OVRP model in this paper, as the review of related literature showed, had not been addressed with a robust optimization approach by now.

There are some approaches to handle uncertainty. Following some significant drawbacks in using a stochastic approach have been pointed out. In stochastic optimization, obtaining the actual probability distribution of the uncertain parameters is difficult because there is no enough historical data for the uncertain parameters in many real cases. Also, the solution could be infeasible in stochastic optimization. Although the probability of this occurrence is small, it can lead to high costs.

Moreover, in the scenario-based stochastic programming approach, increasing the number of scenarios causes a more computational challenge in modeling the uncertainty. Through a framework of handling the uncertainty of parameters in optimization problems, robust optimization can immunize the optimal solution for any realizations of the uncertainty in a given bounded uncertainty. The proposed robust model in handling uncertainty in parameters will generate robust optimal solutions [28]. Regarding the drawbacks mentioned above of stochastic optimization models, the robust model is more efficient compared to other approaches. The circular diagram in Figure 1 demonstrates the result of the literature review.

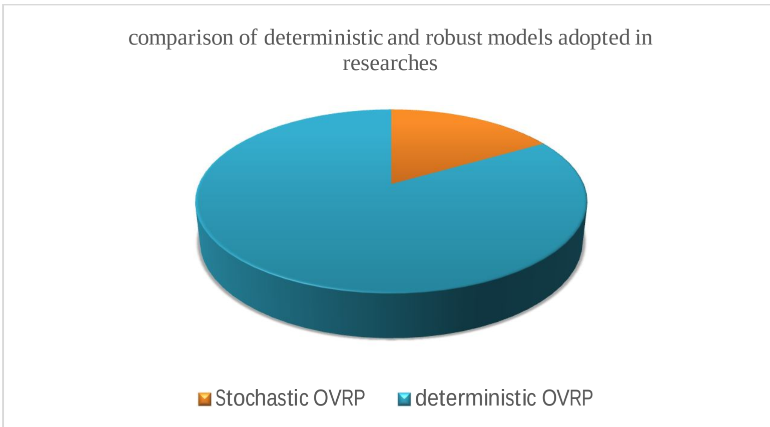

Fig. 1. Comparison of a large amount of primitive research

The remainder of this paper is presented as follows. In Section 2, concepts regarding uncertainty and OVRP have been explained. Also, the robust OVRP with uncertain demand and cost of vehicles uncertainty have been described after that, the robust optimization model, which is based on Bertsimas' model, has been presented. In section 3, some numeric examples solve in different sizes. In Section 4, the obtained results are analyzed. Finally, Section 5 is devoted to the conclusions and suggestions for future studies.

Tab. 1. literature review of OVRP in selected papers

Papers	Objective	Number of objectives	Demand		Modelling		Uncertainty			Solution Method			
			Deterministic	Stochastic	LP	MLP	robust	fuzzy	Probability	exact	heuristic	Meta-heuristic	hybrid
Yannis Marinakis, Georgia-Roumbini Iordaniduo, Magdalene Marinaki (2013) [29]	Minimizes distance (length) of the route traveling of vehicles	1		✓		✓			✓				✓
Cao Erbao, Lai Mingyong (2010) [5]	minimizing total planned travel distance, 2. minimize total additional travel	2		✓		✓		✓					✓
Cao Erbao, Lai Mingyong, Yang Hangming (2014) [19]	minimizing transportation cost, and unsatisfied demands	2		✓		✓	✓						✓
Vahid Baradaran, Amir Shafaei, AmirHossein Hosseinian (2019) [30]	minimize total transportation costs and maximize satisfaction of customers	2	✓		✓				✓			✓	
Yong Shi, Toufik Boudouh,Olivier Grunder(2016) [31]	minimize total distance	1		✓		✓		✓					✓
Yannis Marinakis (2015) [32]	minimize distance	1		✓		✓						✓	
Liang Sun, Bing Wang (2018) [21]	minimize undesirable deviations from a predetermined time window	1		✓		✓							✓

2. Model OVRP with Uncertain Parameters (Demand and Cost of Vehicle)

2.1. Problem description

OVRP is one of the prominent developments in the basic model of vehicle routing problem. Because of its many applications in the real world is one of the most important and challenging combinational optimization problems. The purpose of OVRP is to service many customers with a fleet of vehicles, which are not obligated to come back to the distribution depot [4-5-19]. In the classic OVRP, there is a central distribution depot and a certain number of vehicles with a given capacity. Each route contains only one vehicle to serve several customers on the same route. In this problem, in addition to decreasing transportation costs and unmet demands, we intend to satisfy all constraints. OVRP is an NP-Hard problem; therefore, different heuristics and meta-heuristics have been used to solve it.

Another important development in this area is to bring OVRP closer to the real world by considering uncertain parameters.

Demand and cost of vehicle parameters are considered uncertain parameters and they belong to a bounded set. These parameters are in demand constraints and objective function. In following the robust approach that is proposed by Bertsimas and Sim in 2004 [33], was used. The application of robust optimization is to control the uncertainty in this problem. In this paper, demand constraint and objective function determinately have been presented; after that, the robust approach in the mathematical model is applied. The main contribution of mathematical formulations in this paper is the demand constraint and objective function, which have been modified under uncertain circumstances. Constraint 1, $\sum_j^n q_j\,(\sum_{i=1}^n X_{ij}^k) \leq Q \quad \forall\, k = 1,2,\dots,K$, presents demand constraint in deterministic, which q_j is a determinate parameter. The objective function of the deterministic model is presented by this formula $Min\sum_{k=1}^K W_k Z_k + \sum_{k=1}^k \sum_{i=0}^n \sum_{j=0}^n C_{ij} X_{ij}^k$, which W_k is cost of the vehicle, and it has been considered as a determinate parameter. According to the assumption of this study, q_j and W_k are considered as uncertain parameters. The deterministic objective function and demand constraint have been substituted for the Robust counterpart of the objective function, and robust counterpart constraint while W_k and q_j are uncertain parameters. In continue, we intend to indicate how to apply the robust approach to handle the uncertainty in OVRP. After that, problem assumption, model parameters, decision variables and mathematical model (constraint 7 to 22) have been introduced.

Demand constraint determinately:

$$\sum_j^n q_j\,(\sum_{i=1}^n X_{ij}^k) \leq Q \quad \forall\, k = 1,2,\dots,K \tag{1}$$

In equation (1) q_j is supposed as an uncertain parameter, and it can be changed in a preset interval like this$[q_j - \hat{q}_j\,, q_j + \hat{q}_j]$ at worst all of q_j will be changed, but the probability of such a situation occurring when all parameters reach their maximum value is very low in reality so maximum number of parameters that they can change in the interval set are represented by Γ. Γ is a symbol that it can change in$[0, |J_j|]$. In this interval $|J_j|$ represent number of uncertain parameters in j^{th} row of the coefficient matrix of the constraint. J is a set that it define into $J = \{j|\, \hat{q}_j > 0\}$. In continues, the robust counterpart of this constraint is presented.

$$\sum_{j=1}^n q_j\,(\sum_{i=1}^n X_{ij}^k) + \max_{\{S \mid S \subset J, |S| \leq \Gamma_k\}} \left\{\sum_{j=1}^n \hat{q}_j \sum_{i=1}^n X_{ij}^k\right\} \leq Q \quad ,\forall\, k = 1,2,\dots,K \tag{2}$$

For the linearization of the above constraint, the conservatism function $\beta(\Gamma_k)$ have been considered as follows:

$$\beta(\Gamma_k) = \max_{\{S \mid S\subset J\,, |S| \leq \Gamma_k\}} \left\{\sum_{j=1}^n \hat{q}_j \sum_{i=1}^n X_{ij}^k\right\} \tag{3}$$

The conservatism function can be expressed as the following, model 1

$$\beta(\Gamma_k) = max \sum_{j=1}^n \hat{q}_j\, \mu_j \sum_{i=1}^n X_{ij}^k$$

Model 1

$$\sum_{j=1}^n \mu_j \leq \Gamma_k$$

$0 \leq \mu_j \leq 1 \quad ,\forall j \in J$

The considered dual variables are θ_k , P_j then dual of the model 1 has been presented as follows:

$$min \ \theta_j \Gamma_k + \sum_{j=1}^{n} P_j$$

Model 2

$$\theta_j + P_j \geq \hat{q}_j \sum_{i=1}^{n} X_{ij}^k \qquad \forall j, k$$

$$\theta_j \geq 0 \quad , \quad p_j \geq 0 \qquad \forall j \in J$$

According to the dual strong theory, model 1 and its dual for all $\Gamma_k \in [0 , |J|]$ are boundaries and feasible, so they have the same objective function value in optimal like as follows:

$$\beta^*(\Gamma_k) = \theta_j^* \Gamma_k + \sum_{j=1}^{n} P_j^* \tag{4}$$

$$so\ that \quad \theta_j^* + P_j^* \geq \hat{q}_j \sum_{i=1}^{n} X_{ij}^{k\,*} \tag{5}$$

Then model 2 has to be substituted for equation (2). The robust counterpart constraint will be as follows.

$$\sum_{j=1}^{n} q_j \sum_{i=1}^{n} X_{ij}^k + \theta_j \Gamma_k + \sum_{j=1}^{n} P_j \leq Q$$

Model 3

$$\theta_j + P_j \geq \hat{q}_j \sum_{i=1}^{n} X_{ij}^k \qquad \forall j, k$$

$$\theta_j \geq 0 \quad , \quad P_j \geq 0$$

Then a variable like t_{jk} has been defined and substituted $\sum_{i=1}^{n} X_{ij}^k$ for t_{jk} , after that a constraint like $t_{jk} \leq \sum_{i=1}^{n} X_{ij}^k$ has been added. So, the robust counterpart constraint will be as follows:

$$\sum_{j=1}^{n} q_j \, t_{jk} + \theta_j \Gamma_k + \sum_{j=1}^{n} P_j \leq Q$$

Model 4

$$\theta_j + P_j \geq \hat{q}_j t_{jk} \qquad \forall j, k$$

$$t_{jk} \leq \sum_{i=1}^{n} X_{ij}^k \qquad \forall j, k$$

$$-y_{jk} \leq t_{jk} \leq y_{jk} \qquad \forall j, k$$

$$\theta_j \geq 0 \quad , \quad P_j \geq 0$$

Similar to the uncertain demand constraint, the uncertainty for the cost of the vehicle (W_k) in the objective function will be considered. The objective function of the deterministic model has been shown in equation 6.

$$Min \sum_{k=1}^{K} W_k Z_k + \sum_{k=1}^{k} \sum_{i=0}^{n} \sum_{j=0}^{n} C_{ij} X_{ij}^k \tag{6}$$

The Robust counterpart of objective function when W_k is an uncertain parameter will be as follows:

$$Min \sum_{k=1}^{K} W_k Z_k + \Gamma_0 Z_0 + \sum_{k} PP_k + \sum_{k=1}^{k} \sum_{i=0}^{n} \sum_{j=0}^{n} C_{ij} X_{ij}^k \qquad \text{Model 5}$$

$$Z_0 + PP_k \geq \widehat{W}_k Z_k$$

$$Z_0 \geq 0 \quad , \quad PP_k \geq 0$$

2.2. Assumptions

In this section, characteristics of robust OVRP considered in this study is presented as follows:

(1) Type of customers' demand is delivery.
(2) Location of each customer is considered as a node.
(3) The cost depends on both the travelling distance and the number of active vehicles.
(4) The number of vehicles is specified, that the maximum number of them is N.
(5) The capacity of each vehicle is limited, and the vehicles are not homogeneous.
(6) In this paper planning horizon of OVRP is a single-period.
(7) The problem has only one distribution depot.
(8) Both the customers' demand and the cost of vehicles are uncertain.
(9) The objective function of the problem in this paper is single-objective without time constraints.

2.3. OVRP formulation

The mathematical model of OVRP with demand uncertainty and cost of vehicles uncertainty is presented as follows:

Model parameters:

- N in the graph G(V, A) is the number of total nodes, including depot and customer nodes
- K number of total vehicles
- q_i demand of i-th customer
- Q capacity of each vehicle
- C_{ij} travel cost from i-th node to j-th node
- W_k cost of k-th active vehicle

2.4. Decision variables

$$X_{ij}^{k} = \begin{cases} 1 & if\ the\ k-th\ vehicle\ travels\ the\ arc\ between\ i\ ,j \\ 0 & \text{Otherwise} \end{cases} \qquad \forall\ i,j\epsilon V \quad و\ \forall\ k\epsilon K$$

$$Z_k = \begin{cases} 1 & if\ the\ k-th\ vehicle\ is\ active \\ 0 & \text{Otherwise} \end{cases} \qquad \forall\ k\epsilon K$$

$$Vf_u^k = \begin{cases} 1 & if\ the\ u-th\ customer\ is\ last\ to\ visit\ by\ k-th\ vehicle\ is\ active \\ 0 & \text{Otherwise} \end{cases} \qquad \forall\ k\epsilon \mathrm{K}$$

V is a Positive integer variable.
θ_j , P_j Dual variables for linearizing function protective constraint
Z_0 , PP_k Dual variables for linearizing conservatism objective function

So, the developed model of OVRP under demand uncertainty and cost of vehicles uncertainty as follows:

$$Min \sum_{k=1}^{K} W_k Z_k + \Gamma_0 Z_0 + \sum_{k} PP_k + \sum_{k=1}^{k}\sum_{i=0}^{n}\sum_{j=0}^{n} C_{ij} X_{ij}^{k} \tag{7}$$

Subject to:

$$Z_0 + PP_k \geq \widehat{W}_k Z_k \tag{8}$$

$$\sum_{k=1}^{k}\sum_{i=0}^{n} X_{ij}^{k} = 1 \qquad \forall\ j = 1,2,\dots,n \tag{9}$$

$$\sum_{i=0}^{n} X_{iu}^{k} = \sum_{j=1}^{n} X_{uj}^{k} + Vf\ (u,k) \qquad \forall\ k = 1,2,\dots,K \quad \forall\ u = 1,2,\dots,n \tag{10}$$

$$V_i^k - V_j^k + |n| * x_{ij}^{k} \leq |n| - 1 \quad \forall\ i = 0,1,2,\dots,n\ ,\forall\ j = 0,1,2,\dots,n\ \ \forall\ k = 1,2,\dots,K \tag{11}$$

$$\sum_{j=1}^{n} q_j\, t_{jk} + \theta_j \Gamma_k + \sum_{j=1}^{n} P_j \leq Q \qquad \forall\ k = 1,2,\dots,K \tag{12}$$

$$\theta_j + P_j \geq \hat{q}_j t_{jk} \qquad \forall\ \mathrm{j}\ ,\mathrm{k} \tag{13}$$

$$t_{jk} \leq \sum_{i=1}^{n} X_{ij}^{k} \qquad \forall\ \mathrm{j}\ ,\mathrm{k} \tag{14}$$

$$-y_{jk} \leq t_{jk} \leq y_{jk} \qquad \forall\ \mathrm{j}\ ,\mathrm{k} \tag{15}$$

$$\sum_{i=1}^{n} X_{i0}^{k} = 0 \quad \forall\ k = 1,2,\dots,K \tag{16}$$

$$\sum_{j=1}^{n} X_{0j}^{k} \leq 1 \quad \forall\ k = 1,2,\dots,K \tag{17}$$

$$X_{ij}^{k} \leq BigM * z_{k} \quad \forall\, k = 1,2,\dots,K, \quad \forall\, i = 0,1,2,\dots,n \quad ,\forall\, j = 1,2,\dots,n \tag{18}$$

$$X_{ij}^{k} \epsilon \{0,1\} \quad \forall\, k = 1,2,\dots,K, \quad \forall\, i = 0,1,2,\dots,n \quad ,\forall\, j = 1,2,\dots,n \tag{19}$$

$$z_{k} \epsilon \{0,1\} \quad \forall\, k = 1,2,\dots,K \tag{20}$$

$$\mathrm{Vf}_{u}^{k} \epsilon \{0,1\} \quad \forall\, k = 1,2,\dots,K \tag{21}$$

$$\begin{array}{l} Z_0 \geq 0 \quad , \quad PP_k \geq 0 \\ \theta_j \geq 0 \quad , \quad P_j \geq 0 \end{array} \tag{22}$$

The presented model is the robust counterpart of the deterministic mathematical model that it is based on the model proposed by Mir Hassani et al. [15] with some modifications.

3. Computational Experiments

In this section, we intend to make a comparison between the deterministic model and the uncertain model. The model's performance under deterministic and uncertain parameters is compared in several sizes of the problem, results obtained from GAMS are reported in Figures 2 to 11 for each size of the problem. Each figure shows one size of the problem. In each figure, when Γ is equal to 0, means the model has been performed under deterministic and under uncertainty Γ is equivalent to three amounts (0.2, 0.5, and 1) First different sizes of the problem have been selected that minimum and maximum size of them are N7-K2 and N41-K14, respectively. For each size of the problem, the model was performed under three levels of uncertainty (Γ= 0.2, 0.5, 1). When the value of Γ is equal to 0, it means all parameters are deterministic. Robust optimization is used to manage uncertainty. Values of both robust and deterministic parameters are generated by random distributions specified; these values are presented in Table 2 as the nominal data. Then in Table 3, values of objective function under nominal data and each uncertainty level (Γ) is computed for each size of the problem. Also, computing time has been presented for both the robust and deterministic models. The following five random realizations under each uncertainty level and fifteen random realizations under Γ=0 are generated to compare the mean deviation and standard deviation of objective function values under these realizations. Figures 2 to 11 describe objective function values under realizations of each Γ as the interval plot for each size of the problem.

In this problem, the capacity of heterogeneous vehicles (Q_k) was generated from the Uniform Integer distribution function. The customer demand parameter (q_j) has been generated from the Normal distribution function. Other parameters (w_k, $\hat{q}_j$, $\widehat{w}_k$, and C_{ij}) have been generated from the Uniform distribution function.

Tab. 2. Nominal data

Parameter	Distribution of generation random
q_j	$\sim\ Normal(125,20)$
w_k	$\sim\ uniform(300,700)$
$\hat{q}_j$	$\sim\ uniform(4,30)$
$\widehat{w}_k$	$\sim\ uniform(20,80)$
Q_k	$\sim\ uniformint(700,800)$
C_{ij}	$\sim\ uniform(20,120)$

All different instance sizes of the problem in both the deterministic and robust model have tested in GAMS 23.5 software by computational experiment on a personal computer with 2.53 GHz Intel(R) Core(TM) i5 CPU with 4 GB RAM under Windows 8.1 environment.

Tab. 3. Comparison of objective function values under nominal data and computational time

Problem Size	Uncertainty level(Γ)	Objective Function values under nominal data		Computational time (s) under nominal data(s)	
		deterministic	robust	deterministic	robust
N7-K2	0.2	532.132	538.047	0.328	0.122
	0.5		546.918		0.354
	1		561.703		0.330
N9-K3	0.2	601.824	615.795	0.919	0.254
	0.5		636.751		0.567
	1		671.678		0.252
N11-K4	0.2	764.369	771.799	0.354	0.846
	0.5		782.943		0.519
	1		801.518		0.526
N15-K5	0.2	712.807	718.219	0.460	0.666
	0.5		726.337		0.807
	1		739.868		0.968
N23-K8	0.2	982.569	997.770	7.787	7.557
	0.5		1020.572		7.631
	1		1045.811		10.295
N31-K10	0.2	1106.616	1115.337	6.207	7.725
	0.5		1128.517		8.027
	1		1150.417		11.594
N41-K14	0.2	1273.118	1281.935	31.283	27.953
	0.5		1295.159		31.318
	1		1317.199		53.856

4. Comparison of Deterministic and Robust Models

In this section, some different sizes of the numerical examples will be considered to illustrate the comparison between deterministic and uncertain. The considered model is the one in which we have just discussed and showed how to control uncertainty. Ten types of experimental conditions are made based on the size of OVRP. It is assumed that there are six customers and two vehicles for the small size (N7-K2) and 40 customers and 14 vehicles for the large size problem (N41-K14). The other sizes of the problem are N9-K3, N11-K4, N15-K5, N17-K7, N20-K8, N23-K8, N27-K9, N31-K10, which N-1 indicates the number of customers and K depicts the number of the active vehicles. As we know, the active vehicles are the ones that have been utilized to serve customers. In each experiment, the coordinates of all customers and vehicles are randomly considered. In order to evaluate the performance of the results represented in Table 4, outcomes are presented in the interval plot. Figures 2 to 11 display the interval plot under different uncertainty levels for the problem with various sizes. The interval plot indicates the amount of objective function with Regard to the objective function values under realizations for both deterministic and uncertain in Table 4. According to Table 4, it has been seen that in all problem sizes, maximum and minimum values of the objective function of the deterministic model dominate the maximum and minimum values of the objective function in a robust model. It means that the maximum value of the objective function in the deterministic model is more significant than the maximum objective function values of all uncertainty levels in the robust model, and its minimum value is less than minimum values under all uncertainty levels of the robust model. The said feature justifies for all problem sizes except the minimum value of size N11-K4 under Γ=0.5 and maximum value of size N41-K14 under Γ=1, but under other levels, the values are dominated by values of the deterministic model. Although the minimum objective function value under Γ=0.5 in size N11-K4 is less than the value of the deterministic model, its maximum has the lowest value compared to the maximum value of the other levels and the maximum value of the deterministic model. Also, concerning the size N41-K14, the maximum value of an objective function under Γ=1 of robust model is larger than the maximum value of the deterministic model. In contrast, its minimum value has the largest value compared to the same value under other uncertainty and the minimum value of the deterministic model.

That is important to evaluate the model's performance with respect to standard deviation. Standard deviation and model performance have an inverse relation. It means in the lower value of standard deviation, the model's performance will be better and vice versa. In all sizes of problems, the standard deviation of the deterministic model is larger than the robust model. Figure 12 illustrated this issue clearly. Some sizes of problems with low standard deviation values are slight differences between the maximum and minimum objective function values under each uncertainty level. In Table 4 and Figure 2, it can be stated for Γ=0, the mean value of the objective function of the deterministic model is equal to 746.872, and it expresses for Γ=0.2, 0.5, 1, the mean value of the objective function is equal to 653.239, 651.850, 785.194 respectively. Also, the standard deviation values for deterministic and robust model under Γ=0, 0.2, 0.5, 1 are 124.057, 73.365, 86.762, 80,380 respectively. In Figure 2, although the 95 percent confidence interval for the data means for Γ=0 is smallest compared to the obtained solutions by other uncertainty levels in the robust model, it has a more significant standard deviation than the robust model. In this size (N7-K2) of the problem, the mean values in the uncertainty levels Γ=0.2, 0.5 are smaller than the value obtained by the deterministic model. Table 4 and Figure 3, in size(N9-K3) of the problem as the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 729.627, 712.820, 710.568, 756.926 respectively, that mean values for Γ=0.2, 0.5 are minimum compared with Γ=0. Thus, although the interval of the mean value in the deterministic (Γ=0) is smaller than the other mean values, its standard deviation is larger than the standard deviation obtained by the robust model. Respecting Table 4 and Figure 4, as it can be seen in this size(N11-K4) of the problem, the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 764.398, 697.393, 686.376, 787.943 respectively that mean values for Γ=0.2, 0.5 are minimum compared with Γ=0. Table 4 and Figure 5 shows in this size (N15-K5) of the problem, the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 822.977, 798.823, 843.448, 809.234 respectively that mean values in Γ=0.2, 1 are minimum compared with Γ=0. Table 4 and Figure 6 can be expressed in this size (N17-K7) of the problem; the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 853.419, 893.802, 882.978, 890.692 respectively that mean value in Γ=0 is minimum. Regarding Table 4 and Figure 7, it can be asserted in this size(N20-K8) of the problem, mean values of objective function under Γ=0, 0.2, 0.5, 1 are 926.496, 871.189, 993.177, 938.133 respectively that mean value for Γ=0.2 is minimum compared with Γ=0. On the subject of Table 4 and Figure 8, it can be stated in this size(N23-K8) of the problem the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 963.565, 986.135, 953.708, 978.948 respectively that the mean values in Γ=0.5 is minimum compared with Γ=0. Concerning Table 4 and Figure 9, it can be seen in this size(N27-K9) of the problem the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 1037.464, 1012.247, 1080.392, 1077.992 respectively that mean values in Γ=0.2 is minimum compared with Γ=0. With respect to Table 4 and Figure 10, it can be stated in this size(N31-K10) of the problem the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 1108.372, 1117.050, 1153.060, 1157.753 respectively that mean values in Γ=0 is minimum. Although the interval of the mean value in the deterministic (Γ=0) is smaller than the other mean values, its standard deviation is larger than the standard deviation obtained by the robust model. In Table 4 and Figure 11, it can be declared in this size (N41-K14) of the problem the mean values of objective function under Γ=0, 0.2, 0.5, 1 are 1334.516, 1305.577, 1324.974, 1381.754 respectively that mean values in Γ=0.2, 0.5 are minimum compared with Γ=0. Although the interval of the mean values in the deterministic (Γ=0) is smaller than the other mean values, its standard deviation is larger than the standard deviation obtained by the robust model. As the results show, the robust model was investigated compared to the deterministic one concerning standard deviation and mean objective function values. The results imply that the robust model has a better performance for all instance sizes of problems.

Tab. 4. Comparison of mean and standard deviation of objective function values under realizations

Problem Size	Uncertainty level (Γ)	Mean of objective function values under realizations						Standard deviation of objective function values under realizations	
		deterministic	Min of deterministic objective function values under realizations	Max of deterministic objective function values under realizations	robust	min of robust objective function values under realizations	max of robust objective function values under realizations	deterministic	robust
N7-K2	0.2	746.872	550.858	933.202	653.239	559.892	742.812	124.057	73.365
	0.5				651.850	558.073	748.775		86.762
	1				785.194	701.053	891.807		80.380
N9-K4	0.2	729.627	554.715	877.964	712.820	670.432	823.564	84.605	62.635
	0.5				710.568	632.487	806.191		63.427
	1				756.926	665.206	835.115		74.289
N11-K4	0.2	764.398	634.551	928.583	697.393	661.286	769.559	764.398	47.192
	0.5				686.376	619.344	772.523		62.047
	1				787.943	753.200	851.699		38.252
N15-K5	0.2	822.977	711.074	1021.503	798.823	768.472	847.360	822.977	35.933
	0.5				843.448	795.682	910.493		45.395
	1				809.234	756.656	854.505		36.100
N17-K7	0.2	853.419	772.133	1158.459	893.802	851.809	1003.707	94.894	47.707
	0.5				882.978	854.517	917.560		26.373
	1				890.692	851.809	1003.707		63.824

Continue of Tab. 4. Comparison of mean and standard deviation of objective function values under realizations

Problem Size	Uncertainty level (Γ)	Mean of objective function values under realizations						Standard deviation of objective function values under realizations	
		deterministic	Min of deterministic objective function values under realizations	Max of deterministic objective function values under realizations	robust	min of robust objective function values under realizations	max of robust objective function values under realizations	deterministic	robust
N20-K8	0.2	926.496	830.703	1032.026	871.189	834.780	921.937	61.250	35.774
	0.5				993.177	948.910	1014.756		26.069
	1				938.133	913.252	950.506		16.018
N23-K8	0.2	963.565	891.557	1081.159	986.135	950.643	1001.585	61.339	20.889
	0.5				953.708	926.754	975.825		21.079
	1				978.948	972.801	990.113		7.102
N27-K9	0.2	1037.464	940.020	1219.858	1012.247	948.891	1088.724	73.567	43.679
	0.5				1080.392	1064.411	1095.264		11.981
	1				1077.992	1042.904	1122.961		33.338
N31-K10	0.2	1108.372	1048.789	1207.398	1117.050	1071.638	1147.409	40.365	30.691
	0.5				1153.060	1124.996	1183.956		25.802
	1				1157.753	1110.750	1200.586		37.320
N41-K14	0.2	1334.516	1279.180	1407.378	1305.577	1275.593	1348.885	36.759	28.921
	0.5				1324.974	1292.950	1353.450		28.555
	1				1381.754	1361.871	1412.994		21.871

It is worth mentioning that an interval plot elaborates on the confidence interval for the mean of the data. Also, it shows the information on whether the different uncertainty levels have similar mean values and compares the amount of variation presented in each uncertainty level. The following conclusions can be drawn from the interval plot, which is given for each Figure.

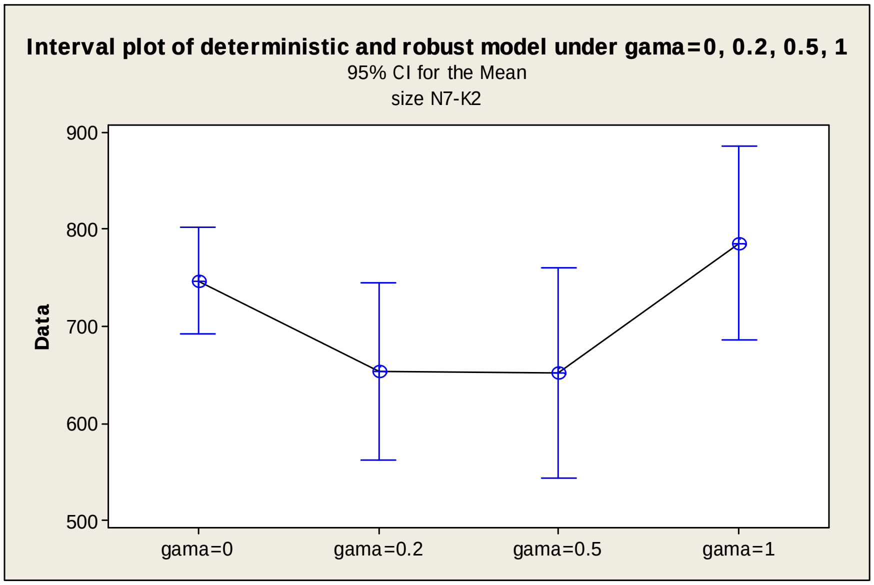

Fig. 2. Interval plot under different uncertainty levels for the problem with size N7-K2

Figure 2 indicates that the amount of objective function for Γ=0 is higher compared to Γ=0.2, 0.5. For Γ=1, although the amount of variation is higher than Γ=0, the standard deviation is significantly lower compared to the Γ=0 and Γ=0.5. Since there is an overlap of the confidence interval for the Γ=0.2 and Γ=0.5, it can be concluded that the performance of them may be similar. In Figure 12, the standard deviation for Γ=0.2 is lower than Γ=0.5. It means the performance of robust approaches for Γ=0.2 is better than other uncertainty levels and also better than the deterministic model.

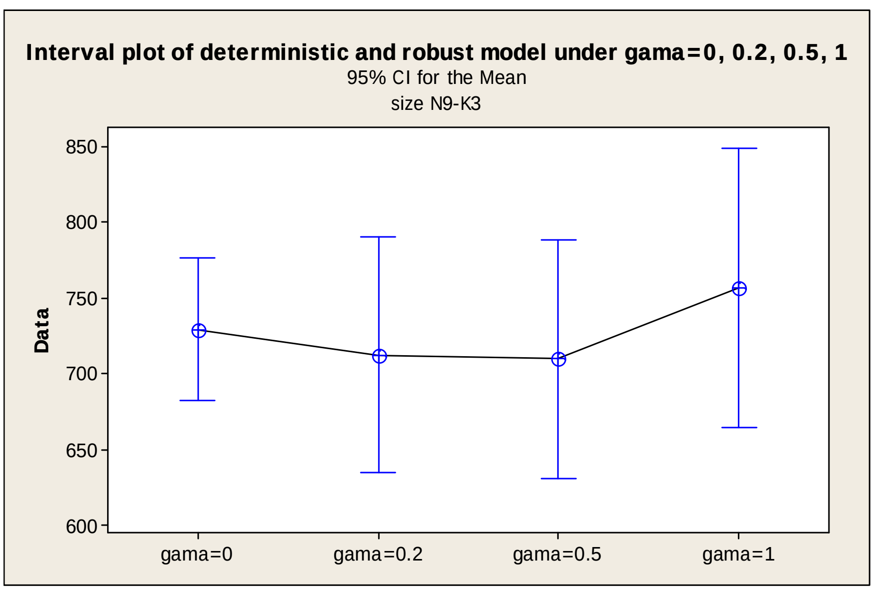

Fig. 3. Interval plot under different uncertainty levels for the problem with size N9-K3

Since there is overlap of the confidence interval for the Γ=0 and other uncertainty levels in Figure 3, it can be concluded that the performance of them may be similar. The amount of the standard deviation based on Figure 12 for Γ=0 is significantly higher than other uncertainty levels.

For Γ=0.2, Γ=0.5 the standard deviation is similar approximately. The performance of robust approach for Γ=0.2, Γ=0.5 is higher than Γ=0 and Γ=1 in this size of the problem. Regarding Figure 12, the highest performance belongs to Γ=0.2.

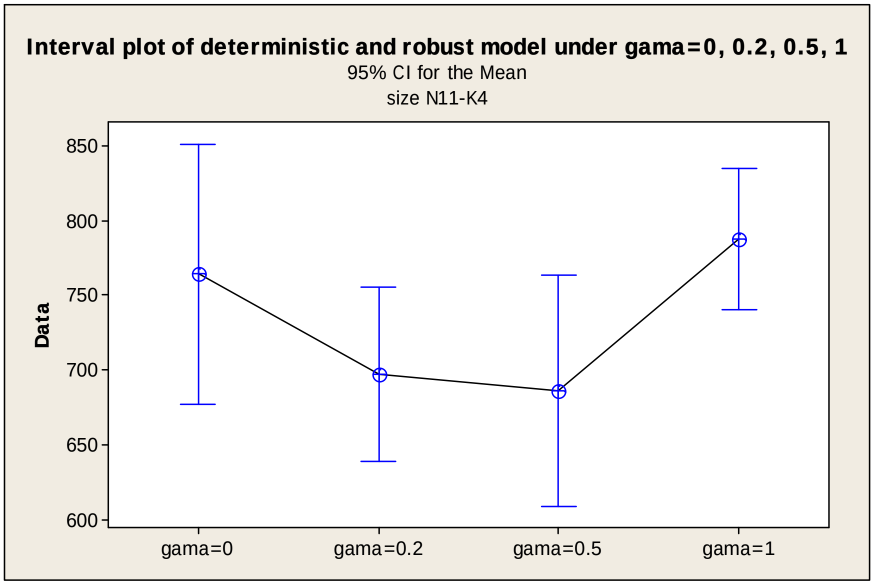

Fig. 4. Interval plot under different uncertainty levels for the problem with size N11-K4

In Figure 4 and Figure 12, the amount of variation and standard deviation for Γ=0 is higher than other uncertainty levels. Since there is an overlap of the confidence interval for the Γ=0 and Γ=1, also for Γ=0.2 and Γ=0.5, as it can be seen, the performance of them may be similar. The amount of objective function for Γ=0.2, Γ=0.5 is lower compared to other uncertainty levels. In Figure 12, it can be concluded that the performance for Γ=0.2 is higher compared to the deterministic model and different uncertainty levels.

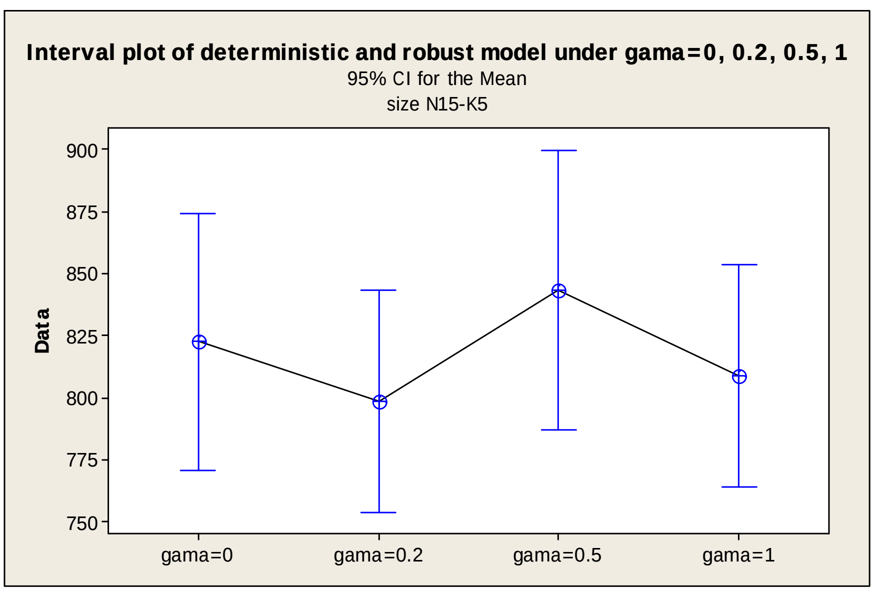

Fig. 5. Interval plot under different uncertainty levels for the problem with size N15-K5

Figure 5 indicates that the amount of objective function for Γ=0.5 is higher compared to other uncertainty levels. Also, this amount is lowest for Γ=0.2 among other uncertainty levels. Although the amount of variation for Γ=0 is lower compared to Γ=0.5, its standard deviation is significantly higher compared to Γ=0.5 and other levels. Figure 12, which presents the lowest standard deviation for Γ=0.2, it can be concluded that it is doing better than others.

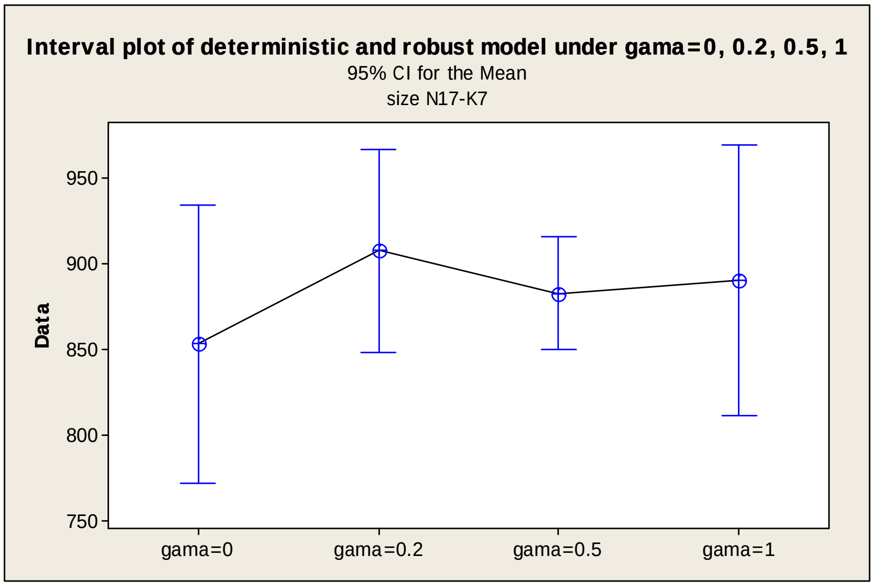

Fig. 6. Interval plot under different uncertainty levels for the problem with size N17-K7

In Figure 6, since there is an overlap of the confidence interval for Γ=0.2, Γ=0.5, Γ=1, it can be concluded that the performance of them may be similar. Although the amount of objective function for Γ=0 in deterministic model is lower compared to other uncertainty levels, the standard deviation is significantly higher than all levels in robust approach. The amount of variation for Γ=0.5 is lower than others though the mean of the objective function is higher than Γ=0. The results show that the highest performance belongs to Γ=0.5.

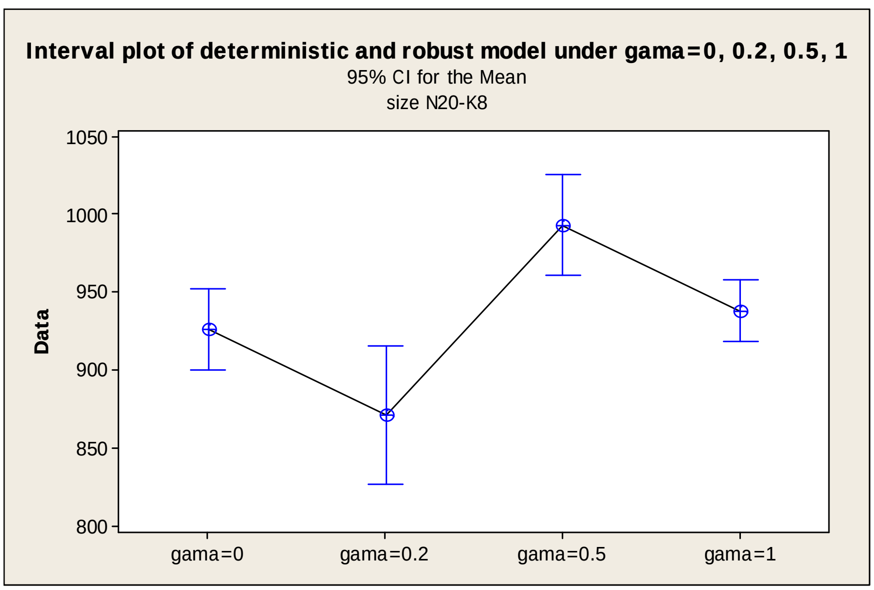

Fig. 7. Interval plot under different uncertainty levels for the problem with size N20-K8

Figure 7 indicates that the amount of objective function for Γ=0.2 is lowest compared to other uncertainty levels. Also, the amount of the variation for Γ=0.2 seems to be highest compared to others. Regarding Figure 12, it can be concluded that the minimum standard deviation belongs to Γ=1. Although the minimum amount of objective function is for Γ=0.2, the best performance belongs to Γ=1, because of the minimum variation and the minimum standard deviation.

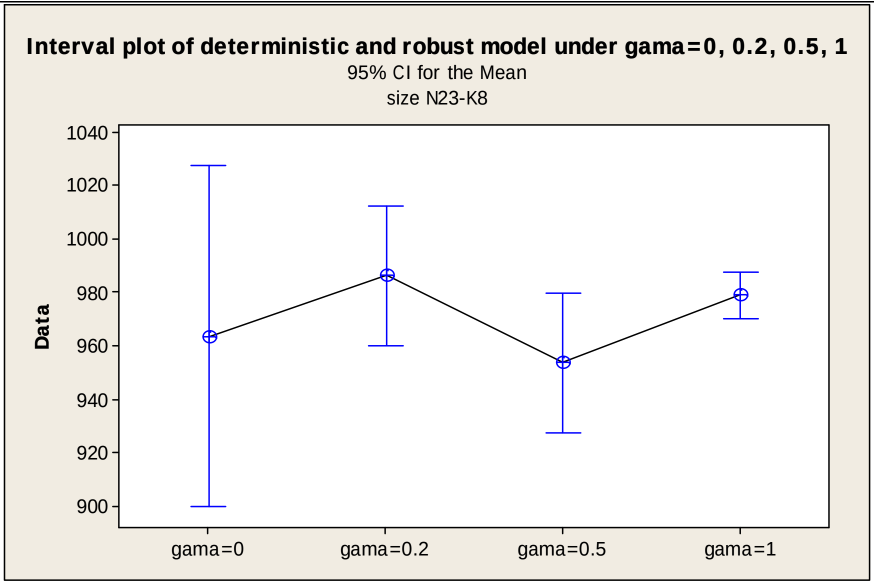

Fig. 8. Interval plot under different uncertainty levels for the problem with size N23-K8

In Figure 8, since there is an overlap of the confidence interval for Γ=0, Γ=0.2, Γ=1, it can be concluded that the performance of them may be similar. The amount of variation and the standard deviation for Γ=0 is the highest. The amount of objective function for Γ=0.5 is lower than other uncertainty levels. The standard deviation for both Γ=0.2 and Γ=0.5 are approximately similar. It can be stated that the robust model has a better performance in Γ=0.5.

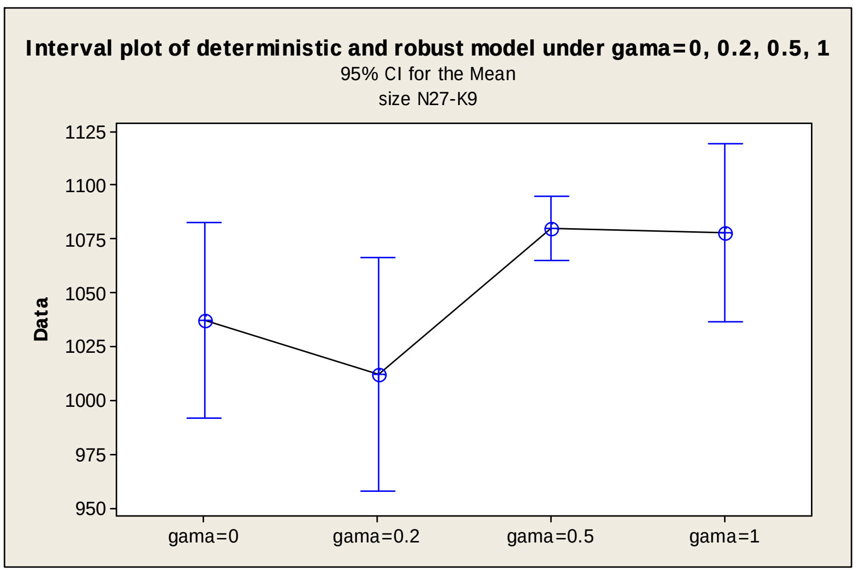

Fig. 9. Interval plot under different uncertainty levels for the problem with size N27-K9

In Figure 9, since there is an overlap of the confidence interval for Γ=0.5 and Γ=1, it can be concluded that the performance of them may be similar. The amount of variation for Γ=0.2 is higher than other uncertainty levels. The standard deviation for Γ=0 is highest than others. Although the amount of objective function for Γ=0.5 is higher compared to Γ=0.2, both its standard deviation and its variation are lowest among all uncertainty levels. Regarding the results, the robust approach has a better performance in Γ=0.5.

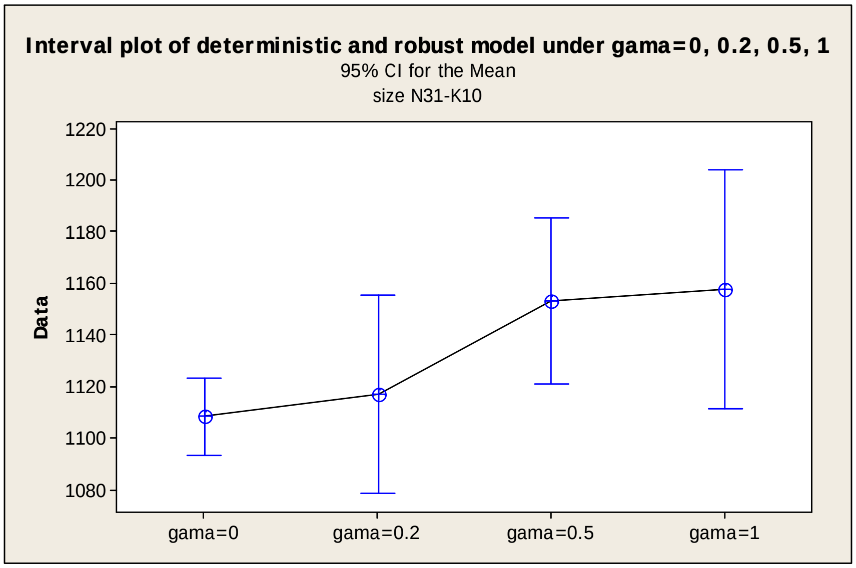

Fig. 10. Interval plot under different uncertainty levels for the problem with size N31-K10

Although the variation for Γ=0 is lower compared to other uncertainty levels, its standard deviation is significantly higher compared to the other levels. Since the variation and standard deviation for Γ=0.5 are lower compared to Γ=0.2 and Γ=1, it has higher performance in the robust model.

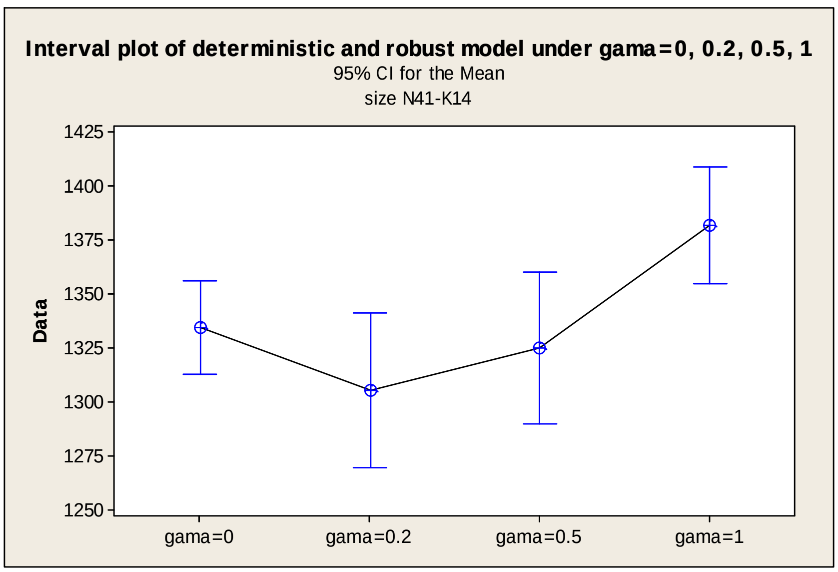

Fig. 11. Interval plot under different uncertainty levels for the problem with size N41-K4

Figure 11 indicates that the amount of objective function for Γ=0.2 is lower compared to other uncertainty levels. Although the amount of objective function for Γ=1 is higher compared to the other uncertainty levels, its standard deviation is lowest. The amount of the objective function for Γ=0.2 is lower compared to others. Also, the standard deviation for all uncertainty levels in robust model is lower compared to the deterministic model. Therefore, for Γ=0.2, the robust approach is doing better than others.

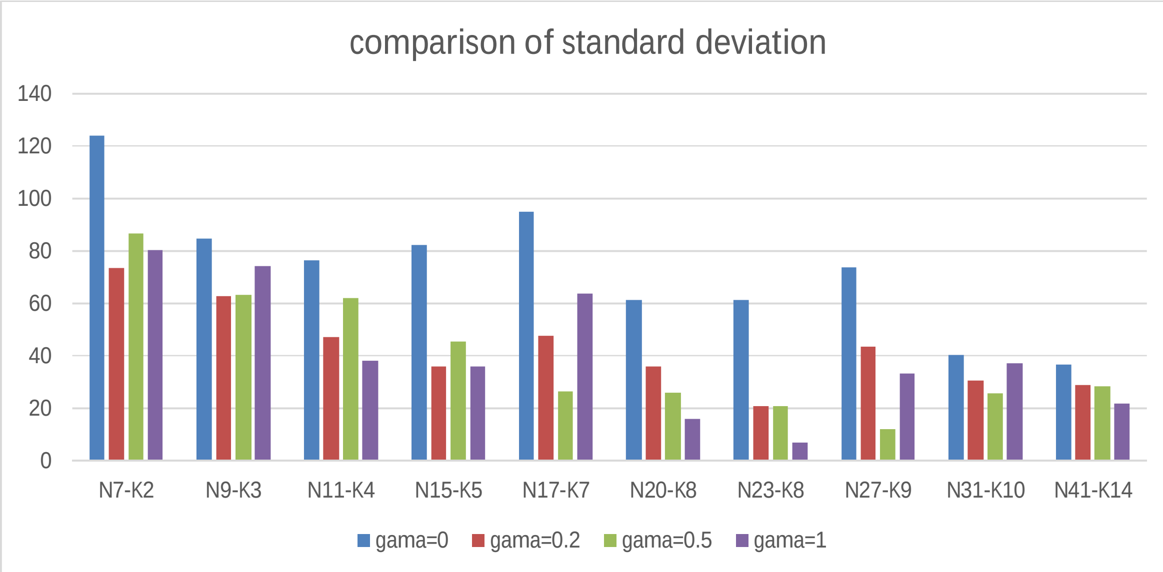

Fig. 12. Comparison of standard deviation for all sizes of numerical experiments under different uncertainty levels

The obtained results in Table 4, Figures 2 to 11, and Figure 12 show that the robust strategy performs better. In other words, the robust model gained the objective function with both higher quality and lower standard deviation compared to the deterministic model.

OVRP, in the real world concerning uncertain parameters, examines how to plan problems in the view of management and decision-making. This approach can propose the managerial implication for management in the transportation section and productive factories who need to manage the cost and time for providing better servicing. Sometimes transportation companies will not be able to pay for vehicles maintenance, so they prefer to apply OVRP concerning the non-deterministic parameters, so this approach of OVRP with uncertain parameters causes a better estimation of the problem in the real world because this problem plays a vital role in decreasing the cost and time.

5. Conclusions and Future Studies

The unpredictability of demand and cost parameters in today's competitive world is challenging for manufacturers or service providers. Factors such as fluctuations of market price and economic downturns have made these parameters uncertain. For this reason, it is essential to provide solutions to adjust the uncertain parameters.

In this paper, the demand and cost of using vehicles in OVRP were considered uncertain parameters. In order to the problem be closer to the real world, the transportation fleet is considered heterogeneous. It means the capacity of the vehicles is various. In dealing with uncertainty, the proposed approach by Bertsimas and Sims was applied, which is a powerful approach in optimization to deal with uncertain parameters. The reasons for the superiority of the Bertsimas model compared to other robust optimization approaches include: the robust counterpart of the linear programming problem itself is linear programming. The decision-maker determines the level of conservation (Γ), and unlike other robust optimization models, it is not necessarily the worst case for the parameters considered non-deterministic. The number of variables and constraints in this model are less than other sustainable models, so the complexity of this model will be less. After presenting the model with the expressed approach, GAMS software utilized to solve the model. Then, ten numerical examples with different sizes were considered. Each size of the deterministic and robust model has been investigated under three different uncertainty levels. The values of objective function and solution time of each size of the problem were presented. Then five random realizations were generated to evaluate the performance of the proposed model, and the deterministic and robust model compared. Numerical results showed that the robust model concerning the mean and standard deviation of the objective function performs better.

For future research, considering other uncertain parameters such as travel time can make the problem more applicable. This approach can also be applied for a wide range of vehicle routing problems with uncertain data, such as OVRP with time windows. On the other hand, this study can be enriched with more than one depot.

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