RESEARCH PAPER

An Optimization Model Based On Decision Support Tools for the Inventory-Routing Rescheduling Problem

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ABSTRACT
The present work addresses Inventory-Routing Rescheduling Problem (IRRP) in case of possible changes in the execution time of pre-planned scheduling of an Inventory-Routing Problem (IRP). Due to the complexity of the process of departing from one pre-planned scheduling IRP to a rescheduling IRP, a decision-support tool was devised in this study to help decision-makers. This complexity derives from the changes occurring in an agreed-upon schedule including the used capacity of the vehicle, total distance, and other factors that need a re-agreement negotiation, which directly relates to the agreed-upon costs, especially when a carrier contractor is responsible for the distribution of goods between customers. Although one may want to stick to the pre-planned scheduling, the changes in predicted data of the problem at the time of execution require a new optimized solution. The proposed approach applies mathematical modeling to the optimization of the rescheduled problem and conducts sensitivity analysis to study the effect of the adjustment of different variables (carried load, distance, etc.).

1. Introduction

To find a solution to the classical Inventory-Routing Problem (IRP), it is assumed that all parameters of the problem including available vehicle capacity, customer's locations, demand, etc. are deterministic and known. In practice, a planner or Decision-Maker (DM) may come to recognize some parameters sooner and realize the significance of some other parameters at some time in the future. Given that the final parameters of a problem are revealed, it is reasonable to have pre-planned scheduling based on known data and the predicted data for unrevealed parameters to avoid high costs of future planning. It is sometimes mandatory to do so, especially when the carrier contractor is responsible for distributing goods between customers. One needs to know the planned schedule and have enough time before going through any price quotation and pre-agreement. A common solution to this problem is to determine a long-term schedule, henceforward Master Schedule (MS), which serves as a long-term guiding schedule over a certain period of time in which multiple deliveries are made [1]. MS works as a timely plan for setting any pre-agreement between the carrier contractor and the distribution manager (decision maker). In the course of revealing the real data of the problem, the MS may not be feasible for DM; therefore, to keep it feasible, the problem needs to be resolved. If a new solution does not match MS and it considerably deviates from it, the pre-agreement should be renegotiated. Although the carrier contractor wants to stick to MS and any deviation from it is unfavorable, the changes in the predicted data of the problem at the time of execution need a new optimized solution. A new agreement based on a new schedule includes extra cost for carrier contractor, which should be paid by DM. It is evident that while lower deviation leads to less carrier contractor charges, greater deviation incurs more charging costs for DM. In negotiation, DM goes through a complex

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decision-making process to keep a balance between the rescheduling cost and MS cost. Herein, decision support tools are required to facilitate this process.

The introduction of IRP dates back to the referenced paper [2] in which only transportation costs were included, demand was stochastic, and customer inventory levels must be met. Henceforth, a number of studies have investigated different variants of the problem. For more details and reviewing related published studies, the interested readers refer to Andersson, Hoff [3] and [4]. To the best of our knowledge, the literature on rescheduling strategies that take into account the deviation costs in IRP is limited in scope. However, on the vehicle routing problem, some authors studied the deviation costs to a greater extent. [1] studied the Vehicle Rescheduling Problem (VRSP) in order to find a new schedule that minimized not only the total traveling costs but also the costs of deviating from the original schedule. Therefore, a mathematical programming formulation of the rescheduling problem was presented and a heuristic solution method was referred to as a two-phase heuristic. Usually, rescheduling is mainly considered in conjunction with designing a master schedule. As a rescheduling method, the master schedule is designed before knowing the demand such that the expected costs incurred

after rescheduling are minimized. Bertsimas [5] used a master schedule until it returns to the depo. Afterwards, it resumes its journey from the last visited customers. Groër, Golden [6] considered the rescheduling in such a way that some companies would want their drivers to develop relationships with customers on a route and have the same drivers visit the same customers at roughly the same time each day that they need service.

The problem under study is very close to IRP when the demand is stochastic or dynamic. The majority of researchers have considered the effect of these uncertainties only on demand [3], [7]; nevertheless, in some rare cases, uncertainty of the other parameters of IRP such as the traveling time [8] and purchase cost [9] has been considered. To clarify this uncertainty in IRP even more, it is worth noticing that IRP is classified as either deterministic or stochastic due mostly to the time of knowing the demand information. In the deterministic IRP, demand is fully available for the decision-maker at the beginning of the planning horizon; however, in stochastic IRP, he knows its probability distribution. The Dynamic IRP (DIRP) is a very close extension of the Stochastic IRP (SIRP), in which the demand is not fully known in advance and is gradually revealed over time [7].
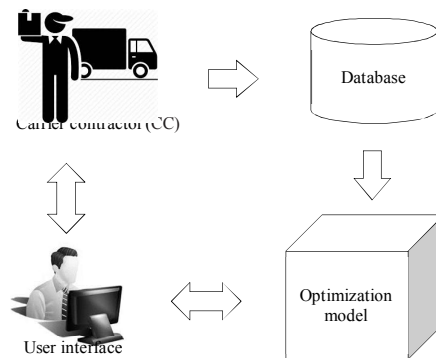
Fig. 1. Proposed decision support system structure for IRRP

This study investigates an IRRP problem with a single-item, single-vehicle, multi-period, one-to-many two-echelon IRP. A mathematical model of the rescheduling problem was also introduced to penalize the deviation from the MS.

In addition, a Decision Support Tool (DST) was proposed that offered an appropriate adjustment to penalty coefficients through mathematical modeling that would assist DM in negotiation with a carrier contractor to go through a new agreement. To this end, this study considered a

master schedule for IRP with the customers' mean predicted demand, which was known in advance, and proposed a scheduling mathematical modeling for IRP when the real demand was revealed prior to the execution of goods distribution.

The objective function of the proposed model was to minimize all costs included in the IRP plus the cost of deviation from the master plan. Furthermore, this study considered the cost of delivery as a step cost function that seemed more

practical while considering rescheduling. Since the rescheduling problem included some price negotiations based on the master schedule, especially when the delivery was outsourced to a carrier contractor, devising a decision support tool to see how IRRP conformed to IRP master schedule would be beneficial for decision-makers.

The sections of the present research are as follows. In Section 2, the relevant research conducted is reviewed. In Section 0, a decision support tool including a mathematical formulation as Mixed Integer Programming for IRRP is proposed. In Section 2, numerical experiments are provided. Finally, Section 5 concludes the study.

2. Solution Approach

In this section, a proposed solution approach to IRRP is introduced and discussed. The architecture of the proposed DSS approach is shown in Fig. **1**. The system includes a Database that provides all necessary data of customers' demands and locations, warehouses and available vehicle capacity of carrier contractor, all cost-related parameters, etc. While the data of the database are partly static, the rest are dynamic, making it more relevant to the data collected from CC. The optimization model comprises a mathematical programming model for IRRP that receives its data from both database and user and offers him/her the optimized solution. The user interface is the final part of the system that directly supports the process of the user's decision-making.

2.1. Mathematical programming model for IRRP

In the following, the concepts applied to the problem modeling are elaborated.

Sets & indexes	Description
O	Vendor's node; $O = \{0\}$
V'	Set of nodes including customers; $V' = \{1, \dots, n\}$
V	Set of nodes including vendors and customers; $V = V' \cup O = \{0, 1, \dots, n\}$
T	Planning horizon; $T = \{1, \dots, p\}$
t,s	Index of each time period; $t, s \in T$
i, j	Index of each node; $i, j \in V$
A	Set of arcs; $A = \{(i, j) : ij \in V, i \neq j\}$
S	Set of steps in the cost rate step function, s = 1,..., NS, where s denotes the corresponding indices.
K	Set of steps in the cost rate step function, s = 1,..., NK, where k denotes the corresponding indices.

Parameters	Description
C_i	Maximum capacity of customer i
e_{ij}	Euclidean distance from vertex i to j, $(i, j) \in A$
b_{ij}	Unit cost of traveling from vertex i to j, $(i, j) \in A$
n	Numbers of customers
p	Number of periods on the planning horizon
d_i^t	Demand rate of customer i in period t
Q	Vehicle capacity
h_0	Unit inventory holding costs of vendor
h_i	Unit inventory holding costs of customer i
π^t	Total load carried by vehicle in period $t \in T$ in master schedule
γ_k	Load variation breakpoint $k \in K$
M_1, M_2	Large numbers
I_i^0	Inventory level at the vertex $i \in V$ at the beginning of the planning horizon
f_{ij}^t	Unit cost of rout deviation corresponding to vertex i to j
m_k	Unit cost of load deviation in step k
Z_{ij}^t	Equal to 1 if customer j immediately follows customer i on the route of the supplier's vehicle in period t in master schedule; otherwise, equal to 0

Variables: Description

$X_{ij}^t =$ { 1 if customer j immediately follows customer i on the route of the supplier's vehicle in period t

0 otherwise

$Y_{ij}^t =$ { 1 if customer j immediately follows customer i on the route of the supplier's vehicle in period t while in MS, customer j immediately does not follow customer i in the same period

0 otherwise

N_k^t { 1 if the load of vehicle in period $t \in T$ is in step $k \in K$

0 otherwise

I_i^t inventory level at the vertex $i \in V$ at the end of period $t \in T$

q_i^t the quantity of products delivered from the vendor to customer i in time period t using his vehicle

g_i^t continuous variables to enforce sub-tour elimination

ψ^t Total load carried by vehicle in period $t \in T$

Min IRRP =

$$\sum_{t\in T} h_0 I_0^t + \sum_{i\in V'}\sum_{t\in T} h_i I_i^t + \sum_{i\in V}\sum_{j\in V}\sum_{t\in T} b_{ij} e_{ij} X_{ij}^t + \sum_{i\in V}\sum_{t\in T} f_{ij}^t Y_{ij}^t + \sum_{t\in T}\sum_{k\in K} m_k N_k^t$$

s.t.

$$I_0^t = I_0^{t-1} + R^t - \sum_{i\in V'} q_i^t, \quad t \in T \tag{1}$$

$$I_0^t \geq 0, \quad t \in T \tag{2}$$

$$I_i^t = I_i^{t-1} + q_i^t - d_i^t, \quad i \in V', t \in T \tag{3}$$

$$I_i^t \geq 0, \quad i \in V', t \in T \tag{4}$$

$$I_i^t \leq C_i, \quad i \in V', t \in T \tag{5}$$

$$q_i^t \leq C_i - I_i^{t-1}, \quad i \in V', t \in T \tag{6}$$

$$q_i^t \leq C_i \sum_{j\in V} X_{ij}^t, \quad i \in V', t \in T \tag{7}$$

$$\sum_{i\in V'} q_i^t \leq Q, \quad t \in T \tag{8}$$

$$\sum_{i\in V} X_{ij}^t = \sum_{i\in V} X_{ji}^t, \quad j \in V, t \in T \tag{9}$$

$$\sum_{i\in V} X_{i0}^t \leq 1, \quad t \in T \tag{10}$$

$$g_i^t - g_j^t + Q X_{ij}^t \leq Q - q_j^t, \quad i \in V', j \in V', t \in T \tag{11}$$

$$q_i^t \leq g_i^t \leq Q, \quad i \in V', t \in T \tag{12}$$

$$X_{ij}^t - Z_{ij}^t \leq Y_{ij}^t, \quad i,j \in V, t \in T \tag{13}$$

$$X_{ij}^t - Z_{ij}^t \geq -Y_{ij}^t, \quad i,j \in V, t \in T \tag{14}$$

$$\psi^t = \sum_{i\in V'} q_i^t, \quad t \in T \tag{15}$$

$$\sum_{k\in K} N_k^t = 1, \quad t \in T \tag{16}$$

$$\gamma_k N_k^t \leq \frac{\psi^t - \pi^t}{\pi^t} \qquad , t \in T, k \in K \tag{17}$$

$$\frac{\psi^t - \pi^t}{\pi^t} < \gamma_{k+1} N_k^t + M_1(1 - N_k^t) \quad , t \in T, k \in K \tag{18}$$

$$q_i^t \in \mathbb{Z}^*, \qquad i \in V', t \in T \tag{19}$$

$$g_i^t \geq 0, \qquad i \in V', t \in T \tag{20}$$

$$X_{ij}^t \in \{0,1\}, \quad i,j \in V, i \neq j, t \in T \tag{21}$$

$$Y_i^t \in \{0,1\}, \qquad i \in V, t \in T \tag{22}$$

$$N_k^t \in \{0,1\}, \qquad k \in K, t \in T \tag{23}$$
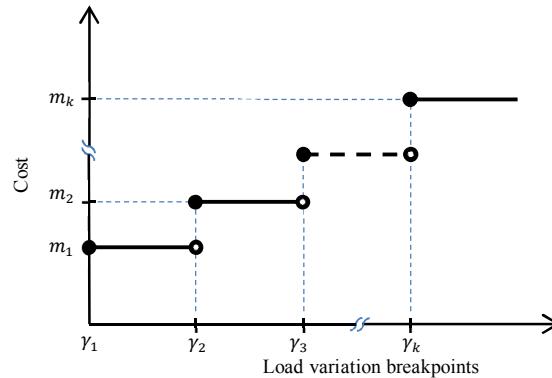

Fig. 2. Cost rate vs. load variation breakpoints

The objective function 0 comprises five parts: (i) inventory holding cost of supplier, (ii) inventory holding cost of customers (iii) routing cost of the supplier's vehicle, (iv) route deviation cost, and (v) load deviation cost. Constraints 0-0 are pertinent to inventory decisions. To be more specific, Constraints 0 calculate the inventory level for the vendor at the end of period $t \in T$. Constraints 0 ensure no shortage of inventory for the vendor at the end of each period. Constraints 0 describe the inventory quantities for each customer at the end of period $t \in T$. Constraints 0 and 0 represent the capacity limitations of the customer warehouse, i.e., the first set of constraints is related to the minimum inventory level, and the second set of constraints is related to the maximum inventory level. Constraints 0 and (8) represent the quantity of vehicles delivered by the vendor's based on the ML policy. Constraint 0 guarantees that the vehicle capacity is respected. Constraints 0 and 0 correspond to the routing of the vendor's vehicles. Constraints 0 and 0 correspond to sub-tour elimination. Constraints 0 and 0 correspond to the rout deviation from MS. Constraints (16) to 0 indicate the load deviation from the MS. Constraints 0 to 0 ensure the integrality and non-

negativity of decision variables. In short, $Z^* = \{0\} \cup Z^+$.

Assuming that cost of load variation is based on a step cost function, Fig. **2** shows a generic schematic of the rate of price breakpoints vs. load variation breakpoints.

2.1. Decision support process

is a flowchart of the decision support process based on IRRP. At the first step, based on the static data extracted from the database and demand prediction, the mathematical programming model IRP is run to determine MS routings and loads. At this step, following the application of the routings and loads derived from the last step, the user begins to negotiate with the carrier contractor and makes a confirmed pre-plan prior to the appearance of the real demand data. After revealing the real demand data and based on the given MS data and results, the mathematical programming model corresponding to IRRP is run to determine the new routing and load amounts (rescheduled plan). At this step, DM checks to make sure whether or not the problem with real demand arises. If it does not match MS, then DM does a what-if analysis by running the mathematical programming at different levels of deviation

penalties in order to offer some acceptable solutions. Based on these solutions and comparison of the costs of applying each solution, DMs can decide on the amount of money that should be paid to match the rescheduling with MS. This can provide DM with enough information to negotiate with CC and execute the rescheduled problem.
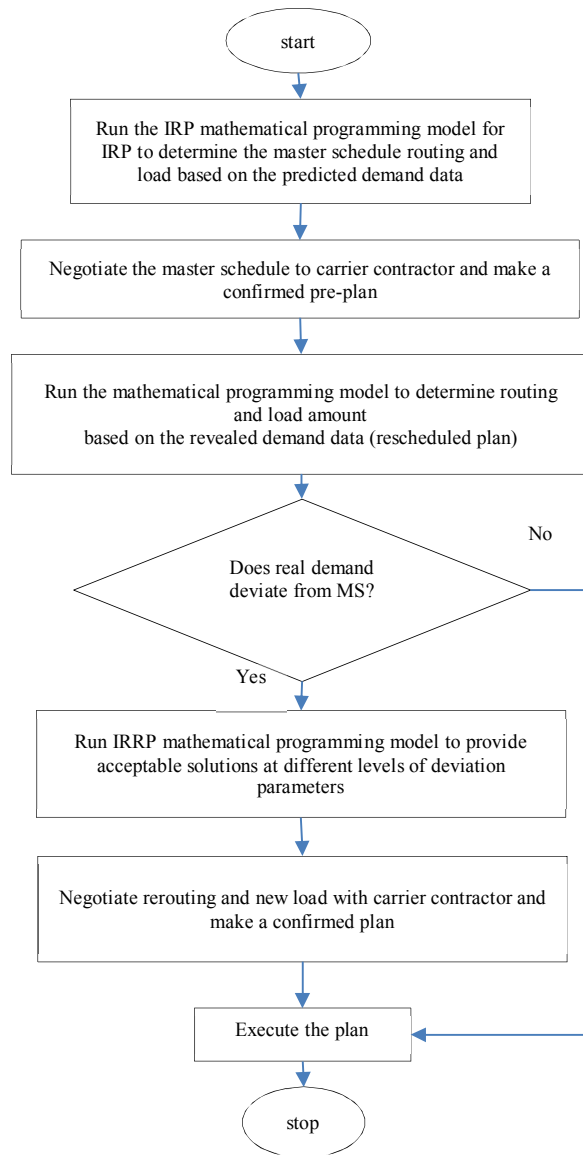
Fig. 3. The process of decision support system for cost change analysis rescheduling

3. Numerical Experiments

This section presents a number of experiments to evaluate the results of the proposed system and its application. The solution approach with GAMS-24.7.3 was implemented and run on a PC, Intel Core i3, CPU 3.70 GHz, Windows 10-64bit, and 8 GB RAM.

Tab. 1. Presumed and actual demand for Scenario 1

Customer #	Presumed demand per period t			Actual demand per period t		
	t=1	t=2	t=3	t=1	t=2	t=3
1	59	59	59	0	130	20
2	35	35	35	35	70	35
3	61	61	61	0	61	0
4	26	26	26	26	26	26
5	10	10	10	0	0	10

Tab. 2. Parameters of sample numerical example

Nodes	Locations		Maximum warehouse capacity	Inventory holding cost
	x_i	y_j	C_i	h_i
depo	154	417	-	0.03
1	172	334	195	0.02
2	267	87	105	0.03
3	148	433	116	0.03
4	355	444	72	0.02
5	38	152	22	0.02

The example benchmark here is based on the following parameters: planning horizon p=3, the number of customers n=5, vehicle capacity of Q=289, the total number of load variation breakpoints k=7, load variation breakpoint [$-100\%; -20\%; 0; 10\%; 25\%; 50\%; 100\%$], the unit cost of traveling from the vertex i to j, $b_{ij} = 1$, Euclidean distance from the vertex i to j,

$$e_{ij} = \left\lfloor \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \right\rfloor,$$ in which sign

$\lfloor \rfloor$ means that the largest integer is lower than or equal to the value inside. The customer demands are presumed, as shown in Table 3, with two columns. The first column is allocated to the predicted or presumed demand per period which is used for MS and actual demands per period t,

applied to IRRP. As observed, while the presumed demand follows a steady trend and does not change during the planning horizon, the actual demand follows different trends. Tab. 2 presents other parameters of the sample numerical example.

While running the IRP optimization model, the master schedule routing cost, inventory cost, and objective function (total cost), i.e., the summation of the routing cost and inventory cost, are 2125.0, 5.05, and 2130.05, respectively. Of note, if the deviation parameters including f_i^t, m_k are considered zero, the IRRP optimization model will be transferred into the IRP. In addition, the application of this model leads to MS.

Tab. 3. Costs and rescheduled edges vs. different levels of deviation coefficient

Deviation coefficient (f)	Total cost		Routing cost		Inventory cost		Deviation cost	Number of rescheduled edges
	Rescheduled	Deviation from MS (%)	Rescheduled	Deviation from MS (%)	Rescheduled	Deviation from MS (%)		
0	2282.1	7.14	2278	7.20	4.09	-19.01	0	7
20	2456.1	15.31	2312	8.80	4.09	-19.01	140	7
50	2555.1	19.95	2401	12.99	4.09	-19.01	150	3
75	2630.1	23.48	2401	12.99	4.09	-19.01	225	3
100	2705.1	27.00	2401	12.99	4.09	-19.01	300	3
150	2855.1	34.04	2401	12.99	4.09	-19.01	450	3
300	3305.1	55.17	2401	12.99	4.09	-19.01	900	3

3.1. Sensitivity analysis

The proposed system assists DM with rescheduling negotiations based on acceptable solutions made available through mathematical programming at different levels of deviation parameters. This is called sensitivity analysis in literature. To this end, in order to decide on the validity of the model, a number of experiments have been proposed to apply a different feasible solution to the rescheduling problem. Then, the IRRP model runs the following objective function 0, which entails the substitution of 0.

$$\text{Min IRP-R} = \sum_{t \in T} h_0 I_0^t + \sum_{i \in V'} \sum_{t \in T} h_i I_i^t + \sum_{i \in V} \sum_{j \in V} \sum_{t \in T} b_{ij} e_{ij} X_{ij}^t + \sum_{i \in V} \sum_{t \in T} f_i^t Y_i^t + \sum_{t \in T} \sum_{k \in K} p_1 m_k N_k^t \tag{24}$$

To ensure the simplicity of studying the deviation parameters, Deviation Coefficient (DC) values of routing deviation and load deviation were analyzed first. This would be beneficial, especially when the DM prefers to adjust any deviation parameter at any moment during the process of decision-making. Finally, these parameters are carefully investigated.

3.1.1. Analysis of routing deviation (rescheduled edges)

By setting the $m_k = 0 \,\forall\, k \in K$, analyzing the effects of different levels of routing deviation cost $\sum_{i \in V} \sum_{t \in T} f_i^t Y_i^t$ on IRRP is made possible. To ensure the simplicity of analysis, it is assumed that $f_i^t = f$ is fixed in each period $t \in T$ and for every customer $i \in V'$. In Table 3, different values of f inputted by the user can be observed in the first column of the rows. For each value, the IRRP optimization console is run, and the results including the total cost, routing cost, inventory cost, deviation cost, and number of rescheduled edges are reported. In cost-related columns, the deviation values of MS are given in percentage. Of note, the higher level of deviation coefficient implies the user's increasing tendency to remain in sync with MS, which is done so using a mathematical model that penalizes the objective function with a higher level of coefficient related to the re-scheduled edges. Coefficient at zero implies that there is no restriction on user to follow MS and that the solution of IRRP model is free from any constraint that enforces compliance with MS. It can be seen that as deviation coefficient rises, the routing deviation from the number of rescheduled edges decreases.

Fig. **4** indicates the depicted data. The routing and deviation costs are shown on the left vertical axis (1st axis), and the inventory cost and the number of re-scheduled edges are shown on the right vertical axis (2nd axis). Increasing the deviation coefficient makes the model offer a schedule that bears a close resemblance to MS. In this respect, although the routing cost and deviation cost rise, the number of re-scheduled edges and inventory cost decline. Considering the value of 50 for f makes the number of rescheduled edges decline from 7 to 3. Further growth of f would no longer be effective in reducing the number of rescheduled edges.
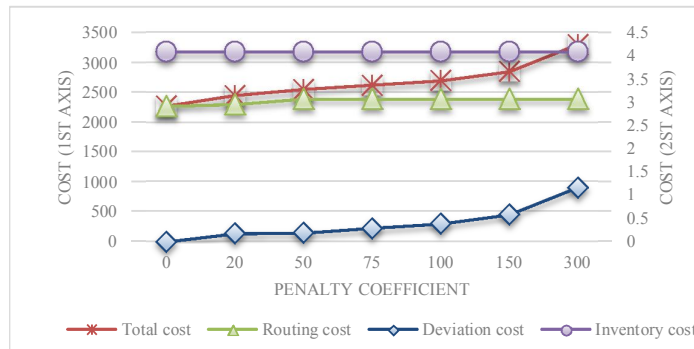

Fig. 4. graphical view of the data in Table 3

The graphical view of the effects of the different f levels on routing, inventory, and deviation costs as well as the summation of all figures representing the total cost is given in Fig. 5. The vertical and horizontal axes show the locations of each customer i numbered from 1 to 5 and those of depo, i.e., the source of distribution of goods, respectively. To precisely analyze this effect on the routing schedule, the routing is depicted in all periods of horizontal planning. In part (a), the

routing schedule for Ms and, in parts (b) and (c), routing schedule for some level of f are presented. A comparison between the graphical view of routing schedule in parts (b) and (c) and that of part (a) shows that as f increases, routing will remain more sync with the MS; in part (c), any increase in f does not make the reschedule match MS routing.
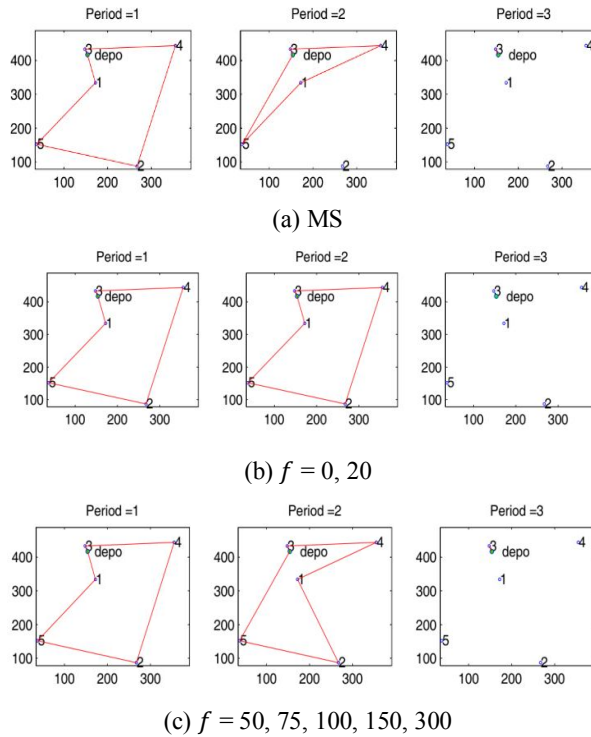

(a) MS

(b) $f = 0, 20$

(c) $f = 50, 75, 100, 150, 300$

Fig. 5. Routing schedule for MS and different levels of PC

3.1.1. Analysis of deviation in load values

Having set the $f_i^t = 0 \; \forall \; i \in V', t \in T$, one can easily analyze the effects of different levels of load deviation cost $\sum_{t \in T} \sum_{k \in K} m_k N_k^t$. It is assumed that the cost of deviation for load is based on the step cost function. However, this assumption in the real world is more practical because considering a linear cost function for deviation in the presence of MS seems unrealistic on two sides of one pre-agreement. Therefore, the data of load variation breakpoints (γ_k) are given by the user, with the technical issues of transportation and CC taken into consideration. To examine the effects of the cost of each breakpoint, the user can add the related data to the model and carry out what-if analysis. Tab. 4 shows the load variation breakpoints at different levels of deviation coefficient Of note, m_k, which corresponds to each load variation breakpoints, is calculated by multiplying the basic cost values by coefficients.

Tab. 4. load variation breakpoints and related deviation coefficients (m_k)

Load variation breakpoints (γ_k)		-100%	-20%	0	10%	25%	50%	100%
Basic cost values		0.5	0.8	0	0.1	0.12	0.3	0.43
coefficients	0	0	0	0	0	0	0	0
	10	5	8	0	1	1.2	3	4.3
	20	10	16	0	2	2.4	6	8.6
	50	25	40	0	5	6	15	21.5
	70	35	56	0	7	8.4	21	30.1
	100	50	80	0	10	12	30	43
	200	100	160	0	20	24	60	86

coefficients	Total cost		Routing cost		Inventory cost		Load deviation cost
	re-scheduled	Deviation from MS (%)	re-scheduled	Deviation from MS (%)	re-scheduled	Deviation from MS (%)	
0	2282.09	7.14	2278	7.20	4.09	-19.01	0
10	2287.31	7.38	2278	7.20	4.31	-14.65	5
20	2292.31	7.62	2278	7.20	4.31	-14.65	10
50	2307.31	8.32	2278	7.20	4.31	-14.65	25
70	2317.31	8.79	2278	7.20	4.31	-14.65	35
100	2318.14	8.83	2312	8.80	6.14	21.58	0
200	2318.14	8.83	2312	8.80	6.14	21.58	0

Tab. 5. Costs including deviation cost vs. different level of deviation coefficient

In the first row of Tab. 4, load variation breakpoints (γ_k) are presented in percentage, as determined by the user. For each breakpoint (γ_k), basic cost values are given by the user, too. For ease of inputting the data, the user is only requested to add a value, which is selected uniformly from [0.00-1.00] cost values at each step. Then, the coefficient is to be placed in the first column by the user and multiplied by the basic cost values, thus achieving m_k values at each step. The IRRP optimization model is run for each m_k.

Tab. 5 shows the effects of the different levels of m_k on routing, inventory, and penalty cost and total cost.

The graphical view of the effects of the different level of DC on routing, inventory, and penalty costs and the total cost is presented in Fig. **6**.
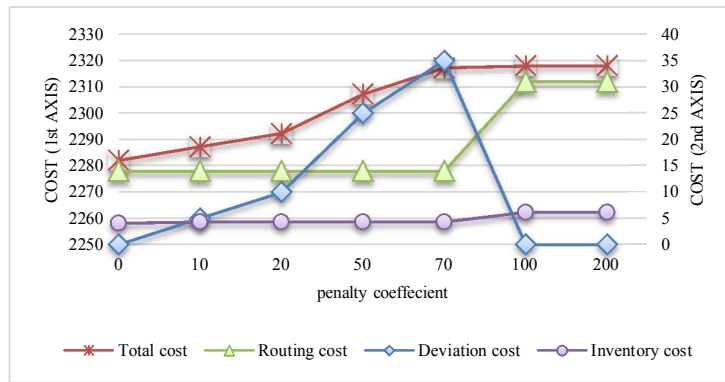

Fig. 6. Trends of costs vs. penalty coeffecient for load change in Case 1

The costs of routing and total costs are given on the left vertical axis (1st axis), while the costs of the deviation and inventory are given on the right vertical axis (2nd axis). It is shown that when DC increases from 0 to 70, no change in routing and inventory costs occurs, whereas this increase can make the load deviation cost rise. In other words, this increase is not enough for the new load plan to match the load offered by MS. When DC surges to almost 100, the routing and inventory costs go up, while the load deviation cost starts to plummet sharply. When the coefficient reaches 100 and more, the the load deviation cost drops to zero, meaning that the master schedule and the rescheduled problem load plan are completely consistent per period and that the increase of coefficient does not affect the problem anymore.

Fig. **7** shows the **Fig. 7**summation of load in different periods (t=1,2,3) that should be carried based on the reschedule program at different load deviation costs. The axis of Periods 2 and 3 is the first axis on the left and Period 3 is the second axis on the right. The solid line is the load that should be carried in different periods (t=1,2,3) based on the master schedule. In Period 1, when the load deviation coefficient is 10, the load amount matches the load amount offered by master schedule. In Periods 2 and 3, the values of the load deviation coefficient would be 20 and 70, respectively.
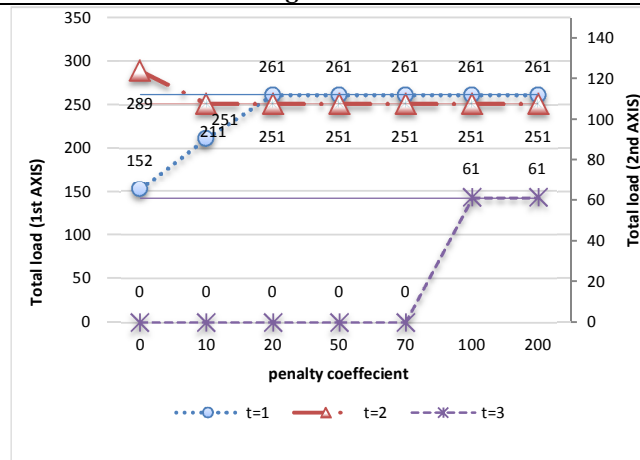
Fig. 7. Summation of loads in different periods

3.1.2. Analysis of combinatorial deviation

In the previous sections, the effects of different levels of deviation coefficients on the load and routing variations were separately analyzed. In real cases, these effects are simultaneously produced; therefore, the effects of different combinatorial levels of deviation coefficient are taken into consideration in this research. In this respect, the user attempts to extract the maximum and minimum values of deviation coefficients, which would make the new planning match MS. To this end, a new measure for total deviation, i.e., the summation of both rout and load deviations, is introduced.

$$\text{Total deviation} = \text{rout deviation} + \text{load deviation} \qquad (25)$$

3.1.3. Tagouchi method for analysis of combinatorial deviation

Since the user is willing to know how the simultaneous changes in deviation affect the total cost and total deviation of rescheduling and, also, to tune the best combination of them, the combinatorial deviation is studied using Tagouchi method. According to Tab. 6, the possible ranges of parameters for routing deviation coefficient (routing D.C.) and load deviation coefficient (load D.C.) are observed. Using MINITAB17 and adding a range of parameters from the table to the Tagouchi design of experiments, one can select the plan L9, where the signal-to-noise ratio is selected to be small. The results of applying the method are shown in Fig. 8.

Tab. 6. Factors, Parameter range, and levels

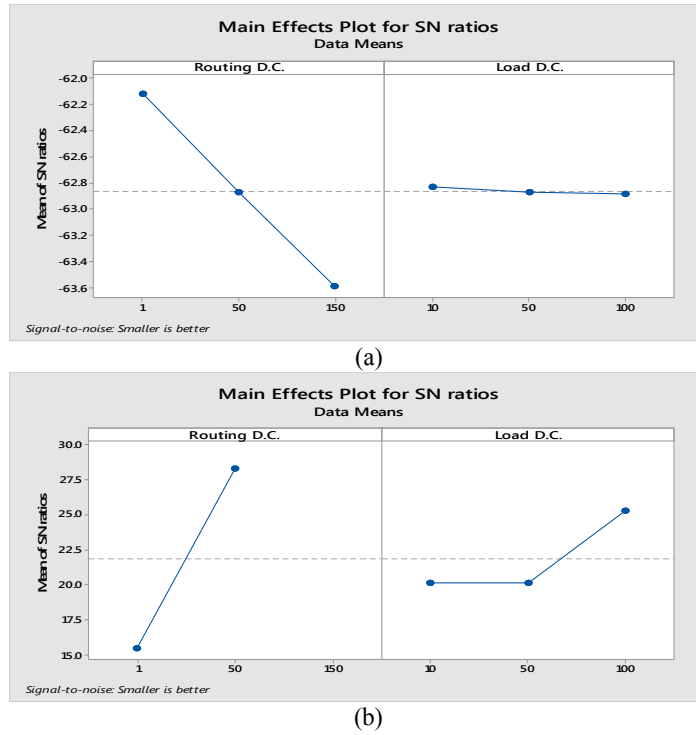| Factor (deviation coefficient)	Parameter range	Level (value)		
		1(low)	2(middle)	3(high)
routing D.C.	0-150	0	50	150
load D.C.	10-200	10	50	100

(a)

(b)

Fig. 8. Main effects plot at SN ratios. (a) response is total cost; (b) response is total deviation
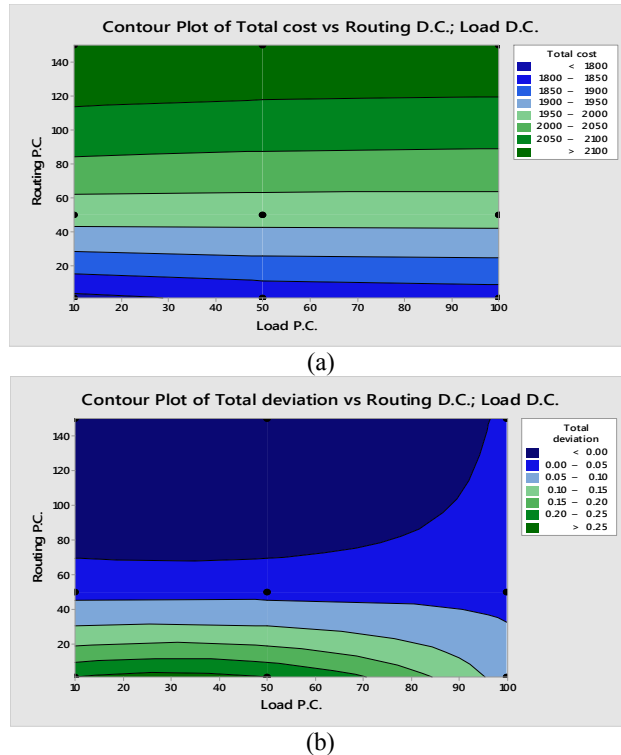

(a)

(b)

Fig. 9. Contour Plots, (a) Total deviation vs Routing P.C. and Load P.C. (b) Total deviation vs Routing P.C. and Load P.C.

Here, in one experiment, the response is selected as "Total deviation" and another time as "Total cost" (Fig. 8). For part (a), the response is chosen to be the "Total cost" and for part (b) the response is chosen to be the "Total deviation". These plots show that lower values of deviation

coefficient result in the lower total cost and higher total deviation level, and vice versa. Load D.C. in comparison to Routing D.C. has lower effects on both Total cost and Total deviation. To be specific, when the total deviation exceeds the value of 50 for both Load PC and Routing PC, it will reach its lowest point. To ensure a better understanding of these effects, the contour plot in Fig. **9** depicts Total cost vs Load D.C. and Routing D.C. in part (a) and Total deviation vs Load D.C. and Routing D.C. in part (b).

4. Conclusion and Future Studies

The present research investigated the Inventory-Routing Rescheduling Problem (IRRP) as a variant of the well-known Inventory-Routing Problem. The aforementioned problem will be solved with the least deviation from the pre-planned scheduling. Based on the available data on customer predicted demand, the decision-maker decides to have pre-planned scheduling for IRP and, then, the real demand appears. Next, rescheduling of IRP is required. As a solution approach, a decision-support tool was devised to help the decision-maker adjust the related deviation parameters. The proposed approach proposed a mathematical modeling to optimize the rescheduled problem and conducted sensitivity analysis in order to study the effects of adjusting different variables (load carrying, distance, etc.). The numerical experiments showed that the sensitivity analysis can assist DM in cost negotiation and determining the cost of a new plan when setting a contract based on the pre-planning. For further study, it is recommended that the future research include heuristic and metaheuristic approaches to provide an optimal or near-optimal solution to large instances.

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