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RESEARCH PAPER

Using Multidimensional Scaling for Assessment Economic Development of Regions

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ABSTRACT
Addressing socio-economic development issues are strategic and most important for any country. Multidimensional statistical analysis methods, including comprehensive index assessment, have been successfully used to address this challenge, but they don't cover all aspects of development, leaving some gap in the development of multidimensional metrics. The purpose of the study is to construct a latent metric space based on the use of multidimensional scaling. Based on statistics showing the economic development of Ukrainian regions, two-dimensional space of latent scales was constructed and Ukrainian's regions were positioned in this space. The results were interpreted meaningfully. This use of multidimensional statistical analysis confirms its usefulness for measuring the economic development of regions and allows their comprehensive assessment and comparison.

KEYWORDS: *Economic development; Multidimensional scaling; Region; Two-dimensional space; Latent scales.*

1. Introduction

1.1. Problem description

The issues of regional development are extremely important for each country, as a lack of sound regional development policy can lead to increase imbalances and exacerbate economic, political, and social problems. In the context of globalization processes, the main goal of national regional policy is to create conditions for dynamic, balanced development of territories, eliminate the asymmetry of region development, activate the involvement of their human capacity to ensure the competitiveness of the region's economy.

The economic development of the regions is characterized by a large variety of indicators, which is why it is multidimensional. This necessitates the use of multidimensional economic-statistical modeling tools, in particular methods of multidimensional scaling (MDS), for assessing the state and level of economic development of regions. The objective of this method is to identify the structure of the set of objects under study (in this case, regions). Identifying the structure means the selection of a set of major factors (parameters) by which objects are separated and a description of each object in terms of these factors is provided. Structuring regions by the level of economic development allows implementing their hierarchy, to establish their differentiation in structural and functional aspects and to study the relevant changes over time, to identify their grouping with similar development trends or factors' reflection, to establish disparities in regional development, which will ultimately contribute to the rational strategic decision-making in the field regional development state policy.

1.2. Literature review

Currently, the toolkit of multidimensional statistical analysis (MSA) to assess the level of regions' economic development is quite widely used. In particular, the papers [1], [2], [3], [4], [5]. [6] considered the technology of cluster analysis to identify gaps in regional development, assessment of the regional

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potential sector. The authors offer recommendations on measures to stimulate sustainable economic growth. The article [7] presents an algorithm for constructing a measure that takes into account multidimensional inequality in European regions in terms of human development. The results of the calculations made it possible to study changes in human development taking into account its inequality, to assess the loss of potential human development due to inequality in society. The study [8] considers the assessment of regional development by combining the methods of factor and cluster analysis with the construction of a composite development index. The authors used the first method to construct a set of partial composite indices of regional development and the second method used to group regions. The full reduction of the multidimensional space of initial indicators and the design of a comprehensive indicator for assessing regional development are considered in studies [9], [10], [11], [12], [13], [14], [15]. This approach simplifies further comparison and ranking of regions, allows you to compare them under some imaginary "ideal point". Separately, in the tools of multidimensional statistical analysis, it is worth highlighting the methods of multidimensional scaling (MDS), which are aimed at studying complex processes and phenomena that aren't descriptive and simulated directly. To this end, the objects under study are deployed in some space of latent features that adequately reflects reality. Its creation is based on data on the pairwise difference (or similarity) or relationship between objects. As a result, based on the relative location of objects in this space, a conclusion is drawn about the internal structure of totality under study.

The use of MDS methods to study various aspects of socio-economic development remains the focus of many scholars. The basic principles of applying this technique to the processing of socio-economic research data and its interrelation with other methods of processing multidimensional data are reflected, in particular, in [16], [17], [18]. There is also considerable interest in using this tool to explore the economic development of regions. For example, in [19] the authors analyzed short-term business cycles of Portugal over the last 150 years and used MDS to visualize the results. The analysis of the MDS cards obtained allowed the authors to reveal the different periods of Portuguese economy development like prosperity and crises, growth, and stagnation. J.J. De Jongh and D.F. Meyer proposed the use of MDS technology to construct MREDI – Multidimensional Regional Development Index [20]. The practical validation of the approach proposed was carried out by the authors for some rural municipal regions of the Northwestern Province of South Africa. The analysis provides detailed trends and changes in the socio-economic strengths and weaknesses of rural areas in the province studied. We agree with the authors' findings and conclude that the successful application of MREDI to rural areas confirms its usefulness for measuring the economic development of regions in other countries and allows for a comprehensive assessment and comparison of ones. the procedure for assessing the social cohesion of the Lower Silesian Region in the period 2005–2015 is reviewed in [21] based on MDS tools combined with linear ordering and Theil decomposition. The application of MDS techniques combined with cluster analysis for processing expert data is presented in an article [22]. This approach was implemented on two samples of farms in Kenya and Zimbabwe. On a global level of MDS use, papers [23], [24] consider the description of the trends of economic growth indicators, globalization, prosperity, and human development of the world economy in the period from 1977 to 2012.

In this study, we propose to use the metric MDS method to identify the vectors of economic development of Ukrainian regions and positioning ones taking into account characteristics obtained.

2. Results and Discussion

Let us denote by m the number of objects under study, n – number of initial indicators, that describe objects and by $X=\{X_1, X_2, \ldots, X_n\}$ a set of these indicators. In our study, we use Torgerson's metric multidimensional scaling algorithm. It is based on the assumption that the initial information about objects' differences is given as a matrix of their pairwise comparisons:

$$\Delta = \begin{bmatrix} \delta_{11} & \delta_{12} & \ldots & \delta_{1m} \\ \delta_{21} & \delta_{22} & \ldots & \delta_{2m} \\ \ldots & & & \\ \delta_{m1} & \delta_{m2} & \ldots & \delta_{mm} \end{bmatrix}, \qquad (1)$$

where δ_{ij} is a measure of the difference between i-th and j-th objects and is interpreted as Euclidean distance between these objects in some unknown scale-space U:

$$\delta_{ij} = d_{ij} = \sqrt{\sum_{k=1}^{p} (u_{ik} - u_{jk})^2} \; , \qquad (2)$$

where m is a number of analyzed objects; $U = (u_{ij})$, $i=1..m$, $j=1..p$, is a matrix of objects' coordinates in the new space; p – dimension of the new scale space.

However, neither the values of the objects' coordinates u_{ij} in this space nor the space dimension p are known. Equality (2) means that the measures of the differences calculated from the initial data correspond to the distances between objects in the new scale space. Therefore, the problem of metric scaling is that based on the known matrix of objects differences like (1), which is calculated by the values of the initial indicators $X_1, X_2, ..., X_n$, it is need to obtain the coordinates of the objects in the new scale-space up to orthogonal transformation its system of coordinated.

Let us use the notation:

$$\Delta^* = UU^T . \qquad (3)$$

Given that matrix Δ is interpreted as a matrix of distances between objects in the new scale-space U, there is the following relation between the elements of matrices Δ and Δ^*:

$$\delta^*_{ij} = -\frac{1}{2}\left(\delta^2_{ij} - \delta^{(2)}_i - \delta^{(2)}_j + \delta^{(2)}\right), \qquad (4)$$

where $\delta^{(2)}_k$ is the average value of the squares of the elements of the k-th column (row) of the matrix Δ:

$$\delta^{(2)}_k = \frac{1}{m}\sum_{i=1}^{m} \delta^2_{ik} \; , \qquad (5)$$

$\delta^{(2)}$ is the average value of the squares of all elements of the matrix Δ:

$$\delta^{(2)} = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m} \delta^2_{ij} \; . \qquad (6)$$

Considering matrix Δ^* as a covariance one for some matrix B and taking into account equation (3), matrix U may be considered as the factor loadings matrix of the principal components of matrix B. Therefore,

$$U = V \cdot \Lambda^{1/2} , \qquad (7)$$

where Λ is a matrix of eigenvalues of the matrix Δ^*; V is a matrix of appropriate eigenvectors.

Thus, taking into account the above considerations, the calculational procedure for constructing latent scales that characterize the economic development of the regions has such form:

1) Choosing a system of indicators X_j, which characterize the economic development of the regions, $j = 1..n$.

2) Standardization of initial values to exclude the impact of measurement units on the calculation results. In paper [25], there are several ways of performing this transformation. We use the following procedure:

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} , \qquad (8)$$

where $\bar{x}_j$ – sample mean of X_j; s_j – sample standard deviation of X_j, z_{ij} – standardized value of X_j; x_{ij} – the initial value of X_j; $i = 1..n$, $j = 1..n$.

3) Calculation of the difference's matrix Δ as a matrix of Euclidean distances between objects in the space of standardized metrics Z.

4) Calculation of the matrix Δ^* by formulas (4) – (6).

5) Calculation of the first p eigenvalues and their corresponding eigenvectors of the matrix Δ^*. The value of p is selected either based on meaningful analysis or based on the sufficiency of the explained variance of the initial indicators.

6) Calculation of the objects' coordinates in a new scale space using the formula (7).

Using the above algorithm, let us identify the latent space of the economic development of Ukraine's regions and carry out their positioning in this space. To make calculation, we choose the following initial indicators,

X_1 –GRP at actual prices per capita, UAH;

X_2 –Volume of capital investments per capita, UAH;

X_3 – Volume of industrial production per capita, UAH;

X_4 – Volume of agricultural products per capita, UAH;

X_5 – Foreign direct investment per capita, USD.

The information base is the data of the State Statistics Service of Ukraine for 2018 0. To simplify the display of the data and the results of the calculations, let's set the code for each region (Table 1). The baseline values of the indicators selected for the study are shown in Table 2. Taking into account the large dimension of the distance matrix Δ between objects-regions studied, let us give it value in block form:

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} \quad (9)$$

Components of the matrix Δ are presented in tables 3-6.

Tab. 1. Relations between the name of regions and their codes

Code	Region	Code	Region
C_1	Vinnytsia	C_13	Mykolaiv
C_2	Volyn	C_14	Odesa
C_3	Dnipro	C_15	Poltava
C_4	Donetsk	C_16	Rivne
C_5	Zhytomyr	C_17	Sumy
C_6	Zakarpattia	C_18	Ternopil
C_7	Zaporizhzhia	C_19	Kharkiv
C_8	Ivano-Frankivsk	C_20	Kherson
C_9	Kyiv	C_21	Khmelnytskyi
C_10	Kyrovohrad	C_22	Cherkasy
C_11	Luhansk	C_23	Chernivtsi
C_12	Lviv	C_24	Chernihiv

Tab. 2. Initial data for calculation

Code	X_1	X_2	X_3	X_4	X_5
C_1	58384	7451.8	45607.7	35829.3	126.2
C_2	49987	6790.8	28099	18691.6	242.3
C_3	97137	13294.8	136028.7	13121.7	1142.8
C_4	39411	4102.5	63147.8	5075.7	265.2
C_5	49737	6244.4	33289.2	22407.3	183.4
C_6	34202	4478.7	17799.3	8832.6	258.9
C_7	75306	9176.2	114071.5	15579.8	526.2
C_8	46312	7055.2	35420.8	12085.9	657.5
C_9	90027	19840.4	64733.6	24038.3	913.8
C_10	55183	7669.2	30779.2	30220.5	73.4
C_11	13883	1529.6	11218.1	5697.7	201.2
C_12	58221	9590.8	36123.2	10694.8	370.0
C_13	60549	9762.3	46799.4	21146.7	180.0
C_14	62701	9394.4	28294.6	13685.2	506.6
C_15	106248	11225.3	136094.3	28247.2	714.2
C_16	42038	5278.8	31561.1	16733.5	115.6
C_17	51419	6331.6	39064.5	25794.4	165.5
C_18	38593	6793.9	20539.6	24930.3	42.7
C_19	69489	7219.2	68469.6	14663.6	238.0
C_20	45532	7012.4	26746.2	29621.8	208.1
C_21	49916	8224.4	31386.1	30277.1	133.8
C_22	59697	6663.8	56034.4	29913.3	274.7
C_23	31509	3308.6	12641.4	13773.6	47.1
C_24	55198	7219.7	46135.5	29586.8	421.9

Tab. 3. Destination matrix Δ_{11} between regions C_1 – C_12

Region	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_10	C_11	C_12
C_1	0.00	2.10	5.71	3.80	1.67	3.51	3.48	3.36	4.90	0.81	4.48	3.06
C_2	2.10	0.00	5.35	2.06	0.52	1.53	3.03	1.68	4.90	1.48	2.75	1.36
C_3	5.71	5.35	0.00	5.40	5.52	6.06	2.77	4.51	3.13	5.82	7.17	4.52
C_4	3.80	2.06	5.40	0.00	2.30	1.40	3.06	2.00	5.91	3.32	2.07	2.07
C_5	1.67	0.52	5.52	2.30	0.00	1.86	3.11	2.07	5.06	1.08	2.95	1.80
C_6	3.51	1.53	6.06	1.40	1.86	0.00	3.84	1.80	5.96	2.88	1.35	1.95
C_7	3.48	3.03	2.77	3.06	3.11	3.84	0.00	2.79	3.75	3.49	4.95	2.52
C_8	3.36	1.68	4.51	2.00	2.07	1.80	2.79	0.00	4.51	2.96	2.90	1.38
C_9	4.90	4.90	3.13	5.91	5.06	5.96	3.75	4.51	0.00	4.96	7.23	4.14
C_10	0.81	1.48	5.82	3.32	1.08	2.88	3.49	2.96	4.96	0.00	3.89	2.52
C_11	4.48	2.75	7.17	2.07	2.95	1.35	4.95	2.90	7.23	3.89	0.00	3.28
C_12	3.06	1.36	4.52	2.07	1.80	1.95	2.52	1.38	4.14	2.52	3.28	0.00

Tab. 4. Destination matrix Δ_{12} between regions C_1: C_12 – C_13: C_24

Region	C_13	C_14	C_15	C_16	C_17	C_18	C_19	C_20	C_21	C_22	C_23	C_24
C_1	1.80	2.96	4.27	2.42	1.25	1.75	2.58	1.13	0.88	0.93	3.20	1.28
C_2	1.16	1.45	4.74	0.76	0.92	1.16	1.56	1.27	1.43	1.58	1.65	1.51
C_3	4.81	4.31	2.41	5.87	5.52	6.27	4.33	5.80	5.73	5.01	6.73	4.86
C_4	2.67	2.47	5.30	1.73	2.62	2.79	2.01	3.11	3.29	3.08	1.97	3.10
C_5	1.18	1.86	4.69	0.83	0.43	0.88	1.68	0.90	1.07	1.23	1.73	1.29
C_6	2.55	2.21	5.88	1.19	2.27	2.10	2.45	2.54	2.82	2.97	1.01	2.86
C_7	2.49	2.55	2.33	3.39	3.10	3.85	1.78	3.54	3.46	2.70	4.30	2.78
C_8	2.25	1.18	4.63	2.07	2.37	2.69	2.11	2.57	2.81	2.59	2.60	2.22
C_9	4.12	3.83	3.36	5.59	5.01	5.51	4.49	5.01	4.82	4.60	6.48	4.33
C_10	1.35	2.50	4.62	1.79	0.77	1.07	2.27	0.70	0.37	1.08	2.56	1.32
C_11	3.79	3.53	7.05	2.22	3.30	2.95	3.66	3.50	3.83	4.02	1.45	3.93
C_12	1.41	0.67	4.39	1.83	2.10	2.38	1.42	2.43	2.45	2.42	2.56	2.28

Tab. 5. Destination matrix Δ_{21} between regions C_13:C_24 – C_1:C_12

Region	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_10	C_11	C_12
C_13	1.80	1.16	4.81	2.67	1.18	2.55	2.49	2.25	4.12	1.35	3.79	1.41
C_14	2.96	1.45	4.31	2.47	1.86	2.21	2.55	1.18	3.83	2.50	3.53	0.67
C_15	4.27	4.74	2.41	5.30	4.69	5.88	2.33	4.63	3.36	4.62	7.05	4.39
C_16	2.42	0.76	5.87	1.73	0.83	1.19	3.39	2.07	5.59	1.79	2.22	1.83
C_17	1.25	0.92	5.52	2.62	0.43	2.27	3.10	2.37	5.01	0.77	3.30	2.10
C_18	1.75	1.16	6.27	2.79	0.88	2.10	3.85	2.69	5.51	1.07	2.95	2.38
C_19	2.58	1.56	4.33	2.01	1.68	2.45	1.78	2.11	4.49	2.27	3.66	1.42
C_20	1.13	1.27	5.80	3.11	0.90	2.54	3.54	2.57	5.01	0.70	3.50	2.43
C_21	0.88	1.43	5.73	3.29	1.07	2.82	3.46	2.81	4.82	0.37	3.83	2.45
C_22	0.93	1.58	5.01	3.08	1.23	2.97	2.70	2.59	4.60	1.08	4.02	2.42
C_23	3.20	1.65	6.73	1.97	1.73	1.01	4.30	2.60	6.48	2.56	1.45	2.56
C_24	1.28	1.51	4.86	3.10	1.29	2.86	2.78	2.22	4.33	1.32	3.93	2.28

Tab. 6. Destination matrix Δ_{22} between regions C_13 – C_24

Region	C_13	C_14	C_15	C_16	C_17	C_18	C_19	C_20	C_21	C_22	C_23	C_24
C_13	0.00	1.54	3.98	1.68	1.20	1.67	1.29	1.54	1.32	1.38	2.65	1.49
C_14	1.54	0.00	4.19	2.08	2.12	2.50	1.65	2.35	2.41	2.30	2.82	2.03
C_15	3.98	4.19	0.00	5.24	4.52	5.38	3.66	4.80	4.62	3.79	6.21	3.88
C_16	1.68	2.08	5.24	0.00	1.19	1.11	1.84	1.59	1.78	1.98	1.01	2.05
C_17	1.20	2.12	4.52	1.19	0.00	0.94	1.79	0.67	0.78	0.88	2.06	1.07
C_18	1.67	2.50	5.38	1.11	0.94	0.00	2.44	0.88	1.01	1.75	1.65	1.81
C_19	1.29	1.65	3.66	1.84	1.79	2.44	0.00	2.38	2.32	1.84	2.75	2.05
C_20	1.54	2.35	4.80	1.59	0.67	0.88	2.38	0.00	0.50	1.11	2.29	1.05
C_21	1.32	2.41	4.62	1.78	0.78	1.01	2.32	0.50	0.00	1.08	2.56	1.17
C_22	1.38	2.30	3.79	1.98	0.88	1.75	1.84	1.11	1.08	0.00	2.88	0.65
C_23	2.65	2.82	6.21	1.01	2.06	1.65	2.75	2.29	2.56	2.88	0.00	2.90
C_24	1.49	2.03	3.88	2.05	1.07	1.81	2.05	1.05	1.17	0.65	2.90	0.00

In the next step, using the components of the matrix Δ we calculate the values of double-centered matrix Δ*. Similarly, we present it in a block form also:

$$\Delta = \begin{bmatrix} \Delta_{11}^{*} & \Delta_{12}^{*} \\ \Delta_{21}^{*} & \Delta_{22}^{*} \end{bmatrix} \quad (10)$$

The results are shown in Tables 7 – 10.

Tab. 7. Matrix Δ_{11}^{*} for regions C_1 – C_12

Region	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_10	C_11	C_12
C_1	3.80	0.00	-3.61	-2.92	0.91	-2.08	-1.48	-2.48	-0.80	2.80	-2.60	-2.00
C_2	0.00	0.62	-3.25	0.58	0.58	1.32	-1.61	0.16	-2.36	0.43	2.04	0.16
C_3	-3.61	-3.25	21.55	-1.41	-4.06	-5.43	9.60	1.88	15.19	-4.94	-9.42	1.34
C_4	-2.92	0.58	-1.41	4.77	0.14	3.58	0.36	1.65	-5.76	-1.91	5.75	1.01
C_5	0.91	0.58	-4.06	0.14	0.80	0.86	-1.76	-0.49	-3.05	1.04	1.58	-0.45
C_6	-2.08	1.32	-5.43	3.58	0.86	4.36	-2.53	1.82	-6.28	-0.73	6.79	1.04
C_7	-1.48	-1.61	9.60	0.36	-1.76	-2.53	5.33	0.04	4.94	-2.21	-4.09	0.27
C_8	-2.48	0.16	1.88	1.65	-0.49	1.82	0.04	2.52	0.40	-1.90	2.57	1.09
C_9	-0.80	-2.36	15.19	-5.76	-3.05	-6.28	4.94	0.40	18.64	-1.73	-11.30	1.53
C_10	2.80	0.43	-4.94	-1.91	1.04	-0.73	-2.21	-1.90	-1.73	2.45	-0.82	-1.18
C_11	-2.60	2.04	-9.42	5.75	1.58	6.79	-4.09	2.57	-11.30	-0.82	11.04	0.93
C_12	-2.00	0.16	1.34	1.01	-0.45	1.04	0.27	1.09	1.53	-1.18	0.93	1.54

Tab. 8. Matrix Δ_{12}^{*} for regions C_1: C_12 – C_13:C_24

Region	C_13	C_14	C_15	C_16	C_17	C_18	C_19	C_20	C_21	C_22	C_23	C_24
C_1	0.62	-1.70	0.70	-0.09	1.64	1.75	-0.80	2.29	2.61	2.20	-0.49	1.73
C_2	-0.03	0.03	-2.99	0.96	0.41	1.02	-0.28	0.53	0.38	-0.21	1.69	-0.18
C_3	-0.47	2.24	15.80	-5.52	-3.93	-7.47	2.04	-5.00	-4.54	-1.02	-9.17	-0.40
C_4	-0.85	0.11	-3.73	1.83	-0.52	-0.12	0.99	-1.42	-1.93	-1.61	3.18	-1.78
C_5	0.05	-0.56	-2.67	1.00	0.84	1.40	-0.37	1.03	0.93	0.39	1.65	0.22
C_6	-0.72	0.50	-7.15	2.41	0.13	1.37	-0.19	-0.03	-0.69	-1.49	4.40	-1.25
C_7	-0.09	0.19	7.87	-2.14	-1.60	-3.38	1.72	-2.57	-2.21	-0.24	-3.86	-0.54
C_8	-0.94	1.33	-1.52	0.06	-1.01	-0.97	-0.33	-1.00	-1.58	-1.37	0.62	-0.56
C_9	1.17	2.75	11.62	-5.37	-2.72	-4.49	-0.11	-2.20	-1.18	-0.51	-8.95	0.58
C_10	0.66	-1.13	-1.54	0.56	1.46	2.04	-0.70	2.01	2.26	1.38	0.70	1.00
C_11	-1.32	0.05	-11.39	4.00	0.60	2.55	-0.54	0.44	-0.70	-1.83	7.20	-1.54
C_12	0.12	1.32	-0.93	0.04	-0.90	-0.67	0.40	-1.16	-1.12	-1.43	0.22	-1.17

Tab. 9. Matrix Δ^*_{21} for regions C_13:C_24 – C_1:C_12

Region	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_10	C_11	C_12
C_13	0.62	-0.03	-0.47	-0.85	0.05	-0.72	-0.09	-0.94	1.17	0.66	-1.32	0.12
C_14	-1.70	0.03	2.24	0.11	-0.56	0.50	0.19	1.33	2.75	-1.13	0.05	1.32
C_15	0.70	-2.99	15.80	-3.73	-2.67	-7.15	7.87	-1.52	11.62	-1.54	-11.39	-0.93
C_16	-0.09	0.96	-5.52	1.83	1.00	2.41	-2.14	0.06	-5.37	0.56	4.00	0.04
C_17	1.64	0.41	-3.93	-0.52	0.84	0.13	-1.60	-1.01	-2.72	1.46	0.60	-0.90
C_18	1.75	1.02	-7.47	-0.12	1.40	1.37	-3.38	-0.97	-4.49	2.04	2.55	-0.67
C_19	-0.80	-0.28	2.04	0.99	-0.37	-0.19	1.72	-0.33	-0.11	-0.70	-0.54	0.40
C_20	2.29	0.53	-5.00	-1.42	1.03	-0.03	-2.57	-1.00	-2.20	2.01	0.44	-1.16
C_21	2.61	0.38	-4.54	-1.93	0.93	-0.69	-2.21	-1.58	-1.18	2.26	-0.70	-1.12
C_22	2.20	-0.21	-1.02	-1.61	0.39	-1.49	-0.24	-1.37	-0.51	1.38	-1.83	-1.43
C_23	-0.49	1.69	-9.17	3.18	1.65	4.40	-3.86	0.62	-8.95	0.70	7.20	0.22
C_24	1.73	-0.18	-0.40	-1.78	0.22	-1.25	-0.54	-0.56	0.58	1.00	-1.54	-1.17

Tab. 10. Matrix Δ^*_{22} for regions C_13 – C_24

Region	C_13	C_14	C_15	C_16	C_17	C_18	C_19	C_20	C_21	C_22	C_23	C_24
C_13	0.62	-0.03	-0.47	-0.85	0.05	-0.72	-0.09	-0.94	1.17	0.66	-1.32	0.12
C_14	-1.70	0.03	2.24	0.11	-0.56	0.50	0.19	1.33	2.75	-1.13	0.05	1.32
C_15	0.70	-2.99	15.80	-3.73	-2.67	-7.15	7.87	-1.52	11.62	-1.54	-11.39	-0.93
C_16	-0.09	0.96	-5.52	1.83	1.00	2.41	-2.14	0.06	-5.37	0.56	4.00	0.04
C_17	1.64	0.41	-3.93	-0.52	0.84	0.13	-1.60	-1.01	-2.72	1.46	0.60	-0.90
C_18	1.75	1.02	-7.47	-0.12	1.40	1.37	-3.38	-0.97	-4.49	2.04	2.55	-0.67
C_19	-0.80	-0.28	2.04	0.99	-0.37	-0.19	1.72	-0.33	-0.11	-0.70	-0.54	0.40
C_20	2.29	0.53	-5.00	-1.42	1.03	-0.03	-2.57	-1.00	-2.20	2.01	0.44	-1.16
C_21	2.61	0.38	-4.54	-1.93	0.93	-0.69	-2.21	-1.58	-1.18	2.26	-0.70	-1.12
C_22	2.20	-0.21	-1.02	-1.61	0.39	-1.49	-0.24	-1.37	-0.51	1.38	-1.83	-1.43
C_23	-0.49	1.69	-9.17	3.18	1.65	4.40	-3.86	0.62	-8.95	0.70	7.20	0.22
C_24	1.73	-0.18	-0.40	-1.78	0.22	-1.25	-0.54	-0.56	0.58	1.00	-1.54	-1.17

The first two eigenvalues of the matrix Δ^* have values: $\lambda_1 = 74.50$; $\lambda_2 = 26.28$. The ratio of the sum of these values to the total of all values is 0.87, so the first two principal components explain 87% of the variance of the original traits, which is a completely acceptable result. The values of eigenvectors V_1 and V_2 for these eigenvalues, as well as the factor loadings of the respective principal components U_1 and U_2, which are the coordinates of the regions in the new scale space, are shown in Table 11. Graphically, the configuration of the regions in the constructed scale space is presented in Figure 1.

Tab. 11. Values of two first eigenvectors and factor loadings for appropriate principal components (coordinates of regions in a new scale space)

Region	V_1	V_2	U_1	U_2	Region	V_1	V_2	U_1	U_2
C_1	-0.02	0.37	-0.19	1.89	C_13	0.01	0.08	0.12	0.43
C_2	-0.08	-0.01	-0.73	-0.03	C_14	0.04	-0.13	0.35	-0.68
C_3	0.50	-0.31	4.34	-1.57	C_15	0.43	0.10	3.72	0.51
C_4	-0.11	-0.34	-0.98	-1.77	C_16	-0.15	-0.04	-1.33	-0.20
C_5	-0.09	0.08	-0.79	0.39	C_17	-0.08	0.15	-0.65	0.77
C_6	-0.19	-0.24	-1.67	-1.22	C_18	-0.16	0.16	-1.41	0.81
C_7	0.22	-0.15	1.89	-0.76	C_19	0.04	-0.09	0.35	-0.47
C_8	-0.01	-0.24	-0.05	-1.25	C_20	-0.09	0.22	-0.76	1.13
C_9	0.44	0.07	3.79	0.34	C_21	-0.06	0.26	-0.55	1.34
C_10	-0.07	0.28	-0.60	1.43	C_22	0.01	0.20	0.07	1.04
C_11	-0.33	-0.33	-2.82	-1.70	C_23	-0.26	-0.10	-2.28	-0.49
C_12	0.01	-0.17	0.10	-0.87	C_24	0.02	0.17	0.14	0.85

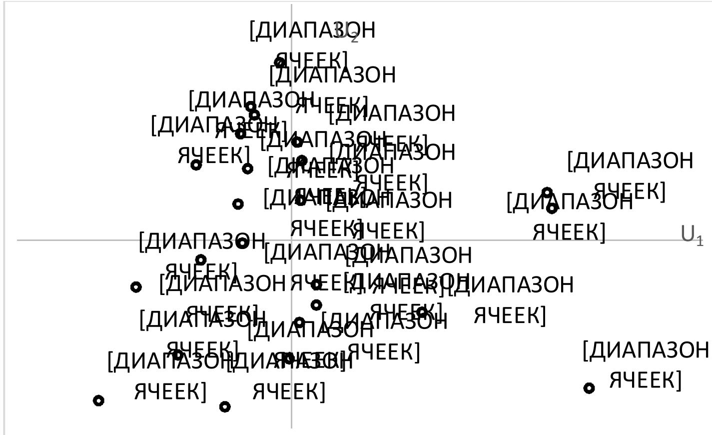

Fig.1. Configure of regions in new scale space

In our opinion, the results obtained don't lend themselves to clear and objective substantive interpretation, so it makes sense to rotate the constructed scale space.

3. Experimental

The results of the calculations made it possible to allocate a subspace of latent features built on new scales in the space of the initial indicators. This determines the ambiguity of the solution obtained. If the factor loadings u_{ij} , i=1..m, j=1..p, have a more or less uniform distribution of values, the search for a title for such a scale is complicated by the lack of its features, structural accents.

One approach that solves this interpretation problem is to use the scale space rotation method. From a theoretical point of view, the rotation of the axes eliminates the uncertainty of the spatial location of the factor coordinate system.

The counter-clockwise rotation matrix has a form:

$$T = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}, \quad (11)$$

where φ – the angle of rotation.

Choosing a rotation angle of 25°, we have a rotation matrix:

$$T = \begin{bmatrix} 0.71 & -0.71 \\ 0.71 & 0.71 \end{bmatrix}, \quad (12)$$

The values of the coordinates of the regions in the space of scales after rotation are shown in Table 12, and the graphical configuration of the regions' location is shown in Figure 2.

Tab. 12. Values coordinates of regions in a scale-space after rotation

Region	U_1	U_2	Region	U_1	U_2
C_1	0.63	1.79	C_13	0.29	0.34
C_2	-0.68	0.29	C_14	0.02	-0.77
C_3	3.27	-3.25	C_15	3.59	-1.11
C_4	-1.64	-1.19	C_16	-1.30	0.38
C_5	-0.55	0.69	C_17	-0.26	0.97
C_6	-2.03	-0.39	C_18	-0.93	1.32
C_7	1.39	-1.49	C_19	0.12	-0.57
C_8	-0.57	-1.12	C_20	-0.21	1.35
C_9	3.58	-1.29	C_21	0.06	1.44
C_10	0.06	1.55	C_22	0.50	0.92
C_11	-3.28	-0.35	C_23	-2.28	0.52
C_12	-0.28	-0.83	C_24	0.49	0.71

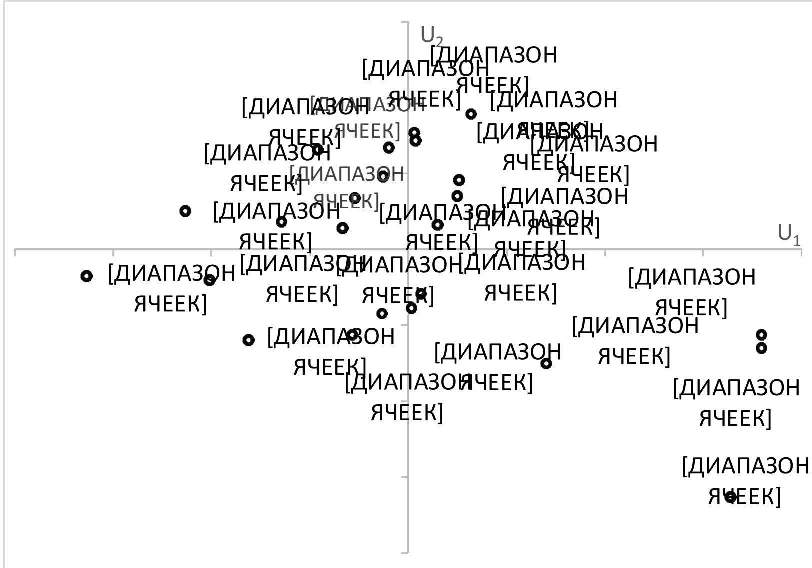

Fig. 2. Configure of regions in scale space after rotation

The analysis of the object configuration allows us to conclude that the U_1 may be interpreted as the axis of economic activity performance and the U_2 as the axis of innovation. It can be stated that the Dnipropetrovsk, Zaporizhia, Kyiv, Odessa, Poltava, and Kharkiv regions have in the frameworks of Ukraine relatively high levels of economic activity efficiency and high level of investment attraction. Vinnytsia, Kirovohrad, Mykolaiv, Khmelnytskyi, Cherkasy, Chernihiv regions have fairly high levels of economic activity performance, but the level of investment for these regions responds to the average value. Similarly, other regions can be interpreted for their positioning in new scale space.

4. Conclusion

Balanced development of the country's regions should be focused on providing conditions that will allow each region of the country to have the necessary and sufficient resources to ensure decent living conditions, integrated development and increase the competitiveness of the economy. Therefore, assessing the economy of the regions in the context of ensuring their sustainable development remains a relevant task both at the state level and for interstate comparisons. Studies have shown the widespread use of multidimensional statistical analysis methodology to solve this problem, in which multidimensional scaling have an important role. In the paper has been used the technology of metric multidimensional scaling to build a space of latent characteristics of economic development of regions and positioning them in this space. Practical testing of the proposed approach has been carried out according to the State Statistics Service of Ukraine. In order to obtain a meaningful interpretation of the results, we rotated the scale space. This allowed to provide an economic interpretation of the scales of the new space and to identify the characteristics of the formed grouping of regions of Ukraine. In particular, it was found that the constructed latent axes of the new space reflect the effectiveness of economic activity and investment. The obtained results are necessary to identify internal and external threats to regional development, and contribute to the development of measures to prevent their negative impact in order to identify development scenarios and develop an optimal strategy for the functioning of regional economic systems. The directions of further research in this area are to expand the set of baseline indicators, supplementing them with indicators of the social sphere, as well as taking into account non-metric indicators in the construction of scale space.

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