RESEARCH PAPER

Operating Room Scheduling Considering Patient Priorities and Operating Room Preferences: A Case Study

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ABSTRACT
Operating rooms have become the most important areas in hospitals because of the scarcity and cost of resources. The present study investigates operating room scheduling and rescheduling, considering the priority of surgical patients in a specialized hospital. The ultimate purpose of scheduling is to minimize patient waiting time, surgeon idle time between surgeries, and penalties for deviations from operating room preferences. A mathematical programming model is presented to solve the problem. Because the problem is strongly NP-hard, two heuristic algorithms are presented. A heuristic algorithm based on a mathematical programming model with local search obtains near-optimal solutions for all the samples. The average relative deviation of this algorithm is 0.02%. In continuous, heuristic algorithms performance has been investigated by increasing the number of patients and reducing recovery beds. Next, a rescheduling heuristic algorithm is presented to deal with real-time situations. This algorithm presents fewer changes resulting from rescheduling than the scheduling problem.

KEYWORDS: *Scheduling; Rescheduling; Operating room; Mathematical programming; Local search.*

1. Introduction

Nowadays, the problems of optimizing healthcare due to high costs and low resources available in hospitals have been considered by many researchers [1], and different methods were applied to reduce the healthcare cost [2]; [3]; [4]. Hospitals are among the main entities providing healthcare services; they are responsible for 36% of healthcare costs. Of this proportion, approximately 40% is designated for operating rooms, making them major units of hospitals [5]. Costs of expert resources, including surgeons, nurses, and anesthesiologists, make up a significant proportion of operating room costs.

Among the medical staff of operating rooms, surgeons and nurses are of great importance [6]. Therefore, resource management associated with these professionals plays an important role in reducing total hospital costs.

Operating rooms contain unusual and very expensive equipment in most hospitals. Effective management of operating room equipment can result in greatly increased profit and improvement in both quality of care and financial condition [7]. This means that resource scheduling, including operating room scheduling, is very important.

Although manufacturing industries have been developing scheduling for many years, it has only recently begun to be implemented in the service sector, including healthcare service centers. Physician scheduling, nurse scheduling, prioritizing and scheduling patient treatment, operating room scheduling, and ward scheduling are issues raised in healthcare service centers [8]. Healthcare service centers often have technical and staffing resource constraints, and many patients cannot be treated immediately. Also, providing correct and on-time services is among the major concerns of healthcare service centers. Therefore, healthcare scheduling is vital to

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acceptable service provision [9]. Rescheduling problem is used when any scheduling disruptions occur. In general, two types of disruptions, including operating room and patient disruptions, have been identified in the operating rooms. Equipment failure and the arrival of a high-priority non-selective patient are examples of disruptions in the operating rooms. Patient disruption occurs when the actual surgery times are longer or less than the specific surgery times. Scheduling may be delayed in both types of disruptions and needs to be updated [10].

This study investigates the scheduling and rescheduling of the operating room at Khanevadeh Hospital in Isfahan city, Iran. In the present paper, the preferences of surgeons, patients, and operating room staff members are considered at a multi-stage integration level for the scheduling and rescheduling problem. The remainder of the paper is organized as follows. Section 2 presents a review of the literature on operating room scheduling. Section 3 describes the problem, notations, assumptions, and complexity associated with the problem. A mathematical programming model is proposed in section 4, and then heuristic algorithms are illustrated in section 5. Computational results are presented in section 6. Finally, section 7 is dedicated to conclusions and suggestions for future research.

2. Literature Review

The operating room scheduling problem has been discussed extensively in the articles. Here, we only review some of the most relevant studies to our article. It should be noted that several articles have reviewed operating room scheduling and planning problems [9]; [10]; [11].

Some previous researchers have investigated only operating room scheduling [12]; [13]; [14], while in the past decade, some have considered operating room scheduling and recovery beds simultaneously as being closer to reality [15]. Some studies have dealt with only one operating room [16]; while others have focused on multiple similar [17] or even distinct operating rooms. Operating rooms are distinguished concerning the requirements for some complex surgeries, such as cardiac and neurological [18]. In some hospitals, the operating rooms have different dimensions but the same equipment. It seems that no attempt has been made in the literature to consider this assumption in the operating rooms.

In scheduling problems, surgery start times for patients are specified according to the designations of the operating rooms, recovery beds, nurses, etc. Wang et al. [16] examined operating room scheduling considering surgeon rest time for one operating room and one day. They considered high-priority and low-priority patient groups; the low-priority patients could only have surgeries when the surgeries of high-priority patients were finished. They specified the sequence of the two patient groups by applying a balance between patient satisfaction and operating room operational costs.

Preferences of stakeholders are among the issues investigated in the literature. The stakeholders involved in operating rooms include patients, surgeons, nurses, and anesthesiologists. A heuristic method was proposed. Zhu et al. [14] solved a dynamic operating room scheduling problem in three steps using two heuristic algorithms by simultaneously considering patient waiting time, surgeon assignment, and operating room overtime. These three steps are (1) determining the number of operating rooms allocated to each specialty, (2) determining the operating room days allocated to each surgeon, and (3) allocating operating room days to all patients in the waiting list and determining the surgery sequence in each operating room days.

Some researchers have combined operational-level problems. Addis et al. [12] considered the operating room scheduling and rescheduling problem to minimize a penalty function based on patients' total waiting time and tardiness. Their problem involved selecting a set of patients from the elective patient list and assigning them to a set of available operating room blocks. In this study, they considered weekly patient visits.

In practice, surgery durations may be longer or shorter than expected. Furthermore, the sequence of elective patients in operating rooms may vary depending on emergency patient visits. This requires updating the scheduling problem. Kamran et al. [13] presented a heuristic algorithm to solve the adaptive allocation scheduling problem for reassigning and rescheduling of patients to the operating room blocks considering the aim of different stakeholder preferences, including minimization of patients' cancellation, patients' tardiness, block overtime, idleness of surgeons, and minimizing the start time of emergency patient's surgery.

The literature review clearly shows that there have been few studies of the rescheduling problem. In contrast, even though the significance of this problem is not negligible in practice due to increases and decreases in surgery durations and visits by emergency patients. The present study aims to fill this gap by investigating

the rescheduling problem. Furthermore, this study is intended to deal with a multi-stage integration level, which is more realistic than previous research that has focused on a single-stage integration level. In addition, the preferences of stakeholders, including patients, surgeons, and operating room staff, are considered simultaneously. Tab. 1 describes the assumptions of previous research and those of the current study.

3. Problem Definition

Operating room scheduling requires special attention, and scheduling problem must be updated to reflect unexpected variations such as emergency patient visits and increases and decreases in surgery durations. The present study investigates operating room scheduling and rescheduling considering operating room preferences (OTSRP) and patient priority in Khanevadeh Hospital in Iran. Patient priority is defined as child patients having the highest surgical priority, while infected patients (e.g., patients with pilonidal cysts) have the lowest priority. This study also investigates stakeholder satisfaction, including patients, surgeons, and operating room staff. Patient satisfaction depends on decreasing wait time. Surgeon satisfaction is related to decreasing the idle time between surgeries.

The case study hospital has three operating rooms with different dimensions but similar equipment and eight similar recovery beds. The differences in the dimensions of the operating rooms make it preferable for some surgeons to perform certain surgeries in certain operating rooms. The preference is to assign surgeries with longer durations to the large operating room. The reason for this preference is the direct relationship between the equipment and size of the operating room (being better equipped and easier to move in the larger operating room than the smaller operating room). However, this preference does not necessarily lead to the assignment of longer surgeries to the large room; rather, this is done only if it is possible. If the large room is occupied, these surgeries are assigned to the medium room; and if the medium room is also occupied, they are assigned to the small room. Taking operating room preferences into account leads to the satisfaction of the surgical team, including the surgeons and operating room staff. The operating rooms are designated as follows: #3 is the large room; #2, the medium room; and #1, the small room.

In this study, operating room scheduling aims to simultaneously minimize weighted normalized patient waiting time, surgeon idle time, and penalties for deviation from operating room preferences. Furthermore, rescheduling aims to minimize variations in the scheduling problem caused by disruptions. A new schedule (rescheduled plan) is carried out due to increases and decreases in surgery durations relative to planned surgery durations. Since emergency cases are rare in the hospital under investigation, patient visit times are assumed to be deterministic, and the only considered variation is surgery durations.

The assumptions of this OTSRP problem are as follows:

- The operating rooms and the recovery beds are investigated. Therefore, this problem is studied at a multi-stage integration level.
- No more than one operating room or recovery bed can be occupied by one patient.
- There is no time lapse between surgery and recovery because the recovery process naturally starts just after the completion of surgery.
- Sixty time slots, each consisting of 15 minutes, are assumed for each working day. The operating rooms work from 7:30 a.m. to 10:30 p.m.
- The recovery stage does not necessarily finish on the day that surgery is scheduled. In other words, the recovery process may start and not finish before the end of the operating room working hours on the day of surgery. However, the recovery stage finishes before the start of the working hours on the following day.
- Preemption is not allowed at any stage; i.e., if the operation has started, it cannot be preempted.
- The patients in this hospital are elective, and the surgeon for each patient is pre-specified.
- No surgeries are late or canceled because of the late arrival of patients. All patients are hospitalized before surgery, e.g., to carry out necessary clinical tests.
- Cleaning time, except for infected patients, and operating room preparation time are counted in surgery durations.
- Since surgeons have different skills and specialties, the duration times of surgeries are different and are known in advance for planning.
- All the patients on the daily list should be operated.
- The patient's surgical priority for each surgeon in order children, normal and infectious is considered.

3.1.   Problem complexity

Suppose there are only one operating room and one recovery bed. The objective function coefficient corresponding to the penalty for deviation from operating room preferences and surgeon idle time between surgeries equals 0, and the coefficient corresponding to patient waiting time equals 1. Patient waiting time equals the time interval between the start of the operating room work hours and the surgery start time. If the duration of surgery and recovery are added to the surgery start time of each patient, the problem's objective function minimizes the completion time of patient recovery. Suppose the preparation time and surgery priority of patients are ignored. In that case, the OTSRP problem is reduced to a two-machine no-wait flow-shop scheduling problem to minimize the total completion time of jobs, such that the patients are jobs and the operating rooms and recovery beds are assumed to be machines. Furthermore, the surgeries and recoveries of patients are performed continuously and without waiting. Therefore, this problem is a no-wait flow-shop scheduling problem. Röck [19] has shown the complexity of the no-wait two-machine flow-shop problem to minimize the completion time of jobs to be strongly NP-hard. Thus, the OTSRP problem is strongly NP-hard.

3.2.   Notations

In this section, the parameters, sets, and decision variables of the OTSRP problem are defined and illustrated.

Parameters:

P :	Number of patients
N_o :	Number of operating rooms
N_b :	Number of recovery beds
S :	Number of surgeons
T :	Number of 15-minute time slots
t_p :	Surgery duration time of patient p, $p=1,...,P$
t'_p :	Recovery duration time of patient p, $p=1,...,P$
t_p^{clean} :	Cleaning duration time of operating room after performing surgery on patient p, $p=1,...,P$
$s_{(p)}$:	Surgeon of patient p, $p=1,...,P$
St_s :	Ready time of surgeon s, $s=1,...,S$
N_r :	Number of patients whose surgical teams have operating room preference r, $r=1,..., N_o$
N_s :	Number of patients of surgeon s, $s=1,...,S$
M_1, M_2 :	large positive numbers
α :	Coefficient of importance of patient waiting time in the objective function
β :	Coefficient of importance of surgeon idle time in the objective function
γ :	Coefficient of the importance of deviations from operating room preferences in the objective function

The value of $\alpha + \beta + \gamma$ in the scheduling problem objective function equals 1.

Sets:

R_r :	Set of patients whose surgical teams have operating room preference r, $r=1,..., N_o$
π^{child} :	Set of child patients
π^{infect} :	Set of infected patients

Decision variables:

X_{prt} : $\begin{cases} 1 \text{ If surgery (recovery) of patient } p \text{ begins in} \\ \text{operating room (recovery bed) } r \text{ at time } t \\ \qquad\qquad 0 \text{ Otherwise} \end{cases}$ $\begin{aligned} & p = 1,...,P \\ & r = 1,...,N_o + N_b \\ & t = 1,...,T \end{aligned}$

u_{pq} : $\begin{cases} 1 \text{ if surgery of patient } p \text{ is performed before} \\ \qquad\quad \text{that of patient } q \\ \qquad\qquad 0 \text{ Otherwise} \end{cases}$ $p, q = 1,...,P$

w_s :	Surgeon s idle time	$s = 1,...,S$
v_p :	Waiting time of patient p	$p = 1,...,P$
C_s :	Completion time of the last surgery of surgeon s	$s = 1,...,S$
S_s :	Start time of the first surgery of surgeon s	$s = 1,...,S$

4. The Mixed-Integer Programming Model

In this section, a mixed integer programming model, called OTS, is presented for the OTSRP problem.

$$Minimize\ Obj_1 = \alpha\left(\frac{\sum_{p=1}^{P} v_p}{\sum_{p=1}^{P}(T-t_p)}\right) + \beta\left(\frac{\sum_{s=1}^{S} w_s}{\sum_{s=1}^{S}\left(T-\sum_{p|s=s_{(p)}} t_p - St_s\right)}\right) + \gamma(\sum_{r=2}^{N_o}\sum_{p\in R_r}\sum_{k=1}^{r-1}\sum_{t=St_{s_{(p)}}}^{T-t_p}\left(\frac{1}{k*N_r}\right)*$$

$$\left(\frac{t_p}{\sum_{p=1}^{P} t_p}\right)X_{pkt})$$

(1)

s.t.

$$\sum_{r=1}^{N_o}\sum_{t=St_{s_{(p)}}}^{T-t_p+1} X_{prt} = 1, p = 1, \dots, P$$

(2)

$$\sum_{r=N_o+1}^{N_o+N_b}\sum_{t=t_p}^{T} X_{prt} = 1, p = 1, \dots, P$$

(3)

$$\sum_{p=1}^{P} X_{prt} \le 1, r = 1, \dots, N_o, t = 1, \dots, T$$

(4)

$$\sum_{p=1}^{P} X_{prt} \le 1, r = N_o + 1, \dots, N_o + N_b, t = 1, \dots, T$$

(5)

$$\sum_{l=1}^{P}\sum_{k=t}^{t+t_p+t_p^{clean}-1} X_{lrk} \le M_1(1 - X_{prt}), p = 1, \dots, P, r = 1, \dots, N_o, t = St_{s_{(p)}}, \dots, T - t_p+1$$

(6)

$$\sum_{l=1}^{P}\sum_{k=t}^{t+t_p'-1} X_{lrk} \le M_1(1 - X_{prt}), p = 1, \dots, P, r = N_o + 1, \dots, N_o + N_b, t = t_p, \dots, T$$

(7)

$$\sum_{r=1}^{N_o}\sum_{t=St_{s_{(p)}}}^{T-t_p+1}(t + t_p)X_{prt} = \sum_{r=N_o+1}^{N_o+N_b}\sum_{t=t_p}^{T} t X_{prt}, \ p = 1, \dots, P$$

(8)

$$\sum_{r=1}^{N_o}\sum_{\substack{l=1 \\ l\notin\pi^{child} \\ s_{(l)}=s_{(p)}}}^{P}\sum_{k=St_{s_{(l)}}|s_{(l)}=s_{(p)}}^{t+t_p-1} X_{lrk} \le M_1(1 - \sum_{r=1}^{N_o} X_{prt})$$

$$p|p \in \pi^{child}, t = St_{s_{(p)}}, \dots, T - t_p + 1$$

(9)

$$\sum_{r=1}^{N_o}\sum_{\substack{l=1 \\ l\notin\pi^{infect} \\ s_{(l)}=s_{(p)}}}^{P}\sum_{k=t}^{T} X_{lrk} \le M_1(1 - \sum_{r=1}^{N_o} X_{prt}),$$

$$t = St_{s_{(p)}}, \dots, T - t_p + 1, p|p \in \pi^{infect}$$

(10)

$$\sum_{t=St_s}^{T-t_p+1}\sum_{r=1}^{N_o}(t + t_p - 1)X_{prt} \le C_s, s = 1, \dots, S, \ p = 1, \dots, P|s_{(p)} = s$$

(11)

$$\sum_{t=St_s}^{T-t_p+1}\sum_{r=1}^{N_o} tX_{prt} \ge S_s, s = 1, \dots, S, p = 1, \dots, P|s_{(p)} = s$$

(12)

$$w_s = C_s - S_s - \sum_{p=1|s=s_{(p)}}^{P}\sum_{r=1}^{N_o}\sum_{t=St_s}^{T-t_p+1} t_p X_{prt}, s = 1, \dots, S$$

(13)

$$v_p = \sum_{t=St_{s_{(p)}}}^{T-t_p+1}\sum_{r=1}^{N_o}(t-St_{s_{(p)}})X_{prt}, p = 1, \dots, P$$

(14)

$$\sum_{r=1}^{N_o}\sum_{t=St_{s_{(q)}}}^{T-t_p+1} tX_{qrt} - \sum_{r=1}^{N_o}\sum_{t=St_{s_{(p)}}}^{T-t_p+1}(t + t_p)X_{prt} + M_2(1 - u_{pq}) \ge 0, p, q = 1, \dots, P|p < q, s_{(p)}=s_{(q)}$$

(15)

$$\sum_{r=1}^{N_o}\sum_{t=St_{s_{(p)}}}^{T-t_p+1} tX_{prt} - \sum_{r=1}^{N_o}\sum_{t=St_{s_{(q)}}}^{T-t_p+1}(t + t_q)X_{qrt} + M_2(u_{pq}) \ge 0, p, q = 1, \dots, P|p < q, s_{(p)}=s_{(q)}$$

(16)

$X_{prt} \in (0,1), p = 1, ..., P, r = 1, ..., N_o+N_b, t=1,...,T$

(17)

$u_{pq} \in (0,1), p, q = 1, ..., P$

(18)

$w_s \geq 0, s = 1, ..., S$

(19)

$v_p \geq 0 \; p = 1, ..., P$

(20)

$C_s \geq 0, s = 1, ..., S$

(21)

$S_s \geq 0, s = 1, ..., S$

(22)

Equation (1) shows the problem's objective function (Obj_1). As can be seen from this equation, the scheduling problem's objective function consists of three weighted normalized terms. These three terms are patient waiting time, surgeon idle time between surgeries, and penalties for deviations from operating room preferences.

The normalization of the components of the objective function is such that the maximum value of each component in the denominator is given. If the patient's surgery starts at the end of the interval, the waiting time will be maximized. Also, if the surgeon's last surgery is at the end of the interval, his idle time will be maximized.

The operating room preferences mean that some surgeries are preferred to surgery in a certain operating room. In the third term, the value of $\left(\frac{1}{k*N_r}\right)$ shows the patient's preferences to the larger operating room. In other words, the more patient is assigned to the larger operating room, the lower the penalty. In this component, the value $\left(\frac{t_p}{\sum_{p=1}^{P} t_p}\right)$ indicates that it is preferred that the patient (among the preferred patients) be assigned to the operating room with a longer surgical duration.

Due to the need to remove the equations of recovery beds in the heuristic algorithms, all operating room and recovery beds equations are written separately. Equation (2) guarantees that each patient's surgery is performed only once in an operating room. This equation also guarantees that all the patients on the daily list are operated on. Equation (3) guarantees that each patient's recovery is performed only once in a recovery bed. Equation (4) shows that the surgery of only one patient is started at each time in each operating room. Also, equation (5) guarantees that the recovery of only one patient is started at each time in each recovery bed.

Furthermore, equation (6) guarantees that no other patient is scheduled for a room during the surgery and recovery of a patient in that room. Equation (7) guarantees that no other patient is scheduled for a recovery bed during a patient's recovery on that bed. Equation (8) guarantees that the recovery stage starts right after the surgery. According to the assumptions, children have the highest priority because of the less tolerance for specific conditions before surgery, so these patients are the first surgeries. In addition, infected patients are the last surgeries due to the need for cleansing after surgery. This reduces the surgeon's idle time between surgeries. In other words, for each surgeon, children's surgery is the first surgery, then normal and ultimately infectious. Equations (9) and (10) are provided to ensure the order of surgery of each surgeon. Equation (9) guarantees that child patients are scheduled as the first surgeries of each surgeon. Also, equation (10) guarantees that no other non-infected surgery is performed if a surgeon is in the middle of surgery for an infected patient. The upper bound for the left side of equation (10) is equal to the number of patients, so the value of M_1 is assumed to be P.

Equation (11) determines the completion time of the last surgery of surgeon s. The left side of this Equation shows the surgery completion time of each patient. Equation (12) determines the start time of the first surgery of surgeon s after the surgeon is ready. Equation (13) denotes the surgeon's idle time from the start of the first surgery until the start of the last surgery. Equation (14) shows the patient waiting time. Since each patient's surgery cannot be started until the surgeon arrives, the patient's waiting time is calculated from the surgeon's arrival until the start of the surgery. Equations (15) and (16) prevent overlapping of the surgeries of one surgeon in different rooms. These two equations guarantee that the two patients p and q

do not overlap if they are both operated on by the same surgeon. The binary variable u_{pq} ensures that only one of the equations is met each time. In these two equations, the first, second, and third terms determine patient $q(p)$'s surgery start time, patient $p(q)$'s surgery completion time, and whether patient q is operated on before patient p, respectively. The value of M_2 is assumed to be T, since the time between two surgeries cannot be more than the total number of time slots. Equations (17) to (22) determine the domain of the decision variables.

The rescheduling problem's objective function is shown in equation (23). This objective function consists of two terms: the first term denotes the scheduling problem's objective function, and the second term minimizes deviations from the start times

acquired from the scheduling problem. Delays in surgery start times lead to waiting times for patients and surgeons. Therefore, start times later than the initial start times are added as penalties in the objective function. Furthermore, patients require some tests before their surgeries. Therefore, surgery start times earlier than the initial start times are also penalized.

$Start_p$:	Surgery start time of patient p in the scheduling problem $p=1,...,P$
λ:	Coefficient of importance of deviations from the scheduling problem start times
δ:	Coefficient of importance of scheduling problem objective function

The value of $\lambda + \delta$ in the rescheduling problem objective function equals 1.

$$Minimize\ Obj_2 = \delta\ Obj_1 + \lambda \left(\frac{\sum_{p=1}^{P}\sum_{t=St_{S_{(p)}}}^{T-tp+1}\sum_{r=1}^{No}|t-Start_p|X_{prt}}{\sum_{p=1}^{P}(T-t_p-Start_p+1)} \right) \tag{23}$$

Equation (24) removes the absolute term from the objective function based on the problem's decision variable type.

$$Minimize\ Obj_2 = \delta\ Obj_1 + \lambda \left(\frac{\sum_{p=1}^{P}\sum_{r=1}^{No}(\sum_{t=St_{S_{(p)}}}^{Start_p-1}(Start_p-t)X_{prt})}{\sum_{p=1}^{P}(T-t_p-Start_p+1)} + \frac{\sum_{t=Start_p}^{T-t_p+1}(t-Start_p)X_{prt}}{\sum_{p=1}^{P}(T-t_p-Start_p+1)} \right) \tag{24}$$

As seen from equation (24), the second term of the objective function splits into two parts. The first part is for patients whose rescheduled start times are earlier than their start time of scheduling problem. In contrast, in the second part, the start times of the rescheduled surgeries are later than the initially scheduled start times. Therefore, equation (24) is utilized to solve the problem instead of equation (23).

5. Heuristic Algorithms

Heuristic algorithms are used for solving the OTSRP problem due to the complexity arising from being strongly NP-hard. Two heuristic algorithms are proposed to solve the scheduling problem, and one algorithm is proposed to solve the rescheduling problem. These heuristic algorithms are described below.

5.1. The heuristic algorithm based on mathematical programming

This algorithm is a constructive heuristic algorithm called heuristic-based mathematical programming (HBMP). As mentioned before, assumptions, including surgeon ready times, patient priority, and a multi-stage integration level, are considered simultaneously in OTSRP. Furthermore, the problem's objective function consists of three different terms, which are considered simultaneously: patient satisfaction, surgeon satisfaction, and operating room staff satisfaction. Therefore, the assumptions and the objective function create a situation in which it is inevitable that a method for acquiring the proper schedule must be utilized. To this end, an HBMP algorithm is proposed. In this algorithm, a pre-specified solution string representing a sequence of patients is produced. Then patient scheduling is acquired by mathematical programming based on the pre-specified solution string sequence. The general structure of the HBMP algorithm consists of the following two steps:

1- Generating the pre-specified solution string
2- Solving the operating room scheduling based on sequence (OTSBS) mathematical programming to obtain the schedule of the pre-specified solution string

The pseudocode of the HBMP algorithm is illustrated in Fig. **1**. The pseudocode of generating the pre-specified solution string is presented in Figure 2.

In the HBMP method, first, a solution string is produced. This string is used as the input of the next step. The patient priority of each surgeon is acquired using the solution string of the first step.

Minimize Obj$_1$

    s.t.

    Equation
    (2) - (14)*,* (17) - (22)

$$\sum_{r=1}^{N_o} \sum_{t=St_{s_{(q)}}}^{T-t_q+1} tX_{qrt} - \sum_{r=1}^{N_o} \sum_{t=St_{s_{(p)}}}^{T-t_p+1} (t + t_p)X_{prt} \geq 0, p, q = 1, \dots, P|p < q, s_{(p)}=s_{(q)}, u_{pq} = 1 \qquad (25)$$

The differences between the OTS and OTSBS models are the u_{pq} transformation to the parameters and changing the corresponding equations. Since the priority of each surgeon's patients is specified in the acquired solution string, this variable turns into a parameter. Equation (25) ensures that patient *q*'s surgery start time is greater than or equal to patient *p*'s surgery finish time, in case both patients *p* and *q* are assigned to the same surgeon and patient *p* is scheduled before patient *q* (u_{pq}=1).

5.2. The heuristic algorithm based on mathematical programming with local search

A heuristic algorithm based on mathematical programming with local search (HBMPLS) is an improvement algorithm. The input solution of an improvement algorithm might be a constructive algorithm solution or a random one. An HBMP solution is used as the initial solution for HBMPLS. This algorithm generates better solutions for the problem by varying the pre-specified string. The general structure of this algorithm consists of two stages. The first is performing the HBMP algorithm and a neighborhood search. The second is solving a mathematical programming model called ORSBS. The pseudocode of the HBMPLS algorithm is presented in Fig. **3**.

The first stage is to acquire an initial solution

This priority is utilized in the mathematical programming model as the initial sequence, and then the mathematical model is solved. In other words, the sequence is specified in the solution string, and schedule determination is done by OTSBS mathematical programming. Therefore, the binary variable corresponding to the patient sequence of each surgeon transforms into a parameter in OTS mathematical programming. Therefore, the solution of the model is simplified and takes less time. The OTSBS mathematical programming is as follows:

using the HBMP algorithm. The objective function value and the string acquired from the HBMP are saved as the best objective function value and best string. In the next stage, the neighborhood of the current string is obtained by a local search method, and the best objective function value and its related string are used as ORSBS input. The local search method utilized in this stage is single-insert. In this method, one of the surgeon's patients is selected each time and inserted before other patients.

In the ORSBS model, an operating room scheduling problem has been solved to simplify and increase the solving speed. For this purpose, the equations and variables corresponding to the recovery stage are removed. In this case, a heuristic procedure is used for assigning the recovery beds. Recovery bed assignments are such that a patient is assigned to one of the empty recovery beds from surgery completion time to recovery completion time. If no empty bed is available, the surgery start time is postponed until one becomes available.

Furthermore, insertion of patients is performed to improve the objective function value. For this purpose, equation (26) is added to the ORSBS model. The ORSBS model is as follows:

Minimize Obj$_1$

    s.t.

Equations
(2),
(4), (6),
(8) - (14), (17) - (22)

$$Obj_1 \leq Z^* \tag{26}$$

As shown above, the ORSBS model is like the OTSBS model, the difference being that the recovery bed equations are removed in ORSBS. Furthermore, equation (26) is added to the model as a redundant equation. Equation (26) ensures that the current iteration objective function value is acceptable if it is lower than the best-found value.

It should be noted that the possibility of insertion is investigated before inserting a patient. This means that placing a normal or infected patient before a child is impossible. Also, an infected patient cannot be placed before a normal one. In other words, the two selected patients are always of the same type. In the following, allowed insertions are defined as insertions of patients with the same surgery types.

Each time the string is varied, the new string is compared with the previous one. If no difference is shown in the string, the next insertion is performed. The obtained objective function value is compared with the best-found value. If the objective function value is better than the best-found value, the current string and best-found objective function value are replaced with the new string and objective function value. This procedure continues until all the allowed insertions are investigated. The pseudocode of the described local search is illustrated in Figure 4.

In this study, six types of local searches are adopted in the HBMPLS, and six variants of this algorithm are investigated. HBMPLS1 is the general case of the algorithm, i.e., the stop criterion is completing all the insertions. In HBMPLS2, besides the general algorithm case, the stop criterion is assumed to be P non-improved iterations. In HBMPLS3, only patients having operating room preferences are inserted. In HBMPLS4, two conditions are considered: inserting preferred patients; and aborting after P consecutive iterations without improvement.

It should be noted that the best value equation, i.e., equation (26), is removed from the ORSBS model in these four algorithms. In order to show the effect of the equation (26) on quality and solution time, it is only considered in HBMPLS5 and HBMPLS6. HBMPLS5 is the same as HBMPLS1, except that equation (26) is added to

the ORSBS model. Finally, HBMPLS6 is the same as HBMPLS1, together with the best value equation (Equation (26)), the stop criterion, and insertion of preferred patients. According to the initial investigations and the results obtained by solving these six algorithms, HBMPLS5 proved to have the best performance compared to the other cases that are shown in Tab. 2. Therefore, this algorithm is used for comparisons in the following.

5.3. Rescheduling heuristic algorithm

The algorithm used for solving the rescheduling problem is called the rescheduling heuristic algorithm (RHA). The initial scheduling is acquired using the best proposed heuristic algorithm for the scheduling problem in this algorithm. Next, the rescheduling is performed three times for the first three scenarios proposed by Van Essen et al. [18]. The difference between the RHA and the Van Essen et al. [18] scenarios is that the latter perform rescheduling only once during each working day, while in the RHA, the rescheduling is performed three times in each working day, such that the scheduled plan and the actual plan are compared for three time slots, 10 (10:00 a.m.), 18 (12:00 p.m.), and 26 (2:00 p.m.), and rescheduling is performed if necessary. In the rescheduling problem, due to the objective functions, especially patient waiting time and surgeon idle time between surgeries, the shorter actual surgery duration time may improve the scheduling. In addition, if the actual surgery duration time is more than the planned surgery duration time, it can also have a great impact on schedule. As a result, rescheduling is required when the planned and actual surgery durations differ. Rescheduling is acquired using the operating room rescheduling based on sequence (ORRBS) model. Finally, the patients are assigned to the recovery beds. The ORRBS model is the same as the ORSBS model. Equation (26) of the best value in the ORRBS model is eliminated with a difference. In addition, the objective function of the problem is also obtained according to equation (24). The ORRBS model is illustrated below:

Minimize Obj_2

s.t.

Equations
(2),
(4), (6),
(8) - (14), (17) - (22), (25)

As an example, for rescheduling in the time slot at 10 (10:00 a.m.), in the ORRBS model, the actual surgery duration time is assumed to be the surgery duration time if the patient's surgery is finished before time slot 10 (10:00 a.m.). A patient's surgery may be finished before time slot 10 (10:00 a.m.) in the initial scheduling, but the surgery is still ongoing in actual time. The surgery duration of this patient is assumed to be the difference between time slot 10 (10:00 a.m.) and the patient's start time. Other patients' surgery durations do not change. The RHA pseudocode is illustrated in Fig. **5**.

6. Computational Results

In this section, first, the data used for testing the proposed methods are presented. Then, the result of solving the problem utilizing these data in the OTS model and heuristic algorithms is presented, and comparisons are performed. The experiments were run on a computer with 6GB of RAM and an Intel Core i7 3.99 GHz CPU. Furthermore, the heuristic algorithms were coded by Visual C#, and the mathematical programming models were solved using CPLEX 12.4.

6.1. Problem data

In this problem, real data for 90 days from Khanevadeh Hospital, located in Isfahan, Iran, are investigated. The daily list is delivered to the operating room at the beginning of the day in this hospital. The operating room scheduling agent schedules the day according to the daily patient list, surgery types, and related surgeons.

The schedule prepared at the beginning of each day is called the Plan of Khanevadeh Hospital (PKH). Only the prepared plan for each day is recorded in the hospital's documentation. Therefore, it is required that the PKH be prepared based on the operating room scheduling agent's opinion and documentation of the performed surgeries. The performed plan for each day is called the Actual of Khanevadeh Hospital (AKH). The AKH and the PKH are prepared according to data available in the operating room and patient recovery offices and the scheduling agent comments. During the preparation of the PKH, the five following points are ensured.

1- The surgery duration of each patient according to the surgeon is evaluated in the PKH based on the opinion of the hospital's operating room scheduling agent.
2- The sequence of patients of each surgeon does not vary in the AKH.
3- If the surgery duration in the PKH is longer

than that in the AKH, because of overestimation, the patient's operating room is changed unless no other option is available. If the other operating rooms are occupied, the next surgeries are performed in the same operating room with a delay.
4- The surgeon's preparation time is the same for both AKH and PKH.
5- Since the surgeon's preparation time is the same in the AKH and the PKH, only the surgeries for patients by the same surgeon are started earlier if the surgery duration in the PKH is shorter than that in the AKH because of underestimation.

Considering the above-mentioned points, the PKH plan is prepared according to the AKH for 90 days. Some specifications of the investigated data are presented in Tab. **3**.

6.2. Scheduling problem results

The values of the objective function coefficients affect the scheduling problem solution. Therefore, 90 existing problems are solved in 10 distinct scenarios, considering the objective function coefficients. An attempt is made to investigate all the possible cases for the impact of the objective function coefficients in these 10 scenarios. The scenarios, along with the corresponding objective function coefficient values, are shown in Tab. **4**.

It should be noted that the OTS results are obtained within a CPU time limit of 3600 seconds. Tab. **5** presents the results obtained by solving the OTS for 10 distinct scenarios. Tab. **5** also shows the improvement percentage of the solution with the OTS relative to the solution with Khanevadeh Hospital (KH). Also shown is the number of problems that reached the optimal solution with OTS in 3600 seconds and the number of problems that are not optimally solved or have reached no feasible solution within 3600 seconds. The number of problems with the same objective values to KH represents the number of solutions in which the objective function value is equal for OTS and KH. It should be noted that "average solution time" corresponds to optimally solved problems via OTS in 3600 seconds.

As shown in Tab. **5**, the minimum improvement percentage of the OTS model over KH is 0%, i.e., some of the KH problems are optimally solved. The minimum average improvement percentage corresponds to scenario G10, yielding the lowest maximum improvement among all scenarios. However, the OTS yields a 26.17% improvement over KH. In other words, the OTS yields optimized results relative to KH even in the worst

cases. According to Tab. **5**, 86% of the problems are optimally solved using the OTS mathematical model. This model also shows relatively high solution time. Therefore, using methods other than the OTS model is justified.

The efficiency of the heuristics is compared using the relative percentage deviation (RPD) of the objective function values. The average RPD value for the ith problem solved by the jth algorithm is calculated using equation (27).

$$RPD_{ij} = \frac{TObj_{ij} - BTObj_i}{BTO_i} \times 100 \qquad (27)$$

In equation (27), $TObj_{ij}$ represents the objective function value of the ith problem solved by the jth algorithm. OTS, HBMP, and HBMPLS5 are numbered 1, 2, and 3, respectively. $BTObj_i$ is the best overall objective function value already obtained for the ith problem. In problems with small dimensions solved by mathematical programming, $BTObj_i$ equals the objective function value obtained by the mathematical model. In problems with larger dimensions that mathematical programming cannot solve, this value equals the best objective function value obtained by heuristics. The results obtained from the solution methods according to the RPD values are shown in Tab. **6**. It should be noted that the RPD values presented in Tab. **6** correspond to problems that are optimally solved by mathematical programming within 3,600 seconds. In addition, the average solution times are compared in Tab. **6**.

According to Tab. **6**, the HBMPLS5 heuristic algorithm shows only 0.02% deviation relative to the best value. This is due to investigating the various solution strings' scheduling. Therefore, this algorithm shows less deviation than the heuristic based on mathematical programming (HBMP). In addition to RPD, the solution time is also important for comparing solution methods. According to Tab. **6**, the solution times of the two heuristic algorithms are approximately the same for all scenarios. Furthermore, although the HBMPLS5 algorithm has more solution time than HBMP, it yields a solution equal to or close to the optimal solution in a shorter time than the OTS.

The other comparison criterion for the solution methods is improvement percentage relative to KH. Since the major goal of this study is to improve the hospital's performance, this criterion is of considerable significance. The average

improvement percentage relative to KH is presented in Figure 6. Additionally, the average percentage improvement of each term of the objective function is shown in Tab. **7**. It should be noted that the average percentage improvement in Tab. **7** and Figure 6 is presented for the problems solved by the OTS model optimally. The improvement percentages of all the solution methods relative to the hospital under investigation show approximately the same trends and equal values.

6.3. Performance of heuristics in cases of increasing the number of patients and reducing the number of recovery beds

This section aims to investigate the performance of the heuristic algorithms for two scenarios: increasing the number of patients and reducing the number of recovery beds. It could happen that not all eight recovery beds are available during the day for some reason, so two cases of 4 and 6 recovery beds are analyzed. The data generation procedure corresponding to the scenario of increasing the number of patients is described below. Then, the generated problems are solved and analyzed using the two heuristic algorithms.

To generate sample problems with larger dimensions, the data of Khanevadeh Hospital are utilized. The number of patients increases by 10% and 25% each day. To increase the number of patients by 10%, all the patients for a day are arranged in a set and numbered in sequence. This set is called "*Patients*." In the following, $0.10 \times P$ denotes the number of patients that must be added. Then $0.10 \times P$ number of random integers are generated within the interval of $[0, P]$. The generated numbers are put in a set called "*SelectedNumber*." After that, one patient is added to "*Patients*" before each "*SelectedNumber*" generates numbers.

According to the previous patient, the surgery and the recovery time corresponding to each added patient are determined. The surgery duration times of these patients are estimated such that the minimum and maximum surgery duration times are calculated for each surgeon according to the available data. These two values are called $MinDuration_i$ and $MaxDuration_i$, in which $i = 1, \ldots, S$. The surgery duration time of each added patient j is estimated within the interval $[MinDuration_{Sj}, MaxDuration_{Sj}]$.

For specifying the patient type and the operating room preferences, three binary parameters are defined: child patient surgery type, infected patient surgery type, and a patient whose surgical

team has an operating room preference. They are denoted by *Child$_p$*, *Infect$_p$*, and *Prefer$_{rp}$*, respectively. *Child$_p$* equals 1 if patient p is a child patient; otherwise, it equals 0. *Infect$_p$* equals 1 if the patient p is an infected patient; otherwise it equals 0. If patient p is not a child or an infected patient, they are considered as a normal patient. *Prefer$_{rp}$* equals 1 if the surgical team of patient p has an operating room preference r; otherwise it equals 0. In the following, integers are generated randomly from $\{0,1\}$ for each parameter. Finally, each of the parameters that equals 1 specifies the priority and preference of each patient. For example, it is assumed that one instance of parameters with 5 patients and 3 operating rooms are generated as follows:

$Child = [0,1,0,0,0]$
$Infect = [0,0,0,0,0]$
$$Prefer = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

As is observed from the above, patient #2 is a child patient. Also, no patient is of the infected type. Furthermore, the surgical team of patient #1 has operating room preference #2, and the surgical teams of patients #3, #4, and #5 have operating room preference #3. It should be noted that the corresponding line of operating room preference #1 is assumed to be 0 for all patients all the time since this preference does not exist in Khanevadeh hospital's documentation.

Some problems may be infeasible due to the random nature of the generated problems, i.e., not all the scheduling of patients could be performed within one day. In other words, the number of patients and their surgery duration times lead to a situation in which the scheduling becomes impossible within the available planning horizon. The number of infeasible problems generated due to increasing the number of patients obtained from the solutions is reported for each algorithm.

Tab. **8** lists the average results obtained by solving the 10 available scenarios using the two heuristic algorithms with increases of 10% and 25% in the number of patients and 4 and 6 recovery beds. The infeasible problem column lists the number of problems that could not be scheduled within the planning horizon. Also, unsolved problems are problems that are not solved within the CPU time limit. It should be noted that solved problems are assumed to be solved feasibly by the OTSBS and ORSBS models. Furthermore, the average solution time corresponds to problems solved by both

algorithms.

As shown in Tab. **8**, the HBMPLS5 algorithm performs more efficiently for all cases than the HBMP algorithm. This is because more than one string is scheduled in the HBMPLS5 algorithm. This scheduling leads to an increase in solution time for the HBMPLS5 algorithm compared with HBMP. However, this algorithm showed the best results for all cases. That is, the HBMPLS5 algorithm was more efficient than the other algorithm. Hence, the HBMPLS5 algorithm could be used for the two scenarios of increasing the number of patients and reducing the number of recovery beds.

6.4.   Rescheduling problem results

Since the objective function coefficients affect the problem solution, five scenarios are considered for solving the rescheduling problem. Tab. **9** lists these scenarios.

The scheduling problem objective function coefficient value within the second term of the rescheduling objective function equals the best value of the objective function importance coefficient. The best objective function coefficients correspond to the scenario in which the highest improvement percentage of the case study hospital's sample problems occurs. According to the obtained results, the best value of the objective function coefficient corresponds to the first scenario (G1). Among the 10 defined scenarios in the previous sections, this scenario yielded the highest average and maximum improvement relative to Khanevadeh Hospital. Therefore, the objective function coefficients of the G1 scenario are used for solving the rescheduling problem in the following.

In Tab. **10**, the average (max) solution time is given according to 3 time slots and 5 proposed scenarios. As shown in Tab. **10**, except GR1, the average solution time in all groups is approximately the same and less than one second.

Tab. **11** presents the results of the rescheduling problem solved by the RHA. The HBMPLS5 algorithm solution is used as the initial scheduling for solving by the RHA algorithm. Multiple criteria are utilized for comparing the initial scheduling and the rescheduling. The number of deviations of patient start times from the initial scheduling and the number of deviations in the number of operating rooms in relation to the initial scheduling are among the criteria used for comparing the two schedules. Furthermore, the number of patients whose

surgical teams had operating room preferences that are not operated on in the preferred operating rooms is considered as another comparison criterion.

In the first scenario (GR1), only the scheduling objective function matters. Therefore, according to Tab. **11**, this scenario has the highest deviation relative to start times among all the scenarios. In contrast, this scenario yields the lowest deviation relative to the number of operating rooms and operating room preferences. This is due to the high value of the coefficient of the scheduling objective function. In the fifth scenario (GR5), only the deviation from the start times matters. Therefore, the GR5 scenario yields the minimum deviation from the start times, except for time slot 10 (10:00 a.m.), and the highest deviation from the number of operating rooms and operating room preferences. The deviation is approximately similar for the three scenarios of GR2, GR3, and GR4, in which both objective function terms are important.

7. Conclusions and Suggestions

In this paper, the operating room scheduling and rescheduling problems are analyzed to minimize the normalized weighted sum of patient waiting time, surgeon idle time, and penalties for deviation from operating room preferences for a hospital in Iran. This study simultaneously considered the satisfaction of patients, surgeons, and operating room staff. Furthermore, surgery time is assumed to be different for each surgeon. A mixed-integer programming model and heuristic algorithms are proposed for solving the scheduling problem. The heuristic algorithm based on mathematical programming with local search yielded optimal or near-optimal results within a shorter time relative to mathematical programming, such that the relative deviation percentage of 1.34% decreased to 0.02%. The algorithm's performance was investigated for the two scenarios of increasing the number of patients and reducing the number of recovery beds. The HBMPLS5 algorithm performed the best among all the solution methods according to the obtained results. This algorithm yielded a nearly zero average relative deviation for all cases of the two investigated scenarios.

Future research on this topic could examine operating rooms that multi-task in crisis conditions, such as reducing recovery beds due to disruptions. Furthermore, some recovery processes could be performed in the operating room, leaving the remaining part for the recovery process. In the OTSRP problem, only the scheduling of operating rooms and recovery beds is performed. The scheduling of the surgeries could also be performed weekly. According to the variations from the beginning of the week, rescheduling could be performed at the beginning of each day. The patients' schedule could also be updated during each day, in addition to the rescheduling performed at the beginning of the day.

8. Acknowledgments

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Appendix

Tab. 1. Some articles in the operating room scheduling literature

Author(s)	Patient Integration				Model		Constraint			Objective function				Solution method				Decision making		
	elective	non-elective	single-stage	multi-stage	deterministic	stochastic	operating room	deadline	source	waiting time	makespan	cost	other	exact	heuristic	metaheuristic	simulation	planning	scheduling	rescheduling
Van Essen et al. [18]	✓	✓		✓	✓		✓					✓					✓			✓
Stuart and Kozan [10]	✓	✓	✓	✓									✓	✓						✓
Zhao and Li [17]	✓		✓	✓			✓					✓		✓					✓	
Xiang et al. [20]	✓		✓	✓						✓	✓					✓			✓	
Wang et al. [16]	✓		✓			✓				✓	✓	✓				✓			✓	
Addis et al. [12]	✓		✓			✓		✓		✓						✓			✓	✓
Heydari and Soudi [15]		✓	✓				✓				✓						✓	✓	✓	
Kroer et al. [21]	✓	✓	✓				✓	✓				✓				✓		✓	✓	
Vali-Siar et al. [22]	✓			✓			✓	✓		✓	✓			✓	✓	✓	✓		✓	✓
Akbarzadeh, et al. [23]	✓		✓		✓		✓	✓				✓		✓	✓				✓	✓
Kamran et al. [13]	✓	✓	✓		✓		✓	✓	✓	✓					✓				✓	✓
Zhu et al. [14]	✓		✓		✓					✓		✓			✓				✓	
This research	✓		✓	✓			✓				✓			✓	✓	✓			✓	✓

```
1   Input Number of surgeons, Number of Patients, Patients of each
2   S = Generate Pre-specified String ();
3   Solve OTSBS (S)
4   S* = S
5   Z* = f(S)
6   Output Z*, S*
```

Fig. 1. Pseudocode of HBMP

```
1    Input number of surgeons, number of patients, patients of each surgeon
2    Surgeon = Sort surgeons based on ascending order of their preparation time
3    For i=1 ….. S
4        Choose Surgeons_i (Surgeons_i is i-th surgeon in Surgeons)
5        Child_i = Sort child patients of surgeon i based on ascending order of
         surgery duration
6        Add Child_i to the last of Pre-specified String
7        Normal_i = Sort normal patients based on ascending order of surgery
8        Add Normal_i to the last of Pre-specified String
9        Infect_i = Sort infect patient based on ascending order of surgery duration
10       Add Infect_i to the last of Pre-specified String
11   End For
```

Fig. 2. Generating the pre-specified solution string

```
1    Input: number of surgeons, number of patients, patients of each surgeon
2    Pre-specified String = Generate Pre-specified String ();
3    Solve OTSBS (Pre-specified String)
```

4    $S^* = $ *Pre-specified String*
5    $Z^* = f(Initial\_String)$
6   *LocalSearch*(Z^*, S^*);
7   Output: Z^*, S^*

Fig. 3. Pseudocode of HBMPLS

1   Input: planned surgery duration, number of patients (P)

2   For $i=1,…,P$

3     $DurationTime_i = PSD_i$; ($PSD_i = $ Planned Surgery Duration time of i-th patient)

4   End for

5   Solve *HBMPLS5 Algorithm* (*Duration Time*);

6   *Set Time For Rescheduling*={10,18,26};(10=10 a.m. , 18=12 p.m. & 26=2 p.m.)
   (TimeForRescheduling$_i$=i-th time in TimeForRescheduling)

7   For $t=1,…,3$

8   Set *Rescheduling*=false;

9   For $i=1,…,P$

10   If $Start\ time_i + ASD_i < TimeForRescheduling_t$ (ASD_i=Actual Surgery Duration time of i-th patient)

11       $DurationTime_i = ASD_i$;

12   *Rescheduling*= true;

13   End If

14   If $Start\ time_i + PSD_i < TimeForRescheduling_t$ & $Start\ time_i + ASD_i > TimeForRescheduling_t$

15       $DurationTime_i = TimeForRescheduling_t - Start\ time_i$;

16   *Rescheduling*= true;

17   End If

18   End For

19   If *Rescheduling*= true

20   Solve *ORRBS(DurationTime)*;
      Assign recovery beds

21   Calculate ND_t (*ND*=Number of Deviation from *InitialScheduling*, ND_t=t-th *ND*)

22   End If

23   End For

24   Output: ND_t

Fig. 4. Pseudocode of local search

Tab. 2. Results from solving HBMPLS algorithms

Average	HBMPLS 1	HBMPLS 2	HBMPLS 3	HBMPLS 4	HBMPLS 5	HBMPLS 6
Solution times (second)	45.36	22.57	19.40	19.12	18.25	17.90
The number of problems that are equal to the best value in five scenarios	90.0	89.8	89.6	89.4	90.0	89.5

1   Input: planned surgery duration, number of patients (P)

2   For $i=1,…,P$

3     $DurationTime_i = PSD_i$; ($PSD_i = $ Planned Surgery Duration time of i-th patient)

4   End for

5   Solve *HBMPLS5 Algorithm* (*Duration Time*);

6 *Set Time For Rescheduling={10,18,26};(10=10 a.m , 18=12 p.m. & 26=2 p.m.)*
   (TimeForScheduling$_i$=i-th time in TimeForRescheduling)

7 For *t*=1,…,3

8   Set *Rescheduling*=false;

9   For *i*=1,…,*P*

10   If *Start time$_i$* + *ASD$_i$* < *TimeForRescheduling$_t$(ASD$_i$*=Actual Surgery Duration time of *i*-th patient)

11   *DurationTime$_i$* = *ASD$_i$*;

12   *Rescheduling*= true;

13   End If

14   If *Start time$_i$* + *PSD$_i$* < *TimeForRescheduling$_t$* &
           Start time$_i$ + *ASD$_i$* > *TimeForRescheduling$_t$*

15   *DurationTime$_i$* = *TimeForRescheduling$_t$*-*Start time$_i$*;

16   *Rescheduling*= true;

17   End If

18   End For

19   If *Rescheduling*= true

20   Solve *ORRBS(DurationTime);*
           Assign recovery beds

21   Calculate *ND$_t$* (*ND*=Number of Deviation from *InitialScheduling*, *ND$_t$*=*t*-th *ND*)

22   End If

23 End For

24 Output: *ND$_t$*

Fig. 5. Pseudocode of RHA

Tab. 3. Specifications of the investigated data

Day No.	Holiday	The number of						Day No.	Holiday	The number of					
		Surgeries	Surgeons	surgery type			room preference				Surgeries	Surgeons	surgery type			room preference	
				Normal	Child	Infected	#3	#2					Normal	Child	Infected	#3	#2
1	-	12	7	12	0	0	1	2	46	✓	13	8	13	0	0	2	1
2	-	16	10	15	0	1	3	5	47	-	13	8	13	0	0	2	3
3	-	11	6	11	0	0	2	3	48	-	16	8	16	0	0	2	6
4	-	10	7	10	0	0	1	2	49	-	15	9	14	1	0	3	1
5	✓	17	5	15	2	0	3	0	50	-	9	4	9	0	0	1	6
6	-	15	10	15	0	0	4	2	51	-	12	8	11	1	0	3	2
7	-	11	3	11	0	0	5	2	52	-	11	6	11	0	0	3	2
8	-	17	10	17	0	0	3	4	53	✓	14	10	13	0	1	4	1
9	-	13	11	13	0	0	2	2	54	-	15	7	14	1	0	3	2
10	-	13	9	12	1	0	3	5	55	-	17	9	15	2	0	5	1
11	✓	14	9	14	0	0	6	5	56	-	14	7	13	1	0	3	2
12	✓	13	8	10	2	1	1	1	57	-	13	8	13	0	0	2	1
13	-	17	12	17	0	0	3	4	58	-	15	6	15	0	0	2	7
14	-	17	7	16	0	1	4	2	59	✓	29	10	17	11	1	3	1
15	-	16	9	16	0	0	5	3	60	-	10	6	10	0	0	2	1
16	-	12	6	11	1	0	6	2	61	-	12	5	12	0	0	3	5

17	-	14	8	14	0	0	1	4	62	-	12	6	12	0	0	5	1
18	-	11	4	11	0	0	3	7	63	-	17	8	14	1	2	0	6
19	✓	15	5	11	4	0	0	0	64	-	12	9	12	0	0	2	1
20	-	19	11	19	0	0	4	3	65	-	6	4	6	0	0	3	0
21	-	14	5	14	0	0	4	2	66	✓	26	5	17	9	0	3	1
22	-	14	7	12	0	2	3	4	67	-	17	8	15	2	0	2	3
23	-	16	9	16	0	0	4	3	68	-	10	5	10	0	0	5	3
24	-	12	8	12	0	0	2	3	69	-	14	7	14	0	0	6	4
25	✓	11	8	11	0	0	3	0	70	-	14	7	13	1	0	0	6
26	✓	11	6	11	0	0	2	2	71	-	11	7	10	1	0	1	2
27	-	10	7	10	0	0	2	2	72	-	10	8	8	0	2	0	1
28	-	10	5	10	0	0	3	4	73	✓	20	9	12	8	0	1	0
29	-	14	7	13	0	1	5	3	74	-	15	10	15	0	0	2	3
30	-	6	5	6	0	0	2	3	75	-	15	7	15	0	0	2	4
31	-	13	8	12	0	1	2	3	76	✓	13	11	12	0	1	1	1
32	-	16	8	16	0	0	0	6	77	-	15	9	14	0	1	2	4
33	✓	22	9	17	5	0	0	2	78	-	12	8	12	0	0	1	3
34	-	13	8	13	0	0	3	3	79	-	10	7	10	0	0	0	4
35	-	18	8	18	0	0	3	4	80	✓	23	7	14	9	0	4	0
36	-	15	10	15	0	0	3	3	81	-	15	9	14	1	0	3	2
37	-	10	6	10	0	0	2	2	82	-	13	7	13	0	0	4	1
38	-	15	9	15	0	0	2	0	83	-	14	9	14	0	0	1	8
39	-	17	8	16	0	1	4	5	84	-	15	9	12	0	3	0	5
40	-	14	9	14	0	0	3	2	85	✓	11	7	11	0	0	2	3
41	✓	13	8	13	0	0	7	1	86	✓	10	7	9	0	1	0	1
42	-	12	6	11	1	0	3	4	87	-	12	6	12	0	0	2	4
43	-	11	7	11	0	0	3	2	88	-	17	7	17	0	0	3	0
44	-	11	5	11	0	0	1	6	89	-	12	5	11	0	1	2	6
45	-	10	8	10	0	0	1	3	90	-	13	9	13	0	0	2	5

Tab. 4. Different scenarios based on the objective

Scenario	Coefficient of patient waiting time (α)	Coefficient of surgeon idle time (β)	Coefficient of operating room preferences (γ)
G1	0.15	0.35	0.50
G2	0.15	0.50	0.35
G3	0.25	0.25	0.50
G4	0.25	0.50	0.25
G5	0.33	0.34	0.33
G6	0.35	0.15	0.50
G7	0.35	0.50	0.15
G8	0.50	0.15	0.35
G9	0.50	0.25	0.25
G10	0.50	0.35	0.15

Tab. 5. Results from solving OTS

Scenario	Number of problems			Average solution time (s)	Improvement percentage relative to *KH*			Number of problems with the same objective
	Optimal	Feasible	Unsolved		Min	Max	Average	
G1	74	4	12	125.07	0.00	92.23	50.77	4
G2	74	1	15	102.81	0.00	89.41	46.18	4
G3	76	3	11	112.28	0.00	87.91	44.50	5
G4	79	3	8	125.46	0.00	78.92	35.55	6

G5	77	0	13	108.85	0.00	78.92	35.86	6
G6	77	3	10	90.74	0.00	84.04	40.49	5
G7	79	3	8	111.18	0.00	63.35	28.16	6
G8	81	2	7	171.01	0.00	75.02	32.98	6
G9	79	2	9	100.65	0.00	66.45	29.44	6
G10	78	5	7	91.80	0.00	62.50	26.17	8
Average	77.4	2.6	10.0	113.99	-	-	37.01	5.6

Tab. 6. The average solution times and the RPD value of OTS, HBMP and HBMPLS5

| Scenario	RPD OTS			Average solution time (s)	RPD HBMP			Average solution time (s)	RPD HBMPLS5			Average solution time (s)
	Min	Max	Avg.		Min	Max	Avg.		Min	Max	Avg.	
G1	0.00	0.00	0.00	125.07	0.00	60.87	3.07	5.30	0.00	0.00	0.00	11.95
G2	0.00	0.00	0.00	102.81	0.00	42.64	2.39	5.26	0.00	0.00	0.00	11.65
G3	0.00	0.00	0.00	112.28	0.00	35.93	2.16	5.13	0.00	0.00	0.00	11.67
G4	0.00	0.00	0.00	125.46	0.00	22.23	1.02	5.04	0.00	5.51	0.07	11.29
G5	0.00	0.00	0.00	108.85	0.00	22.22	1.17	5.03	0.00	5.52	0.07	11.21
G6	0.00	0.00	0.00	90.74	0.00	24.42	1.40	5.06	0.00	0.00	0.00	11.30
G7	0.00	0.00	0.00	111.18	0.00	20.00	0.62	4.91	0.00	0.00	0.00	10.55
G8	0.00	0.00	0.00	171.01	0.00	12.46	0.62	5.03	0.00	1.05	0.01	11.14
G9	0.00	0.00	0.00	100.65	0.00	15.32	0.59	4.86	0.00	0.00	0.00	10.34
G10	0.00	0.00	0.00	91.80	0.00	11.06	0.33	4.91	0.00	0.00	0.00	10.16
Average	-	-	0.00	113.99	-	-	1.34	5.05	-	-	0.02	11.13
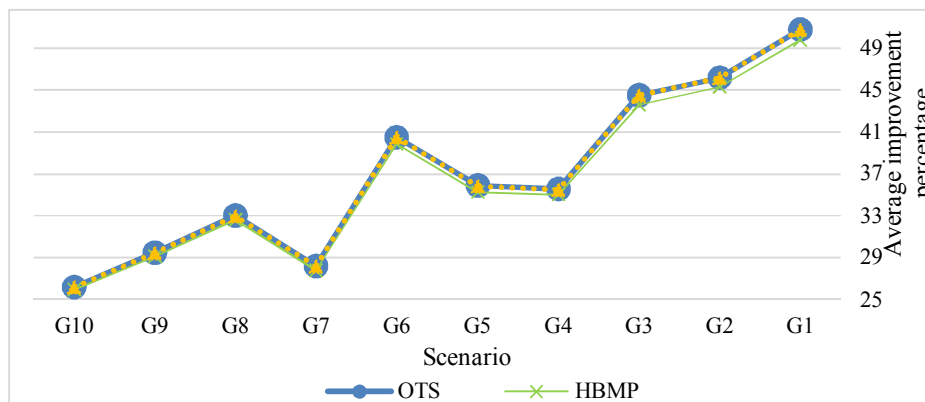
Fig. 6. Average improvement percentage relative to KH based on different scenarios

Tab. 7. Average improvement percentage for each term of the objective function relative to KH based on different scenarios

scenario	OTS			HBMP			HBMPLS5		
	patient waiting time	surgeon idle time	operating room preferences	patient waiting time	surgeon idle time	operating room preferences	patient waiting time	surgeon idle time	operating room preferences
G1	-4.90	0.00	28.64	-7.19	0.00	5.67	-4.90	0.00	20.54
G2	-3.70	0.00	24.24	-4.16	0.00	7.83	-3.70	0.00	14.78
G3	-0.34	0.00	30.85	-0.84	0.00	10.92	-0.34	0.00	16.38
G4	9.13	0.00	31.25	8.49	0.00	15.08	9.14	0.00	21.20

G5	9.46	0.00	28.77	9.06	0.00	16.77	10.00	0.00	25.67
G6	7.93	0.00	22.66	7.70	0.00	13.42	7.93	0.00	22.66
G7	15.81	0.00	26.99	15.84	0.00	18.90	15.81	0.00	23.19
G8	15.72	0.00	34.36	15.79	0.00	15.95	15.91	0.00	15.94
G9	15.70	0.00	24.78	15.72	0.00	16.48	15.70	0.00	17.19
G10	19.09	0.00	29.05	19.48	0.00	19.29	19.09	0.00	20.08

Tab. 8. Average results from solving heuristic algorithms in larger dimensions in 10 existing scenarios

Scenarios		HBMP				HBMPLS5				
			Number of problems		Solution time (s)			Number of problems		Solution time (s)
	RPD	Solved	Infeasible	Unsolved		*RPD*	Solved	Infeasible	Unsolved	
Increasing number of patients	10%	2.92	88.3	1.0	0.7	11.71	0.00>	89.0	1.0	0.0	41.44
	25%	2.16	82.7	5.0	2.3	22.84	0.00>	83.7	5.0	1.3	70.73
Reducing number of recovery beds	6	1.43	90.0	0.0	0.0	10.85	0.00	90.0	0.0	0.0	20.72
	4	12.40	89.8	0.0	0.2	24.63	0.00>	89.8	0.0	0.2	34.15

Tab. 9. The coefficients values of the rescheduling problem objective function

Scenario	Coefficient value of	
	deviation from start time (λ)	scheduling (δ)
GR1	0.00	1.00
GR2	0.25	0.75
GR3	0.50	0.50
GR4	0.75	0.25
GR5	1.00	0.00

Tab. 10. The average (max) solution time from solving RHA

Time slot to reschedule	GR1	GR2	GR3	GR4	GR5
10	0.85 (5.67)	0.58 (4.97)	0.51 (5.09)	0.58 (5.12)	0.60 (8.34)
18	0.93 (4.98)	0.66 (5.39)	0.60 (5.43)	0.64 (5.41)	0.65 (8.43)
26	0.99 (5.43)	0.68 (5.39)	0.61 (5.62)	0.68 (5.57)	0.70 (9.11)

Tab. 11. The number of deviations from the rescheduling problem solved by RHA

Type of deviation	Time slot to reschedule	Average number of rescheduling deviation relative to the initial scheduling in the scenario				
		GR1	GR2	GR3	GR4	GR5
Number of operating rooms	10	1.48	1.53	1.46	1.64	3.67
	18	0.89	1.16	1.14	1.20	2.33
	26	0.42	0.61	0.71	0.63	1.13
Start time	10	2.22	1.13	0.98	1.08	1.12
	18	1.77	0.77	0.79	0.76	0.75
	26	0.91	0.36	0.43	0.34	0.31
Operating room preferences	10	0.54	0.82	0.75	0.87	1.89
	18	0.39	0.72	0.77	0.80	1.31
	26	0.23	0.37	0.43	0.40	0.60