RESEARCH PAPER

A Modified ICA Based Process Monitoring Strategy-Case Study of A Steel Billet Manufacturing Unit

Bhagwan Kumar Mishra[1] & Anupam Das[2*]

ABSTRACT
The article highlights the development of a Non-Gaussian Process Monitoring Strategy for a Steel Billet Manufacturing Unit (SBMU). The non-Gaussian monitoring strategy being proposed is based on Modified Independent Component Analysis (ICA) which is a variant of the widely employed conventional ICA. The Independent Components(IC) being extracted by modified ICA technique are ordered as per the variance explained akin to that of Principle Component Analysis (PCA). Whereas in conventional ICA the variance explained by the ICs are not known and thereby causes hindrance in the selection of influential ICs for eventual building of the nominal model for the ensuing monitoring strategy. Hotelling T^2 control chart based on modified ICA scores was used for detection of fault(s) whose control limit was estimated via Bootstrap procedure owing to the non-Gaussian distribution of the underlying data. The Diagnosis of the Detected Fault(s) was carried out by employment of Fault Diagnostic Statistic. The Diagnosis of the Fault(s) involved determination of the contribution of the responsible Process and Feedstock characteristics. The non-Gaussian strategy thus devised was able to correctly detect and satisfactory diagnose the detected fault(s).

KEYWORDS: *Non-Gaussian; Process monitoring; Strategy; Modified ICA; Steel billet; Manufacturing unit; Hotelling T^2 control chart; Fault diagnostic strategy.*

1. Introduction

Most of the manufacturing processes or facilities are of complex nature and are associated with a large number of Process and Feedstock Characteristics (PFC) which need to be monitored to produce good quality end products. Process monitoring is mainly associated with the detection of faults and their subsequent diagnosis. With a large number of PFCs associated in complex manufacturing processes or facilities, the Multivariate Statistical Process Monitoring (MSPM) [1-5] strategies has been preferred for the detection of fault and their subsequent diagnosis. Multivariate projection based techniques such as Principal Component Analysis (PCA) [6-8] and Partial Least Squares Regression (PLSR) [9] has been found to be useful in the development of process monitoring strategies for

processes associated with large number of PFCs but the traditional MSPM strategies are based on the assumption that the underlying data corresponding to the observations of the PFCs are normally distributed. It has been highlighted that in many process industries [10] laden with large number of PFCs, the normality assumption may not hold good. And as such devising a monitoring strategy for such cases taking into consideration the Normal distribution assumption may lead to production of erroneous results. The last decade has witnessed the development and implementation of the non-Gaussian variant of MSPM strategies to address the detection of fault and their subsequent diagnosis of complex manufacturing processes or facilities. The non-Gaussian counterparts of the traditional MSPM strategies include Independent Component Analysis (ICA) [11-14], Support Vector Data Description (SVDD) [15-17] and Gaussian Mixture Model (GMM) [18-20]. ICA which has been abundantly employed as a non-Gaussian counterpart of the traditional MSPM based strategy, gels well with non-Gaussian data by decomposing the original variables into

*
 Corresponding author: *Anupam Das*
   Anupam.das@ncsu.edu

1. *Department of Mechanical Engineering, National Institute of Technology Patna, Bihar, India.*
2. *Department of Mechanical Engineering, National Institute of Technology Patna, Bihar, India.*

Independent Components and then perform the fault detection and subsequent diagnosis akin to that of traditional PCA based strategies.

ICA is a non-Gaussian technique for transforming observed multivariate statistical data into statistically independent components, which are expressed as a linear combination of observed variables. The notion of ICA may actually be seen as an extension of the PCA, which can only impose independence up to the second order and, consequently, defines directions that are orthogonal. The ICA may reveal more meaningful information in the non-Gaussian data than PCA, because ICA not only decorrelates the data based on second-order statistics but it also reduces higher order statistical dependencies. GMM is a probabilistic modeling approach for non-Gaussian MSPM, which is based on the assumption that the underlying data describing the concerned manufacturing process or any system can be aptly described by a multiple of local linear models. An iterative Expectation-Maximization (EM) algorithm is used for estimating the parameters of the GMM model. SVDD is a non-Gaussian MSPM technique that does not require any assumption regarding the distribution of the underlying data. SVDD aims to find a minimum-hypersphere that covers most of the data and thereafter this hypersphere is used to monitor whether the given data is in control or not.

The majority of the non-Gaussian process monitoring strategies are mainly based on ICA and its various variants. ICA have a simple model structure and its mathematical formulation is easy to understand as compared to the other non-Gaussian techniques. On the other hand, in GMM it is difficult to determine the optimum number of local linear models used for describing the data associated with the manufacturing process or the system under consideration. Whereas in SVDD determining the suitable kernel function remains an issue and it is also prone to produce false alarms pertaining to fault detection owing to establishment of tighter control limit.

The fundamental problem associated with ICA is that the extracted ICs (Independent Components) are not arranged as per the amount of variance explained in opposition to conventional PCA where the components arranged as per decreasing order of variance explained. Thereby it becomes difficult to identify the ICs containing the maximum information which is the one with greater variance. As a consequence less significant ICs may get included in building of

the nominal model of the ensuing process monitoring strategy thereby reducing the efficacy of the monitoring strategy as a whole. A variant of ICA termed as Modified ICA [21] employs PCA to extract the components and reduce the dimension of the underlying data and thereafter, conventional ICA is used to extract the ICs which are arranged in decreasing order of variance explained. Thereby in building of the process representation or nominal model, the significant ICs get included which in turn will eventually lead to greater efficacy of the nominal model and thereby also improve the overall efficacy of the monitoring strategy as a whole.

In this article an attempt has been made to develop a process monitoring strategy for a Steel Billet Manufacturing Unit (SBMU). Any typical steel making unit is a complex manufacturing facility associated with multiple process, feedstock and quality characteristics. The characteristics in turn may display a myriad of properties including high cross-correlation, non-Gaussian distribution to name a few. The monitoring strategy being devised encapsulates the properties such as high cross-correlation and non-Gaussian distribution associated with the PFCs for building the process representation or nominal model. The monitoring strategy being devised is based on modified ICA technique whose advantages over the widely employed conventional ICA technique is documented in the preceding paragraph. Hotelling T^2 chart [22] based on modified ICA scores was used for identifying the out-of-control observation(s) which is a representation of the fault. The establishment of the control limit of the Hotelling T^2 chart was determined by Bootstrap procedure [23, 24]. And appropriate fault diagnostic statistic was used for diagnosis of the detected fault.

2. Steel Billet Manufacturing Unit

The case being considered in a Steel Billet Manufacturing Unit (SBMU) engaged in the production of Steel Billets situated in eastern India. Steel billets can be considered as a semi-finished square or round cross-section steel shape with undefined specific length, produced from caster or rolled from a bloom. It can be further rolled to form bar stocks and wire rods. Figure 1 show the process flow diagram of the SBMU which is composed of six major workstations or stages viz. Coke Oven, Sintering Plant, Blast Furnace (BF), Ladle Refining Furnace (LRF) and Continuous Casting Machine (CCM). Coke is produced by coke oven and sintered Iron ore produced by sintering plant are fed

simultaneously in the BF which convert the molten mixture into molten pig Iron. Thereafter, the molten Iron is transferred to LRF which produced desired grade of molten steel by addition of alloys and fluxes. In the next stage the molten metal from LRF is then transferred to the CCM for the production of rectangular ingots or the blooms and it further processed in Rolling Mill to obtain desired steel billets.

A total of 25 PFCs associated with the stages of the SBMU were considered for development of the proposed monitoring strategy. Table1 shows the PFCs involved with different work stages and their description. 180 observations pertaining to the selected PFC characteristics were considered for subsequent analysis.
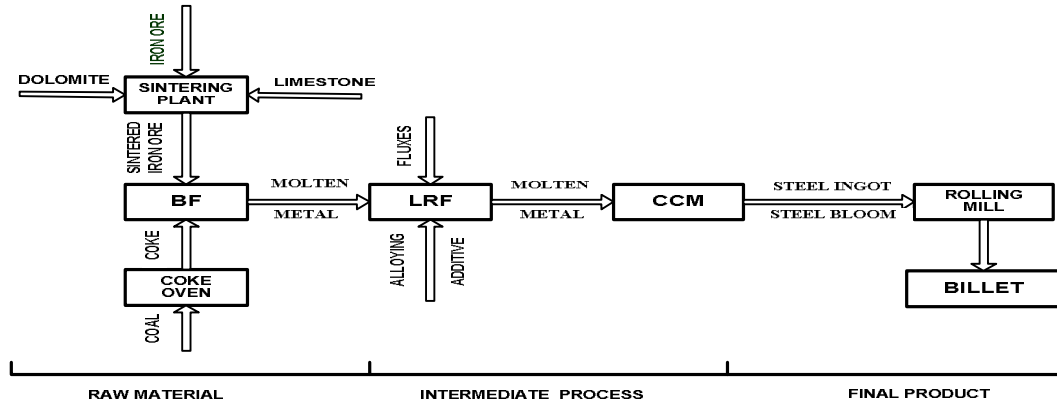

Fig. 1. Process flow diagram of the SBMU

Tab. 1. PFCs involved with major workstation of the SBMU

Workstation	Process and feedstock characteristic	Abbreviation
Sintering Plant(SP)	Heat (Mcal/T) supplied in SP	HSSP
	Compressed air(cum/T) supplied in SP	CASP
	Steam (Kg/T) supplied in SP	SSSP
	Iron ore fines(in%) supplied in SP	IFSP
	Lime stone(in%) supplied in SP	LSSP
	Raw dolomite(in%) supplied in SP	RDSP
	Wet coke (in%) supplied in SP	WCSP
Blast furnace (BF)	Temperature generated in BF	TGBF
	Hot air(m^3/sec) supplied in BF	HABF
	Skip coke(in%) supplied in BF	SCBF
	Nut coke(in%) supplied in BF	NCBF
	Coal dust(in%) supplied in BF	CDBF
	Sintered iron (in %) supplied in BF	SIBF
	Calcium oxide (in%) supplied in BF	COBF
LRF	Carbon (in %) supplied in LRF	CPLR
	Manganese (in %) supplied in LRF	MPLR
	Sulfur (in %) supplied in LRF	SPLR
	Phosphorus (in %) supplied in LRF	PPLR
	Silicon (in %) supplied in LRF	SILR
CCM	Heat (Mcal/T) supplied in CCM	HCCM
	Oxygen (cum/T) supplied in CCM	OCCM
Bloom and Billet	Temperature (in ^{0}C) receive in bloom and billet	TRBB
	Rolling mill speed(RPM) in bloom and billet	RMSB
Continuous Billets	Temperature (in ^{0}C) receive in continuous billets	TRCB
	Rolling mill speed(RPM) in continuous billets	RMSB

3. Modified ICA Based Monitoring Strategy

The article delved into the development of a monitoring strategy based on modified ICA. Modified ICA is an advanced version of conventional ICA which eliminates the associated drawbacks of conventional ICA. In modified ICA algorithm, PCA is employed to estimate the variance captured by the components of the modified ICA termed as Independent Components(IC) and then the ICs are arranged as per the descending order of variance. The main aim of modified ICA is to find a demixing matrix whose form is such that the element of extracted vector become as independent of each other as possible and have been ordered by their variances that are the same as the variances of the corresponding PCs (components of the employed PCA). ICA and its variants are well suited for cases were the data displays non-Gaussian distribution. The normality test for the associated data was carried out via Multivariate Central Limit Theorem (MCLT) which test the hypothesis whether the underlying data is normally distributed. Figure 2 depicts the probability plot and the corresponding Anderson-Darling statistic of the underlying data which is basically the mean of the PFCs observations. As evident from Figure 2, the underlying data is highly non-Gaussian which is validated by the corresponding p-value (< 0.005). In order to determine the correlation between PFCs a correlation analysis was carried out and related p-value were calculated. Table 2 depicts the important correlation coefficient values amongst the PFCs with the corresponding in the parenthesis. As evident from Table 2 appreciable correlation exists amongst various pairs of PFCs thus justifying the employment of modified ICA for development of the proposed monitoring strategy.
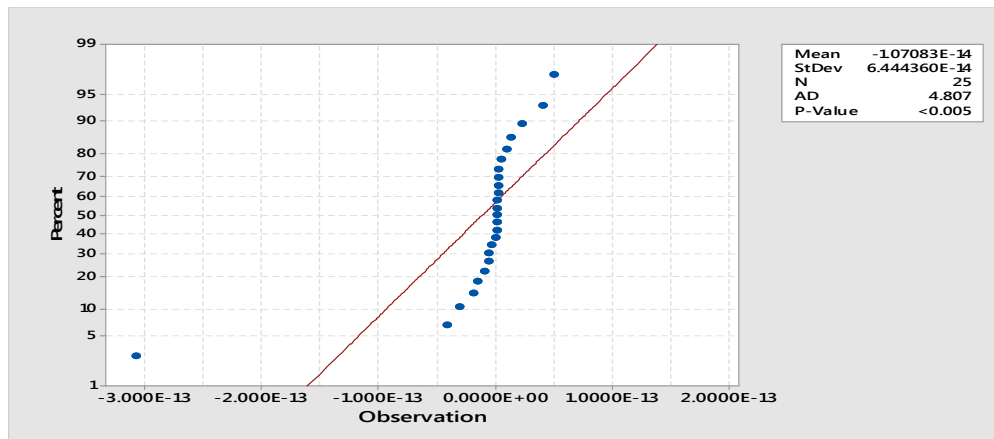

Fig. 2. Probability plot of the mean values of PFCs.

Tab. 2. PFCs with major correlation coefficient with P-values in parentheses

	IFSP	RDSP	SCBF	SIBF	COBF	TRCB
IFSP						
RDSP	0.007 (0.03)					
SCBF	-0.046 (0.055)	0.051 (0.023)				
SIBF	-0.011 (0.892)	-0.103 (0.028)	0.104 (0.0211)			
COBF	-0.254 (0.048)	-0.088 (0.032)	0.686 (0.686)	0.055 (0.469)		
TRCB	-0.066 (0.0495)	0.005 (0.968)	0.016 (0.858)	0.172 (0.019)	0.12 (0.061)	

3.1. Nominal model building

For building of the process representation or the nominal model, modified ICA was employed. The mathematical formulations of modified ICA for estimation of ICs are highlighted below

Consider that the d is a measured variables $x_1, x_2, \ldots, x_d$ can be expressed as a linear combination of m ($\leq$d) unknown independent components and the measured variables have zero mean. The relationship between measured variables and independent components is given below:

$$X = AS \tag{1}$$

Where, $X = [x_1, x_2, \ldots, x_n] \in R^{d \times n}$ is the data matrix.
$A = [a_1, a_2, \ldots, a_m] \in R^{d \times m}$ is the mixing matrix.
$S = [s_1, s_2, \ldots, s_n] \in R^{m \times n}$ is the independent component matrix.

Here the mixing matrix A and independent component S is computed from only the data matrix X, which is a problem of ICA for this need to finding a demixing matrix W whose form is such that the elements of the reconstructed vector Y becomes as independent of each other as possible. The mixing matrix A is the precursor for estimation of the demixing matrix which explains the linear relationship between Independent Components represented by vector Y and Original variables.

$$Y = WX \tag{2}$$

For calculating W, first extract all score components from PCA technique.

$$t = U^T X \tag{3}$$

Where $t \in R^d$ is a score vector, $x \in R^d$ is a column data matrix and $U \in R^d$ is an Eigen vector of covariance matrix $E(xx^T) = U \wedge U^T$, and $E(tt^T) = \wedge$. The scores are actually the weighted linear combination of the original variables.

For the removal of cross-correlation between the variable, the whitening process is used in ICA. The whitening transformation is expressed as bellow

$$Z = \wedge^{-1/2} t = \wedge^{-1/2} U^T X = Q X \tag{4}$$

Where
$Q = \wedge^{-1/2} U^T$ and Z is the normalised score vector, given that $E(ZZ^T) = I$. After the transformation from equation (1) and equation (4)

$$Z = QX = QAS = BS \tag{5}$$
Where

B = QA is an orthogonal matrix and given that $E(ZZ^T) = B E(SS^T) = BB^T = I$.
After that calculate Y from equation (5)

$$Y = B^T Z = B^T QX \tag{6}$$

With the help of equation (2) and (6) the relation between W and B can be expressed as

$$W = B^T Q \tag{7}$$

To calculate B, each column vector bi is randomly initialised and then updated so that the ith independent component $Y_i = (b_i)^T$ has maximum non-Gaussian.

The modified ICA find dominant ICs that satisfied $E(YY^T) = D$ is a diagonal matrix of the eigenvalues such that the elements of Y becomes as independent of each other as possible.

$$Y = C^T Z \tag{8}$$

Where $C \in R^{d \times m}$ and $C^T C = D$ refers that the variance of each element of Y is the same as that of score in PCA. Thus the ICs are arrange according to their variance captured.

3.2. Fault detection and diagnosis

Detection of faults represented by out-of-control observations has been carried out by employment of Hotelling T^2 control chart based on modified ICA scores. The monitoring statistic for the Hotelling T^2 control chart is depicted in equation (9).

$$T^2 = Y^T D^{-1} Y \tag{9}$$

Where Y representing the ICs is obtained from equation (2) and D is the diagonal matrix of the eigenvalues associated with the retained dominant ICs. Bootstrap procedure is used for estimating the control limit of the ensuing T^2 control chart whose overview is depicted in Figure 3. A total of 120 observations were used to estimate the control limit and 60 observations were treated as new observation. Figure 4 depicts the control chart for monitoring of the new observations. As evident, observations numbers 3, 9 and 31 were detected as out-of-control observations which is an indicative of an abnormal behaviour of fault of the ensuing process. Thereafter, diagnosis of the detected faults represented by the out-of-control observations were carried out. The diagnosis of fault is about determination of the contribution of the responsible PFC or combination of PFCs to a

particular out-of-control observations. The diagnosis of fault is achieved by employment of the Fault Diagnostic Statistic [18]. Figures 5(a), 5(b) and 5(c) depicts the contribution plots of the responsible PFCs to the respective out-of-control observations.

The expression of the fault diagnostic statistic representing the contribution of the responsible

PFCs to the out-of-control T^2 statistic measurement is given by question (10).

$$C_j (T^2) = Y^T D^{-1} W_j X_j \qquad (10)$$

Where $C_j (T^2)$ is the contribution of the jth variable to the T^2 statistic X_j is the jth element of X, were X represent the observation matrix of the PFCs and W_j is the jth row of the demixing matrix W.
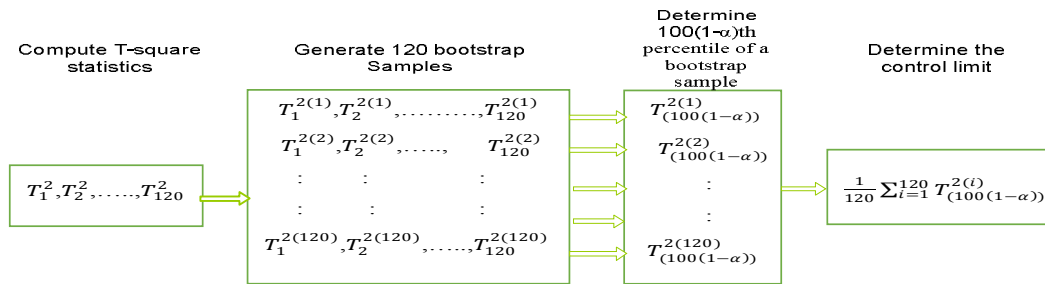

Fig. 3. An overview of the bootstrap procedure for estimation the control limit of T^2 control chart
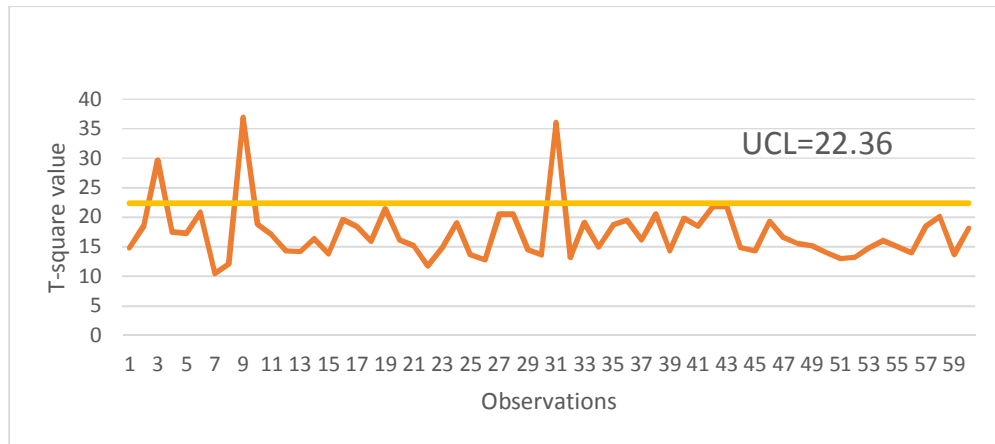

Fig. 4. T^2 control chart for new observations

In Figure 5(a), the major contributors to observation number 3 include PFC SILR (Percentage of Silicon supplied in LRF) whose contribution is 20% followed by SPLR (Percentage of Sulfur supplied in LRF) contributing 17 % and COBF (Percentage of Calcium oxide supplied in BF) whose contribution amount to 16%. Silicon increases tensile strength, hardness, magnetic permeability and electrical resistance of steel. With increase in percentage value (> 0.3%) of silicon in LRF, hardness of steel increases but machinability decreases. An increase in sulfur content on the other hand tends to increase the brittleness and reduce the weldability of the ensuing steel. The recommended percent content of sulfur in steel is

limited to 0.05%. Calcium oxide helps to remove impurities such as phosphorus and sulfur from the steel. However if the Calcium oxide content increases beyond a certain extent it would end up in extracting heat from the BF.

In Figure 5(b), the major contributors to observation number 9 include PFC MPLR (Percentage of Manganese supplied in LRF) whose contribution is 22% followed by CPLR (Percentage of Carbon supplied in LRF) whose contribution amount to 20 %. Manganese increases strength, stiffness, hardness, toughness, hardenability, wear resistance as well as forging and rolling quality of steel. Most steels contain 0.15 to 0.8% manganese, if amount of manganese content exceeds 0.8%, the steel becomes brittle.

Steel is categorized according to its carbon content viz. low carbon steel, medium carbon steel and high carbon steel. Increase in carbon content of the steel increases hardness and tensile strength but reduces ductility and weldability.

As per Figure 5(c), the major contributors to observation number 31 include PFC SCBF (Percentage of Spike coke supplied in BF) whose contribution is 22% followed by CDBF (Percentage of Coal dust supplied in BF) whose contribution amount to 18 %. Coke act as a fuel

in BF and it effect the BF operation and hot metal quality. It help to keep the impurity level present in the molten metal as low as possible. At low coke rate, the BF works efficiently but as coke rate increases its working efficiency decreases. Coal dust act as a supplemental carbon source in BF which is used to speed up the production of metallic iron and reducing the need for coke production. And lastly, Coal Dust help in substantial reduction of coke consumption in BF.
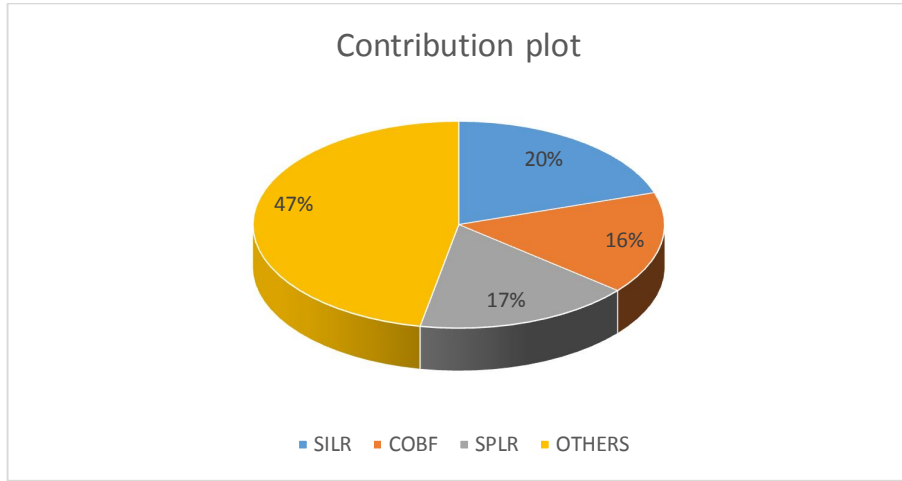

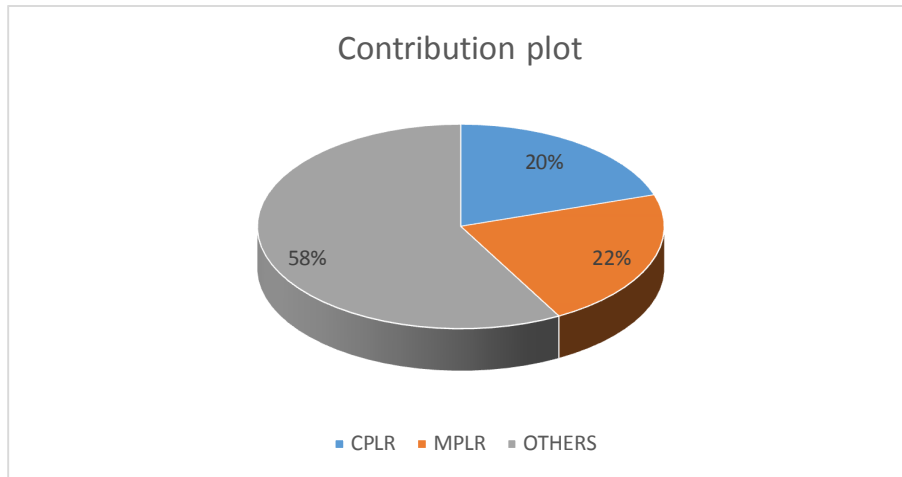**Fig. 5. (a) Contribution plot of out-of-control observation no 3**

Fig. 5. (b) Contribution plot of out-of-control observation no 9
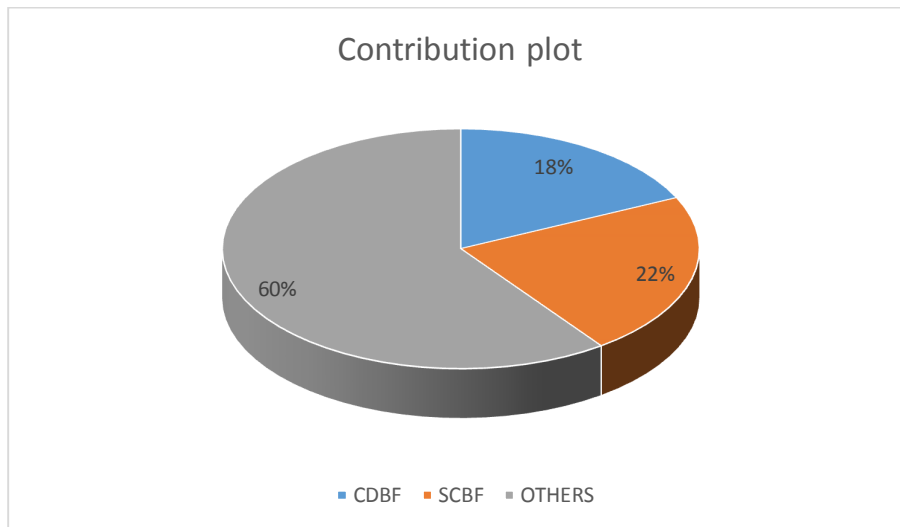
Fig. 5. (c) Contribution plot of out-of-control observation no 31

4. Conclusions

This article proposes a modified ICA based non-Gaussian process monitoring strategy. The drawbacks of conventional ICA based process monitoring strategies are investigated and a modified ICA algorithm is used for elimination of the associated drawbacks of conventional ICA based strategy. The MCLT test was carried out for finding the distribution of the underlying data and subsequent test result indicated that the underlying data is highly non-Gaussian duly validated by the corresponding p-values. The basic idea of the proposed methodology is to first use PCA to extract the PCs arranged in descending order of variance explained and then employ conventional ICA to convert the PCs into ICs. Thus the proposed methodology based on modified ICA has the ability to extract a few dominant ICs according to the amount of variance explained and produces consistent solution. The developed monitoring strategy was applied for fault detection and diagnosis of a SBMU. Bootstrap procedure was used for estimating of the control limit of ensuing Hotelling T^2 control chart. A total of three out-of-control observations were detected. The diagnosis of the first out-of-control observation revealed SILR, SPLR and COBF as the chief contributors. For the second out-of-control observation MPLR and CPLR were the chief contributors whereas SCBF and CDBF were the major contributors for the third out-of-control observation. A closer consultation with the concerned process engineer regarding the major contributors would aid in formulation of corrective steps to arrest the recurrence of similar faults. The proposed methodology has successfully being engaged in developing a process monitoring strategy for SBMU. But the application of proposed methodology is not confined to a case only. In the industrial parlance the proposed methodology may find a diversified application. To be precise such methodology may be easily modified to device a process monitoring strategy for various process industries viz. Cement plant, Oil refineries, Petrochemical industries etc. where the data in consideration may be highly non-Gaussian in nature.

References

[1] W Peres FA, Fogliatto FS., "*Variable selection methods in multivariate statistical process control: A systematic literature review,*" Computers & Industrial Engineering, Vol. 115, (2018), pp. 603-19.

[2] Luo L, Wang J, Tong C, Zhu J., "*Multivariate fault detection and diagnosis based on variable grouping,*" Industrial & Engineering Chemistry Research, Vol. 59, (2020), pp. 7693-705.

[3] Noorossana, R., & Khalili, S., "*Phase II monitoring of auto-correlated linear profiles using multivariate linear mixed model,*" International Journal of Industrial Engineering & Production Research, Vol. 32, (2021), pp. 1-11.

[4] Soleimani, P., & Noorossana, R., "*Investigating effect of autocorrelation on monitoring multivariate linear profiles,*" International Journal of Industrial

Engineering & Production Research, Vol. 23, (2012), pp. 1-9.

[5] Noorossana, R., & Najafi, M., *"Change Point Estimation in High Yield Processes in the Presence of Serial Correlation,"* International Journal of Industrial Engineering & Production Research, Vol. 28, (2017), pp. 367-376.

[6] Rato T, Reis M, Schmitt E, Hubert M, De Ketelaere B., *" A systematic comparison of PCA based statistical process monitoring methods for highdimensional, timedependent processes,"* AIChE Journal, Vol. 62, (2016), pp. 1478-93.

[7] Feng YT, Zhao T, Wang M, Owen DR., *"Characterising particle packings by principal component analysis,"* Computer Methods in Applied Mechanics and Engineering, Vol. 340, (2018), pp.70-89.

[8] Tran NM, Burdejová P, Ospienko M, Härdle WK., *"Principal component analysis in an asymmetric norm,"* Journal of Multivariate Analysis, Vol. 171, (2019), pp. 1-21.

[9] Xie Y, Zhang Y, Jia Q, Zhai L., *"Fault detection based on probabilistic kernel partial least square regression for industrial processes,"* Journal of Chemical Engineering of Japan, Vol. 51, (2018), pp. 89-99.

[10] Xu Y, Shen SQ, He YL, Zhu QX., *" A novel hybrid method integrating ICA-PCA with relevant vector machine for multivariate process monitoring,"* IEEE Transactions on Control Systems Technology, Vol. 27, (2018), pp. 1780-7.

[11] Li Z, Yan X., *"Fault-relevant optimal ensemble ICA model for non-Gaussian process monitoring,"* IEEE Transactions on Control Systems Technology, Vol. 28, (2019), pp. 2581-90.

[12] Chen, M. C., Hsu, C. C., *"Malhotra, B., & Tiwari, M. K. An efficient ICA-DW-SVDD fault detection and diagnosis method for non-Gaussian processes,"* International

Journal of Production Research, Vol. 54, (2016), pp. 5208-5218.

[13] Wang B, Yan X, Jiang Q., *"Independent component analysis model utilizing de-mixing information for improved non-Gaussian process monitoring,"* Computers & Industrial Engineering, Vol. 94, (2016), pp.188-200.

[14] Hsu CC, Cheng CY, Tien FC., *"Fault detection based on ICA-GLR for non-Gaussian industrial processes,"* International Journal of Applied Science and Engineering, Vol. 18, (2021), pp. 1-7.

[15] Li Z, Yan X., *"Ensemble model of wastewater treatment plant based on rich diversity of principal component determining by genetic algorithm for status monitoring,"* Control Engineering Practice, Vol. 88, (2019), pp. 38-51.

[16] Kim Y, Kim SB., *"Optimal false alarm controlled support vector data description for multivariate process monitoring,"* Journal of Process Control, Vol. 65, (2018), pp. 1-4.

[17] Xia B, Jian Z, Tao N., *"An effective combined multivariate control chart based on support vector data description,"* Journal of Ambient Intelligence and Humanized Computing, Vol. 10, (2019), pp. 4819-35.

[18] Fan, M., Ge, Z., & Song, Z., *"Adaptive Gaussian mixture model-based relevant sample selection for JITL soft sensor development,"* Industrial & Engineering Chemistry Research, Vol. 53, (2014), pp. 19979-19986.

[19] Liu J, Liu T, Chen J., *"Sequential local-based Gaussian mixture model for monitoring multiphase batch processes,"* Chemical Engineering Science, Vol. 181, (2018), pp. 101-13.

[20] Zheng J, Wen Q, Song Z., *"Recursive Gaussian mixture models for adaptive process monitoring,"* Industrial & Engineering Chemistry Research, Vol. 58, (2019), pp. 6551-61.

[21] Tong C, Lan T, Shi X., "*Ensemble modified independent component analysis for enhanced non-Gaussian process monitoring,*" Control Engineering Practice, Vol. 58, (2017), pp. 34-41.

[22] Mostajeran A, Iranpanah N, Noorossana R., "*An explanatory study on the non-parametric multivariate T^2 control chart,*" Journal of modern applied statistical methods, Vol. 17, (2018), p. 12.

[23] Phaladiganon P, Kim SB, Chen VC, Baek JG, Park SK., "*Bootstrap-based T^2 multivariate control charts,*" Communications in Statistics-Simulation and Computation®, Vol. 40, (2011), pp. 645-62.

[24] Mostajeran A, Iranpanah N, Noorossana R., "*A new bootstrap based algorithm for Hotelling's T^2 multivariate control chart,*" Journal of Sciences, Islamic Republic of Iran, Vol. 27, (2016), pp. 269-78.