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RESEARCH PAPER

Effect of Revenue-Sharing Contracts in the Points Supply Chain of Coalition Loyalty Programs with Stochastic Advertising-Dependent Demand

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ABSTRACT

A coalition loyalty program (CLP) is a business strategy employed by for-profit companies to increase or retain their customers. One of the operational challenges of these programs is how to choose the mechanism of coordination between business partners. This paper examines the role of revenue-sharing contracts in the loyalty points supply chain of CLP with stochastic advertising-dependent demand where the program operator (called the host) sells loyalty points to the partners of the program. The purpose of this study is to examine the effect of this coordination mechanism on the decisions and profits of the members of the chain using the Stackelberg game method and determine whether the presence of revenue-sharing contracts benefits the chain members when the advertising is done by the host and when the advertising cost is shared between the host and its partners. The results show that when the host gives bonus points to end customers (advertising), revenue-sharing contracts become a powerful incentive for the profitability of the host and its partners. The findings provide new insights into the management of CLPs, which can benefit business decision-makers.

KEYWORDS: *Coalition loyalty program; Supply chain management; Advertising; Uncertainty; Revenue-sharing contract; Loyalty point; Stackelberg game.*

1. Introduction

In many highly competitive markets, companies do everything they can to retain their customers as this is simply far less expensive than attracting new customers. Loyal customers tend to be less sensitive to price changes and are more likely to talk positively about the brand with their friends and acquaintances [31].

In recent years, many companies have started using loyalty programs (LPs), frequent reward programs, and customer clubs, forums, blogs, and magazines for customer retention purposes [26]. LP is a marketing process that rewards customers for frequent purchases or frequent interaction with the brand. LPs are among the most important customer relationship management (CRM) tools that marketers use to identify, reward, and retain profitable customers [27].

Depending on the context, LPs are also referred to as loyalty rewards programs, reward programs, repeat purchase programs, or frequent flyer programs. Regardless of the name, such programs deal with two major concepts: loyalty and reward. Basically, “loyalty” is the primary goal of these programs, and “reward” is the primary means of achieving this goal. Psychological research has shown that rewards can have a great impact on a person’s decision-making and behavior [9]. In LPs, rewards can be offered in a variety of forms, such as discounts, cash, free merchandise, or special services. Regardless of their forms, these rewards are designed to encourage customers to continue doing business with the company or its sponsors and partners rather than competitors [12].

LPs can be divided into three broad categories. The first category is programs that belong to a single company and is called single-sponsor programs (e.g., the LP of a store). In the second category of programs, which is an extension of

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the first category, several companies form a partnership or coalition to create a joint LP. In this category, programs do not belong to any single company. These programs are called coalition LPs (CLPs) or multi-sponsor LPs (e.g., AIR MILES LP). In the third category, LP is formed by a group of companies that unite to create a joint program. These programs are called cross-company programs [21]. Unlike other types of LPs, CLP is wholly owned and operated by a company called the "host," which runs the program as a business entity independent of all coalition partners and cultivates customer loyalty around its own reward system rather than a product or service of a particular partner [6]. CLPs offer businesses and customers a wide range of benefits. For example, CLP allows customers to earn points (currency) much faster with just one membership card and give them more choices for spending their points compared to single-sponsor LPs. It also gives coalition members access to a richer (and more diverse) database of customer information and more efficient marketing and advertising platform. Like other marketing programs, CLP can benefit from advertising as a means to increase sales and target potential customers. The most common type of advertising in the supply chain literature is vertical cooperative advertising, which refers to an interactive arrangement whereby the manufacturer pays some or all of the local advertising costs of the retailer [19]. Manufacturers often use cooperative advertising for immediate sales stimulation at the retail level and to strengthen the brand image [19]. One of the major problems of host companies of CLPs is how to set up the contracts that will govern their relationship with each business partner (coalition member). In cases where the host company wants to have a simple relationship with the partners, the best choice is to use a wholesale price contract [5]. However, if the host company wants a closer relationship with partners, it might be better to use other types of contracts. For instance, the host company and coalition partners may decide to sign some types of revenue-sharing contracts. These contracts tend to give the host more flexibility in managing points and rewards. However, they also tend to impose higher management costs and require more management effort than wholesale price contracts.

Therefore, it might be interesting to investigate how revenue-sharing contracts affect the decisions and profits of coalition members when the outcomes are also dependent on advertising efforts. So far, very few studies have been conducted on the subject of the loyalty points supply chain in CLPs with revenue-sharing mechanisms when demand is uncertain and advertising-sensitive. To fill this research gap, this study explores this subject in a decentralized supply chain with uncertain ad-sensitive demand, in which the host sells the points to its business partners.

The findings of the study provide novel managerial insights that can guide business managers interested in this subject. In summary, this research answers the following questions:

1. What is the optimal point pricing strategy in the loyalty points supply chain of CLP with revenue-sharing contracts according to the game theory?
2. Is the existence of revenue-sharing contracts in the loyalty points supply chain beneficial to the partners of CLP when demand is uncertain and ad-sensitive?
3. How do revenue-sharing and no revenue-sharing contracts affect supply chain decisions and profitability?

The plan of the paper is as follows: In the second section of this article, studies on the subject of LPs, advertising, and contracts in supply chains are reviewed. In the third section, the problem is defined, and the assumptions are explained. In the fourth section, advertising strategies of the host in the presence and absence of revenue sharing are discussed and compared. The fifth section examines the advertising strategy in a scenario where not only a revenue-sharing mechanism is in place but also the advertising cost is shared with partners. The sixth section discusses a scenario where in addition to revenue sharing and advertising cost sharing, there is also a policy of sharing shortage costs. The seventh section presents the results of parametric and numerical analyses, followed by managerial insights drawn from the findings. Finally, the eighth section presents the conclusions and discusses the implications for management practice.

2. Literature Review

For a more organized review of the literature, this section is structured into three subsections as follows: (i) LPs and point and reward supply chains, (ii) revenue-sharing mechanisms and contracts in supply chains, and (iii) advertising in supply chains with LPs.

2.1. LPs and point and reward supply chains

With increased emphasis on the role of customer relationships in marketing, most businesses and organizations now recognize the importance of adopting customer-centered measures and strategies for their success. Over the past three decades, loyalty reward programs (LRPs) have become commonplace in almost every industry, especially those involving consumer goods and services. The goal of LRPs is to establish, cultivate, and maintain long-term relationships with customers [29], [32], [13].

In the study by [24] on point and reward programs, a market consisting of two segments (heavy- and light-user segments) was defined. An important feature of this model is that it considers different price sensitivities for heavy- and light-user segments. They reported that companies would benefit from reward programs as long as light-user segments are not too price-sensitive. Also, heavy-user segments often want to extract more value from reward programs. In the study by [11], a competitive LP was designed, and a stochastic game theory model was presented for better selection of the reward structure for this program. The numerical analysis of this study showed that the choice of reward structure becomes more important as the magnitude of strategic decisions increases. [8] tried to answer the question of whether the rewards are commensurate with the efforts of the members of LPs. This investigation showed that the average price of points was higher than the market price, which is not good for the sustainability of LP.

[11] used a game theory to examine whether competing companies should offer "buy one, get one free" LPs. They defined a game in which equilibrium is pursued through four sub-games. The results showed that setting up LPs is a superior strategy when customers value rewards rather than time. [5] proposed a stochastic linear modeling method for reward planning in LRPs, as well as a method for solving the model. They formulated this problem as a two-stage stochastic linear program with simple resources and developed a stochastic heuristic to solve it. [35] introduced an inventory-based model for predicting redemption and liability in LRPs. They proposed a redemption and liability prediction model to support short-, medium-, and long-term planning and operational decision-making in LRPs.

In the study by [33] on the benefits and limitations of advertising sales in the presence of point-sharing policies, a model consisting of two retailers was developed with fixed retail prices under general demand assumptions, where the retailers' equilibrium decisions were governed by a pure point-sharing policy. They also proposed a rebate contract for two retailers to improve their profits under the point-sharing policy. They stated that this contract could maximize the total profit of the two retailers and split it between them.

[28] used the Stackelberg non-cooperative game to model supply chain coordination in supply chains with stochastic demand and disruptive technologies, specifically those affected by blockchain technologies.

Previous studies in the field of LPs have explored this subject from various perspectives, based on which they can be divided into categories of marketing-oriented, management-oriented, and economics-oriented studies. The economics-oriented studies have been focused on understanding the performance of LPs and their infrastructure mechanisms, such as switching costs, point economics, etc. [47]. The marketing-oriented studies have pursued three general objectives. The first objective is to provide managerial insights for the design and implementation of LPs [2]. The second objective is to examine the short- or long-term impact of LPs on customer behavior, attitude, and purchasing decisions [8]. The third objective is to examine the impact of LPs on the market competition of companies [24] and [11].

Although all of these studies are related to general LP management, most of the models proposed in these papers do not provide any tool for operational planning and decision-making. Therefore, LP managers have to rely on their own experiences and observations rather than analytical methods to overcome the challenges of LP management in terms of revenue or cost optimization, reward planning, point demand prediction, point redemption prediction, etc. Also, as LPs grow in size and complexity, it becomes more difficult for LP managers to conduct proper operational decision-making without analytical tools. Among modeling-based studies, some studies have used game theory models to analyze the impact of LPs on market competition [24] and [40]. In recent years, several studies in the LP literature have attempted to address some specific issues in the operations of LPs [10] and [33].

2.2. Revenue-sharing mechanisms and contracts in supply chains

In the last decade, supply chain coordination has been the subject of many studies to align the policies of chain members for profit maximization purposes. This coordination can be achieved through various mechanisms, one of the most important of which is contracts. In general, supply chain coordination contracts are used when a variable of one member of the supply chain affects the profits of other members [34]. A supply chain contract specifies the parameters that regulate the retailer-supplier relationship. In addition to clearly expressing the terms of the retailer-supplier relationship, contracts have a significant impact on the behavior and performance of all phases in a supply chain [4]. These contracts are supposed to coordinate the decisions of supply chain members and optimize them for maximizing the chain's total profit. Coordinating contracts have two main purposes [45]:

1- Proportional distribution of the expected profit of the whole chain among the members

2- Proportional distribution of risk among the members of the chain

In revenue-sharing contracts, the supplier offers the retailer a wholesale price but takes a fixed percentage of the retailer's sales revenue.[3] examined the benefits and limitations of this contract for price-sensitive demand. A major premise of revenue-sharing contracts is the ability of members (especially suppliers) to monitor sales revenue, especially when part of the revenue comes from scrapping surplus (scrap revenue). This restriction does not apply to businesses such as video rental, CD production, editing and newspaper services, and sports leagues [25]. The features of revenue-sharing contracts are as follows:

• The supplier offers the retailer a lower price, provided that the retailer shares part of its revenue with the supplier.

• This type of contract encourages cooperation between members to determine the best order quantity.

• In this contract, the supplier earns from two sources (direct sales and a percentage of revenue).

[3] examined the strengths and weaknesses of revenue-sharing contracts and compared them with other contracts, such as buy-back, quantity-flexibility, or sales-rebate contracts. They showed that such contracts were generally very attractive and increased the integration of the supply chain. However, while one may think that these contracts should be widely used in all industries, this study showed that because of their extra management costs, they might not be profitable under some circumstances. They recommended using profit-sharing contracts in industries such as video broadcasting, where management costs are low, and it is easy to track and monitor sales through barcodes.

[37] studied the application of virtual transshipment and revenue-sharing contracts in small-scale supply chains. In this study, the problem was modeled as a Stackelberg game where the party with more than half of the revenue is the leader, and the other party is the follower. After comparing revenue sharing and revenue swap contracts, it was found that the revenue sharing approach is suitable for both parties.

[18] modeled a supply chain with revenue-sharing contracts under uncertainty. They found that there was an optimal supply size for the supplier, above which its profit became a non-increasing function of the buyer's order size, and otherwise, the supplier's profit would be a non-decreasing function of the buyer's order size. After comparing this model with the supply chain model in which only demand uncertainty was taken into account, they found that the model with revenue sharing worked better for the supplier in terms of revenue sharing when the wholesale price was constant and offered the supplier a better wholesale price when the revenue sharing ratio was constant. [7] presented a two-level Stackelberg game model for a decentralized supply chain. In this model, the planning horizon was multi-period, revenue-sharing contracts were in place, products were perishable, and demand was defined as a multivariate function. The objective of this model was to maximize the profit of the supply chain in both cooperative and non-cooperative modes. Analyses of this study showed that the benefits of the cooperative approach could lead to significant profitability improvements.

[36] studied the subject of supply chain coordination with profit-dependent revenue-sharing contracts to understand why revenue-independent revenue-sharing contracts are (or are not) preferred to their profit-independent counterparts. They reported that while supply chains could be fully coordinated using both types of revenue-sharing contracts, there are situations in which profit-dependent contracts work better than profit-independent contracts. A revenue-sharing contract can play an important role in coordinating the distribution of profits among upstream and downstream members of a supply chain and improving its overall performance [41]. In the study by [38], supply

chain coordination was discussed using revenue-sharing contracts and how it is affected by corporate social responsibility and demand information. [16] introduced a revenue-sharing contract for coordinating a reverse supply chain where returned products were of random quality, and the remanufacturing capacity was uncertain. [49] studied the subject of collusion and information sharing in a two-level supply chain with a revenue-sharing contract. [39] developed a model for supply chain coordination with the objective of optimizing the capacity procurement decisions of the manufacturer through a commitment-based approach with penalty and revenue sharing.

2.3. Advertising in supply chains and LPs

Many studies have tried to determine the role of cooperative advertising and the coordination of the chains of suppliers and retailers [46], [19], [22] . For example, [19] and [48] both introduced a solution to optimize the cost of cooperative advertising for suppliers, manufacturers, and retailers in a simple one-to-one setting (e.g., one manufacturer and one supplier). Going one step further, [48] also tried to optimize the cost of cooperative advertising in a situation where the manufacturer offers a direct price reduction instead of buying and selling.

[46] investigated the effect of having lower retail prices rather than manufacturer price discounts on customer demand in a cooperative advertising model.[22] developed a supply chain model consisting of two suppliers and two retailers to examine the effect of a cooperative advertising scheme by comparing scenarios in which no manufacturer, only one manufacturer, or both manufacturers offer cooperative advertising opportunities to retailers. Unlike previous studies, [14] studied the cooperative advertising decisions of manufacturers and retailers using an analytical model with a random sales response function rather than a nonrandom function. In the study by [43] on cooperative advertising under demand uncertainty, a number of models for a manufacturer-retailer chain were developed, in which market demand was random, and retail prices could be exogenous or endogenous. They used these models to predict and compare the optimal behavior of the manufacturer and retailer under the no incentive (NI) policy and the manufacturer promotional cost-sharing (MPCS) policy.

[47] studied the joint optimization of ordering and advertising strategies with a focus on the choice between discounts and daily pricing. [20] explored a situation where retailers' advertising efforts had a positive impact on demand but a negative effect on the manufacturer's brand image. Malekian and Rasti-Barzoki (2019) studied the effect of reference price on advertising costs in a two-level supply chain.

Few studies have been conducted on the association between advertising and LPs. In one of these studies, [23] investigated the performance of advertising for CLPs and the implication of CLP design for capacity management. [42] showed how advertising and LPs simultaneously affected market share. They also discussed how the budget should be divided between LPs and advertising. [5] examined the problem of reward planning in CLP, in which not only the demand for point accumulation and redemption by LP members is unknown, but also the host company offers bonus points to members. Also, very few studies have been performed on the subject of revenue-sharing mechanisms in the presence of advertising [17], [41], [38], [16], [49], [39], [1]. proposed an advertising cost and revenue-sharing mechanism with a two-part tariff contract for sustainable supply chain coordination. However, they did not consider demand uncertainty.

It is a common practice in LPs that LP members can receive "bonus points" when they purchase specified products or services from some LP partners during a certain time period. This kind of advertising/promotion activity is offered by the host as a type of cooperative advertising between the host and its partners.

This refers to an advertising contract in which the host of CLP rewards additional points when members purchase certain products from LP partners or participate in the promotional activities of specific partners. For example, AIR MILES may allow members to earn triple points when shopping online through the AIR MILES website.

This scheme is a double-edged sword for the hosts of CLPs. In the short term, bonus points can directly contribute to the growth of LP value and encourage the partners to promote the LP in addition to their own products. However, in the long term, these bonus points increase the redemption costs and liabilities associated with the points, which may create some risks for the management of LP.

Therefore, the host must manage the ads with great care. Unlike in traditional supply chains, in the LP system, the host can communicate directly with the members of the program (customers) through its media network (e.g., email, website, or call centers) and does not require partners to advertise the program. In other words, the host can estimate and monitor the sales' response to ads by tracking the purchase history of the members. Therefore, the host can control the effect of advertising by adjusting the number of awarded bonus points. Like other advertising strategies, the purpose of bonus points is to increase end-user demand (e.g., the demand of members to accumulate points) for certain products or services. From a marketing perspective, although LPs can improve market share and retention by encouraging frequent purchases, advertising can increase a brand's market share by encouraging brand switching, i.e., attracting new customers from other brands, as well as frequent purchases. Table 1 provides a summary of the hypotheses and assumptions of the studies similar to the present work.

Tab. 1. Comparisons with other recent researches

study	Revenue Sharing	Promotion sensitive Demand	Uncertain Demand	Loyalty programs	Game Theory
Canbulut et al. (2021)	✓		✓		✓
Raza (2018)	✓		✓		✓
Tsao and Lee (2020)	✓	✓	✓		✓
Cao et al. (2015)		✓	✓	✓	
Li et al. (2019)	✓			✓	✓
Hu and Feng (2017)	✓	✓			✓
Malekian and Rasti-Barzoki (2019)	✓	✓	✓		✓
Tsao (2015)	✓		✓		✓
current study	✓	✓	✓	✓	✓

2.4. Research gap and innovations

Despite the great number of studies on the mechanisms of revenue sharing in supply chains, very few of these works have considered the sharing of advertising costs in cases where demand is uncertain and ad-sensitive [44], [30]. Also, despite the widespread use of LPs in the business world, their economic impact, and the increasing complexity of managing these programs, very few academic models have been specifically developed for operational planning and decision support in LPs. Most articles in this field are limited to covering the management problems of marketing-centered LPs. Meanwhile, many operational problems of LPs, such as contract design for host-partner coordination and revenue evaluation, have not yet been fully explored. This motivated us to focus this research on the operational problems in the management of LPs. Since previous studies have not considered the effect of advertising efforts and demand uncertainty in CLPs with revenue-sharing contracts, this article intends to cover this gap and discuss how revenue-sharing contracts may work in combination with advertising policies in the loyalty points supply chain of LP. In this study, we consider a decentralized loyalty points supply chain in CLP with uncertain ad-sensitive demand, in which the host sells to its partners the points that may award to the members (customers). To the best of our knowledge, the subject of this article has not been explored in any other study, and this is the first time that revenue-sharing policy and advertising in CLP with uncertain ad-sensitive demand are considered in a quantitative study.

The innovations of this article are as follows:

1- Investigating the effect of the revenue-sharing mechanism in the decisions of the loyalty points supply chain of CLP
2- Considering cooperative advertising in the loyalty points supply chain of CLP with a revenue-sharing mechanism
3- Using a Stackelberg game to solve the loyalty points supply chain model
4- Investigating the effect of the sharing of advertising costs and shortage costs in addition to revenue on the supply chain performance
5- Providing managerial insights for the management of CLPs

3. Problem Definition and Notations

CLP is a program founded by a coalition of brands to increase their collective sales and revenue by offering customers a range of opportunities to accumulate loyalty points and redeem them for discounts and rewards. This concept was first introduced in 1992 by the Canadian airline industry (Air Miles). Later, this

idea was used by a British company to launch a program called Nectar (2002), which became one of the most successful LPs worldwide.
Following the approach of [5], we model a problem as a supply chain of non-storable goods (i.e., points), consisting of two parallel chains: one for point accumulation and the other for point redemption. The host is a focal company with an independent supplier-buyer relationship with each business partner. In this relationship, the host may act as a supplier or buyer. The conceptual model of the loyalty points supply chain with this definition is illustrated in Figure 1.

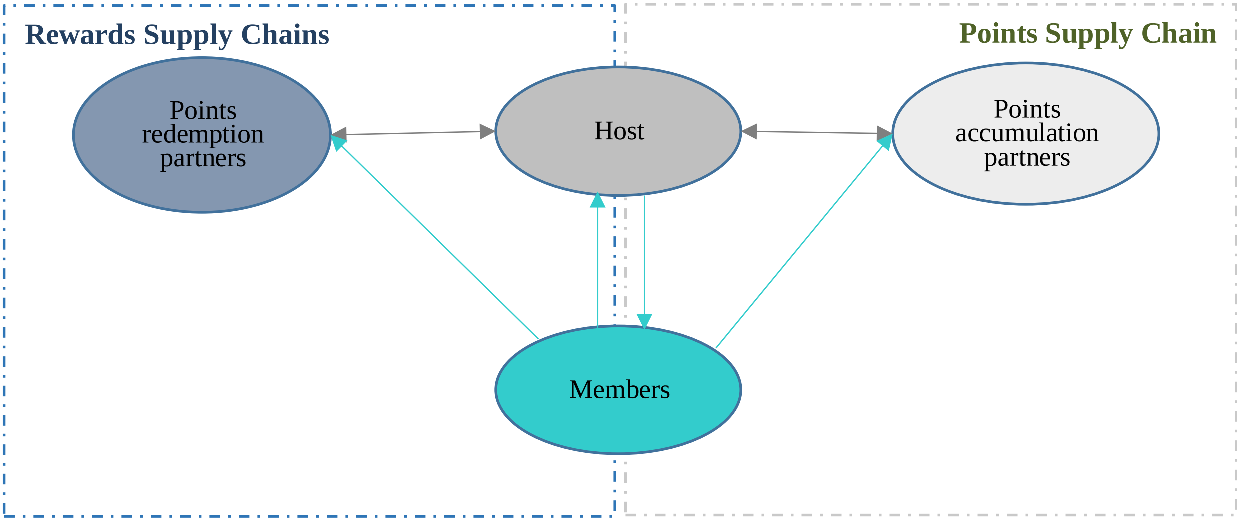

Fig. 1. Conceptual model of the loyalty points supply chain in a coalition loyalty program

Customers earn loyalty points for purchasing products or services from a network of partners, and they can redeem these points to get certain rewards based on a predetermined reward structure.
In this conceptual model, the host company works with a set of redemption partners (i.e., firms that join the program to provide members with redemption options) and accumulation partners (i.e., firms that join the program to provide members with accumulation options). In this loyalty points supply chain, loyalty points are the common currency of all components of the program (host, partners, and member customers).
When a customer buys a product or service from an accumulation partner, the acquired value for the customer is the loyalty point that he earns by this purchase (in addition to the product/service he has purchased). For the accumulation partner, the acquired value is the revenue it has earned from the sale of the product/service. It should be noted that the accumulation partner does not issue the loyalty points directly but rather buys them from the host company (the sole issuer of loyalty points). This is the way the host generates revenue.
When a customer wants to redeem his points to receive a reward, the host buys the reward from the redemption partners and gives it to the customer in exchange for a certain amount of loyalty points.
In this chain, customers have two types of demand: accumulation demand (which is the total demand for accumulating points from accumulation partners) and redemption demand (which is the total demand for redeeming points for rewards offered by redemption partners).
In this study, we consider a decentralized coalition loyalty points supply chain, in which the host sells the points to the accumulation partners, there is a revenue-sharing contract between the host and partners, and the host and accumulation partners advertise cooperatively to increase the accumulation demand of customers. The purpose of this study is to determine how revenue-sharing contracts affect the decisions and profits of both host and accumulation partners when such an advertising policy is in place. The study also aims to determine whether revenue sharing benefits the members under this advertising policy. To determine the effects of revenue sharing, we defined two scenarios: revenue sharing and no revenue sharing. It should be noted that since replacing multiple similarly behaving partners with one partner does not change the outcome, the model is formulated for one partner.

Notations

Indices

Accumulation partners; allow customers to collect loyalty points through their purchasing of products or services	A
Host; runs the loyalty program as the profit center.	H
Scenario of advertising with a revenue-sharing contract in place	α
Scenario of advertising without a revenue-sharing contract in place	β
Scenario of revenue sharing contract with an advertising cost sharing arrangement in place	ϕ
Scenario of revenue sharing contract with an advertising and shortage cost sharing arrangement in place	δ
Parameters	
Price of product/service that if purchased earns the customer 1 loyalty point	p
Per unit shortage penalty cost of loyalty points	v
Marginal cost of the host company	c
Basic demand without advertising	ξ
Advertising cost	k
The fraction of the revenue of the accumulation partner that is shared with the host	λ
Decision variables	
The wholesale price of each point for the accumulation partner A	w
Promotional effort (bonus points)	ρ
Number of points ordered by the accumulation partner A from the host	q
Optimal revenue-sharing ratio	${\lambda_R}^*$
A random variable with continuous uniform distribution in $[-\rho\xi, +\rho\xi]$	U
Dependent variables	
Point demand function for the accumulation partners (A)	X
Profit function of the host	π_H
Profit function of the accumulation partners	π_A

3.1. Problem assumptions

1- The relationship between the host and the partners is governed by contracts.
2. The accumulation demand is uncertain but has a known probability distribution and is price-independent.
3- Accumulation demands of different accumulation partners are independent of each other.
4- Accumulation partners will be penalized if they encounter a point shortage in the middle of the planning period.
5- There is a revenue-sharing contract acting as the legal mechanism between the host and the accumulation partners.
6- The model is formulated based on the constrained single-period newsvendor model.

3.2. Demand function

Following the approach of [44] and [6], we assume that the demand function consists of two components: one fixed and one random, which are formulated as follows:

$$X = \rho\varepsilon + U \quad (1)$$

In this formulation, the first term is a function of the advertising effort and refers to the additional demand resulting from the host's advertising effort. The host's cooperative advertising effort for the accumulation partners is represented by bonus points. Here, U is assumed to be a continuous random variable with uniform distribution in $[-\rho\xi, +\rho\xi]$. Therefore, the stochastic demand function (X) has a uniform distribution in $[0, 2\rho\xi]$. In general, there is no interaction between bonus points offered through point accumulation partners, as the competition between them is so weak that it can be ignored. Advertising effort (ρ) has no impact on the basic demand (ξ), but it does affect the ad-dependent part of demand ($\rho\varepsilon$).

4. Advertising by the Host Under the Revenue-Sharing and no Revenue-Sharing Policies

This section is dedicated to the scenarios in which advertising is done by the host with and without a revenue-sharing mechanism in place.

4.1. Host-funded advertising under the revenue-sharing policy

In this scenario, the host sells the loyalty points to accumulation partners, in which loyalty points are awarded to customers (members) upon purchases, there is a revenue-sharing contract between the host and accumulation partners, and the host performs advertising to increase the accumulation demand of customers and bears all advertising costs. The profit function of the host is formulated as follows:

$$\max \pi_H^{\alpha}(\mathrm{w}_\alpha, \rho_\alpha) = (w_\alpha - c)q_\alpha + \lambda_\alpha \mathrm{p} \int_0^{q_\alpha} \frac{\mathrm{x}}{2\rho_\alpha\xi}\mathrm{dx} + \lambda_\alpha \mathrm{p} \int_{q_\alpha}^{2\rho_\alpha\xi} \frac{q_\alpha}{2\rho_\alpha\xi}\mathrm{dx} - k(\rho_\alpha - 1)^2 \quad (2)$$

In Equation (2), the first term is the revenue earned by the host company from the sale of loyalty points at the price of w_α minus its marginal cost (e.g., software development), the second and third terms show the fraction of revenue of accumulation partners, shared with the host according to the revenue-sharing contract, and the last term is the cost of advertising efforts (bonus points), paid by the host. In previous studies, the last cost term has been mostly convex. The idea of considering advertising costs was first proposed by [44]. However, we consider the advertising effort as a decision separate from the pricing of loyalty points.
The profit function of the accumulation partners is formulated as follows:

$$\max \pi_A^{\alpha}(q_\alpha) = (1-\lambda_\alpha)\mathrm{p}\int_0^{q_\alpha} \frac{\mathrm{x}}{2\rho_\alpha\xi}\mathrm{dx} + (1-\lambda_\alpha)\mathrm{p} \int_{q_\alpha}^{2\rho_\alpha\xi} \frac{q_\alpha}{2\rho_\alpha\xi}\mathrm{dx} - \mathrm{w}_\alpha q_\alpha - v\int_{q_\alpha}^{2\rho_\alpha\xi} \frac{x-q_\alpha}{2\rho_\alpha\xi}\,\mathrm{dx} \quad (3)$$

In Equation (3), the first term is the revenue earned by the accumulation partners from giving points to members (when demand is lower than the order size), the second term is this revenue (when demand is higher than the order size), the third term is the cost incurred by purchasing points, and the fourth term is the penalty incurred when demand exceeds the order size.
According to the game theory and specifically the Stackelberg game (backward induction), first, the host (as the leader) determines the wholesale price of the point (w) and the advertising effort (ρ) with the goal of maximizing its own profit, and then the accumulation partners (as the followers) determine the order size (q) that maximizes their profits according to host decisions. To solve the problem, we first obtain the closed-form expression of the order size of the accumulation partners and then determine the wholesale price and the advertising effort by substituting the order size function into the profit function of the host.
Lemma (1) shows how the optimal wholesale price and advertising effort for the host and the optimal order size for the accumulation partners are computed under the revenue-sharing policy (with a revenue-sharing contract in place) when the advertising is done entirely by the host.

Lemma (1):
(a) The host's optimal wholesale price;

$$w^*_\alpha = \frac{((1-\lambda_\alpha)p+c+v)((1-\lambda_\alpha)p+v)}{(2-\lambda_\alpha)p+2v} \quad (4)$$

(b)The host's optimal promotional effort;

$$\rho^*_\alpha = \frac{(p+v-c)^2\xi}{2((2-\lambda_\alpha)p+2v)k} + 1 \quad (5)$$

(c)The accumulation partner's optimal order quantity;

$$q^*_\alpha = \frac{(\xi(p+v-c)^2 + 2k((2-\lambda_\alpha)p+2v))\xi(p+v-c)}{((2-\lambda_\alpha)p+2v)^2 k} \quad (6)$$

4.2. Host-funded advertising under no revenue-sharing policy

To determine the impact of revenue-sharing contracts, we compare the two models with and without the revenue-sharing mechanism (revenue sharing and no revenue sharing).

Putting λ = 0 in Equations (2) and (3) gives the profit function of the host and partners under no revenue-sharing policy. Lemma (2) shows how the optimal wholesale price, advertising effort, and order size are computed under no revenue-sharing policy (without a revenue-sharing contract) when the advertising is done entirely by the host.

Lemma (2):

(a) The host's optimal wholesale price;

$$w^*_\beta = \frac{p+c+v}{2} \tag{7}$$

(b) The host's optimal promotional effort

$$\rho^*_\beta = \frac{\xi(p+v-c)^2}{4(p+v)k}+1 \tag{8}$$

(c) The accumulation partner's optimal order quantity

$$q^*_\beta = \frac{\xi(p+v-c)(\xi(p+v-c)^2+4k(p+v))}{4(p+v)^2k} \tag{9}$$

5. Advertising Cost Sharing Under Revenue Sharing Policy

Similar to the revenue and promotional sharing contract described by [1], in this scenario, not only revenue but also advertising cost is shared between the host and accumulation partners. For this scenario, the profit function of the host is calculated as follows:

$$\max \pi_A^\phi(q_\phi) = (1-\lambda_\phi)p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx + (1-\lambda_\phi)p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx - v\int_{q_\phi}^{2\rho_\phi\xi}\frac{x-q_\phi}{2\rho_\phi\xi}dx - w_\phi q_\phi - (1-\lambda_\phi)k(\rho_\phi-1)^2 \tag{10}$$

The terms of the above equation are similar to those in Equation (2), with the difference that in the last term, the advertising cost is shared between the host and partners at the same ratio as for revenue. The profit function of the accumulation partners is formulated as Equation (11):

$$max\pi_H{}^\phi(w_\phi,\rho_\phi) = \lambda_\phi p\int_0^{q_\phi}\frac{x}{2\rho_\phi\xi}dx + \lambda_\phi p\int_{q_\phi}^{2\rho_\phi\xi}\frac{q_\phi}{2\rho_\phi\xi}dx + (w_\phi - c)q_\phi - \lambda_\phi k(\rho_\phi-1)^2 \tag{11}$$

The terms of this equation are also similar to those of Equation (3), except that in the last term, the advertising cost is shared between the host and partners at the same ratio as for revenue.

Lemma (3) shows how the optimal wholesale price, advertising effort, and order size are computed under revenue sharing policy (with a revenue-sharing contract) when $\lambda_\phi \neq 0$ (the advertising cost is shared between the host and the partners).

Lemma (3):

(a)The Host's optimal wholesale price;

$$w^*_\phi = \frac{((1-\lambda_\phi)p+c+v)((1-\lambda_\phi)p+v)}{(2-\lambda_\phi)p+2v} \tag{12}$$

The Host's optimal promotional effort;

$$\rho^*_\phi = \frac{(p+v-c)^2\xi}{2((2-\lambda_\phi)p+2v)k\lambda_\phi}+1 \tag{13}$$

(b)The Accumulation partner's optimal order quantity;

$$q^*_\phi = \frac{(\xi(p+v-c)^2+2k\lambda_\phi((2-\lambda_\phi)p+2v))\xi(p+v-c)}{((2-\lambda)p+2v)^2k\lambda_\phi} \tag{14}$$

6. Advertising and Shortage Cost Sharing Under the Revenue-Sharing Policy

In this section, we examine the scenario where in addition to revenue and advertising cost, the shortage cost is also shared between the host and accumulation partners at the same ratio as for revenue. The profit functions of the host and accumulation partners in this scenario are as follows:

$$\max \pi_A^\delta(q_\delta) = (1-\lambda_\delta)p\int_0^{q_\delta}\frac{x}{2\rho_\delta\xi}dx + (1-\lambda_\delta)p\int_{q_\delta}^{2\rho_\delta\xi}\frac{q_\delta}{2\rho_\delta\xi}dx - (1-\lambda_\delta)v\int_{q_\delta}^{2\rho_\delta\xi}\frac{x-q_\delta}{2\rho_\delta\xi}dx - w_\delta q_\delta - (1-\lambda_\delta)k(\rho_\delta-1)^2 \tag{15}$$

$$max\,\pi_H{}^{\delta}(w_\delta,\rho_\delta) = \lambda_\delta p\int_0^{q_\delta}\frac{x}{2\rho_\delta\xi}dx + \lambda_\delta p\int_{q_\delta}^{2\rho_\delta\xi}\frac{q_\delta}{2\rho_\delta\xi}dx + (w_\delta - c)q_\delta - \lambda_\delta v\int_{q_\delta}^{2\rho_\delta\xi}\frac{x-q_\delta}{2\rho_\delta\xi}dx - \lambda_\delta k(\rho_\delta - 1)^2 \quad (16)$$

The terms of these equations are similar to those of Equations (2) and (3), except that in addition to revenue, advertising and shortage costs are also shared between the host and accumulation partners at the same ratio. Lemma (4) shows how the optimal wholesale price, advertising effort, and order size are obtained in this scenario.

Lemma (4):

(a) The Host's optimal wholesale price;

$$w^*{}_\delta = \frac{\left((1-\lambda_\delta)(p+v)+c\right)(1-\lambda_\delta)}{2-\lambda_\delta} \quad (17)$$

The Host's optimal promotional effort;

$$\rho^*{}_\delta = \frac{(p+v)\lambda_\delta\left(-\frac{1}{2}v\xi+k\right)(2-\lambda_\delta)+(p+v-c)^2\xi}{(2-\lambda_\delta)(p+v)k\lambda_\delta} \quad (18)$$

(b) The Accumulation Partner's optimal order quantity;

$$q^*{}_\delta = 2\rho\xi\left(\frac{p+v-c}{(2-\lambda_\delta)(p+v)}\right) \quad (19)$$

7. Analysis of Results and Managerial Insights

The analysis of the results is presented in two sections: parametric sensitivity analysis and numerical analysis.

7.1. Parametric analysis

In the parametric analysis, we compare the changes of some advertising-related variables under revenue sharing and no revenue sharing policies. From this comparison, it can be concluded that the sharing of revenue between the host and accumulation partners decreases the wholesale price. It also makes the host intensify its advertising efforts and makes accumulation partners order more points. The following proposition can be derived from the results of this comparison:

Proposition 1:

(a) The wholesale price decreases when accumulation partners share revenue with the host, i.e., $w^*{}_\alpha < w^*{}_\beta$.

(b) The host makes a more promotional effort when accumulation partners share revenue with the host, i.e., $\rho^*{}_\alpha > \rho^*{}_\beta$.

(c) The order quantity increases when accumulation partners share revenue with the host, i.e., $q^*{}_\alpha \geq q^*{}_\beta$.

Proposition 2:

(a) The host's profit is higher when the accumulation partner shares revenue with the host, i.e., $\pi_H^\alpha > \pi_H^\beta$.

(b) When accumulation partner's profit is higher when the accumulation partner shares revenue with the host, i.e., $\pi_A^\alpha > \pi_A^\beta$.

Proposition 2(a) argues that the host benefits under a revenue-sharing contract. In Proposition 2(b), we find the threshold of λ_α, where the accumulation partner benefits under a revenue-sharing contract. When the fraction of λ_α revenue shared by the accumulation partner with the host is smaller than the upper bound of λ_α, the accumulation partner is willing to share revenue with the host because this will lead to an increase in the accumulation partner's profit. Therefore, revenue sharing could be an effective incentive policy for increasing channel members' profits in a decentralized points supply chain of CLPs under a host promotion.

7.2. Numerical analysis and managerial insights

In this section, we use numerical examples to examine the problem and further analyze the variables of supply chain members.

7.2.1. Effect of the revenue-sharing ratio on decision variables and profit functions of supply chain members

Table 2 presents a summary of the effect of the revenue-sharing ratio (λ) on supply chain decision variables and profit functions of the host and accumulation partners in two scenarios with and without revenue sharing. These results show that as λ increases, the wholesale price of loyalty points decreases, although it remains slightly higher than marginal costs, and at the same time, the advertising effort and the order size both increase. They also show that the profit of accumulation partners is concave and decreases with increasing λ.

The revenue-sharing mechanism has a greater impact on the profit of the host company than that of the accumulation partners. As shown in Table 2, this coordination mechanism can be profitable for accumulation partners as long as the revenue-sharing ratio is low. This means that the λ value can be set such that accumulation partners profit more under the revenue-sharing policy (with profit sharing) than under the no revenue-sharing policy (without profit sharing). In this example, the maximum profit of the accumulation partner when λ = 0.1 is $\pi_A{}^\alpha =$

75.581. These results confirm the proposition provided in the previous section.

As can be seen, the total profit of the supply chain and host increases with the increase of the revenue-sharing ratio. Thus, it can be stated that, in general, a revenue-sharing contract will improve the performance of the whole supply chain.

Tab. 2. Effects of on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5, v = 0.5$)

λ	Wholesale price		Promotion effort		Points Order quantity		Host Profits		Accumulation partner Profit		Total Profit	
	No RS	RS	No RS	RS	No RS	RS	No RS	RS	No RS	RS	No RS	RS
0	10.250	10.250	1.178	1.178	39.894	39.894	193.633	193.633	75.276	75.276	268.909	268.909
0.1	10.250	9.017	1.178	1.187	39.894	42.244	193.633	204.323	75.276	75.581	268.909	279.904
0.2	10.250	7.812	1.178	1.197	39.894	44.883	193.633	216.255	75.276	75.272	268.909	291.527
0.3	10.250	6.641	1.178	1.208	39.894	47.865	193.633	229.655	75.276	74.109	268.909	303.764
0.4	10.250	5.510	1.178	1.220	39.894	51.261	193.633	244.810	75.276	71.753	268.909	316.563
0.5	10.250	4.425	1.178	1.234	39.894	55.162	193.633	262.087	75.276	67.723	268.909	329.810

Table 1 also incurs an interesting problem, considering the revenue sharing ratio (λ) as a decision variable. Substituting Equations (4), (5), and (6) into Equation (3) gives the accumulation partners profit function of (π_A) with only one decision variable. Therefore, the optimal revenue sharing ratio ($\lambda_R{}^*$) from the perspective of accumulation partners is obtained by maximizing the function (π_A^{α}) as shown below:

$$\lambda_R{}^* = \frac{(p^2 - 2pc + (c-v)^2)\xi + 2k(p+v) - \sqrt{4k^2(p+v)^2 + 2k\xi(p+v)(p+v-c)^2 + \xi^2(p+v-c)^2(-2v^2 - (p+2c)v + (p-c)^2)}}{p(2k - \xi v)} \quad (20)$$

7.2.2. Effect of the optimal sharing ratio ($\lambda_R{}^*$) on shortage costs, marginal costs, and initial demand

As Figures 2, 3, and 4 demonstrate, $\lambda_R{}^*$ is inversely related to costs and is directly related to the demand for point accumulation. In other words, as demand increases, so does $\lambda_R{}^*$. However, any increase in the host's marginal cost or shortage cost will decrease $\lambda_R{}^*$. Therefore, it can be concluded that such sharing contracts encourage the supply chain members to lower their costs to generate more revenue, thereby improving the overall performance of the supply chain.

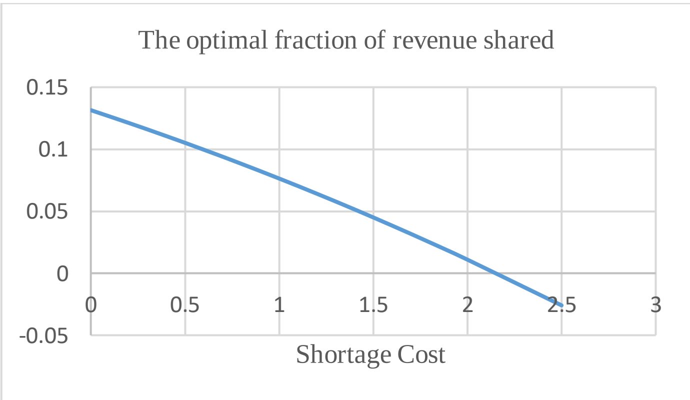

Fig. 2. Relationship between $\lambda_R{}^*$ and the shortage cost

The optimal fraction of revenue shared ($\lambda_R{}^*$) increases as the accumulation partner's shortage cost (v) decreases.

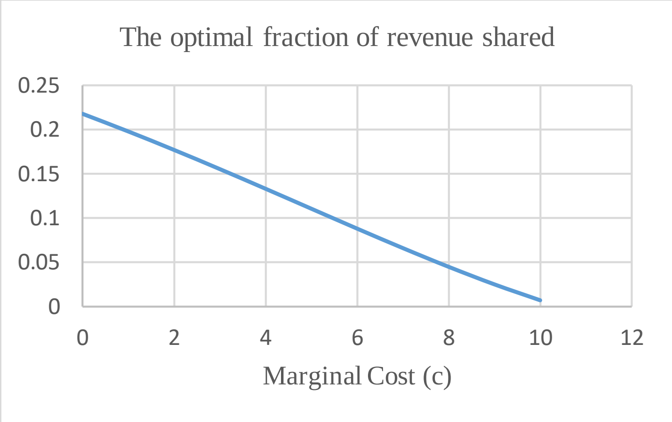

Fig. 3. Relationship between $\lambda_R{}^*$ and the marginal cost of the host

The optimal fraction of revenue shared ($\lambda_R{}^*$) increases as the host's marginal cost (c) decreases.

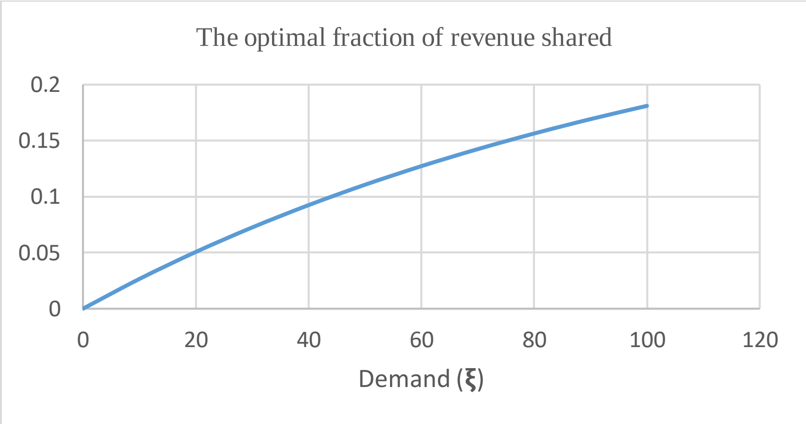

Fig. 4. Relationship between $\lambda_R{}^*$ and the basic demand

The optimal fraction of revenue shared increases as the basic demand (ξ) increases.

7.2.3. Effect of the revenue-sharing ratio and advertising cost on decision variables and profit functions of supply chain members

As shown in Table 3, sharing of the advertising cost between the host and accumulation partners significantly increases the host's profit and the performance of the entire supply chain but decreases the profit of the accumulation partners (compared to when only the host pays for the ads or there is no revenue-sharing mechanism in place at all). This is because, in this arrangement, the entire shortage cost will be borne by the partner. Thus, it might be better to adjust the sharing mechanism so that the host also pays a percentage of the partner's shortage cost.

Tab. 3. Effects of λ_ϕ on decisions and profits ($K = 500$; $\xi = 50$; $p = 15$; $c = 5, v = 0.4$)

λ_ϕ	Wholesale price		Promotion effort		Points Order quantity		Host Profits		Accumulation partner Profit		Total Profit	
	No RS	RS	No RS	RS	No RS	RS	No RS	RS	No RS	RS	No RS	RS
0	10.200	10.200	1.878	1.878	60.819	60.819	210.323	210.323	5.872	5.872	216.105	216.105
0.40	10.200	5.458	1.878	1.545	60.819	64.797	210.323	277.505	5.872	7.649	216.105	285.154
0.50	10.200	4.373	1.878	1.464	60.819	65.355	210.323	285.975	5.872	32.071	216.105	318.046
0.60	10.200	3.346	1.878	1.413	60.819	67.431	210.323	299.357	5.872	40.482	216.105	339.839
0.70	10.200	2.390	1.878	1.380	60.819	70.729	210.323	317.97	5.872	39.440	216.105	356.537
0.80	10.200	1.519	1.878	1.359	60.819	75.210	210.323	339.377	5.872	30.609	216.105	369.986

7.2.4. Effect of the revenue-sharing ratio, advertising cost, and shortage cost on decision variables and profit functions of supply chain members

Considering the reduced profit of accumulation partners in the previous scenario, Table 4 examines the scenario in which, in addition to revenue and advertising costs, the shortage cost is also shared between the host and partners. By comparing Tables (3) and (4), it can be seen that when not only advertising cost but also shortage cost is shared between the host and partners, the profit of partners increases. While this decreases the host's profit, it also improves the overall performance of the supply chain. Thus, it can be concluded that if the program is intended to have a shortage cost mechanism in addition to revenue sharing, to improve the profit of accumulation partners and the entire supply chain, it is best to make the host pay a fraction of this shortage cost. This will make the partners more willing to participate in such a program.

The comparison of Tables 3 and 4 shows that when revenue- and advertising-cost-sharing mechanisms are in place, making the host pay a fraction of the shortage cost will not make much change in the wholesale price and advertising effort. However, the overall outcome will be a slight improvement in all of these parameters, leading to an overall improvement in supply chain performance.

Tab. 4. Effects of λ_δ on decisions and profits ($K = 500;\ \xi = 50;\ p = 15;\ c = 5, v = 0.4$)

λ_δ	Wholesale price		Promotion effort		Points Order quantity		Host Profits		Accumulation partner Profit		Total Profit	
	No RS	RS	No RS	RS	No RS	RS	No RS	RS	No RS	RS	No RS	RS
0	12.7	12.7	1.826	1.826	61.119	61.119	201.224	201.224	16.172	16.172	217.396	217.396
0.40	12.7	5.340	1.826	1.530	61.119	64.523	201.224	267.385	16.172	23.618	217.396	291.003
0.50	12.7	4.233	1.826	1.448	61.119	65.201	201.224	274.340	16.172	48.307	217.396	322.647
0.60	12.7	3.188	1.826	1.398	61.119	67.439	201.224	286.370	16.172	57.320	217.396	343.690
0.70	12.7	2.220	1.826	1.366	61.119	70.956	201.224	302.990	16.172	56.869	217.396	359.859
0.80	12.7	1.347	1.826	1.346	61.119	75.738	201.224	324.472	16.172	48.298	217.396	372.770

From the above results, the following managerial insights can be derived.

1- Revenue sharing between the host and accumulation partners decreases the selling price of loyalty points, makes the host intensify its advertising efforts, and makes the accumulation partners order more points. Overall, this indicates that the presence of revenue-sharing mechanisms benefits the members of the loyalty points supply chain in CLPs.

2- Revenue-sharing contracts can significantly improve the profits of the host and can also be profitable for accumulation partners as long as the revenue sharing ratio is low. In general, these contracts improve the overall performance of the loyalty points supply chain in CLPs.

3- The existence of contracts such as revenue sharing in the loyalty points supply chain encourages chain members to spend less and earn more revenue, which improves the overall performance of the supply chain.

4- In supply chains with revenue-sharing contracts, making the chain members share advertising and shortage costs will make the partners more willing to cooperate with the host in the program.

8. Conclusions and Suggestions

Today, the management of CLPs is an important part of customer-centric business strategies. Revenue sharing is commonly used as a marketing strategy to increase the profits of members in the B2B environment. Since previous studies have not considered the effects of advertising efforts and demand uncertainty in CLPs with revenue-sharing mechanisms, in this study, we considered a decentralized supply chain of loyalty points in CLP with uncertain advertising-sensitive demand where the host sells loyalty points to its business partners, which then they reward to their customers. This study aimed to determine whether sharing revenue and advertising cost are in the best interest of program members. The results showed that revenue sharing between the host and accumulation partners decreases the wholesale price of loyalty points. It also makes the host put more effort into advertising and makes partners order more loyalty points from the host. This suggests that when the advertising of bonus points is funded by the host, revenue sharing will be an effective incentive to increase the profit of the host and accumulation partners. It was also found that in the presence of a revenue sharing policy, sharing of the advertising cost between the host and accumulation partners significantly increases the profit of the host and the overall supply chain performance. However, it will decrease the profit of partners compared to when only the host pays for advertising, or there is no revenue-sharing mechanism in place at all. This is because, in this arrangement, the entire shortage cost will be borne by partners. Therefore, we also examined another scenario where, in addition to revenue and advertising cost sharing, the host also bears a fraction of the partners' shortage costs.

From the results, it can be concluded that when the demand is unknown, revenue sharing could be an attractive option for the entire supply chain only when the costs imposed on the downstream part of the supply chain (accumulation partners) are shared with upstream parts. This will increase the profitability of the downstream part of the supply chain and improve its overall performance. In future studies, it is recommended to consider the behavioral aspects of this type of coordination mechanism, such as fairness, integrity, and customer satisfaction in the model. In addition, since this study discussed whether revenue sharing would benefit the members of the program, it might be worthwhile to investigate how other policies affect their behavior. It might also be beneficial to develop a model for the cases where the host can provide loyalty points at any time. Finally, in this study, λ is not considered a decision variable and is assumed as a parameter; thus, this issue can be interesting for further investigation in future studies.

Appendix

Proof of Lemma 1:

First, we calculate the first and second-order derivatives of the partners' profit function with respect to q:

$$\frac{\partial {\pi_A}^{\alpha}(q_\alpha)}{\partial q_\alpha} = \frac{(1-\lambda_\alpha)p(2\rho\xi-q)}{2\rho\xi} + \frac{v(2\rho\xi-q)}{2\rho\xi} - w$$

$$\frac{\partial^2 \pi_A(q_\alpha)}{\partial {q_\alpha}^2} = -\left(\frac{(1-\lambda_\alpha)p}{2\rho\xi} + \frac{v}{2\rho\xi}\right)$$

Based on the problem assumptions, since $0 < \lambda_\alpha < 1$, therefore $\frac{\partial^2 \pi_A(q_\alpha)}{\partial {q_\alpha}^2} < 0$. Thus, we will have:

$$\frac{\partial {\pi_A}^{\alpha}(q_\alpha)}{\partial q_\alpha} = 0 \ \rightarrow \ q_\alpha = \frac{2\rho\xi((1-\lambda_\alpha)p+v-w)}{(1-\lambda_\alpha)p+v}$$

Substituting the optimal value of orders in the host's objective function gives the optimal wholesale price and advertising effort as follows:

$$\frac{\partial {\pi_H}^{\alpha}(w_\alpha,\rho_\alpha)}{\partial w_\alpha} = \frac{2\rho\xi((1-\lambda_\alpha)^2p^2+\big((1-\lambda_\alpha)(c+2v)-(2-\lambda_\alpha)w\big)p+v(c+v-2w))}{\big((1-\lambda_\alpha)p+v\big)^2}$$

$$\frac{\partial^2 {\pi_H}^{\alpha}(w_\alpha,\rho_\alpha)}{\partial {w_\alpha}^2} = -\frac{2\rho\xi((2-\lambda_\alpha)p+2v)}{((1-\lambda_\alpha)p+v)^2}$$

Since $\frac{\partial^2 {\pi_H}^{\alpha}(w_\alpha,\rho_\alpha)}{\partial {w_\alpha}^2} < 0$, therefore:

$$\frac{\partial {\pi_H}^{\alpha}(w_\alpha,\rho_\alpha)}{\partial w_\alpha} = 0 \quad \rightarrow \quad {w_\alpha}^* = \frac{\big((1-\lambda_\alpha)p+v+c\big)((1-\lambda_\alpha)p+v)}{(2-\lambda_\alpha)p+2v}$$

Now we can determine the optimal advertising coefficient;

$$\frac{\partial \pi_H(w_\alpha,\rho_\alpha)}{\partial \rho_\alpha} = \frac{\xi\lambda_\alpha p((1-\lambda_\alpha)p+v-w)^2}{\big((1-\lambda_\alpha)p+v\big)^2} + \frac{2\xi\lambda_\alpha pw\big((1-\lambda_\alpha)p+v-w\big)}{\big((1-\lambda_\alpha)p+v\big)^2} + \frac{2\xi(w-c)\big((1-\lambda_\alpha)p+v-w\big)}{((1-\lambda_\alpha)p+v)} - 2k(\rho-1)$$

$$\frac{\partial^2 \pi_H(w_\alpha,\rho_\alpha)}{\partial {\rho_\alpha}^2} = -2k$$

Since $\frac{\partial^2 \pi_H(w_\alpha,\rho_\alpha)}{\partial {\rho_\alpha}^2} < 0$, therefore the function will be concave with respect to ρ_α. The optimal value of ρ_α is given by:

$${\rho_\alpha}^* = \frac{1}{2k((1-\lambda_\alpha)p+v)^2}\Big(\xi\lambda(1-\lambda_\alpha)^2p^3 + 2(1-\lambda_\alpha)p^2((1-\lambda_\alpha)(k+w\xi-c\xi)) + \xi\big((w-v)^2 - 2(c(w-2v)+wv)\big)\lambda_\alpha - 4(v-\tfrac{1}{2}w)(c-w)) + 4kv(1-\lambda_\alpha))p - 2v((v-w)(c-w)\xi - kv)\Big)$$

Putting the obtained ${w_\alpha}^*$ in ρ_α gives:

$${\rho^*}_\alpha = \frac{\xi\big(p+(v-c)\big)^2}{2((2-\lambda_\alpha)p+2v)k} + 1$$

After putting the optimal values of $w_\alpha{}^*$ and $\rho^*{}_\alpha$ in q_α, we arrive at:

$q^*{}_\alpha = \frac{\xi(p+v-c)(\xi(p+v-c)^2+2k((2-\lambda_\alpha)p+2v))}{((2-\lambda_\alpha)p+2v)^2k}$

Proof of Lemma 2:

By putting λ = 0 in Equations (1) and (2) and following the procedure described for the proof of Lemma 1, it can be proved that:

$w^*{}_\beta = \frac{(p+c+v)}{2}$

$\rho^*{}_\beta = \frac{\xi(p+v-c)^2}{4(p+v)k} + 1$

$q^*{}_\beta = \frac{\xi(p+v-c)(\xi(p+v-c)^2+4k(p+v)}{4(p+v)^2k}$

Proof of Proposition 1:

(a) From $w_\beta{}^* - w_\alpha{}^* = \frac{(p+c+v)}{2} - \frac{((1-\lambda)p+c+v)((1-\lambda)p+v)}{(2-\lambda)p+2v} = \frac{(p+v)^2+\lambda(1-\lambda)p^2+c(p+v)}{2((2-\lambda)p+2v)}$ and $0 < \lambda < 1$, it is proved that $w_\beta{}^* - w_\alpha{}^* > 0$.

(b) From $\rho_\alpha{}^* - \rho_\beta{}^* = \frac{\xi(p+v-c)^2\lambda p}{2((2-\lambda)p+2v)(2p+2v)k}$ and $0 < \lambda < 1$, it is proved that $\rho_\alpha{}^* - \rho_\beta{}^* > 0$.

(c) From $q_\alpha{}^* - q_\beta{}^* = \frac{\xi(p+v-c)(\xi(p+v-c)^2(\lambda(4-\lambda)p^2+4\lambda pv))+4k(p+v)((2-\lambda)p+2v)\lambda p}{4(p+v)^2\,((2-\lambda)p+2v)^2k}$ and $0 < \lambda < 1$, it is proved that $q_\alpha{}^* - q_\beta{}^* > 0$.

Proof of Proposition 2:

(a) From $\pi_H^\alpha - \pi_H^\beta = \frac{(-c+p+v)^2\xi\lambda_\alpha((4-\lambda_\alpha)(-c+p+v)^2\xi-8k(p+v)(2-\lambda_\alpha))}{4k(-2(p+v)(2-\lambda_\alpha))^2}$ and $0 < \lambda < 1$ we know that $\pi_H^\alpha - \pi_H^\beta > 0$

From $\pi_A^\alpha - \pi_A^\beta = \frac{1}{16}\left(\frac{\lambda_\alpha(-c+p+v)^2\left((4+\lambda_\alpha{}^2-6\lambda_\alpha)(-c+p+v)^2\xi^2-4k\xi(2-\lambda_\alpha)(\lambda_\alpha(p+v))\right)}{k(p+v)^2\xi(2-\lambda_\alpha)}\right)$ We know that $\pi_A^\alpha - \pi_A^\beta > 0$ when $\lambda_\alpha(-c+p+v)^2\left((4+\lambda_\alpha{}^2-6\lambda_\alpha)(-c+p+v)^2\xi^2-4k\xi(2-\lambda_\alpha)(\lambda_\alpha p)\right) > 0$

Then

$$\frac{\left(3(-c+p+v)^2\xi+(4k(p+v))- \ (16k^2(p+v)^2+k(p+v)(-c+p+v)^2 8\xi^1+5(-c+p+v)^4\xi^2)^{\frac{1}{2}}\right)}{4k(p+v)+(-c+p+v)^2\xi} < \lambda_\alpha < \frac{\left(3(-c+p+v)^2\xi+(4k(p+v))+ \ (16k^2(p+v)^2+k(p+v)(-c+p+v)^2 8\xi^1+5(-c+p+v)^4\xi^2)^{\frac{1}{2}}\right)}{4k(p+v)+(-c+p+v)^2\xi}$$

From $0 < \lambda < 1$ should be satisfied, we get the range of threshold:

$$0 < \lambda_\alpha < \frac{\left(3(-c+p+v)^2\xi+(4k(p+v))+ \ (16k^2(p+v)^2+k(p+v)(-c+p+v)^2 8\xi^1+5(-c+p+v)^4\xi^2)^{\frac{1}{2}}\right)}{4k(p+v)+(-c+p+v)^2\xi}$$

Proof of Lemma (3):

Similar to the proof of Lemma 1, we first calculate the first and second-order derivatives of the partners' profit function with respect to q:

$\frac{\partial\pi_A{}^\phi(q_\phi)}{\partial q_\phi} = \frac{(1-\lambda_\phi)p(2\rho\xi-q)}{2\rho\xi} + \frac{v(2\rho\xi-q)}{2\rho\xi} - w$

$\frac{\partial^2\pi_A(q_\phi)}{\partial q_\phi{}^2} = -(\frac{(1-\lambda_\phi)p}{2\rho\xi} + \frac{v}{2\rho\xi})$

According to the problem assumptions, since $0 < \lambda_\phi < 1$, therefore $\frac{\partial^2\pi_A(q_\phi)}{\partial q_\phi{}^2} < 0$. Thus:

$\frac{\partial\pi_A{}^\phi(q_\phi)}{\partial q_\phi} = 0 \rightarrow q_\phi = \frac{2\rho\xi((1-\lambda_\phi)p+v-w)}{(1-\lambda_\phi)p+v}$

The optimal wholesale price and advertising effort are then obtained as follows:

$$\frac{\partial \pi_H{}^{\phi}(w_\phi,\rho_\phi)}{\partial w_\phi}=\frac{2\rho\xi((1-\lambda_\phi)^2p^2+\big((1-\lambda_\phi)(c+2v)-(2-\lambda_\phi)w\big)p+v(c+v-2w))}{\big((1-\lambda_\phi)p+v\big)^2}$$

$$\frac{\partial^2 \pi_H{}^{\phi}(w_\phi,\rho_\phi)}{\partial w_\phi{}^2}=-\frac{2\rho\xi((2-\lambda_\phi)p+2v)}{((1-\lambda_\phi)p+v)^2}$$

Since $\frac{\partial^2 \pi_H{}^{\lambda}(w_\phi,\rho_\phi)}{\partial w_\phi{}^2}<0$, therefore:

$$\frac{\partial \pi_H{}^{\phi}(w_\phi,\rho_\phi)}{\partial w_\phi}=0 \quad\rightarrow\quad w_\phi{}^*=\frac{\big((1-\lambda_\phi)p+v+c\big)((1-\lambda_\phi)p+v)}{(2-\lambda_\phi)p+2v}$$

Now the optimal advertising coefficient can be determined;

$$\frac{\partial \pi_H(w_\phi,\rho_\phi)}{\partial \rho_\phi}=\frac{\xi\lambda_\phi p((1-\lambda_\phi)p+v-w)^2}{\big((1-\lambda_\phi)p+v\big)^2}+\frac{2\xi\lambda_\phi pw\big((1-\lambda_\phi)p+v-w\big)}{\big((1-\lambda_\phi)p+v\big)^2}+\frac{2\xi(w-c)\big((1-\lambda_\phi)p+v-w\big)}{\big((1-\lambda_\phi)p+v\big)}-2k\lambda_\phi(\rho-1)$$

$$\frac{\partial^2 \pi_H(w_\phi,\rho_\phi)}{\partial \rho_\phi{}^2}=-2\lambda_\phi k$$

Since $\frac{\partial^2 \pi_H(w_\phi,\rho_\phi)}{\partial \rho_\phi{}^2}<0$, the function will be concave with respect to ρ_ϕ. The optimal value of ρ_ϕ is given by:

$$\rho_\phi{}^*=\frac{1}{2k\lambda_\phi((1-\lambda_\phi)p+v)^2}\Big(\xi\lambda_\phi\big(1-\lambda_\phi\big)^2p^3+2\big(1-\lambda_\phi\big)p^2\Big((c+v-w)\lambda_\phi+w-c\Big)\xi+k\lambda_\phi(1-\lambda_\phi))p^2+\big(\big(\,(v^2+(4c-4w)v-2cw+w^2)\lambda_\phi-4(v-\tfrac{1}{2}w)(c-w)\big)\xi+4kv\big(1-\lambda_\phi\big)\big)p-2v((v-w)(c-w)\xi-kv\lambda_\phi)\Big)$$

Putting the obtained $w_\phi{}^*$ in ρ_ϕ gives:

$$\rho^*{}_\phi=\frac{(p+v-c)^2\xi}{2((2-\lambda_\phi)p+2v)k\lambda_\phi}+1$$

Finally, substituting the optimal values of $w_\phi{}^*$and ρ_ϕ in q_ϕ gives:

$$q^*{}_\phi=\frac{(\xi(p+v-c)^2+2k\lambda_\phi((2-\lambda_\phi)p+2v))\xi(p+v-c)}{((2-\lambda_\phi)p+2v)^2k}$$

Proof of Lemma (4):

Similar to the proof of Lemma 1, we start with calculating the first and second-order derivatives of the partners' profit function with respect to q:

$$\frac{\partial \pi_A{}^{\delta}(q_\delta)}{\partial q_\delta}=\frac{(1-\lambda_\delta)p(2\rho\xi-q)}{2\rho\xi}+\frac{(1-\lambda_\delta)v(2\rho\xi-q)}{2\rho\xi}-w$$

$$\frac{\partial^2 \pi_A(q_\delta)}{\partial q_\delta{}^2}=-(\frac{(1-\lambda_\delta)p}{2\rho\xi}+\frac{(1-\lambda_\delta)v}{2\rho\xi})$$

Based on the problem assumptions, since $0<\lambda_\delta<1$, therefore $\frac{\partial^2 \pi_A(q_\delta)}{\partial q_\delta{}^2}<0$. Thus, we have:

$$\frac{\partial \pi_A{}^{\delta}(q_\delta)}{\partial q_\delta}=0 \;\rightarrow\; q_\delta=2\rho\xi(1-\frac{w}{(1-\lambda_\delta)(p+v)})$$

Then, the optimal wholesale price and advertising effort can be obtained as follows:

$$\frac{\partial \pi_\delta{}^{\delta}(w_\delta,\rho_\delta)}{\partial w_\delta}=\frac{2\rho\xi((1-\lambda_\delta)^2(p+v)+c(1-\lambda_\delta)-(2-\lambda_\delta)w)}{(p+v)(1-\lambda_\delta)^2}$$

$$\frac{\partial^2 \pi_H{}^{\delta}(w_\delta,\rho_\delta)}{\partial w_\delta{}^2}=-\frac{2\rho\xi\,(2-\lambda_\delta)}{(p+v)(1-\lambda_\delta)^2}$$

Since $\frac{\partial^2 \pi_H{}^{\lambda}(w_\delta,\rho_\delta)}{\partial w_\delta{}^2}<0$, we have:

$$\frac{\partial \pi_H{}^{\delta}(w_\delta,\rho_\delta)}{\partial w_\delta}=0 \quad\rightarrow\quad w_\delta{}^*=\frac{((1-\lambda_\delta)(p+v)+c)\,(1-\lambda_\delta)}{2-\lambda_\delta}$$

Next, the optimal advertising coefficient must be determined.

$$\frac{\partial \pi_H(w_\delta,\rho_\delta)}{\partial \rho_\delta}=\frac{1}{(1-\lambda_\delta)^2(p+v)}\Big(\big(\xi p+2k(1-\rho)\big)(p+v)\lambda_\delta{}^3-2\big((p+c-w)\xi+2k(1-\rho)\big)(p+v)\lambda_\delta{}^2+\big((p^2+(4c+v-4w)p-4(w-c)v-2cw+w^2)\xi+2k(1-\rho)(p+v)\big)\lambda_\delta+2\xi(w-c)(p+v-w)$$

$$\frac{\partial^2 \pi_H(w_\delta,\rho_\delta)}{\partial \rho_\delta{}^2}=-2\lambda_\delta k$$

Since $\frac{\partial^2 \pi_H(w_\delta,\rho_\delta)}{\partial \rho_\delta{}^2} < 0$, the function will be concave with respect to ρ_δ. The optimal value of $\rho_\delta$is determined as follows:

$$\rho_\delta{}^* = \frac{1}{2k\lambda_\delta(1-\lambda_\delta)^2(p+v)}\Big((p+v)(\xi p+2k)\lambda_\delta{}^3 - 2((p+c-w)\xi+2k)(p+v)\lambda_\delta{}^2 + \big((p^2 + (4c+v-4w)p - 4(w-c)v - 2cw + w^2)\xi + 2k(p+v)\big)\lambda_\delta + 2\xi(w-c)(p+v-w)\Big)$$

By substituting the obtained $w_\delta{}^*$ in ρ_δ, we arrive at:

$$\rho^*{}_\delta = 1 + \frac{(p+v-c)^2\xi}{(2-\lambda_\delta)(p+v)k\lambda_\delta} - \frac{v\xi}{2k}$$

Finally, putting the optimal values of $w_\delta{}^*$and $\rho_\delta$in q_δ gives:

$$q^*{}_\delta = 2\rho\xi\left(\frac{p+v-c}{(2-\lambda_\delta)(p+v)}\right)$$

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