

RESEARCH PAPER

# Fuzzy Vehicle Routing Problem with Split Delivery: Trade-Off Between Air Pollution and Customer Satisfaction

Masoud Rabbani<sup>1\*</sup> & Elham Abutalebi<sup>2</sup>

Received 13 April 2022; Revised 2 May 2022; Accepted 11 May 2022;  
© Iran University of Science and Technology 2022

## ABSTRACT

*In large-scale emergency, the vehicle routing problem focuses on finding the best routes for vehicles. The equitable distribution has a vital role in this problem to decrease the number of death and save people's lives. In addition to this, air pollution is a threat to people's life and it can be considered to omit other kinds of disasters happens because of it. So, a new MINLP model presented is going to face a real situation by considering real world assumptions such as fuzzy demands and travel time, multi depots and items, vehicle capacity and split delivery. The first objective function is to minimize the sum of unsatisfied demand which follows a piecewise function and the second one is to minimize the cost which depends on the fuel consumption. In order to solve the multi-objective problem with fuzzy parameters, nonlinear function has been linearized by convex combination and a new crisp model is presented by defusing fuzzy parameters. Finally, NSGA-II algorithm is applied to solve this problem and the numerical results gained by this procedure demonstrate its convergence and its efficiency in this problem.*

**KEYWORDS:** Vehicle routing problem; Equity; Air pollution; Uncertainty; Split delivery; NSGA-II.

## 1. Introduction

Terrorist attacks such as those occurred in September the 11<sup>th</sup>, or natural disasters like earthquakes in different urban areas, hurricanes, tsunamis, floods and various epidemic diseases could seriously threaten human life [1]. These occurrences are basically unpredictable and recognized as disasters and fall into this category. Providing people with relief resources such as medicine, food, tent, blanket, water and other medical supplies can save their lives, but the problem is that, these resources are usually scarce and beyond reach. Improving the performance of emergency systems can obviously decrease the number of deaths in large-scale state. For example, it is necessary to deliver medication in a specific time period but it is considerable that the objective is by no means conveying and delivering the aids to the damaged areas. Regarding the mentioned problem, we could

certainly state that time is valuable and the least delay in carrying and delivering process can increase the death rate. It is quite necessary to manage the distribution of supplies in a way to reach an equitable distribution and speed up the process. Hence, planning and controlling the route of vehicles could decrease the number of deaths, delay times and consequently result in saving people's lives.

In addition to this, air pollution is a threat to people's lives especially in big cities. When a disaster happens, help aids increases for a specified period and it may result in more air pollution. The people's lives are more important and no attention to this problem can menace a human's lives especially in big cities with high level pollution. As pointed the green supply chain management and sustainability viewpoint in VRP are vital because the production of carbon dioxide can make the global warming and a kind of disaster in the future [2]. The killing number for air pollution is more than any disaster such as war, drought, and etc [3]. In addition to this, the most dangerous pollution is CO<sub>2</sub> which can effect human's health [4]. There is a relation between CO<sub>2</sub> emission and fuel consumption. Any reduction in amount of CO<sub>2</sub> emissions is

\* Corresponding author: Masoud Rabbani  
[mrabbani@ut.ac.ir](mailto:mrabbani@ut.ac.ir)

1. Industrial Engineering Department, College of Engineering, University of Tehran, Tehran, Iran.
2. Industrial Engineering Department, College of Engineering, University of Tehran, Tehran, Iran.

beneficial to environment and decreasing the Carbone dioxide or air pollution can result in economic growth, too. Hence, considering this subject in this problem can be effective and realistic.

As seen, VRP is a known and applicable problem. Many different researches have been carried out in this field. It can be extended by considering different assumptions and objective functions. Some of the objective functions mentioned in paper [1] are the minimization of transportation costs, the number of vehicles, total travel time, total delay time, sum of route duration, sum of completion time and unmet demand. The kinds of objective functions are determined by their application and the type of problem. As pointed in disaster one of critical consideration is equitable aid distribution [5] and the equity between customer's nodes has been applied by minimizing the maximum unmet rate for each node [6]. Another research has been studied by Huang et al. [7] and performance metrics has been used to focus on equity, efficacy and efficiency. As mentioned, the amount of unsatisfied demand follows a nonlinear penalty function and the gradient for higher values is greater than the low values. Hence, considering equity conception in objective function can be realistic to reach the minimum amount of unsatisfied demand ratio.

In the case of various assumptions used in vehicle routing problems, different studies have already been done and listed [8]. Some of these assumptions are time windows, split delivery, time dependent, discrete demand, stochastic demand and travel time, service time in each node, capacity constraint for each vehicle and depot, multiple depots, pick-up and delivery, giving priority to some customers, multi period and road capacity. In this article, a new model with some real assumptions such as multiple depots, split delivery, multi commodities, fuzzy demand and travel time has been presented. Equity and air pollution, the most effective issues in real world which can reduce the number of death caused by the lack of equitable distribution and decrease air pollution and its related costs. In recent researches, these issues considered in different types of modeling without considering some of assumptions are mentioned.

At all, this research is concentrated on these most important conceptions. The next section, the literature of VRP, its application and what has been studied in air pollution and GHG VRP are expressed. New MINLP model for this problem is discussed in section three. In section four, the

procedures for solving are presented and Numerical results of this problem are shown in section five. The sixth section includes the conclusion and future results.

## 2. Literature

Sustainable management of the relief activities is one of the most important actions after disasters. It is obvious that, sending and transporting some required and emergency commodities such as medical supplies, relief equipment, foods and water are important challenge in post-disaster to decrease the number of deaths. Plan and controlling this process can provide relief to affected people.

Vehicle routing problem is a known problem presented by Dantzing and Rasmer [9]. In recent decades, different studies with Different assumptions and objective functions have been studied and various papers have been published. Variant assumptions of real world problems presented by Coltorti et al. [10] can be added to reach a real world problem. Some of them are multi depot, multi item, multi period, split delivery, backhauling, time windows, heterogeneous vehicles, identical vehicles, vehicle capacity, pickup and delivery, fuzzy demands and travel times. Hence, VRP is categorized to different groups such as SDVRP, MDVRP, VRPTW, CVRP, SVRP, and FVRP. Some of assumption is explained separately at the next paragraphs.

The first type, SDVRP, is a VRP with split delivery assumption. The conception of split delivery is that each customer can be served by one or several vehicles. Dror et al. [11] presented an integer linear programming for VRP with split delivery. The research of Archetti et al. [12] is a survey of split delivery VRP, too. The objective of SDVRP is usually to minimize the total travel cost. As mentioned in this survey, SDVRP can incorporate different assumptions such as time windows, pick-up delivery, inventory and production, stochastic, arc routing, discrete demands and heterogeneous fleet. In addition to this, split delivery assumption can help to achieve the best value of objective function. As seen in Tavakoli et al.'s paper [13], the objective is to minimize the total distance travelled and the fleet cost. So considering split assumption results in decreasing the number of vehicles and the fleet cost. Recently, a new model of VRP is presented by Ferreira et al. [14], and it is integrated split delivery assumption with green requirements which concerns the emission of CO<sub>2</sub>. As pointed, the split delivery can result in reducing the cost

of routes and emitting less CO<sub>2</sub> emissions. As pointed split delivery is effective assumption because results show an economic advantage over the non-split delivery [15]. Although kinds of problems are different, considering this assumption can be logical.

The second kind of VRP is MDVRP. In the real world, one depot is not sufficient. Usually, multi depots are located in different places to give the best service. A new survey for the MDVRP and different kinds of models has been studied by Montoya-Torres et al. [16]. Ho et al. [17] studied multi depots VRP and solved it by a hybrid Genetic algorithm. In continuous, Gulczynski et al. [18] presented a new research which is different from previous researches and split delivery and multiple depots have been combined together. This problem is a practical one. It means multiple depots deliver goods to customers and they can be served by one or more vehicles. At extension, MDGVRP with the time-varying vehicle speed and soft time windows is presented by minimizing total cost including fixed costs, fuel costs and time window penalty cost [19]. Also, their attention is to design and solve by hybrid genetic algorithm with variable neighborhood search. Furthermore, different kinds of items or commodities are needed to be delivered to required places and can be considered in models. For example, one of these multi depots VRP has been done by adding multi items [20].

Another type of vehicle routing is VRPTW. Vehicle routing problem with time window and multiple routes has been studied by Macedo et al. [21]. In this research, assumptions are single depot, similar vehicles, vehicle capacity constraint, service time and time window for each node. But, What Montoya et al. [22] studied is a little different. In this model, Capacity of vehicles is not considered so it is called a classical VRP. In this research, each customer requires a demand and it cannot be served before the deadline so lower bound is used for the start time. In following, Narasimha et al. [23] studied on a single depot vehicle routing problem with time window by considering different objective functions which is the minimization the maximum distance. Abderrahman et al. [24] studied this kind of model by minimizing the transportation cost and penalties of delays. Recently, one of new researches is a multi-depot, multi trip VRP with time windows and release times which is solved by a hybrid particle swarm optimization algorithm and a hybrid genetic algorithm [25]. It is noteworthy, as pointed [26]

deadline parameter can be considered uncertain too. But in this article, it is assumed certain.

The next type of VRP is CVRP. The vehicle capacity is one of the real world assumptions because every vehicle has a deterministic capacity. In this field, Christiansen et al. [27] surveyed the capacitated vehicle routing problem with stochastic demands. Luo et al. [28] extended the capacitated VRP with stochastic demands and time windows. It is assumed that the demand of customers is equal or less than the vehicle capacity. If customer's demand exceeds the vehicle capacity, it must return to the depot and load supplies, and then it must visit the previous customer in order to give the rest of service. It is obvious this assumption can be added in green vehicle routing problems. Recently, a capacitated GVRP by considering time-varying vehicle speed and the soft time windows is developed with minimizing carbon emission and maximizing customer satisfaction [29].

The last types of vehicle routing problem are called SVRP and FVRP. After disaster unexpected events happens on routes and roads accessibility changes so the travel time cannot be certain. The first fuzzy VRP with fuzzy due times was done by Teodorovic et al. [30]. As mentioned there are two types of non-deterministic VRPs, stochastic VRP and fuzzy VRP [31]. In stochastic VRP, some elements are random while in fuzzy VRP, elements are vague. In disaster, demands and travel times are not deterministic, so they considered the demand as a fuzzy parameter and Zheng et al. [32] studied VRP with time window and uncertain travel time. In following, Shen et al. [33] extended VRP in disaster by considering uncertain demands and travel times. It is obvious, large-scale emergency is unpredictable and the amount of required demands is high and uncertain. As pointed customer's demands usually depend on the population of that position or the number of injured persons [34]. Additionally, travel time is not certain because natural disasters may usually lead to other accidents and sometimes result in decreasing the speed of response. Recently, Gutierrez et al. [35] studied VRP with time window and stochastic travel time by minimizing the cost of routes. Another research is done by Luo et al. [28] in optimization of large-scale emergency problems. In this paper, new mathematical model is proposed so that the quantity of demands is stochastic and a real-time vehicle control is considered to get informed of the used vehicle location, new demands and route of vehicles. The research for emergency vehicle

routing problem is done by Qin et al. [36] is a little different. The objective of presented model is to minimize costs (the number of vehicles and travel distance) and the penalty cost or operating costs occurred when the actual demand are more or less than supplies. So, because of these kinds of costs, demand is considered uncertain.

It's remarkable the kind of VRP affects on selecting the objective function. In the previous papers, the most important objective is to minimize the total cost, but recently the type of objective function is different. Various objective functions have been studied by Jozefowicz et al. [37]. Some of these objectives are minimizing make span, maximizing profit, minimizing the total length of the tour, minimizing the number of vehicles used or the investment made. But in disaster relief routing, the objective functions are different. Jiang et al. [38] studied a multi objective model of MDVRP by maximizing the total quantity of materials delivered to customers during a time horizon and minimizing the difference of materials received by multiple disaster points. Another research is done by Usefi et al. [39] by considering two objectives. The first objective is to minimize the fixed costs and the second one is to consider the priority of customer satisfaction by the gap between arrival time and ready time. Actually it is a trade-off between the costs and customer satisfaction. These kinds of problems are categorized by different objectives such as minimizing the maximum unsatisfied demand, minimizing the latest arrival time and maximizing the travel reliability which is pointed by Torre et al. [40]. Recently, vehicle routing problem is mainly focused on large scale emergency. Liu et al. [41] studied the distribution of medical supplies at customer's location or required places. As noticed in this paper, the objective is to minimize the total unmet demand and delayed time because saving people's lives is vital. It is extended by considering multiple depots by Zhu et al. [42] in large scale emergency. Recently, Eydi et al. [43] represents VRP with time window by considering two objective functions, minimizing the total traveled distances and maximizing the coverage of potential demands. It is noteworthy, another objective functions are studied by Huang et al. [7] and performance metrics are used to focus on equity, efficacy and efficiency so that the structure of VRP is influenced by these measures and it concentrates on minimizing the total unsatisfied rate which its penalty follows a pieces function. At result, the first objective function of

our study is considered such as objective function pointed in Huang et al. [7].

Nowadays, as mentioned in Bashiri et al.' paper [44] the air pollution produced because of transportation is a most important concern and the survey of social and safety concepts in green house vehicle routing problem can be effective in the global market. Touati et al. [45] pointed that considering environmental and social costs in vehicle routing problem and scheduling is necessary due to growing concerns about public health and global warming; In this regard, El Bouzekri et al. [46] cued each country takes some actions to reduce pollutant emissions such as greenhouse gas because it affects on climate change. Many researches have studied in GHG VRP based on reduction CO2 emission. As seen in Saka et al. [47], VRP is one of the most critical elements in the physical distribution and the minimization the energy costs and greenhouse gas emissions depends on the mitigation of fuel consumption. In continues, Kazemian et al. [48] presented a time dependent model with time windows to decrease disagreeable environmental effects by focusing on the fuel consumption and greenhouse gas emissions. Different techniques to calculate the fuel consumption are disciplined by Toro et al. [49]. Jabali et al. [50] mentioned CO2 emission and fuel consumption related to vehicle speed, load and distance.

Based on previous studies and what is vital in disaster, the most concentration is equitable distribution and decreasing air pollution. The conception of equitable distribution points to customer satisfaction. As pointed in previous articles, customer satisfaction can be expressed in different definitions. Someone defines it as minimizing the maximum ratio of unmet or unsatisfied demand to total demand and others define the sum of these ratios. What is obvious, the unmet demand is occurred when commodities are not delivered before deadline. In other words, customers' requirements are not provided before determined time and this delay results in customers' dissatisfaction. Hence, in this research, based on Huang et al.'s research [7], the objective function is considered as minimizing the sum of unmet demand's ratio which follows a nonlinear function. The minimization objective function and the incremental slope of function ensure that this ratio tends to be minimal, in other words, it makes the tendency to delivery before deadline increases and disparity between nodes or customers decreases. In addition to this, another goal or concentration of this research is to control pollution or decrease the cost which depends on

the fuel consumption and the amount of emission CO<sub>2</sub>. At result, as seen in table1, these two important subjects have rarely been studied and the first contribution of this research is related to these defined objective functions which are integrated and studied simultaneously.

In this way, another research gap is correlated to assumptions. As pointed in previous paragraphs and seen in table1, some of realistic assumptions have not been studied together. In disaster, determining and calculating the accurate amount

of demands and travel times are not possible, so considering them as fuzzy parameters is realistic. Furthermore, the problem can be expressed generally by considering multi depots, multi items and vehicles with different capacity. Hence, a new model of VRP is presented by considering real and effective assumptions such as fuzzy demand and fuzzy travel time between nodes, multi depots, multi items, split delivery and vehicle capacity. All of these assumptions are pointed and illustrated at the next section.

**Tab. 1.A summary of the selected literature**

Author/ Reference	Objective functions	Features/ some of assumptions							Solution	
		Allowable delay time	Time window	Single depot	Multiple depot	Split delivery	Multi commodity	Fuzzy or Stochastic demand	2 dimensional-loading Fuzzy or Stochastic travel time	
Xu et al. [29]	Min: Total expected fuel consumption Max: Customer satisfaction under soft time windows	√	√							Improved NSGA--II
Gutiérrez-Padilla et al. [4]	Min: Cost total distance, cost total emission	√	√							CPLEX
Gutierrez-Pailla et al. [35]	Min: Cost of routs and fixed cost	√	√					√		Multi population Memetic algorithm
Jabali et al. [50]	Min: Total emission and total time on the road	√	√							Tabu search
Bin et al. [15]	Min: Transportation cost			√		√			√	An enhanced neighborhood search and packing heuristic
Ferreira et al. [14]	Min: CO2 emission			√		√			√	Exact solution, Branch and cut method
Eydi [43]	Min: Total travel distance Max: The coverage of demands	√	√				√			NSGA—II with different modification in mutation operator
Wan et al. [55]	Min: Carbon emission Min: Operation cost	√			√					Combination of WSHA, SWA and MOPSO
Fan et al. [19]	Min: Total fixed cost, fuel consumption, penalty cost	√			√					A Hybrid GA with VNS
Kazemian et al. [48]	Min: Fuel consumption and GHG emission	√	√							Simulated annealing
Zhu et al. [42]	Minimize the maximum unsatisfied rate				√		√			LP relaxation algorithm based on LP-rounding technique
Huang et al. [7]	Min: Total travel cost Min: Total arrival time Min: Total the percentage of unsatisfied demand based on non-linear function	√		√		√				CPLEX+ Greedy adaptive search procedure
New research	Min: Unsatisfied demand (non-linear function) Min: Cost of fuel consumption	√			√	√	√	√	√	NSGA—II

### 3. Proposed Model

The multiple depots vehicle routing problem is defined as a graph. This graph  $G = (V, E)$  is defined by V for vertex and E for edges. The set of vertex (V) is categorized under two subsets: one subset for customer nodes and another for depots and the weight of edges is the travel time

between vertexes. This new model and our assumptions are presented as follows:

#### Assumptions:

Our assumptions considered in this problem, are mentioned as follows:

1. Multiple depots are considered.

2. The number of vehicles is not the same in each depot and vehicles are heterogeneous with different capacities.
3. There are deterministic supplies of commodities in each depot.
4. Each node or customer wants fulfillment of its demands before the defined deadline.
5. The demand of each node may be satisfied by several vehicles. Giving services are not limited to only one vehicle, because of split delivery assumption.
6. The demand of each node is not certain, it is a fuzzy parameter.
7. Travel time between nodes is considered a fuzzy parameter.
8. All of the vehicles are available in each depot at the start time.
9. Vehicles leave the depot and return there after delivering the goods to the customers.
10. The maximum limit for overall CO2 intensities is defined.

#### Indexes:

$i, j$  represents node  $i$  or customer  $i$  ( $i=0$ ; represent depots), ( $i=1 \dots N$ )

$m$  Index for each depot

$k$  Index for vehicles [ $A_1, A_2 \dots A_m$ ]

$p$  Index for kind of commodities

All of the vehicles in a depot are shown with an index  $k$ , so that the number of  $A_1$  column represents vehicles of depot1, the number of  $A_2$  column is for depot 2 and etc.

#### Parameters:

$M$  The number of depots

$A_m$  The number of vehicles for depot

$A$  The sum of available vehicles is equal  $\sum_{m=1}^M A_m$

$C_k$  Capacity of vehicle  $k$

$S_{m,p}$  Amount of available commodity

$p$  in depot  $m$

$dl_i$  Deadline is determined by node  $i$

#### Constraints:

$$T_{0,k} = 0$$

$$\sum_{j=1}^N X_{0,j,k}$$

$$\sum_{k=1}^A \sum_{i=0}^N X_{i,j,k} \geq 1$$

$$\sum_{i=0}^N X_{i,j,k} = \sum_{p=0}^N X_{j,p,k}$$

$$\sum_{j=1}^N X_{0,j,k} = \sum_{p=1}^N X_{p,0,k}$$

$$\sum_{i=1}^N \sum_{j=1}^N X_{i,j,k} \leq N - 1$$

$$T_{i,k} + \widetilde{T}_{i,j} - T_{j,k} \leq (1 - X_{i,j,k}) * M$$

$$0 \leq T_{i,k} \leq \sum_{j=0}^N X_{i,j,k} * M$$

$\widetilde{D}_{i,p}$  Required demand of commodity

$p$  for node  $i$

$\widetilde{T}_{i,j}$  Travel time from node  $i$  to node  $j$

$dd_{i,j}$  Distance in kilometers from node  $i$  to node  $j$

$\delta$  Decrement in fuel efficiency for a kilogram of load on the vehicle

$E$  Maximum CO2 emission limit for all transportations

$CF$  Cost for a unit amount of fuel

#### Decision variables:

$U_{i,p}$  The demand of commodity  $p$  is not met in node  $i$

$Y_{i,k,p}$  The amount of commodity  $p$  is carried to node  $i$  by vehicle  $k$  (from depot  $m$ )

$T_{i,k}$  The time, vehicle  $k$  (from depot  $m$ ) visits node  $i$

$\sigma_{i,k}$  Delay time is happened in node  $i$  by vehicle  $k$  (from depot  $m$ )

$X_{i,j,k}$  Flow variable, if vehicle  $k$  (from depot  $m$ ) traversed arc  $\{i, j\}$  from node  $i$  to node  $j$ , it is equal to one, otherwise it is zero

$\theta_{i,k}$  Fuel efficiency of vehicle  $k$  after serving node  $i$

$e_{i,j,k}$  CO2 emission amount by vehicle  $k$  for traveling from node  $i$  to node  $j$

#### Objective functions:

$$\text{Min} \quad \sum_{p=1}^P \sum_{i=1}^N f(w_{i,p})$$

$$f(w) = \begin{cases} \frac{4w}{13} & w < 0.25 \\ \frac{8w-1}{13} & 0.25 \leq w < 0.5 \\ \frac{16w-5}{13} & 0.5 \leq w < 0.75 \\ \frac{24w-11}{13} & 0.75 \leq w \end{cases}$$

$$W_{i,p} = U_{i,p} / \widetilde{D}_{i,p}$$

$$\text{Min} \quad \sum_{k=1}^m \sum_{j=0}^N \sum_{i=0}^N c_{i,j} * X_{i,j,k}$$

$$\forall k = 1 \dots A \quad (1)$$

$$\forall k = 1 \dots A \quad (2)$$

$$\forall j = 1 \dots N, i \neq j \quad (3)$$

$$\forall j = 1 \dots N; k = 1 \dots A \quad (4)$$

$$\forall k = 1 \dots A \quad (5)$$

$$\forall k = 1 \dots A \quad (6)$$

$$\forall i = 0, \dots N; j = 1 \dots N; i \neq j; k = 1 \dots A \quad (7)$$

$$\forall i = 0 \dots N; k = 1 \dots A \quad (8)$$

$$\begin{aligned}
 T_{i,k} - \sigma_{i,k} &\leq dl_i * \sum_{j=0}^N X_{i,j,k} & \forall i = 1 \dots N; k = 1 \dots A & (9) \\
 \sum_{m=1}^M S_{m,p} &\geq \sum_{k=1}^A \sum_{i=1}^N Y_{i,k,p} & \forall p = 1 \dots P & (10) \\
 \sum_{k=1}^A Y_{i,k,p} &\geq \bar{D}_{i,p} - U_{i,p} & \forall i = 1 \dots N; p = 1 \dots P & (11) \\
 C_k &\geq \sum_{p=1}^P \sum_{i=1}^N Y_{i,k,p} * W_p & \forall k = 1 \dots A & (12) \\
 \sum_{p=1}^P Y_{i,k,p} * W_p &\leq C_k * \sum_{j=1}^N X_{i,j,k} & \forall i = 1 \dots N; k = 1 \dots A & (13) \\
 \sum_{k=1}^M \sum_{j=0}^n \sum_{i=0}^n e_{ij}^k &\leq E & & (14) \\
 e_{i,j}^k &= (-13.841 \theta_i^k + 367.91) * dd_{i,j} & \forall i = 1 \dots N; j = 1 \dots N; k = 1 \dots A & (15) \\
 \theta_i^k &= \theta_0^k (1 - \delta * \sum_{j=0}^n x_{i,j}^k * (\sum_{p=1}^M y_{i,k,p} * W_p)) & \forall i = 1 \dots N; k = 1 \dots A & (16) \\
 c_{i,j} &= CF * dd_{i,j} / \theta_i^k & \forall i = 1 \dots N; j = 1 \dots N; k = 1 \dots A & (17) \\
 Y_{i,k,p} \geq 0; T_{i,k} \geq 0; \sigma_{i,k} \geq 0; U_{i,p} \geq 0; X_{i,j,k} &= 0 \text{ OR } 1 & & (18)
 \end{aligned}$$

The function shows the importance of equity concept in disaster. As seen the penalty function for the fraction of unsatisfied demand follows nonlinear function. It means by increasing the fraction of unmet demand, the penalty cost will be increased. The second objective function is to minimize the cost which depends on the fuel consumption. Constraint (1) shows that all of the vehicles are available in depots at time zero. Constraint (2) shows that the numbers of vehicles departure their depots are limited. Constraint (3) shows that at least one vehicle visits each customer node. It means that several vehicles can give services to the same node. It shows split delivery is allowed. As pointed this assumption are rarely seen in GVRP. Constraint (4) represents that each vehicle, enters a node, would leave it. Constraint (5) is for balancing the number of vehicles, each vehicle that travels from a depot to customer nodes, return to its depot at last. Constraint (6) represents that the maximum length of cycle for each vehicle is less than the number of nodes. Constraint (7) shows the sequence of nodes that vehicle  $k$  (from depot  $m$ ) travels. If the vehicle passes from arc  $\{i,j\}$ , the time of node  $j$  will be equal to the sum of the visit time of node  $i$  and the travel time. Constraint (8) represents that if vehicle  $k$  (from depot  $m$ ) does not pass from node  $i$ , the visit time in this node for this vehicle will be zero. Constraint (9) is for considering the delay time and the dead line. If before the dead line, node  $i$  is getting the required services by vehicle, the delay time will be zero. Constraint (10) shows that the sum of commodity  $p$ , delivered to customers by different vehicles is less than the available supplies of commodity  $p$  in depot  $m$ . Constraint (11) create a balance for each node that its satisfied demand and unsatisfied demand is less than the total commodity carried by different vehicles. Constraints (12), (13) show that the amount of commodity carried by each vehicle at each edge cannot be more than the

capacity of that vehicle. Non negative variables are represented in constraint (18).

Constraints (14-17) related to the CO2 emission. The maximum limit for overall CO2 intensities is shown by constraint (14). As mentioned in Yin et al. paper [2], the relationship among the CO2 emission intensity, transportation distance and the vehicle loading is defined in ITRI (<http://auto.itri.org.tw/>) site. Constraint (15) is a regression formula and constraint (16) shows the fuel efficiency. The fuel cost is calculated by constraint (17).

As seen in presented mode, two different objective functions, the sum of unmet demand's ratio and the cost of fuel consumption, are integrated with elaborated assumptions. All of mentioned assumptions including fuzzy parameters, multi depot, multi items, split delivery and vehicle's capacity are handled in a new MINLP and the next section illustrates how this model is transformed to a linear type.

### 3.1. Linearize model

As mentioned in Zhu et al.'s paper [42] VRP is an NP-hard problem even if it is assumed for one depot and one vehicle. In addition to this, one of our objectives is to minimize the fraction of unmet demand for each commodity at each customer node to its demand, so that it is defined as a piecewise linear function. By using a simple technique called convex combination (CC) which aggregates the continuous variables by one auxiliary continues variable  $\lambda_v$ , it is formulated as a linear programming. In this problem, the function has been linearized by adding these constraints.

$$f(x) = m_p(x) + C_p \quad x \in P$$

$$\sum_{v \in v(p)} \lambda_v v = x \quad (19)$$

$$\sum_{v \in v(p)} \lambda_v (m_p(v) + C_p) \leq Z \quad (20)$$

$$\lambda_v \geq 0; \quad \forall v \in v(P) = \cup_{p \in P} v_p; \quad \sum_{v \in v(p)} \lambda_v = 1 \quad (21)$$

$$\lambda_v \leq \sum Y_p; \quad \forall v \in v(P) \quad \sum_{p \in P} Y_p = 1; \quad Y_p \in \{0,1\} \quad (22)$$

As mentioned in assumptions section, the demand and travel time between nodes are considered as fuzzy parameters, because in disaster areas the demand and travel time are vague. Additionally, there is not sufficient time to estimate the function of these parameters so by considering these assumptions the difference between the real answer and the solved model will be minimal. In this paper, the patterns used to represent fuzzy parameters are triangular fuzzy numbers so that three numbers including optimistic, pessimistic and possible values are estimated for the demand and travel times. A membership function is defined for fuzzy sets. It is between zeros and one that shows the degree of values.

For solving this kind of model, it is required to defuse the model to a crisp model. Different ranking methods are applicable for fuzzy decision making. Jimenz method was introduced in 1996. This definition which is presented by Jimenz et al. [51] shows how to defuse the model. It is assumed that  $\tilde{a}$  and  $\tilde{b}$  is fuzzy sets so that  $\tilde{a}$  is bigger than  $\tilde{b}$ . For each fuzzy set two expected values E1 and E2 are calculated; E1 is the expected value of  $[a_o, a_m]$  interval and E2 is the expected value of  $[a_m, a_p]$  interval.

If  $\tilde{a} \geq \tilde{b} \rightarrow \beta * E_1^a + (1 - \beta) * E_2^a \geq (1 - \beta) * E_1^b + \beta * E_2^b$

Finally, a crisp model includes two objective functions, constraint (1)-(6), (8)-(10), (12)-(22) with (23), (24) and (25) constraints.

Constraints:

$$T_{i,k} + (1 - \beta) * E_1^{TT_{ij}} + \beta * E_2^{TT_{ij}} - T_{j,k} \leq (1 - X_{i,j,k}) * M; \quad (23)$$

$$(1 - \beta) * E_1^{D_{i,p}} + \beta * E_2^{D_{i,p}} - U_{i,p} \leq \sum_{k=1}^A Y_{i,k,p} \quad (24)$$

$$U_{i,p} \leq ((\beta) * E_1^{D_{i,p}} + (1 - \beta) * E_2^{D_{i,p}}) * w_{i,p} \quad (25)$$

#### 4. Solution Method

As seen in published papers, different methods are applied for solving vehicle routing problems. Variant applicable heuristics are mentioned in Gendreau et al. [52]. Laporte et al. [53] used classical and modern heuristics such as sweep algorithm, two-phase approach and the saving method for solving the VRP. The MDVRP presented by Montoya-Torres et al. [16] are solved by Meta heuristic and hybrid procedures. Ho et al. [17] applied two hybrid genetic algorithms, HGA1 and HGA2 to solve their problem. In continuous, a new algorithm, pseudo-polynomial network, is applied to solve MVRPTW presented by Macedo et al. [21] and

Ant colony optimization is applied to solve VRPTW by Narasimha et al. [23]. The results given by randomization, the procedure used by Montoya et al. [22] show that it is better procedure than the exact solution. Also, Bin et al. [15] solved a new model of SVRP by an enhanced neighborhood search algorithm.

Multi objective models are different from single objective ones. The reason is that in single objective, there is one search space for decision variables so two solutions can be compared and the best one can be selected. But, in multi objective optimization, in addition to decision space, there is objective space; so, the comparison between two solutions is difficult. As seen for solving multi objective problems, different optimization methods are applicable. Liu et al. [41] solved the multi objective model by global search solution method. Another research was done in this field by Zhu et al. [42] and solved by a local search algorithm and Shen et al. [34] solved stochastic VRP by a tabu search. Some of new researches are focused on exact solution. For example, Rekika et al. [20] presented a two-stage solution method and a heuristic algorithm called exhaustive enumeration algorithm to solve presented problem. Zarandi et al. [54] developed a solving procedure combined Simulated and different kinds of clustering algorithm such as random, best fit decreasing, hard c-means, fuzzy c-means and possibility c-means. Usefi et al. [39] solved VRP in large sized problem by multi choice goal programming approach and SA-GA methods. In continuous, the GVRP presented by Wang et al. [55] solved by a hybrid heuristic algorithm, the combination the Clarke and Wright Savings Heuristic Algorithm (CWSHA), the Sweep Algorithm (SWA) and the Multi-Objective Particle Swarm Optimization algorithm (MOPSO). A hybrid genetic algorithm with variable neighborhood search is applied to solve the MGVRP presented by Fan et al. [19] and an improved NSGA-II with adaptive strategies and greedy strategies is applied to solve the capacitated GVRP proposed by Xu et al. [29]. The survey done by Coello et al. [56] points that the NSGA-II procedure has been demonstrated as one of the most efficient algorithms for solving multi objective problems because of using a more efficient ranking scheme (called non-dominated sorting) and adopting a clever mechanism called crowded comparison operator. Furthermore, as seen in Verma et al.'s [57] survey, the NSGA-II procedure is implemented to obtain the trade-off between two objectives and has been applied in different VRP. At result, based on the previous



studies and NSGA-II's features, this procedure is selected to solve the multi objective problem. It is explained completely at the next section.

#### 4.1. Genetic algorithm NSGA-II procedure

The new model, presented in this paper, is a multi-objective problem, too. Variant procedures are proposed to achieve the best solution for multi-objectives. Here, the procedure which is applied to solve the new model is NSGA-II. It is a non-dominating sorting genetic algorithm that solves non-convex and non-smooth multi-objective optimization problems. It is depicted in below and the results are reported in the experimental section. In multi-objective problems, the purpose is to find an approximation subset of the set of efficient solutions.  $E(P)$  includes an efficient solution and  $\tilde{E}(P)$  is an approximation of  $E(P)$ .

**Domination definition:** The solution  $x$  dominates  $y$  when the objective functions of individual  $x$  are equal or less than the other one ( $y$ ) and at least one of the objective functions of it( $x$ ) is better than the other one( $y$ ) :

1. For  $k=1 \dots K$ ,  $z(x) \leq z(y)$
2. At least for one objective,  $z(x) < z(y)$

In the objectives space  $R$ , the set of points  $\{(Z_1(x) \dots Z_k(x)) | X \in E(P)\}$  are called Efficient Frontiers, so the purpose is to find an approximation of this efficient frontier.

**Min**  $Z_k(x)$  (For  $k=1 \dots K$ )

$$W_k^1 = (W_1^1, W_2^1 \dots W_k^1)$$

$$\sum_{k=1}^K W_k^1 = 1$$

$W^1$ : Different weighted vectors are used to cover all of the feasible space

The weighted sum is applied for scalarizing the objective function; so the scalarizing function is defined  $S(Z, W^1) = \sum_{k=1}^K W_k^1 * Z_k(x)$ .

When there are several objectives, the goal of multi objective optimization problem is to find a set of Pareto-optimum solutions by creating a trade-off relationship between objectives. As mentioned by Watanabe et al. [58], GA is one of the useful optimization tools to find a Pareto-optimum set. In following, as mentioned by Raghuvanshi et al. [59], NSGA is a non-domination based Genetic algorithm for multi objective optimization. It was improved by Deb et al. in 2002 and called NSGA-II. The new version is an elitist multi objective evolutionary algorithm and has a better sorting algorithm than the old one. It includes two kind of sorting, dominated sorting and crowding distance. The steps of this algorithm are mentioned as follows:

1. [start] create initial and feasible population
2. [fitness] evaluate the objectives for each solution
3. [sorting] sorting of population based on two metrics:
  - Dominated sorting
  - Crowding distance
4. [new population]
  - Selecting the best parents
  - Crossover
  - Mutation
5. [exchange]
6. Repeat ...
7. [loop] step2

As pointed, the initial population is sorted by non-dominated algorithm and Dominated sorting algorithm finds efficient individuals of the set of  $P$ , categorize them based on their rank into separate fronts. Another parameter for measuring is crowding distance. It is a sharing method applied to preserve the diversity of solutions. After assigning individuals to each front, the crowding distance is used to sort the individuals which have the same rank and are put at the same front. The crowding distance is calculated based on the Euclidian distance for each objective and between two individuals which belong to a front. The crowding distance value for each individual is equal to the sum of crowding distance values calculated for each objective. The step of this algorithm is described as follows. It is obvious it is sorted descending because the large amount assures its diversity at a solution set.

#### 5. Numerical Results:

As mentioned NSGA-II procedure is applied to solve the multi objective problem. In this section, at first some related parameters of this procedure is described and then the results of experiments and related analyses are presented. Determining the initial parameters for NSGA-II methods is important to give the best solution. For the solution presented in below, some of parameters such as the population size, maximum iteration, keep percentage and mutation percentage are defined 90, 30, 2/3 and 1/3. This program is codified and run by MatlabR2014a program on a Pentium 5 with a 2.4 GHz CPU processor and 4G of RAM and the best initial parameter for each method is determined.

Different experiments are carried out in the scenarios with various customers, vehicles and depots. For example, the range of customers' number is considered 10, 30 and 50 and the number of vehicles and depots are set from 5 to 16 with 2 or 3 depots. The number of Product

types is fixed 3. It is obvious the purpose of vehicle routing problem is to determine the best rout-sequence for each vehicle based on conditions and constraints. The solution for one of scenarios which includes 50 customers, 16 vehicles and 2 depots is presented in table2 so that vehicles for depot one are determined by number 1 to 8 and vehicles for depot 2 are determined by number 9 to 16. As seen, the routing sequence for vehicle#1 is <0-44-18-4-39-30-0> where 0 show its start from its depot and

return to it. It covers 3 types of commodities are required for customers by numbers 44,18,4,39 and 30 but the first and second type of demand for customer 44 is prepared by vehicle#1 and the third one are provided by vehicle#3 because of allowed split delivery. The routing sequence for vehicle#12 is <0-27-45-31-0>, it means vehicle#12 which belongs to second depot provides customers' demand defined by numbers 27, 45 and 31.

**Tab. 2. Vehicle's routs**

Depot No.	Vehicle No.	Vehicles routing sequence
1	1	0-44-18-4-39-30-0
1	2	0-34-50-25-0
1	3	0-23-10-44-0
1	4	0-20-46-7-0
1	5	0-5-1-8-47-0
1	6	0-13-40-11-0
1	7	0-15-36-49-0
1	8	0-29-42-4-0
2	9	0-7-11-19-39-0
2	10	0-3-32-25-0
2	11	0-22-35-38-0
2	12	0-27-45-31-0
2	13	0-14-12-44-0
2	14	0-47-9-0
2	15	0-37-2-8-0
2	16	0-21-24-13-0

This procedure, NSGA-II is run several times for different scenarios in large-scales and the gained results of objective functions including the average of lower bounds, upper bounds and the best values of objective functions are reported in table3. Furthermore, the values of objective functions are calculated in two cases, with split delivery and without split delivery. Results show the best values of objective functions are reduced by considering split delivery. The column named NS1/S1 is defined the ratio of the first objective's best value without split to first objective's best value with split assumption. The average of these values is calculated 1.12, which indicates that the effect of the split delivery on the first objective function is approximately 12%. This definition can also be expressed for column named NS2/S2

so that the average of these values is computed 1.052, which indicates that by considering split delivery, an average reduction of 5% on the second objective function or cost of fuel consumption can be achieved.

In addition to this, there is a relation between the objective functions and the number of depots. The graph of the first and second objective functions' best values for each scenario are plotted in the figure1. The results show that increasing the number of depots results in decreasing objective functions. It makes the amount of unsatisfied demands be diminished and the cost of fuel consumption is dropped, too. At last, considering these assumptions can be effective in real world problems.

**Tab. 3. Results table for different scenario**

Customers # Depots	Mean Lower bound1	S1:Mean Best_value1 (with split)	NS1: Mean Best_value1 (no split)	NS1/S1	Mean Upper bound1	Mean Lower bound2	S2: Mean Best_value2 (with split)	NS2: Mean Best_value2 (no split)	NS2/S2	Mean Upper bound2
10#2	12.21	12.36	14.83	1.19	19.49	9.52	11.25	11.54	1.025	35.39
10#3	9.68	10.03	11.33	1.13	18.27	9.35	9.96	10.85	1.08	30.15
30#2	24.91	26.41	29.2	1.11	37.56	22.07	27.63	28.18	1.02	51.66
30#3	21.87	26.03	24.49	0.94	38.69	17.74	24.5	26.46	1.08	46.83
50#2	27.22	30.64	36.76	1.19	42.08	23.13	37.04	38.89	1.049	55.82
50#3	21.83	23.22	27.39	1.17	37.62	20.51	29.02	30.76	1.06	47.68

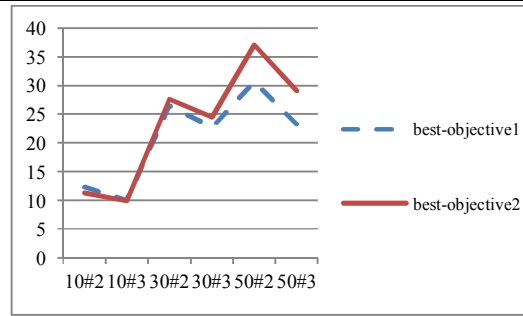


Fig. 1. Results diagram

As explained, in presented MINLP model, two objective functions are defined. It is clear that the focus of applied NSGA-II is to give an optimal solution by balancing between two objective functions which are plotted in figure 2. It shows that the trend of both values is nearly reversed and the value of objective1 increases with decreasing the value of objective2. These values are not possible at the minimum value based on trade-off conception so it is required the performance of this algorithm is appraised. To

appraise, the difference between the best values and lower bounds are compared. The results of table3 show that the average difference between the optimal value of the first objective function and its lower bound is about 7% and this difference for the second objective function is about 31%. It means the NSGA-II procedure converges to optimality with a decrease in the sum of unmet demands and the cost of fuel consumption.

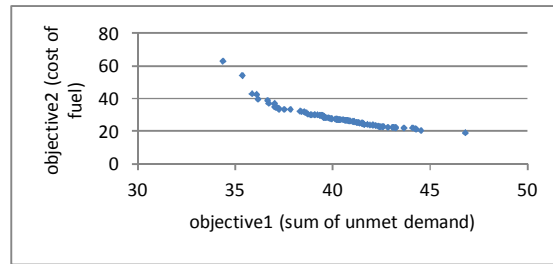


Fig. 2. Trade-off between unsatisfied demand and the cost of fuel consumption

Moreover, to validate the performance of this algorithm some metrics such as diversification metric and mean of ideal metric which has been adopted by Rabiee et al. [58] are computed. These metrics can be computed as the following formulation illustrated below by EQ (26) and EQ (27). As seen, diversification metric measures the diversity of non-dominated solutions and the mean ideal distance is a metric that measures the average of distances between the Pareto solutions and ideal point. The results of this problem show the amount of 49.59% for the diversification metric and 54% for the mean ideal distance metric. The large amount of these metrics expresses the procedure's effectiveness for these kinds of problems.

Parameters  $f_{1i}^{max}, f_{1i}^{min}$  are the maximum and minimum amount for objective 1 (Z1) of the best Pareto front

Parameters  $f_{2i}^{max}, f_{2i}^{min}$  are the maximum and minimum amount for objective 2 (Z2) of the best

Pareto front and n presents the number of non-dominated solution.

$$\text{Diversification} = \frac{\sum_{i=1}^N \sqrt{(f_{1i}^{max} - f_{1i}^{min})^2 + (f_{2i}^{max} - f_{2i}^{min})^2}}{N} \quad (26)$$

$$\text{Mean ideal distance} = \frac{\sum_{i=1}^{n-1} \sqrt{z_{1i}^{max^2} + z_{2i}^{max^2}}}{n} \quad (27)$$

## 6. Managerial Implications

In today's competitive environment, planning for transportation and distribution system has been the focus of many organizations because a good plan has an important role in saving people's life especially in emergency situation. There are two major and global concerns that should be considered in this issue. One of them is increasing the customer satisfaction or decreasing the delay time. It is clear that the customers and enhancing the customer's satisfaction is of great importance. Another one is related to air pollution and decreasing the CO2 emission and

fuel consumption. This study points out both of major issues and tries to balance between these significant concerns. Accordingly, the efficient procedure is applied to create a trade-off between them and make a good decision.

Besides, in a real world some parameters such as demands and travel times cannot be estimated accurately so this study takes into account these kinds of uncertainties in decision making to deal with the least deviation from a real problem. As a result, based on the obtained results, it can be concluded that we can make the best decision and reach our goals, the most appropriate level of customer satisfaction and air pollution, with considering the more realistic conditions and creating a trade-off between them.

### 7. Conclusion and Future Research

Planning and controlling distribution system has an important role in large-scale emergency to achieve the equitable distribution and decrease the number of death. In addition to this, air pollution is another concern that threatens people's life and can make a kind of disaster in the real world. Hence, the kind of problems can affect on the type of objective functions. Here, the new MINLP model is proposed by different real world assumptions such as split delivery and fuzzy demand and travel time with two objective functions. The first objective of this model is to minimize the sum of unmet demand's rate which follows a piecewise function to reach an equity conception and equitable distribution and the second one is to minimize the cost of fuel consumption and decrease the co2 emissions. A new multi-objective problem is presented and the purpose is to trade-off between equity and decreasing disagreeable environmental effects of the cost of fuel consumption. NSGA-II is Meta heuristic algorithms applied to solve this problem. For presented model, NSGA-II is a convergent procedure and the results approve its accuracy because there is a trade-off between two different objectives and the metrics used demonstrate its performance and efficiency in this model.

It is obvious, when a disaster occurs; the distribution of commodities is not limited to ground distribution. For example, helicopter is one of usual equipment to service the customers. So, one of the future researches can be an extendable model by using this assumption and studying different kinds of service equipment. In addition to this, travel time is considered uncertain but when an earthquake is happening the road may be disturbed and the road capacity

is changes completely. So, it can be studied in the future. Addition to this, some algorithms improves the performance of NSG-II so applying some methods in this algorithm can be effective and can be surveyed.

### References

- [1] Rabbani M., & Abutalebi E., "*Vehicle Routing Problem for Medical Supplies in Large-scale Emergency with Multiple Depot on Split Deliveries*", paper presented at the 12th International Conference on Traffic and Transportation Engineering Tehran, Iran, 21-22 Feb, (2013).
- [2] Yin P.Y., Lyu S.R., & Chuang Y.L., "*Cooperative coevolutionary approach for integrated vehicle routing and scheduling using cross-dock buffering*", Engineering Applications of Artificial Intelligence, Vol. 52, (2016), pp. 40-53.
- [3] Figliozzi M., "*Vehicle routing problem for emissions minimization*", Transportation Research Record: Journal of the Transportation Research Board, Vol. 2197, (2010), pp. 1-7.
- [4] Gutiérrez-Padilla M.V., Morillo-Torres D., & Gatica G., "*A Novel Mathematical Model for a Discrete Speed Pollution Routing Problem with Time Windows in a Colombian Context*", IFAC-PapersOnLine, Vol. 54, No. 1, (2021), pp. 229-235.
- [5] Balcik B., Beamon B.M., & Smilowitz K., "*Last mile distribution in humanitarian relief*", Journal of Intelligent Transportation Systems, Vol. 12, No. 2, (2008), pp. 51-63.
- [6] Zhu J., Huang J., & Liu D., "*Equitable resource allocation problem with multiple depots in emergency management*", Emergency Management and Management Sciences (ICEMMS) IEEE International Conference on (2010), pp. 37-40.
- [7] Huang M., Smilowitz K., & Balcik B., "*Models for relief routing: Equity, efficiency and efficacy*", Transportation research part E: logistics and

- transportation review, Vol. 48, No. 1, (2012), pp. 2-18.
- [8] Laporte G., "What you should know about the vehicle routing problem", Naval Research Logistics (NRL), Vol. 54, No. 8, (2007), pp. 811-819.
- [9] Dantzig G.B., & Ramser J. H., "The truck dispatching problem", Management science, Vol. 6, No.1, (1959), pp. 80-91.
- [10] Coltorti D., & Rizzoli A.E., "Ant colony optimization for real-world vehicle routing problems", ACM SIGEVolution, Vol. 2, No. 2, (2007), pp. 2-9.
- [11] Dror M., Laporte G. & Trudeau P., "Vehicle routing with split deliveries", Discrete Applied Mathematics, Vol. 50, No. 3, (1994), pp. 239-254.
- [12] Archetti C., & Speranza M.G., "Vehicle routing problems with split deliveries", International Transactions in Operational Research, Vol. 19, No. 1-2, (2012), pp. 3-22.
- [13] Tavakkoli-Moghaddam R., Safaei N., Kah M.M.O. & Rabbani M., "A new capacitated vehicle routing problem with split service for minimizing fleet cost by simulated annealing", Journal of the Franklin Institute, Vol. 344, No. 5, (2007), pp. 406-425.
- [14] Ferreira K.M., Queiroz T.A.d., & Toledo F.M.B., "An exact approach for the green vehicle routing problem with two-dimensional loading constraints and split delivery", Computers & Operations Research, Vol. 136, (2021), pp. 0305-0548.
- [15] Bin J., Saiqi Z., Samson S.Y., & Guohua Wu., "An enhanced neighborhood search algorithm for solving the split delivery vehicle routing problem with two-dimensional loading constraints", Computers & Industrial Engineering, Vol. 162, (2021), pp. 0360-8352.
- [16] Montoya-Torres, J.R., Franco J., Isaza S., Jimenez S., & Herazo-padilla N., "A literature review on the vehicle routing problem with multiple depots", computer and industrial engineering, Vol. 79, (2015), pp. 115-129.
- [17] Ho W., Ho G.T.S., Ji P., & Lau H.C.W., "A hybrid genetic algorithm for the multi-depot vehicle routing problem", Engineering Applications of Artificial Intelligence, Vol. 21, No. 4, (2008), pp. 548-557.
- [18] Gulczynski D., Golden B. & Wasil E., "The multi-depot split delivery vehicle routing problem: An integer programming-based heuristic, new test problems, and computational results", Computers & Industrial Engineering, Vol. 61, No. 3, (2011), pp. 791-804.
- [19] Fan H., Zhang Y., Tian P., Lv Y., & Fan H., "Time-dependent multi-depot green vehicle routing problem with time windows considering temporal-spatial distance", Computers & Operations Research, Vol. 129, (2021), pp. 0305-0548.
- [20] Rekika M., Renauda J., & Berkounea D., "Two-Stage Solution Methods for Vehicle Routing in Disaster Area", (2012), pp. 1-30.
- [21] Macedo R., Alves C., De Carvalho V., Manuel J., Clautiaux F., & Hanafi S., "Solving exactly the vehicle routing problem with time windows and multiple routes using a pseudo-polynomial model", European Journal of Operational Research, Vol. 214, No. 3, (2011), pp. 536-545.
- [22] Montoya-Torres J.R., Alfonso-Lizarazo E.H., Franco E.G., & Halabi A.X., "Using randomization to solve the deterministic single and multiple vehicle routing problems with service time constraints", Winter Simulation Conference (WSC), in Proceedings of the 2009 Winter Simulation Conference, pp. 2989-2994.
- [23] Narasimha V., Koushik S., & Kumar M., "Ant colony optimization technique to solve the min-max Single Depot Vehicle Routing Problem", In American Control

- 
- Conference, ACC, IEEE, (2011), pp. 3257-3262.
- Expert Systems with Applications, Vol. 37, No. 3, (2010), pp. 2405-2411.
- [24] Abderrahman A., Karim E.L.B., Hilali E.L., Ahmed A., & Adil B., “A hybrid algorithm for vehicle routing problem with time windows and Target time”, Journal of theoretical and applied information technology, Vol. 95, No. 1, (2016), pp. 0-8.
- [25] Zhen L., Ma Ch., Wang K., Xiao L., & Zhang W., “Multi-depot multi-trip vehicle routing problem with time windows and release dates”, Transportation Research Part E: Logistics and Transportation Review, Vol. 135, (2020), p. 101866.
- [26] Zhang D., Li D., Sun H., & Hou L., “A vehicle routing problem with distribution uncertainty in deadlines”, European Journal of Operational Research, Vol. 292, (2021), pp. 311-326.
- [27] Christiansen C.H., Lysgaard J., & Wøhlk S., “A branch-and-price algorithm for the capacitated arc routing problem with stochastic demands”, Operations Research Letters, Vol. 37, No. 6, (2009), pp. 392-398.
- [28] Luo J.Y., Wang J.Y., & Yu H., “A dynamic vehicle routing problem for medical supplies in large-scale emergencies”, Information Technology and Artificial Intelligence Conference (ITAIC), 6th IEEE Joint International, Vol. 1, (2011), pp. 271-275.
- [29] Xu Z., Elomri A., Pokharel S., & Mutlu F., “A model for capacitated green vehicle routing problem with the time-varying vehicle speed and soft time windows”, Computers & Industrial Engineering, Vol. 137, (2019), p. 106011.
- [30] Teodorović D., & Pavković G., “The fuzzy set theory approach to the vehicle routing problem when demand at nodes is uncertain”, Fuzzy sets and systems, Vol. 82, No. 3, (1996), pp. 307-317.
- [31] Cao E., & Lai M., “The open vehicle routing problem with fuzzy demands”,
- [32] Zheng Y., & Liu B., “Fuzzy vehicle routing model with credibility measure and its hybrid intelligent algorithm”, Applied Mathematics and Computation, Vol. 176, No. 2, (2006), pp. 673-683.
- [33] Shen Z., Ordóñez F., & Dessouky M.M., “The minimum unmet demand stochastic vehicle routing problem”, University of Southern California, (2006).
- [34] Afshar A., & Haghani A., “Modeling integrated supply chain logistics in real-time large-scale disaster relief operation”, Socio-Economic Planning Sciences, Vol. 46, No. 4, (2012), pp. 327-338.
- [35] Gutierrez A., Dieulle L., Labadie N., & Velasco N., “A multi population memetic algorithm for the vehicle routing problem with time windows and stochastic travel and service times”, IFAC-Papers OnLine, Vol. 49, No. 12, (2016), pp. 1204-1209.
- [36] Qin J., Ye Y., Cheng B.R., Zhao X., & Ni L., “The emergency vehicle routing problem with uncertain demand under sustainability environments”, Sustainability, Vol. 9, No. 228, (2017).
- [37] Jozefowicz N., Semet F., & Talbi E.G., “Multi-objective vehicle routing problems”, European Journal of Operational Research, Vol. 189, No. 2, (2008), pp. 293-309.
- [38] Jiang J., Li Q., Wu L., & Tu W., “Multi-objective emergency material vehicle dispatching and routing under dynamic constraints in an earthquake disaster environment”, ISPRS Int. J. Geo-Information, Vol. 6, No. 5, (2017).
- [39] Yousefi H., Tavakkoli-Moghaddam R., Taheri Babil Oliaei H., Mohammadi M., & Mozaffari A., “Solving a bi-objective vehicle routing problem under uncertainty by a revised multi choice goal programming approach”, International Journal of Industrial Engineering

- Computations, Vol. 8, No. 3, (2017), pp. 283-302.
- [40] Torre L.E., Dolinskaya I.S. & Smilowitz K.R., "Disaster relief routing: Integrating research and practice", Socio-Economic Planning Sciences, Vol. 46, (2012), pp. 88-97.
- [41] Liu D., Han J., & Zhu J., "Vehicle routing for medical supplies in large-scale emergencies", Lecture Notes in Operations Research, Vol. 8, (2007), pp. 412-419.
- [42] Zhu J., Yang W., Huang J., Liu D., & Han J., "Vehicle Routing for Medical Supplies with Multi-Depots in Large-Scale Emergencies", In the First World Congress on Global Optimization in Engineering & Science, (2009).
- [43] Eydi A., Ghasemi-Nezhad S.A., "A bi-objective vehicle routing problem with time windows and multiple demands", Ain Shams Engineering Journal, Vol. 12, (2021), pp. 2617-2630.
- [44] Bashiri M., "Green Vehicle Routing Problem with Safety and Social Concerns", Journal of Optimization in Industrial Engineering, Vol. 10, No. 21, (2017), pp. 93-100.
- [45] Touati N., & Jost V., "On green routing and scheduling problem", arXiv preprint arXiv:1203.1604, (2012).
- [46] El Bouzekri E.I., Idrissi E., & Elhilali Alaoui A., "Evolutionary algorithm for the bi-objective green vehicle routing problem", International Journal of Scientific & Engineering Research, Vol. 5, No. 9, (2014), pp. 70-77.
- [47] Saka O.C., "Local search heuristics for pollution routing problem with multiple vehicle types and deadlines", Middle east technical university, (2013).
- [48] Kazemian I., & Aref S., "A green perspective on capacitated time-dependent vehicle routing problem with time windows", International Journal of Supply Chain and Inventory Management, Vol. 2, No. 1, (2017), pp. 20-38.
- [49] Toro O., Eliana M., Escobar Z., Antonio H., & Granada E., "Literature review on the vehicle routing problem in the green transportation context", Luna Azul, Vol. 42, (2016), pp. 362-387.
- [50] Jabali O., Van Woensel T., & De Kok A.G., "Analysis of travel times and CO2 emissions in time-dependent vehicle routing", Production and Operations Management, Vol. 21, No. 6, (2012), pp. 1060-1074.
- [51] Jiménez M., Arenas M., & Bilbao A., "Linear programming with fuzzy parameters: an interactive method resolution", European Journal of Operational Research, Vol. 177, No. 3, (2007), pp. 1599-1609.
- [52] Gendreau M., Potvin J.Y., Bräumlaysy O., Hasle G., & Løkketangen A., "Meta heuristics for the vehicle routing problem and its extensions: a categorized bibliography", The Vehicle Routing Problem: Latest Advances and New Challenges, Springer, Vol. 43, (2008), pp. 143-169.
- [53] Laporte G., Gendreau M., Potvin J.Y., & Semet F., "Classical and modern heuristics for the vehicle routing problem", International transactions in operational research, Vol. 7 ,No. 4-5, pp. 285-300.
- [54] Zarandi M.H.F., Hemmati A., & Davari S., "The multi-depot capacitated location-routing problem with fuzzy travel times", Expert Systems with Application, Vol. 38, No. 8, (2011), pp. 10075-10084.
- [55] Wang Y., Assogba K., Fan J., Xu M., Liu Y., & Wang H., "Multi-depot green vehicle routing problem with shared transportation resource: Integration of time-dependent speed and piecewise penalty cost", Journal of Cleaner Production, Vol. 232, (2019), pp. 12-29.

- [56] Coello Coello C.A., González Brambila S., Figueroa Gamboa J., Castillo Tapia M.G., & Hernández Gómez R., “*Evolutionary multiobjective optimization: open research areas and some challenges lying ahead*”, *Complex & Intelligent Systems*, Vol. 6, No. 2, (2020), pp. 221-236.
- [57] Verma S., Pant M., & Snasel V., “*A comprehensive review on NSGA-II for multi-objective combinatorial optimization problems*”, *IEEE Access*, Vol. 9, (2021), pp. 57757-57791.
- [58] Watanabe S., Hiroyasu T., & Miki M., “*NCGA: Neighborhood cultivation genetic algorithm for multi-objective optimization problems*”, *Proceedings of the Genetic and Evolutionary Computation Conference, GECCO*, (2002), pp. 458-465.
- [59] Raghuwanshi M.M., & Kakde O.G., “*Survey on multi objective evolutionary and real coded genetic algorithms*”, *Proceedings of the 8th Asia Pacific symposium on intelligent and evolutionary systems*, (2004), pp. 150-161
- [60] Rabiee M., Zandieh M., & Ramezani P., 2012. “*Bi-objective partial flexible job shop scheduling problem: NSGA-II, NRGa, MOGA and PAES approaches*”, *International Journal of Production Research*, Vol. 50, No. 24, (2012), pp. 7327-7342.

Follow This Article at The Following Site:

Abutalebi E, Rabbani M. Fuzzy Vehicle Routing Problem with Split Delivery: Trade-off between Air Pollution and Customer Satisfaction. *IJIEPR*. 2022; 33 (2) :1-16  
 URL: <http://ijiepr.iust.ac.ir/article-1-1469-en.html>

